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## Investigating teaching strategies in mathematics classrooms : a CD-ROM resource enabling teachers to explore teaching strategies in mathematics education

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# Intestigating aching strategies in mathematics classrooms mumber

Tony Herrington, Len Sparrow, Jan Herrington, Ron Oliver

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# Investigating Teaching Strategies

in Mathematics Classrooms

A CD-ROM resource enabling teachers to explore teaching strategies in mathematics education **Investigating Teaching Strategies in Mathematics Classrooms** 

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## Investigating teaching strategies in mathematics classrooms

## A new vision

Teaching mathematics has traditionally followed a standard pattern across the world. Students are placed in classrooms where they are seated individually, are required to listen passively and observe the teacher demonstrating mathematical procedures and then spend extensive time practising the newly acquired skills. Generally the content being taught comprises mathematical facts and skills taught sequentially in the order presented by a textbook. Students are then assessed by unseen pencil and paper tests for the purpose of grading and ranking. This cycle is then repeated with another mathematical topic which is viewed by students as discrete from the previous one. The National Council of Teachers of Mathematics offers a new vision of mathematics teaching and learning.

The image of mathematics teaching needed includes elementary and secondary teachers who are more proficient in ---

- · selecting mathematical tasks to engage students' interests and intellect;
- providing opportunities to deepen their understanding of the mathematics being studied and its applications;
- orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas;
- using, and helping students use, technology and other tools to pursue mathematical investigations;
- seeking, and helping students seek, connections to previous and developing knowledge;
- guiding individual, small-group, and whole-class work. (NCTM, 1991, p. 1)

#### The need for change

These new demands for teaching are a result of major changes in society. Equity and justice for members of society demand opportunities for all students to learn mathematics successfully. The shift from industrial to information-based societies has resulted in the need to utilise relevant technologies. With technologies like calculators and computers, the requirements for students to perform operations that machines can do faster and more accurately are no longer needed. The accepted view of mathematics as basic arithmetic skills has given way to a broader view of mathematics that emphasises mathematics as

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general processes, or ways of thinking and reasoning (NCTM, 1989), as an important form of communication (DES, 1982), and as a science of patterns (AEC, 1991).

The teacher presenting information for students to absorb is a metaphor that reflects student learning as transmission rather than transformation of knowledge. If understanding and applying mathematical ideas are to become primary goals of teaching then teachers need to gain an appreciation of students' learning, not simply as a replication of that which exists in the teacher's mind or the textbook but as a personalised construction of knowledge achieved through active engagement with mathematical ideas (von Glasersfeld, 1991).

#### What is needed

There is no single, suitable method for teaching mathematics (DES, 1992). Mathematics consists of a range of topics, containing a range of mathematical outcomes, taught over a range of year levels. In any mathematical topic one can identify important mathematical facts, skills and concepts. Learning and understanding these outcomes requires the learner to engage in important mathematical process such as pattern generating and generalising. Processes such as planning and communication are developed alongside attitudes and dispositions. It is not surprising then, that a single teaching approach is inadequate. A variety of strategies for teaching mathematics is required and the use of any one, or a combination of methods, will depend upon the nature of the mathematics being developed, the capabilities and experiences of the learners and an appreciation by the teacher of how learning occurs (AEC, 1991).

#### **Teaching approaches**

Successful student learning occurs in situations where students are motivated to learn, where tasks build upon previous experiences, where students can construct their own understandings. Tasks need to be challenging, provide multiple perspectives and representations, and allow avenues for reflection and communication that are developed in social and supportive environments. Some or all of these general principles may be reflected in a variety of teaching approaches.

As well as the traditional approaches of teacher exposition followed by student practice or 'seatwork', current curriculum documents highlight the need for teachers to be familiar with, and able to implement, other equally important approaches. These includes such strategies as problem solving, investigations, practical activity, discussion (DES, 1982); group work, projects (NCTM, 1989); and applications of relevant technologies (AEC, 1991). In addition, to these broad approaches more fine-grained strategies can be employed that require additional refinements and considerations. For example, problem solving can include approaches for teaching real-world applications and modelling (Burkhardt, 1981) and strategies that emphasise problem posing (Brown & Walter, 1993).

#### The CD-ROM program

The complex nature of mathematics and the complex nature in which mathematics is learnt requires teachers to be aware of and able to select from a large range of alternative teaching strategies. The program will enable you to explore the variety of approaches that are commonly advocated across the K - 12 mathematics curriculum. As you interact with the program you will recognise many of the complex issues highlighted and the benefits to be gained by using different teaching approaches. The program does not attempt to provide a definitive approach for you and your students. What it does hope to do is enable you to develop an awareness of the breadth of strategies available and a consideration of how and when these strategies may be implemented in particular circumstances so that you will be able to use this knowledge in your school, with your students and for your particular needs.

## The program

The interface of the program has been to designed to simulate the front part of a classroom, and to allow teachers to access resources intuitively, almost in the same way they would if they were physically present in the room. Seven types of resource are available to users and these are described below.

#### Video scenes and interviews

By clicking on the video cassettes pictured under the television screen, teachers can view short video sequences of:

The scenario: A scene of the strategy being used in the classroom.

Teacher: A short interview with the teacher on his or her use of the strategy.

Student: A short interview with a student from the class on the strategy.

#### Filing cabinet resources

Each filing cabinet drawer contains a written resource which students can examine:

Descriptions:	A description of the teaching strategy.	
Samples:	Samples of students' work or teachers' records, or other documents.	
Reflections:	Advice given by a preservice teacher on his or her experience of using the strategy on professional practice in schools.	
Interviews:	Expert commentary on the use of the teaching strategy.	

The following page gives a map of the interface, together with a short list of instructions on how to get started using the program. The page is a useful guide for new users of the program and can be photocopied for distribution to teachers. For details of setting up the software and troubleshooting, see pages 48-49.

## The elements in the program and how to access them

## Quick guide to getting started

- 1. Wait till the EXIT sign turns green.
- Click to select a strategy from the whiteboard to investigate.
- 3. Click on any element on the screen to watch a video or read some text. For example, click on a video cassette or one of the filing cabinet drawers.
- Copy items or write your own reflections in the notebook.
- 5. When you have finished, choose another strategy from the whiteboard to investigate.

Click here to read a description and discussion of the strategy

Click here to see scanned images of samples of students' work and other material

Click here to read a preservice teacher's comments and advice on the strategy

Click here to read an interview with a mathematics education expert on the strategy

Click here to use the electronic notebook, which includes space for reflections and pastes from other documents. It also includes investigations and problems Choose a teaching strategy to investigate by clicking on one of the categories shown in black on the whiteboard

Click here to play a video clip of the teaching strategy being used in the classroom



Click here for help

Click here to play a video clip of a student's comments on the teaching strategy

Click here to play a video clip of a teacher's comments on the teaching strategy

## **Electronic notebook**

In addition to providing documents and videos for users to examine, the resource also provides an electronic notebook to enable users to write their own reflections and ideas as they explore the various elements provided. They can also select text from the documents in the filing cabinet drawers (all documents except the scanned items in the samples drawer) and cut and paste text into their notebook.



