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ABSTRACT

The worldwide impact of the Global Financial Crisis on stock markets, investors and fund managers has lead to a renewed interest in tools for robust risk management. Quantile regression is a suitable candidate and deserves the interest of financial decision makers given its remarkable capabilities for capturing and explaining the behaviour of financial return series more effectively than the ordinary least squares regression methods which are the standard tool. In this paper we present quantile regression estimation as an attractive additional investment tool, which is more efficient than Ordinary Least Square in analyzing information across the quantiles of a distribution. This translates into the more accurate calibration of asset pricing models and subsequent informational gains in portfolio formation. We present empirical evidence of the effectiveness of quantile regression based techniques as applied across the quantiles of return distributions to derive information for portfolio formation. We show, via stocks in Dow Jones Industrial Index, that at times of financial setbacks such as the Global Financial Crisis, a portfolio of stocks formed using quantile regression in the context of the Fama-French three factor model, performs better than the one formed using traditional OLS.

Keywords: Factor models; Portfolio optimization; Quantile regression

1. INTRODUCTION

From the introduction of Modern Portfolio Theory (MPT) by Markowitz, (1952), the analysis of historical series of stock returns has been extensively used as the basis of investment decisions. Diversification, as proposed by MPT, has been used for minimizing risk, which works on the analysis of the covariance matrix of the chosen universe of stock returns. Prior to the development of modern computing technology, this was computationally demanding and short cuts were developed, such as Sharpe's single index model (1963). A heuristic which focuses on the empirical estimation of systematic risk, which has a parallel focus in the modern finance's central paradigm: the capital asset pricing model (CAPM). Independently developed by Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966).

Fama and French (1992, 1993) extended the basic CAPM to include two additional factors; size and book-to-market as explanatory variables in explaining the cross-section of stock returns. SMB, which stands for Small Minus Big, is designed to measure the additional return investors have historically received from investing in stocks of companies with relatively small market capitalizations. This additional return is often referred to as the "size premium." HML, which is short for High Minus Low, has been constructed to measure the "value premium" provided to investors for investing in companies with high book-to-market values (essentially, the book value of the company's assets as a ratio relative to the market value reflecting investor's valuation of the company, commonly expressed as B/M).

Ordinary Least Squares regression analysis, has been the work-horse for all the regression forecasting estimates used to model CAPM and its variations; such as the Fama-French three factor model or other asset pricing models. With the introduction of alternative robust risk measures such as Value at Risk (VaR) or Conditional Value at Risk (CVaR), which are now standard in risk management, more emphasis has been laid on the lower tails of the return distributions. The way in which OLS is constructed requires it to focus on the means of the covariates. It is unable to account for the boundary values, or to explore values across the quantiles of the distribution. It is also a Gaussian technique, with an assumption of normality of the covariates, which does not sit well with the abundant evidence of fat tails and skewness encountered in financial asset return distributions. This

feature of asset returns is even more acute in times of severe financial distress like the Global Financial Crisis (GFC). Quantile Regression, as introduced by Koenker and Basset (1978), has gained popularity recently in finance as an alternative to OLS, as this robust regression technique can account for the lower and also the upper tails of the return distribution and automatically accounts for outliers, or extreme events in the distribution, and hence quantifies more efficiently for risk.

In this paper, we introduce quantile regression as a tool for investment decision making and also show the applicability of this technique to robust risk management. We show the effectiveness of quantile regression in capturing the risk involved in the tails of the distributions which is not possible with OLS. We also use a basic portfolio construction exercise using the Fama-French three factor model, on the components of the Dow Jones Industrial 30 stocks index from a period running from 2005-2008 and show how quantile regression based risk estimates can reduce the losses which we can incur when using OLS based methods as portfolio construction tools.

2.   QUANTILE REGRESSION

Linear regression represents the dependent variable, as a linear function of one or more independent variables, subject to a random 'disturbance' or 'error' term. It estimates the mean value of the dependent variable for given levels of the independent variables. For this type of regression, where we want to understand the central tendency in a dataset, OLS is a very effective method. OLS loses its effectiveness when we try to go beyond the mean value or towards the extremes of a data set by exploring the quantiles.

Quantile regression as introduced in Koenker and Bassett (1978) is an extension of classical least squares estimation of conditional mean models to the estimation of an ensemble of models for conditional quantile functions. The central special case is the median regression estimator that minimizes a sum of absolute errors. The remaining conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors. Taken together the ensemble of estimated conditional quantile functions offers a much more complete view of the effect of covariates on the location, scale and shape of the distribution of the response variable.

In linear regression, the regression coefficient represents the change in the response variable produced by a one unit change in the predictor variable associated with that coefficient. The quantile regression parameter estimates the change in a specified quantile of the response variable produced by a one unit change in the predictor variable.

The quantiles, or percentiles, or occasionally fractiles, refer to the general case of dividing a dataset into parts. Quantile regression seeks to extend these ideas to the estimation of conditional quantile functions - models in which quantiles of the conditional distribution of the response variable are expressed as functions of observed covariates.

In quantile regression, the median estimator minimizes the symmetrically weighted sum of absolute errors (where the weight is equal to 0.5) to estimate the conditional median function, other conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors, where the weights are functions of the quantile of interest. This makes quantile regression robust to the presence of outliers.

We can define the quantiles through a simple alternative expedient as an optimization problem. Just as we can define the sample mean as the solution to the problem of minimizing a sum of squared residuals, we can define the median as the solution to the problem of minimizing a sum of absolute residuals. The symmetry of the piecewise linear absolute value function implies that the minimization of the sum of absolute residuals must equate the number of positive and negative residuals, thus assuring that there are the same number of observations above and below the median.

The other quantile values can be obtained by minimizing a sum of asymmetrically weighted absolute residuals, (giving different weights to positive and negative residuals). Solving

$$\min_{\xi \in \mathcal{R}} \sum \rho_\tau (y_i - \xi) \tag{1}$$

where $\rho_\tau(\cdot)$ is the tilted absolute value function as shown in Figure 1, this gives the τth sample quantile with its solution. To see that this problem yields the sample quantiles as its

solutions, it is only necessary to compute the directional derivative of the objective function with respect to ξ, taken from the left and from the right.
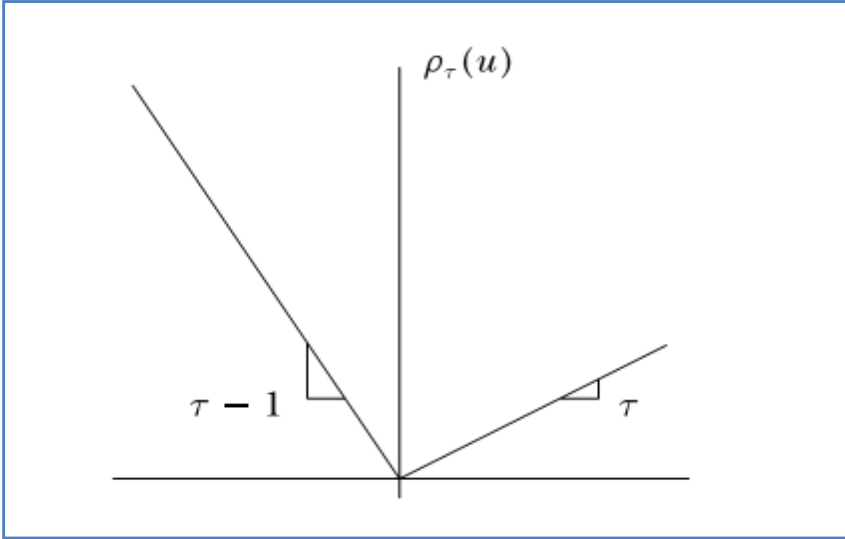
Figure 1: Quantile Regression ρ Function

After defining the unconditional quantiles as an optimization problem, it is easy to define conditional quantiles in an analogous fashion. Least squares regression offers a model for how to proceed. If, we have a random sample,$\{y_1, y_2, \ldots, y_n\}$, we solve

$$\min_{\mu \epsilon \mathcal{R}} \sum_{i=1}^{n}(y_i - \mu)^2 \tag{2}$$

we obtain the sample mean, an estimate of the unconditional population mean, *EY*. If we now replace the scalar μ by a parametric function $\mu(x, \beta)$ and solve

$$\min_{\mu \epsilon \mathcal{R}^p} \sum_{i=1}^{n}(y_i - \mu(x_i, \beta))^2 \tag{3}$$

we obtain an estimate of the conditional expectation function $E(Y|x)$.