At the end of a work session, users can save copies of their notes to their own disks, then format them using their regular word-processing program. If required, the notebook file can be saved and reloaded in plain text form at a later work session, by clicking on the Load button, and nominating the saved file. Note that you cannot reload the document once it has been saved as a word processor file, unless it is returned to Text Only form.

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## Using the program

The program is designed to be used with preservice and practising teachers of mathematics. It has been designed according to principles of situated learning to provide a learning environment which provides for the following conditions:

- Authentic context that allows for the natural complexity of the real world
- Authentic activities
- Access to expert performances and the modelling of processes
- Multiple roles and perspectives
- Collaboration to support the cooperative construction of knowledge
- · Coaching and scaffolding which provides links to complete the task
- · Reflection to enable abstractions to be formed
- Articulation to enable tacit knowledge to be made explicit
- Integrated assessment of learning within the tasks.

Note that not all these elements can be incorporated into the program itself. Many, such as collaboration and articulation, need to be provided at the time of the implementation of the program and they have also been allowed for in the investigations provided with the program. One such investigation places teachers in a situation where they are required to investigate alternative teaching strategies. Critical features are highlighted in the diagram opposite.

## Critical features of the investigations

## **Multiple** perspectives

A problem situation is described which requires investigation of the resource from a variety of perspectives

## Authentic task

A realistic task is set, requiring a written response

## Articulation

An opportunity to articulate and defend findings is given

## **Collaborative learning**

All activities are addressed to a group rather than an individual to allow for collaborative learning

Memo to: The Mathematics Coordinating Committee

From: Chris Miles, Principal

Reference: Calculators and computers

I have enclosed the attached 'Letter to the Editor' as an indication of my concern about parents' possible response to our recent policy decision regarding calculators and computers. I am particularly concerned that many parents will not understand our intentions, and will view our decision to allow calculators and computers to be used in all year levels and for all activities, as nothing more than spoon feeding or condoning cheating

After consultation with teachers and experts in the area, can you please prepare for me a one page document for parents on the school's approach to using calculators and computers in mathematics classrooms. Please explain why the decisio made, the benefits at both junior and senior levels of the school, and above all, attempt to alleviate any concerns the parents might have.

Please provide ms with a single page, ready to be copied (include pictures or examples to make it very readable) and I will print it on the back of the next newsletter to go to parents in two weeks'time. I'd also like you to present a short explanation to all teachers at our next staff meeting.

Chris Miles, Principal

## Authentic context

A supplementary document is provided, often the source of the problem that requires action

Back to Basics Bast When are schools going to realise that there is no substitute for learning your tables, and learning them well, as early as passible? My son recently returned home from school and told me that they are allowed so use calculateres in the classroom. In fact it is home from school and told me that they are allowed to use calcularors in the classroom. In fact h is encouraged! I never had access to calculators when it was young and I have always been good at calculation an stranded on a desert island, i calculation the sums. Pity halo my son. He will be dependent on calculatore ferover, Let's det bask to basks below these crary new ideas in schools spole a whole

Loscustors rerever. Lot's get back to basics beil these crazy new ideax in achoods spoil a whole genaration, Fed Up (Rame and eddress supplied)

## **Problems and investigations**

There are two types of activities provided in the program: *Problems* and *Investigations*. *Problems* are questions which allow the user to explore the resource with a single issue in mind and are more limited than *Investigations* which require a more open-ended and sustained examination of the resource.

Users will gain the most from the resource if it is used with the *Investigations*, under the following conditions:

Focus of investigation:	The resource is best examined in depth, from a number of different perspectives, for example, completing two or more investigations
Length of time:	Best used over a sustained period of 2-3 weeks rather than for a single session
No. of teachers:	Teachers or pre-service teachers working in pairs or small groups around each computer, rather than individually
Facilitator support:	Facilitator present during use to provide 'scaffolding' and support, rather than as an independent study activity
Setting the task:	Facilitator demonstrates the resource by thinking-aloud as an investigation is modelled. Students then choose an investigation from those provided, or one of their own choice.

## Using the investigations

Five investigations have been included in the *Investigations* section of the electronic notebook. The investigations are all in the form of two related documents which set the task in a realistic and authentic manner.

The five investigations have been reproduced on the following pages.

## **Investigation 1: Parent information evening**

The first investigation asks teachers to prepare a presentation for a parent information evening on an enquiry-based approach to mathematics teaching. The resource provides information on aspects of problem solving, investigations, and discussion which can then be contrasted with more traditional approaches.

#### Memo to: The Mathematics Sub-Committee

From: Leslie Moore, Principal

Reference: Parent information evening

Thanks for offering to lead the Parent Information evening on the enquiry approach to mathematics. I've allowed 15-20 minutes for your group which will be directly after the welcome given by me. The draft program is:

7:00 - 7.15 Introduction and welcome: Principal

7:15 - 7.20 An overview of enquiry approaches to mathematics. Maths Sub-committee

7:20 - 8.30 Question Time Panel

Can we meet early next week to discuss your presentation? I suggest you have ready for the meeting:

- Your notes about enquiry approaches especially what you regard as the main points. (Remember, most parents will be familiar with drill and practice. Can you compare and contrast enquiry with this and highlight the strengths of enquiry?)
- Any resources you intend to use, such as overhead projector transparencies.
- A brief of four or five typical questions that might arise at Question Time (please provide answers as well).

I look forward to our meeting and a very informative evening."

Principal

Parent Information Evening: Mathematics As part of our continuing movement to As part or our commung movement to develop effective ways for your children to learn, we are to incorporate an enguiny to rearry, we are to incorporate an enquin-based philosophy into our mathematics Dased prilosophy into our mathematic teaching. An enquiry approach is one where students are required to use mathematics to solve problems and Investigate mathematical situations. in an effort to keep you informed, you are invited to a Darent information evening next Thursday week. At this evening next intractory week. At Units meeting, the Mathematics Sub-Committee will give a brief overview of the enquiry-based approach. There will also be an opportunity for you to ask questions. I hope you are able to join to for what promises to be a very informative and interesting evening. Principal

## **Investigation 2: Back to basics**

The second investigation asks teachers to address community concerns about aspects of mathematics teaching, in particular, learning basic facts and skills. Teachers will consider how these concerns can be addressed using appropriate teaching strategies, and present their findings to a staff meeting.

#### Memo to: New teachers

From: Kim Simpson, Principal

Reference: Basic facts and skills planning in mathematics

Welcome to the school. I hope you will be happy here and will be most productive and effective in the education of the children.

There is some concern in the parent body and community generally about education in basic facts and skills. I have attached a typical letter to the Editor on this matter. Such letters frequently appear in the community newspapers. To that end, and so I can have some indication of your thinking on this matter would you please send me an outline of how you propose to ensure/ your class know their basic facts and skills.

Please remind me with which classes you are working. Outline the teaching strategies you will be using and offer your reasons for selecting these and rejecting others. You could show a couple of examples of how you would put your principles into practice. (You can do this as a group, I don't want individual responses from each of you.) I'll also get you to present a short report at our next staff meeting.

I look forward to your speedy response. Once again welcome to the school.