We proceed exactly the same way in quantile regression. To obtain an estimate of the conditional median function, we simply replace the scalar ξ in the first equation by the parametric function $\xi(x_t, \beta)$ and set τ to $\frac{1}{2}$. To obtain estimates of the other conditional quantile functions, we replace absolute values by $\rho_\tau(\cdot)$ and solve

$$\min_{\xi \epsilon \mathcal{R}^p} \sum \rho_\tau (y_i - \xi(x_i, \beta)) \qquad\qquad (4)$$

The resulting minimization problem, when $\xi(x, \beta)$ is formulated as a linear function of parameters, can be solved very efficiently by linear programming methods.

This technique has been used widely in the past decade in many areas of applied econometrics; applications include investigations of wage structure (Buchinsky and Leslie 1997), earnings mobility (Eide and Showalter 1999; Buchinsky and Hunt 1996), and educational attainment (Eide and Showalter 1998). Financial applications include Engle and Manganelli (1999) and Morillo (2000) to the problems of Value at Risk and option pricing respectively. Barnes, Hughes (2002), applied quantile regression to study CAPM, in their work on the cross section of stock market returns.

3.   THE FAMA-FRENCH THREE FACTOR MODEL

Volatility is widely accepted measure of risk, which is the amount an asset's return varies through successive time periods. Volatility is most commonly quoted in terms of the standard deviation of returns. There is a greater risk involved for asset whose return fluctuates more dramatically than another other. The familiar beta from the CAPM equation is a widely accepted measure of systematic risk; whilst unsystematic risk is captured by the error term of the OLS application of CAPM. Beta is a measure of the risk contribution of an individual security to a well diversified portfolio as measured below;

$$\beta_A = \frac{cov(r_A, r_M)}{\sigma_M{}^2} \qquad\qquad (5)$$

where

r_A is the return of the asset

r_M is the return of the market

$\sigma_M{}^2$ is the variance of the return of the market, and

$cov(r_A, r_M)$ is covariance between the return of the market and the return of the asset.

Jack Treynor (1961, 1962), William Sharpe (1964), John Lintner (1965) and Jan Mossin (1966) independently, proposed Capital Asset Pricing Theory, (CAPM), to quantify the relationship between beta of an asset and its corresponding return. CAPM stands on a broad assumption that, that only one risk factor is common to a broad-based market portfolio, which is beta. Modelling of CAPM using OLS assumes that the relationship between return and beta is linear, as given in equation (2).

$$r_A = r_f + \beta_A(r_M - r_F) + \alpha + e \qquad\qquad (6)$$

where

r_A is the return of the asset

r_M is the return of the market

r_f is the risk free rate of return

α is the intercept of regression

e is the standard error of regression

Fama and French (1992, 1993) extended the basic CAPM to include size and book-to-market as explanatory factors in explaining the cross-section of stock returns. SMB, which stands for Small Minus Big, is designed to measure the additional return investors have historically received from investing in stocks of companies with relatively small market capitalization. This additional return is often referred to as the "size premium." HML, which is short for High Minus Low, has been constructed to measure the "value premium" provided to investors for investing in companies with high book-to-market values (essentially, the value placed on the company by accountants as a ratio relative to the value the public markets placed on the company, commonly expressed as B/M).

SMB is a measure of "size risk", and reflects the view that, small companies logically, should be expected to be more sensitive to many risk factors as a result of their relatively undiversified nature and their reduced ability to absorb negative financial events. On the other hand, the HML factor suggests higher risk exposure for typical "value" stocks (high B/M) versus "growth" stocks (low B/M). This makes sense intuitively because companies need to reach a minimum size in order to execute an Initial Public Offering; and if we later observe them in the bucket of high B/M, this is usually an indication that their public market value has plummeted because of hard times or doubt regarding future earnings.

The three factor Fama-French model is written as;

$$r_A = r_f + \beta_A(r_M - r_F) + s_A SMB + h_A HML + \alpha + e \qquad (7)$$

where s_A and h_A capture the security's sensitivity to these two additional factors.

Portfolio formation using this model requires the historical analysis of returns based on the three factors using regression measures, which quantifies estimates of the three risk variables involved in the model, i.e. β_A , s_A, h_A , and the usual regression analysis using OLS gives us the estimates around the means of the distributions of the historical returns and hence doesn't efficiently quantify the behaviour around the tails. Modelling the behaviour of factor models using quantile regression gives us the added advantage of capturing the tail values as well as efficiently analysing the median values.

4.   DATA & METHODOLOGY

The study uses daily prices of the 30 Dow Jones Industrial Average Stocks, for a period from January 2005-December 2008, along with the Fama-French factors for the same period, obtained from French's website to calculate the Fama-French coefficients.[1] Table 1, gives the 30 stocks traded in the Dow Jones Industrial Average and used in this study.

Table 1 : Dow Jones Industrial 30 Stocks used in the study.

3M	EI DU PONT DE NEMOURS	KRAFT FOODS
ALCOA	EXXON MOBILE	MCDONALDS
AMERICAN EXPRESS	GENERAL ELECTRIC	MERCK & CO.
AT&T	GENERAL MOTORS	MICROSOFT
BANK OF AMERICA	HEWLETT-PACKARD	PFIZER
BOEING	HOME DEPOT	PROCTER&GAMBLE
CATERPILLAR	INTEL	UNITED TECHNOLOGIES
CHEVRON	INTERNATIONAL BUS.MCHS.	VERIZON COMMUNICATIONS
CITIGROUP	JOHNSON & JOHNSON	WAL MART STORES
COCA COLA	JP MORGAN CHASE & CO.	WALT DISNEY

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[1] (Available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#International)

The approach here is to study the behaviour of the return distribution along the quantiles, using quantile regression. The coefficients for all the three factors of the model are calculated both by virtue of their means using OLS and in their quantiles applying quantile regressions. While OLS calculates the coefficients around the mean, quantile regression calculates the values for the .05, .25, .50, .75 and .95 quantiles, at 95 percentile confidence levels.[2] After studying the behaviour of the returns along the quantiles of the distribution, we use the three factor model for portfolio formation. We use a simple Sequential Quadratic Programming routine with the help of MATLAB, to minimize risk and mazimise return for portfolio formation. A hold out period of one year is taken to roll over the weights calculated from the previous year's returns to the stock returns of next year to explore the outcomes of portfolios selected using this method and to compare their effectiveness with portfolios formed using OLS.

## 5.   QUANTILE ANALYSIS OF FAMA-FRENCH FACTORS

We use OLS regression analysis and quantile regression analysis to calculate the three Fama-French coefficients. Figure 2, gives an example of the Bank of America stock's actual and fitted values obtained from the two regression methods for the year 2008. Exhibit-a from Figure 2 shows how the actual and fitted values run through the mean of the distribution for OLS and the next two exhibits, b and c shows the use of quantile regressions in efficiently capturing the lower and upper tails of the return distribution.

Figure-3, Figure-4, and Figure-5 provide a three dimensional area plot for the quantile estimates for all the stocks for the year 2007, these figures show how the values are non uniform across the quantiles and the effect can increase in the lower and upper quantiles, a feature that is ignored by OLS. The figures present the quantile estimates of beta, the size effect and the value or book to market effect respectively.

This analysis shows that the three-factor model can provide even more useful risk information, if it is used in combination with quantile regressions, as we display in the next stage of our analysis in which we form portfolios.