Principal

Schools neglect the beaks Haven't we neglected our oblidiers's education Prevent We registre our ormorers exections for long enough? isn't it time we abandoned the rame seaching ideas which are so The root and the root of the r The new teaching ideas which are so disadvantaging ideas which are so schools also to the todards? The under omphasis achieves in the task of action are simplestic disadvantaging dus studenter The underemphasis schools give to the basics of education is appainted childran nam on known and in a children is appainted. A constant where a subscript shall be been and subscriptions of the subscription of the subscrip Children can no kongor sool or do arthmetic A recent visit to a school fabric fabric arthmetic discusses and wrigh she same are available for marked and and and articles are available and and and articles and and available of a school fabric fabric fabric fabric and and available of a school fabric fabric fabric fabric and available of a school fabric fabri A recent visit to a school fair test into shocked disputed with the current state of schoolsed constances hand on two a contralision to schoolsed. Dispussed with the current state or equivalence Students had to use a calculator to work out askes as at an a same i reach a same do work out out to work out of the out Students had to use a calculator to work of a sum of a su Canes at 51.50, 8 sum / count easily so in any head, When are schools going to return to the Anative? basics? Back to Basics (Name and eddress supplied)

## Investigation 3: The superintendent's visit

The third investigation asks teachers to prepare and document a rationale for the way they teach mathematics. Evidence to support their views can be found in the resource. Their findings will be conveyed to the district superintendent.

#### Memo to: The Mathematics Coordinating Committee From: Con Macrides, Principal **Reference: Visit by District SuperIntendent** The District SuperIntendent for mathematics will be visiting the school next week (see attached letter). As you know we do not have any document or even a verbal policy in this area. Would you please prepare a briefing sheet for me so that I will have something to discuss. Please consult with other teachers and experts and provide me with: Education Department Once of the District Superintendent · up to ten main points related to acknowledged effective PO Box 999 practice in teaching mathematics Central Post Office - an appraisal of our present strategy of working through the Deer Principal, local text book series supplemented with sets of worksheets Reference; School Inspection Visite The District Superintencient for Mathematics wishes to · your advice and recommendations as to what we should be doing/ Vian your achool within the next month. During her visit she wishes to talk to you at some length about your • a couple of lesson outlines that show these strategies. I will one warnes to task to you at some wright about your policy for teaching mathematics. She is perticutarly bring the Superintendent along to your classroom to see these in keen to see documentary evidence of the use of a action. Variety of different feeching strategies, please advise eppropriate staff and have available any relevant I need you to present this to me in our meeting. I will be forever in your debt. policy documents for inspection. Thanks Sally James Office of the District Superintendent **Con Macrides, Principal**

## **Investigation 4: Calculators and computers**

The fourth investigation addresses the community concern about using new technologies and the effects these might have on students' learning. Teachers are asked to provide a rationale for the use of calculators and computers in their mathematics classrooms, and to present their findings at a staff meeting.



## Investigation 5: School of the year

The fifth investigation asks teachers to document and justify innovative approaches for teaching mathematics. Teachers may wish to outline the benefits of these approaches and contrast them with traditional methods.



## Using the problems

Ten problems have also been listed in the *Problems* section of the electronic notebook. These are less comprehensive than the investigations and may require the exploration of only part of the resource. They could also be done in a much shorter time than the investigations, if necessary.

#### The problems listed in the Problem section of the notebook

- 1. You wish to introduce the concept of a fraction to a class of mixed ability students who have little or no formal experience of fractions.
  - (a) Design two introductory lessons each using a different teaching strategy.
  - (b) Comment on these lesson plans and offer a justification for selecting one of them to use.
- 2. You are about to begin a series of lessons using an investigative approach with students. Write a list of DOs and DON'Ts and comment on them.
- 3. Offer, and comment upon, two situations that could be used for role play in the classroom. (Do not use the classroom shop example.)
- Offer suggestions for making drill and practice sessions less boring and anxiety ridden. Give an example to illustrate this, based on a number or algebraic algorithm.
- 5. Your teaching practice teacher says that you have to work with students in groups. Design a group-work lesson. Describe and justify your role and that of the students.
- 6. List a series of themes, titles or subjects that are useful for project work. Give a set of steps for planning lessons using a theme.
- 7. Offer thoughts and a variety of general methods for starting mathematics lessons. Illustrate with an example showing one of these methods.
- 8. Describe two game situations that are suitable to support the teaching of probability or chance processes. Comment on your selection and provide a justification.
- 9. Describe a rationale for using manipulatives as a teaching strategy.
- 10. What situations outside the classroom are suitable for developing mathematical ideas. Show how you would use one in a lesson plan format.

## The teaching strategies

A review of the literature on teaching, and extensive reading of current issues in the field, enabled the identification of 28 teaching strategies which were relevant to both primary and secondary mathematics classrooms. These categories formed the basis of the teaching strategies which are featured on the CD-ROM.

The strategies have been grouped into several sections:

Consolidation & practice	Drill and practice Homework Textbooks Worksheets
Discussion	Group work Role play Explaining Peer tutoring
Practical work	Manipulatives Game playing Outdoors Mathematics centres
Investigating	Guided discovery Projects Open-ended tasks
Teacher centred	Exposition Beginning a lesson Team teaching Involving others
Problem solving & application	Puzzles Modelling Applications Themes Problem solving Problem posing
Using technology	Calculators Graphic calculators Computers

Each strategy is described in detail in the following pages.

## STRATEGY: Drill and practice

#### **Description:**

Drill and practice in many classrooms forms the main strategy for a mathematics lesson. Students are shown a method often broken into smaller steps, for instance, how to subtract using the decomposition method. They then practise the method by completing a series of examples of the same kind of calculation in the same designated way. The intention is that they will copy the method exactly, memorising the steps and repeating them. This is the mainstay of many textbooks and teacher-designed worksheets or blackboard work.

Influential reports on mathematics education have noted that all students need opportunities to practise skills and routines which have recently been acquired, as well as consolidate those that they already possess. They also point out that the amount of practice will vary from student to student depending upon the degree of fluency shown by the child. Drill is usually used to produce an automatic response or the immediate recall of specific facts such as remembering of number facts and 'tables' knowledge. A major limitation of both drill and practice is that by the very nature of repetition, topics can become dull, aimless and boring for many students. Teachers have explored and used a variety of ways to help students overcome this dullness. Technology, especially calculators, and game playing are useful techniques.

#### Samples:

An example of a drill and practice activity

## **Reflections:**

Alice: A preservice teacher's reflections on using the strategy



An interview with Nerida Ellerton

#### Scenario:

Video clip of a teacher using a dice activity

## **Teacher:**

The teacher's comment on drill and practice activities

## Student:

A student's opinion of the activity

## STRATEGY: Homework

#### **Description:**

There can be a number of reasons for teachers to use homework: for students to complete unfinished work and thus keep pace with the class; for students to consolidate and practise skills, thus allowing classroom time to be spent on other, possibly more productive things; to involve and inform parents of classroom mathematics; to gather information that will be subsequently used in the classroom, for example, prices of household products, the heights of family members and so on; to develop independent study habits.

Generally, there needs to be a reason or purpose for the work other than simply providing 'busy work'. Coupled with this is the need for a legitimate use of the homework back in the classroom, again to give purpose to the work. Too often students see their work consigned to the rubbish bin or totally ignored. Systems for collecting, marking and discussing need to be carefully planned so that they don't encroach too far into class time and produce the dreary episode of 'going through the homework'. One needs to be aware that students have different levels of support and resources at home. Adequate provision should be given to the range of situations possible and to allow flexibility so that the family may easily fit the homework around other legitimate demands.



#### Scenario:

Video clip of a teacher giving advice on homework

## **Teacher:**

The teacher's comment on the benefits of homework

### Student:

A student's views on homework

#### **Reflections:**

Heather: A preservice teacher's reflections on using the strategy

## Interviews:

An interview with Deborah Ball

## STRATEGY: Textbooks

#### **Description:**

Most schools and most teachers employ a textbook in some form as part of their teaching methodology. Some teachers use a text book as the only method for teaching mathematics with students working from page to page. Others, however, select only certain aspects of the book for the students to use. Exponents of textbook use might argue that the person who wrote the book was an expert in the field of mathematics education and could provide a good sequence and suitable scope of material, especially if the book was linked closely with the local curriculum. On the other hand, many teachers would argue that it is impossible for an author, no matter how expert a person in mathematics, to know what particular children in the class needed at that moment to develop their understanding or to remediate their misconceptions.

Some discretion is needed in the use of textbooks. Generally textbooks can provide the teacher with a useful resource of problems, puzzles, exercises and alternative topics and approaches for both practice and enrichment. However, textbooks should not be viewed as a curriculum to be slavishly followed, and cannot substitute for important social interactions between teachers and students.



## STRATEGY: Worksheets

#### **Description:**

Worksheets and workcards may come from a number of sources, present a variety of styles and be used for a range of purposes by teachers. In some cases the worksheet is photocopied from a commercially produced black-line master. At other times they are produced by the teacher, either handwritten or computer generated. They may be used to convey information, to practise a new idea, to act as a recording vehicle or a combination of all three.

As with the use of textbooks, care needs to be taken to ensure that the task and purpose suit the particular child and that the worksheet forms part of a variety of teaching strategies and learning experiences. Too often most students' education is based purely on the completion of worksheets-a limited, repetitive and boring experience for many of them.

Ideally there will have been some initial teaching of the idea using materials and discussion before the worksheet is used by the students.

## Samples:

An example of a teacher-developed worksheet

## **Reflections:**

Jeffrey: A preservice teacher's reflections on using the strategy



Interviews:

An interview with Deborah Ball

#### Scenario:

Video clip of students using a worksheet

## Teacher:

The teacher's advice on using worksheets

#### **Student:**

A student's comment on worksheets

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## STRATEGY: Group work

#### **Description:**

Teachers use different categories and styles of groups within a classroom to achieve a range of goals. Teachers who wish to develop the discussion aspects of teaching and learning will use groups, as this enables students to interact more easily with each other. Another reason for using groups is that it allows teachers to teach specific aspects or techniques to the exact group of students who need it. Groups may be composed according to a number of criteria. They could be constructed because of the needs of particular students. Often the general ability of the students will suggest their grouping. Classrooms may have three groups corresponding to three general ability bands. Groups of mixed ability may be used to develop the interaction between the more able students and others in the class. Friendship may be another criterion used. Students may select their own group within certain parameters, for example, no more than six members. Interest is yet another factor for selection. That is, all the people who wish to work on Problem A have a common interest and work together.

Group size may vary but commonly it is between 3 and 8 students with 4 being the norm. Teachers should also monitor the interactions to ensure that no one is always dominant, with others making little or no contribution.



Alan Bishop

#### Scenario:

Video clip of a teacher organising students into groups

#### **Teacher:**

The teacher's comment on ways to group students

#### Student:

A student's comment on working in groups

#### 20

## STRATEGY: Role play

#### **Description:**

Teachers use role play as a vehicle to add as far as is possible some realism to the mathematics being learned. In a shop role play, typically students would adopt the roles of shopkeeper and customer, or waiter and diner in a restaurant. Often there would be a dramatic episode requiring the players to use real (preferable) or token money which might involve the students in adding prices and giving change. This requires a very different way of working from the textbook money calculations. It is also a good situation for the teacher to observe and see how the students apply their knowledge of money to the 'real situation'. For older students, situations may require them to be a point on a graph and then to react accordingly as the values or equation changes.

Conceptual learning through drama and movement, especially with the young child, ties learning with language use where the environment is constructed purposely to provide opportunities for this to happen informally.

### Samples:

A student's shopping list

## **Reflections:**

Joshua: A preservice teacher's reflections on using the strategy



An interview with Nerida Ellerton

## Scenario:

Video clip of a class role playing a shopping activity

## **Teacher:**

The teacher's comment on the value of role playing

### **Student:**

An interview with a student on the activity

## STRATEGY: Explaining

#### **Description:**

This is one aspect of the much larger strategy of discussion. In this instance the discussion is not short answers to teacher questions but a more extended use of talk. The emphasis is on the role of students using talk to develop their ideas and understandings in mathematics. Students bring into play their own language and, through discussion with their peers, try to make sense of the mathematical ideas they are using as part of the activity. The students become an audience for one another's comments which they can guestion or challenge or present a counter argument. The ability to explain and offer a justification for one's point of view grows out of these opportunities to talk about mathematics-opportunities that are missing from silent work on textbook exercises.

The teacher needs to organise the classroom so that the desk arrangement allows for and encourages discussion. The culture of the classroom must also value this form of behaviour as a means of understanding mathematics. Providing for classroom discussion also gives the teacher an opportunity to hear how students are thinking and the reasoning behind their thoughts-a good chance to spot any misunderstandings that may exist and would be hidden in a purely written response.



## Scenario:

students explaining and justifying their

describing the

## STRATEGY: Peer tutoring

#### **Description:**

The general practice here is to pair one student with another of greater ability and knowledge in the subject. The pairings may be from within the class which presents less of an organisational concern for the teacher. At other times, older students are paired with younger ones thus requiring the cooperation of more than one class.

The system provides advantages for both the participants, not just the less able or younger child. Obviously the child being tutored gains from having constant and immediate attention, instant feedback to their question or method as well as having the idea or task explained in childspeak. The tutor needs to understand the material or idea well but will gain further insight and understanding from trying to help the partner.

Training of the tutors is important as they need to help rather than tell their partner how to do the task. Some teachers use the pause, prompt and praise reminders to help the tutors with their interactions. By freeing themselves from the actual teaching task the teacher is now able to move around the classroom to monitor and asses students' understandings by listening and observing the conversations.