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[2] GRETL an open source software is used for OLS and Quantile Regression estimates plus STATA.
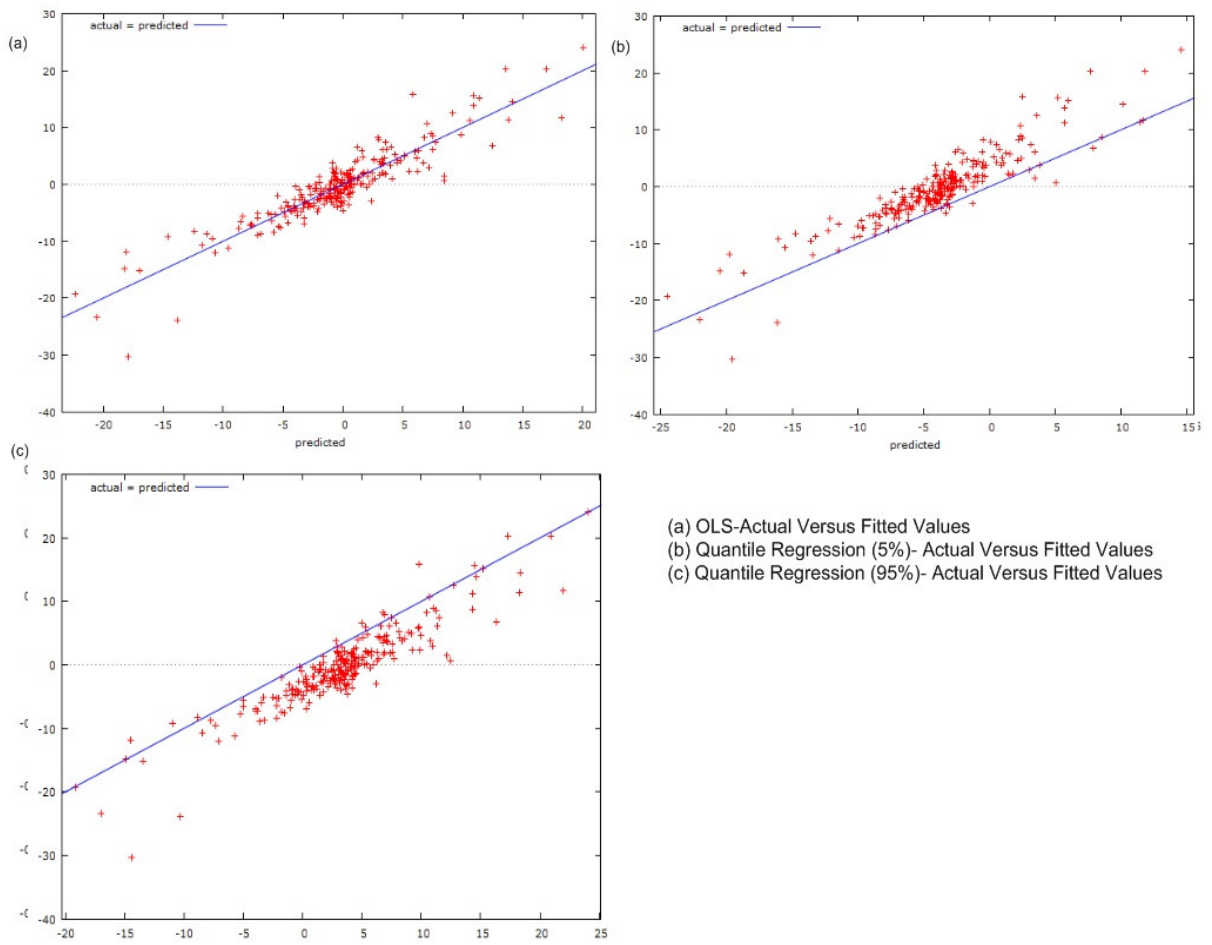
(a) OLS-Actual Versus Fitted Values
(b) Quantile Regression (5%)- Actual Versus Fitted Values
(c) Quantile Regression (95%)- Actual Versus Fitted Values

Figure 2 : OLS and Quantile Regression Fitted Versus Actual Values
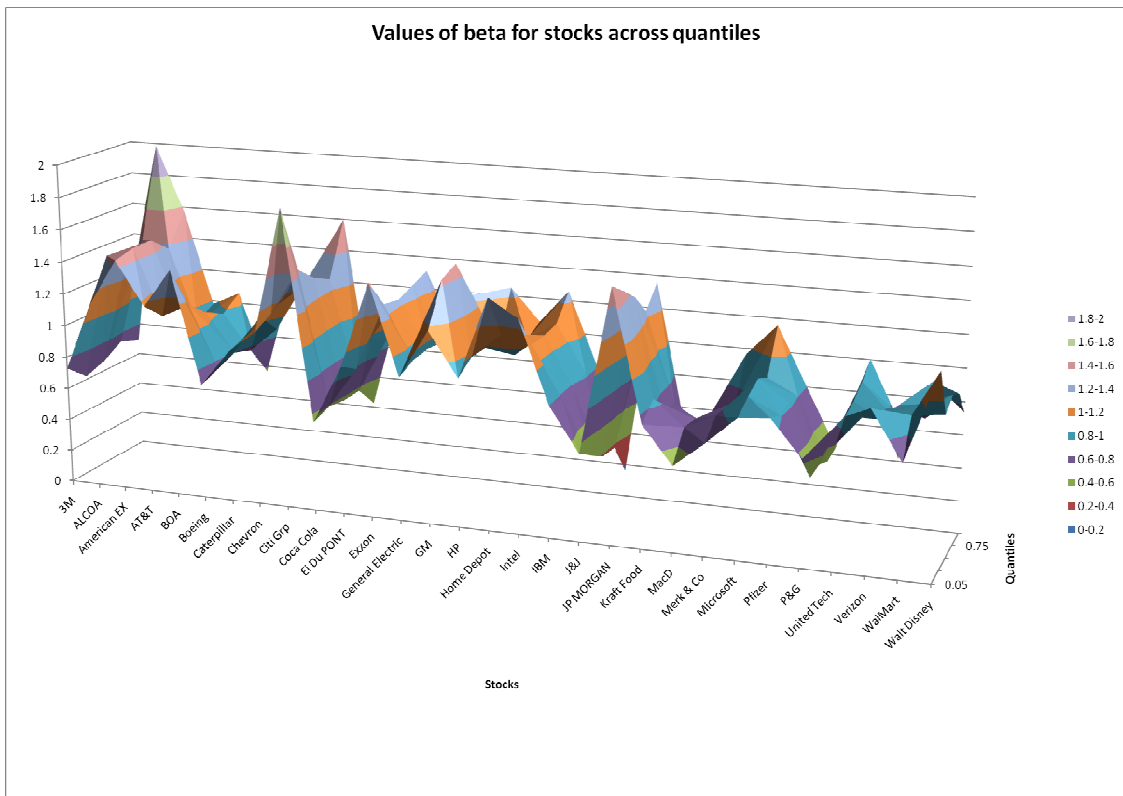
Figure 3 : Beta for stocks across quantiles