#### An interview with Nerida Ellerton

## Scenario:

Video clip of a peer tutoring activity

## **Teacher:**

The teacher's comment on the benefits of peer tutoring

#### Student:

A student's comment on helping others

23

## STRATEGY: Manipulatives

#### **Description:**

Most teachers would acknowledge the learning potential of using materials with their classes. In general terms the materials can be classified into structured and unstructured. Structured materials include Dienes' Multibase Arithmetic Blocks (MABs) and Cuisenaire rods, where there is a relationship between the pieces. Unstructured materials include multilink cubes, shells and matchsticks. Materials-based teaching is considered fundamental to working in the lower age range of the primary school but reports and research suggest that it is useful in helping learners of all ages, and that the stigma of babyishness should be removed and countered by the teacher. The visual and the tactile aspects of using materials are mathematical representations that add to the students' understandings as much as the written and spoken word.

One must be aware that the use of materials needs to be developed alongside discussion and reflection. For example, many students using MABs to perform calculations may in fact learn by rote the manipulation of the materials and gain little or no understanding of the calculation. Materials-based learning can take more time in the short term but, with the understanding gained, work at later levels can be faster and more secure.



### Scenario:

Video clip of students working with algebra blocks

discussing the benefits of using manipulatives

comment on using algebra blocks

## STRATEGY: Game playing

#### **Description:**

Games provide a useful teaching and learning strategy that can be used in a variety of ways. They are useful for practice of facts and skills, development of concepts, strategy building and problem solving. The games do, however, need to be relatively simple, have few rules, and have some worthwhile mathematical purpose.

With students engaged in an educationally worthwhile game, the teacher is free to observe and assess the use students make of their mathematical knowledge in a motivating context. If the game requires students to talk and justify their choices or moves, then it can provide a good vehicle for language as well as mathematical development. The strategy can be extended to allow students to develop their own games.



## Scenario:

Video clip of students playing the game of dominoes

## Teacher:

The teacher's comment on the benefits of playing games

## Student:

A student's comment on the game

#### **Reflections:**

Nicole: A preservice teacher's reflections on using the strategy

## STRATEGY: Outdoors

#### **Description:**

The major advantage of working outside, apart from the excitement of variety, is that of space: space for wet, possibly messy activities such as pouring water and sand for volume and capacity work. One of the most fruitful ways of engaging students in mathematics and in developing their enjoyment of the subject is to design and use a mathematics trail. There are many examples to be found in professional journals in mathematics education. This can also make a good connection between the often abstract school mathematics and the mathematics of the real world.

Some exciting work for both teachers and students requires the students being involved in kinaesthetic ways with mathematical ideas. The main thing needed here is space to move, something that is often in short supply in the classroom. Activities such as spirolateral walk, human graphs and human hundred squares all require lots of room to move, making an outside location ideal. As with all activities away from the enclosed nature of the classroom consideration of safety and careful preparation are needed.



#### Scenario:

Video clip of a sandpit activity

## **Teacher:**

The teacher's comment on the benefits of working outdoors

#### Student:

A student's comment on the sandpit activity

## STRATEGY: Mathematics centres

#### **Description:**

Mathematics or task centres are generally based around puzzles, problems and investigations that can be of a general mathematical nature or related to specific mathematical content, such as 2D shapes. Everything necessary for the activity is prepared by the teacher and placed in a stacking container or resealable plastic bag. These are coded and stored within easy reach for the students. Codes may refer to the level of difficulty or the main mathematical content so that students can be guided or informed in their selection of which task to complete.

The physical storage of these materials may present problems to some schools. They can be used by one class or year group but, if appropriately coded, can be more profitably used by the whole school. Many teachers would opt for one session a week with the materials. They would need to assign levels and routes to the students so that a reasonable match of difficulty and progression was established. Recording systems of work completed need to be designed, and students can monitor their own progress. Often they are difficult to integrate with the ongoing class work and may be viewed as a set of disconnected activities. However, much of the richness of these situations comes from the discussions between participating students and the opportunities for them to gain pleasure from solving the puzzle.

Teaching Strategies

Maths Centres

Role Play

Explaining Poor Tutoring PRACTICAL WORK Henipulatives Game Playing Outdoors

Maths Centras

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REFLECTIONS

INTERVIEWS.

- 14

## Scenario:

Video clip of a maths centre in action

## Teacher:

TEACHIN

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The teacher describing the organisation of the centre

#### **Student:**

A student describing a maths centre

### **Reflections:**

Neil: A preservice teacher's reflections on using the strategy

#### Interviews:

An interview with Charles Lovitt

## STRATEGY: Guided discovery

#### **Description:**

While there is a designated endpoint or learning envisaged by the teacher, a guided discovery approach will see the teacher working in a very different way to the exposition method often practised. The teacher plans and sets up a situation and task that has embedded within it the desired learning outcome. For example, the teacher presents groups of students with a set of 2D shapes and asks them to examine them and note any patterns and relationships they find. The students have to use their problem solving techniques to help them find the patterns and relationships. During the work the role of the teacher is to ask questions that probe students' understandings and direct them to useful processes such as making a table or chart.

Generally, a guided discovery lesson consists of three phases: (1) The teacher sets the problem or activity, (2) Students explore the situation, (3) Teacher and students discuss the situation and draw their conclusions. Students in a guided discovery activity have to take much more of the initiative acting like mathematicians and solving problems rather than as passive receivers of knowledge. The teacher on the other hand is more of a guide and questioner rather than a provider of knowledge.



## STRATEGY: Projects

#### **Description:**

Projects can take a number of forms ranging from finding out about squares and building up a file of drawings, relationships, and photographs, to researching famous mathematicians and their discoveries, or designing, costing and building a swimming pool. Many projects can relate mathematics to the real world and can involve techniques needed in the later lives of the students. Some of the more effective projects exploit the integrated nature of their subject matter.

Most projects would involve the students in investigating, solving problems, researching and applying mathematics. Projects are usually completed with the help of a group of students involved in negotiation, group work skills, identification and collection of resources, and presentation skills (both oral and written). From a teaching point of view, such work also gives an insight into how the students select and apply their various mathematical tools.



## Scenario:

Video clip of students displaying their projects

## **Teacher:**

The teacher's comment on the benefits of project work

## Student:

A student's comment on the project

## STRATEGY: Open-ended tasks

#### **Description:**

Good open-ended tasks allow teachers to challenge a mixed-ability class, since such tasks can be answered at a variety of levels allowing an element of success for all students. Open-ended tasks not only allow for a number of levels of answers but also are approachable from a range of directions. An example of an open-ended question related to mental computation could be: *The answer is 12. What could be the question?* 

Criteria for selecting and developing open-ended tasks could include the following:

- Tasks should challenge all students
- Individuals or groups should be able to work on the task
- Tasks should be easily extended
- Tasks should require only minimal direction by the teacher
- More than one solution and more than one path to a solution should be possible

There are excellent opportunities here for the teacher to assess the students at work and to see the mathematics they select to use and the understandings they exhibit in its use. Here again the responsibility is with the students to think about what mathematics is relevant, design how they will approach the situation and decide on the appropriateness of the solution.



## STRATEGY: Exposition

#### **Description:**

This is the traditional teaching approach used by many mathematics teachers. The main purpose of this strategy is to transmit information from the teacher to the students as quickly as possible. It tends to be favoured in classes with older students and in situations where factual information or specific methods have to be acquired in a short time. In this context students can easily become passive receivers of information making little or no contribution to the lesson. Students may attempt to rote learn knowledge with minimal understanding taking place.

At worst the approach can be overly long, boring for many, and poorly presented, with little thought given to individual differences and mixed abilities.

More effective use of the exposition strategy will incorporate stimulating questions and demonstrations that build on the prior knowledge of students, and motivate them to make new and important connections.



## STRATEGY: Beginning a lesson

#### **Description:**

The main purpose of beginning a lesson in an interesting way is to grab the attention of the students and establish motivation which will take them through into the body of the lesson. A good beginning should set the scene for the learning, link ideas with previous and future learning and motivate the students to learn. Ideally it should be short, not too intricate or involved and engender curiosity.

Unfortunately, for many students their introduction to a mathematics lesson is more along the lines of 'Take out your books/worksheets and do the next section'. Alternatively, students may be asked to answer ten quick questions. The practice of giving a quick mental arithmetic test as a warm up is a highly questionable practice, as it is often not related to the body of the lesson and can reinforce the already negative feelings of many students.

More appropriate examples might be to involve the students physically, read some relevant literature or pose an interesting problem found in the newspaper.



## STRATEGY: Team teaching

#### **Description:**

This is a situation where two or more teachers work with a group of students. Usually the team would plan and prepare the work for the students—this aspect is vitally important to the ultimate success of the strategy. The delivery of the tasks might vary in format with sometimes one teacher taking a leading role while the other acts as an aid or support. At other times, they would both take equal responsibility for the activities. A similar style of team is the cooperative situation where teachers plan together but take responsibility for a particular group to whom they teach the material.

An advantage of the system is the flexibility it allows for teachers to work in concentrated ways with groups or individual students. One teacher can work with the group while the other one takes responsibility for the remaining students and supervises their activities. Students receive more than one perspective on the work they are undertaking, and benefit from the talents and social interactions of more than one person. With more than one person working with the large group there is a better chance that successful learning will take place for more students.

Teaching Strategies

Team Teaching





Video clip of a lesson involving team teaching

### **Teacher:**

TEA

Two teachers describing how they team teach

#### **Student:**

A student's comment on the advantages of having two teachers

## **Reflections:**

Olivia: A preservice teacher's reflections on using the strategy

## Interviews:

An interview with Nerida Ellerton

## STRATEGY: Involving others

#### **Description:**

Others, in this sense, may mean parents of students in the school or the particular class, and visitors with special talents or areas of expertise. The usual 'other' is a parent helper, and this will be the focus of the consideration here. Volunteer helpers bring to the classroom, and the education of the students within it, a different face and personality and often special talents. For the teacher they are often an extra pair of hands which is particularly important with younger students. Motivation and a reduction of anxiety for some students is an advantage. There is more chance of an interaction and conversation for each child with extra people in the room.

Helpers in the classroom who are not trained teachers may need some help and advice with responding and helping students to learn. Suggestions for good questions and ways to react to students' own questions are useful. Often helpers in the classroom are used to supervise small groups of students working on tasks established by the teacher. This allows the teacher to move around the classroom and observe and assess how the students are reacting and involving themselves in the task.



Nerida Ellerton

### Scenario:

Video clip of a parent helper

#### **Teacher:**

The teacher describing parent involvement in a lesson

## Student:

Student and parent discussing the benefits of helping in the classroom

#### **Reflections:**

Tim: A preservice teacher's reflections on using the strategy

## STRATEGY: Puzzles

#### **Description:**

Puzzles form part of the larger area of problem solving and application in mathematics. They are a useful teaching strategy in that generally they don't have an obvious set procedure which can be applied to solve them. Good puzzles—and it is important to distinguish the good from the trivial, such as word searches and other low level busy work-require students to think in mathematical ways. If puzzles are used in a cooperative way, they become excellent vehicles for discussion. The involvement of a challenge is very motivational for most students. It is especially important that the level of difficulty of the puzzle is appropriately matched to the child so that frustration and negative feelings from failure are not established.

Puzzles can be found to challenge students in particular areas of mathematics, for example, number puzzles, and they can also be used to foster more general mathematical processes, such as the Tower of Hanoi which require students to think through the situation and employ pattern searching to arrive at generalisations.

#### Samples:

An example of a mathematical puzzle: the frogs puzzle

## **Reflections:**

Damon: A preservice teacher's reflections on using the strategy



## Interviews:

An interview with Alan Schoenfeld

## Scenario:

Video clip of students engaged in a puzzle activity

## **Teacher:**

The teacher's comment on the benefits of using puzzles

## Student:

A student's comment on problem solving with puzzles

## STRATEGY: Modelling

#### **Description:**

Mathematical modelling is the term used to describe processes involved in using mathematics to solve real-world problems. Many problems that students face in schools are chosen by the teacher (or the textbook) as practice for recently learned procedures. Mathematical modelling is quite different in that once students have understood the nature of the problem, part of the task is to select the appropriate mathematics that will lead towards a solution. Generally, mathematical modelling problems involve a range of suitable solutions and for this reason alone they are particularly useful with mixed-ability classes. An example of a mathematical modelling problem would be to ask students to plan and cost a family trip. Such an activity requires students to gather information on hotel accommodation, the costs of meals, the costs of transport, the availability of transport and so on. Students then need to select and represent mathematically the necessary costs and compare alternatives leading to possible solutions. The results can then form the basis of a report for the class (and family) to discuss. In this way students develop important modelling processes that include constructing a mathematical model, solving the model for mathematical solutions, interpreting these solutions in the context of the real situation and refining the model to produce better solutions.



#### Scenario:

Video clip of teacher and student discussing their modelling projects

#### **Teacher:**

The teacher explaining the difference between modelling and the traditional approach

## **Student:**

A student's comment on modelling

## STRATEGY: Applications

#### **Description:**

The distinction between pure and applied mathematics has a long history. For many students the abstract nature of pure mathematics begs the question 'Where are we going to use this?' Applying the mathematics to situations that are real to the students provides some of this sought-after relevance.

For example, the standard procedure for calculating the average or arithmetic mean can be learnt by the manipulation of symbols without any thought given as to a context in which finding an average might be useful, as in say sporting averages. A theme such as sport may also provide teachers and students with a context to apply a range of mathematical tasks including skills, such as scoring different sports, and problem solving, such as predicting track times in 50 years' time.

Often word problems describe the tasks in which students will apply previously learnt procedures, and frequently these are the areas where students can indicate their ability to display not only their knowledge of the procedures but also their ability to choose the most appropriate ones.

#### Samples:

Diagram representing the application activity

## **Reflections:**

Sally: A preservice teacher's reflections on using the strategy



Interviews:

An interview with

Helen Mansfield

## Scenario:

Video clip of an application in trigonometry

## Teacher:

The teacher justifying the use of applications

### **Student:**

A student explaining the benefit of applications

37

## STRATEGY: Themes

#### **Description:**

One way to show the relevance of a mathematical idea is to apply it in a realistic situation. Conversely, a different teaching approach is to begin with the context or theme and investigate the related mathematical ideas. Making connections is an important process in learning and understanding. The connections that can be made include mathematical ideas across different content areas like measurement and geometry as well as across other disciplines such as music and art. Themes can be chosen that have significance for the children being taught.

Once a theme has been chosen the teacher can determine suitable mathematical ideas that are related and prepare activities for investigation. It is a good idea to include student questions about the theme as these may lead to mathematical ideas that had not been considered by the teacher. Often the theme is a realistic situation such as 'the circus' or 'cars' but the context could just as easily be mathematical such as 'the cube.' The mathematical ideas that can be linked to a cube theme might include the ideas such as nets, tessellations, rotational symmetry, planes of symmetry, Euler's law, volume, and surface area. A thematic approach can be motivating for many students and can result in classroom displays and project work that show the relevance and connectedness of mathematics.

#### Samples:

The students' survey results

## **Reflections:**

Fleur: A preservice teacher's reflections on using the strategy



#### Interviews:

An interview with Alan Bishop

#### Scenario:

Video clip of students presenting a television advertisement

#### **Teacher:**

The teacher describing related ideas within the theme

#### Student:

A student describing thematic activities

## STRATEGY: Problem solving

#### **Description:**

Working with students in problem solving ways has become one of the main recommendations of current curriculum documents. A problem in this context is a situation that does not have an immediate and obvious method of solution for the solver. It has been suggested that problem solving is what you do when you don't know what to do. Thus, many of the things that have in the past been considered problems are in fact, according to this definition, not a problem. Problem solving is a fundamental way of working in mathematics and should be part of the mathematical diet of all students. By using problem solving as a teaching and learning strategy, there is a change in teaching style that emphasises students working mathematically, with the teacher as a helper and guestioner rather than as a transmitter of facts and skills. Students have to bring into play mathematical thinking in order to solve the problem presented. Wrong methods and dead ends are accepted, evaluated and used on the way to a solution. In fact there may be more than one acceptable solution and more than one route to its discovery. Students will usually, though not exclusively, be organised into groups so that the power of talk and discussion as well as cooperative learning may be used as a vehicle for others to learn important mathematical processes necessary for solving problems.

## Samples:

The milk bottle problem

#### **Reflections:**

Jai: A preservice teacher's reflections on using the strategy



#### Interviews:

An interview with Alan Schoenfeld

## Scenario:

Video clip of students describing their strategies for solving a problem

## **Teacher:**

A teacher's views of teaching problem solving

### Student:

A student describing the way she solves problems

## STRATEGY: Problem posing

#### **Description:**

Problem posing is an important strategy to utilise in the classroom—particularly one that has within it a range of abilities and interests. Initially this may come directly from the instigation of the teacher but eventually the intention is that students will pose their own problems. The teacher question starter: 'What happens if ... ?' is a useful one in that it allows aspects of a problem situation to be changed easily. For example one may ask 'What happens if I change the whole numbers to fractions?' This then develops the original situation into a new one to be explored.

Problem posing will develop the inquiry approach and frame of mind which is important in mathematical exploration. The students' work becomes self-generating from their questions which in turn add intrinsic motivation and develops the idea of ownership.

Other areas worthy of exploration come from incidental questions posed by the students in everyday situations. Rich data-handling activities can arise from the problems posed by the students rather than imposed by the teacher or the text book.

#### Samples:

A list of posed problems

## **Reflections:**

Jian: A preservice teacher's reflections on using the strategy



An interview with Alan Schoenfeld

## Scenario:

Video clip of students posing problems

## **Teacher:**

The teacher suggesting ways to pose problems

## **Student:**

A student commenting on her problem posing

## STRATEGY Calculators

#### **Description:**

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Considerable controversy surrounds the use of calculators especially in the primary or elementary classroom. Most of the objections, for example, that students will become mindless button pushers who are unable to think, are based on myths that have no basis in educational research. As a teaching and learning aid, calculators are invaluable in every classroom. They have their place as a calculator of large numbers, and as an aid to learning. They can assist students' understanding of critical concepts such as place value and decimals as well as with powerful processes in the area of number patterns and relationships. Using the calculator to explore different methods to calculate is a way to use them sensibly. For example, use the constant function to find the remainder in a division problem.

Teachers should, however, avoid the trivial activities which have little if any mathematical value such as using the calculator to make words or to fill out crossnumbers or to check calculations already done by another method. According to major curriculum recommendations, calculators should be used in all classes K-12 and integrated into the normal teaching activity rather than as an add-on or an early-finisher task.

## Samples:

A calculator problem

#### **Reflections:**

Desiree: A preservice teacher's reflections on using the strategy



#### Interviews:

An interview with Susie Groves

## Scenario:

Video clip of students using a calculator to solve a problem

## **Teacher:**

The teacher outlining the benefits of using calculators

### Student:

A student's comment on why he likes using calculators

## STRATEGY: Graphic calculators

#### **Description:**

Essentially, graphic calculators are hand-held computers that allow the user to carry out calculations that normal calculators perform, as well as operations that involve algebra, matrices, statistics, programming and, of course, graphing. Potentially, the availability of such tools will have major implications both in terms of teaching and assessment. For example, the time and effort spent on teaching and learning procedures for factorising algebraic expressions will be seriously questioned in a society that has ready access to graphic calculators. Similarly, the traditional ways of testing students' memory for mechanical procedures will have no place in examination settings in which students can use graphic calculators.

The challenge for teachers and curriculum developers will be to find ways of using calculators to explore challenging mathematics which result in greater mathematical understanding, and which are not simply a quicker way of arriving at an answer.

#### Samples:

An example of a graphic calculator activity

## **Reflections:**

Aidan: A preservice teacher's reflections on using the strategy



#### Interviews:

An interview with Susie Groves

## Scenario:

Video clip of students using a graphic calculator ē

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#### **Teacher:**

The teacher's comment on the benefits of using a graphic calculator

#### Student:

A student describing the way she uses a graphic calculator

## STRATEGY: Computers

#### **Description:**

Computers are becoming more prominent in classrooms as well as in designated computer rooms. In some schools, portable computers are becoming available for each child. For most schools, however, this is still some way off. Computers can be used effectively where they can: do something more quickly, do new things that were not possible without them, and facilitate learning. Teachers can use them as electronic blackboards that may, for example, quickly display sets of graphs or introduce a motivating problem or investigation. Students can use them as a tool to think with and to set and solve problems. As with calculators, they need to be integrated into the session rather than used as a reward for early finishers.

There are a number of powerful and useful packages available to the classroom teacher. Usually these are 'content free' and comprise spreadsheets, databases, graphing packages, wordprocessors, microworlds and computer languages such as LOGO. Many of the mathematical adventure games and simulations are worthwhile and provide a useful strategy to promote student discussion and group problem solving. Beware of the use of the computer for rote learning basic facts, and repetition of mechanical methods for calculations that can often be done more easily and cheaply in other ways.

## Samples:

An example of a computer activity

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**TERUCTIONS** 

## **Reflections:**

Zane: A preservice teacher's reflections on using the strategy



Interviews:

Charles Lovitt

An interview with

## Scenario:

Video clip of students using a computer graphing package

## Teacher:

The teacher's comment on the benefits of using computers

## Student:

A student explaining why she likes using computers

## Assessment advice for facilitators

If you are a facilitator using the program with teachers or preservice teachers, you may wish to assess participants after they have completed one of the investigations.

Authentic assessment is a very important aspect of the situated learning model upon which this multimedia program is based. It would be inconsistent with the approach to present a resource based on authentic contexts and tasks and then to assess participants with a paper and pencil test or essay.

The following pages give a suggested method of assessing teachers' use of the program and the learning that has occurred in a meaningful, enjoyable and authentic manner. A sample handout is reproduced on the following page suggesting an authentic assignment which could be given to preservice teachers, consisting of two parts: an oral presentation to the class and a written report.

Participants can work on the investigation for an agreed number of hours. An arrangement can be made for a class presentation to be done by each group with a written report to follow.

If different groups in the class are completing different activities, an announcement will need to be made at the start of each presentation about the context of the presentation and the role the audience is required to assume. A suggested context for each activity is given below:

Investigation 1:	The presentation is made at a Parent Information Evening called to discuss enquiry-based approaches to teaching mathematics.
Investigation 2:	The presentation is made to a staff meeting where a group of teachers is asked to outline approaches for the teaching and learning of basic facts and skills.
Investigation 3:	The presentation is made to a district superintendent who wishes to discuss the school's policy on mathematics teaching,
Investigation 4:	The presentation is made to parents via a newsletter and a staff meeting called to discuss the school's policy on the use of calculators and computers in mathematics classrooms.
Investigation 5:	The presentation is made to the principal and to editors of an educational journal that is offering prizes for innovative approaches to teaching.

## A sample assignment

#### Groupwork

Students are invited to work in small groups

## Authentic task

A realistic task is set, requiring users to interact with the program

## Oral presentation

An oral presentation is to be given to the class

## Details

Due dates, scores and other administrative details are provided

Written report

A written report is to be completed and submitted

## Assignment details

Part 1

You will be given an investigation to do using a multimedia program on teaching in mathematics. You will need to work in small groups of 2-3 people.

The activity will require you to imagine that you are a mathematics teacher in a school and to observe teachers using teaching strategies in their classrooms, to talk with them and their students, and to consult with experts and student teachers. The interactive multimedia program will allow you to do this and you will work on the program for .... hours.

At the conclusion of this time, depending on the activity, you need to present a report to teachers, parents, journal editors, the district superintendent or the principal about your findings, and to make recommendations on aspects of mathematics teaching.

As well as presenting your report, you will also be part of the audience for other groups' presentations. You will need to imagine that you are a member of the school's staff, board of directors or other group and then evaluate each group's findings and recommendations from a real-life perspective.

Part 1 requirements:

Dates:	T

Weeks x-y: Multimedia program/compilation of report

Week z: P

Presentation to class

Length of presentation: 10 minutes

Score: 40% (group score)

#### Part 2

Part 2 of the Assignment requires you to write up the presentation given to the class and present it as you would a formal written report in a real-life situation.

Part 2 requirements:

Due date: Length of report: Score:

2-3 pages 60% (group score)

Due on ...

## Peer assessment of presentations

Teachers and participants can use peer assessment to evaluate each others' presentations. However, they need to be informed of the criteria for assessment before they present. A sample evaluation form is given below:

	Evaluation Form	
Your name:		
G	roup no.	
Presenters' names:		
	Criteria:	Score out of
,	Effectiveness of argument	/=
	How persuasive was the group's proposal? Were you convinced of the value of the suggestions?	
1	<b>Proposal's practicality</b> Were the suggestions practical and able to be implemented? How convinced were you that the suggestions would work?	/5
/	Argument well supported Was there sufficient evidence to support the proposal? Did you feel they had researched the problem we	/5 ell?
1	<b>Presentation skills</b> How well did the group present the report? Did the presentation hold your attention?	/5

## References

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- Department of Education & Science. (1982). Mathematics counts: Report of the committee of inquiry into the teaching of mathematics in schools under the chairmanship of Dr W H Cockcroft. London: HMSO. (The Cockcroft Report).
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- National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.
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## Setting up the software

#### Minimum computer configuration

Macintosh	PC
In order to use the program you will need: • A 68040 or Power Macintosh Computer	In order to use the program you will need: • An 80486 or better
<ul><li>running System 7.0 or greater</li><li>640 x 480 colour monitor</li></ul>	<ul> <li>640 x 480 colour monitor</li> <li>Windows 95 or Windows NT</li> </ul>
12 Megabytes of RAM	<ul> <li>16 Megabytes of RAM</li> </ul>
Double-speed CD-ROM drive	Double-speed CD-ROM drive
Thousands of colours	Thousands of colours
Fonts: Geneva and Times	Fonts: Arial and Times New Roman

#### Loading the software

Macintosh	PC
1. Place the CD into the CD-ROM player	1. Place the CD into the CD-ROM player
2. Double click on the icon to view the files on the CD	<ol><li>Double click on the icon to view the files on the CD</li></ol>
<ol> <li>If you need to install QuickTime 2.5, (a program which enables you to play the videos in the program on your computer) click on the installer in the file included with the program. It is named QuickTime Installer. The program will automatically install but you will need to restart your computer before you use the program.</li> <li>Double click the Teaching Strategies icon to run the program.</li> </ol>	<ol> <li>If you need to install QuickTime 2.5, (a program which enables you to play the videos in the program on your computer) click on the installer in the file included with the program. It is named QT32. The program will automatically install but you will need to restart Windows before you use the program.</li> <li>Double click the Teach icon to run the program.</li> </ol>

## Running the program

- 1. Wait until the EXIT sign turns green
- 2. Click on the Help icon if necessary
- 3. Use the scrolling box on the whiteboard to view the available teaching strategies
- 4. Click one of the strategies printed in black to choose a strategy to investigate
- 5. Click within the interface to select the various information elements contained for each teaching strategy.

#### Quitting the program

Click on the EXIT sign in the top right hand corner to quit from *Investigating Teaching* Strategies in Mathematics Classrooms.

## Trouble shooting guide (for Macintosh and PC)

Problem	Solution
The Teaching Strategies program does not run.	Check that your machine is using the correct operating system (OS). For example, this program will not run under Windows 3.1.
Nothing happens when elements on the screen are clicked.	Wait until the EXIT sign turns green.
The video doesn't play (on PC).	Check that you have run the QuickTime Installer program.
The image on the screen is pixelated or banded.	You may be using a computer which uses only 256 colours or less. The program will run best on a computer capable of using 'thousands of colours'.
Documents from the Samples drawer won't copy.	You cannot copy from the Samples drawer.
You cannot copy to the notebook.	The notebook may be full (the total amount is about 15 A4 printed pages). Save the full notebook, delete the text and save the new one with a different name.
You cannot find the cursor in the notebook, or the cursor is not flashing.	Scroll through the notebook a line or two. You will notice the cursor flashing.
You cannot reload your notebook file.	You may have worked on your notebook material and saved it in a Word Processor file. These files cannot be loaded into the program, only plain text files can. Save the file as a Text Only file, and load back into the program.

#### For comments or feedback on the resource, contact:

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