Figure 4: Size effect for stocks across quantiles

Figure 5: Value(HML) effect for stocks across quantiles

6. PORTFOLIO FORMATION USING THE FAMA-FRENCH THREE FACTOR MODEL

We now proceed to portfolio analysis using the three factor model and OLS and quantile regression estimates. As stated earlier; quantile regression provides better estimates along the tails of the distribution and hence accounts for risk more efficiently than OLS. We now introduce an additional ad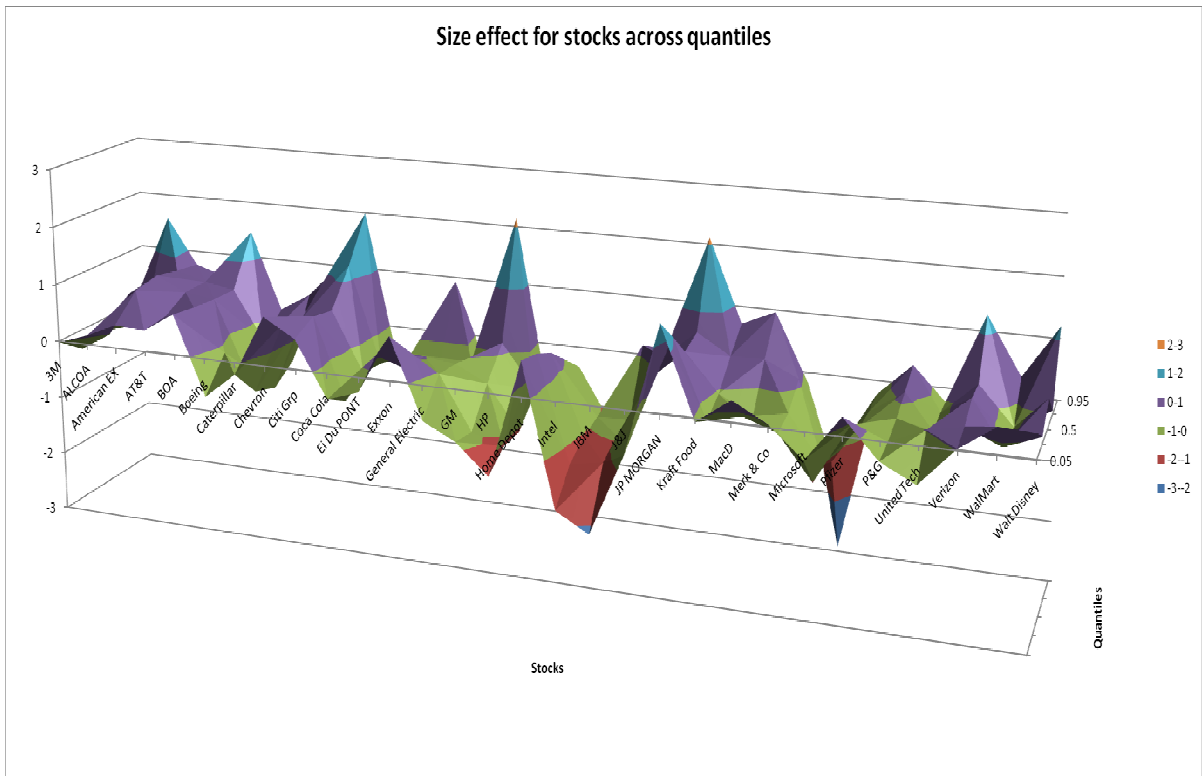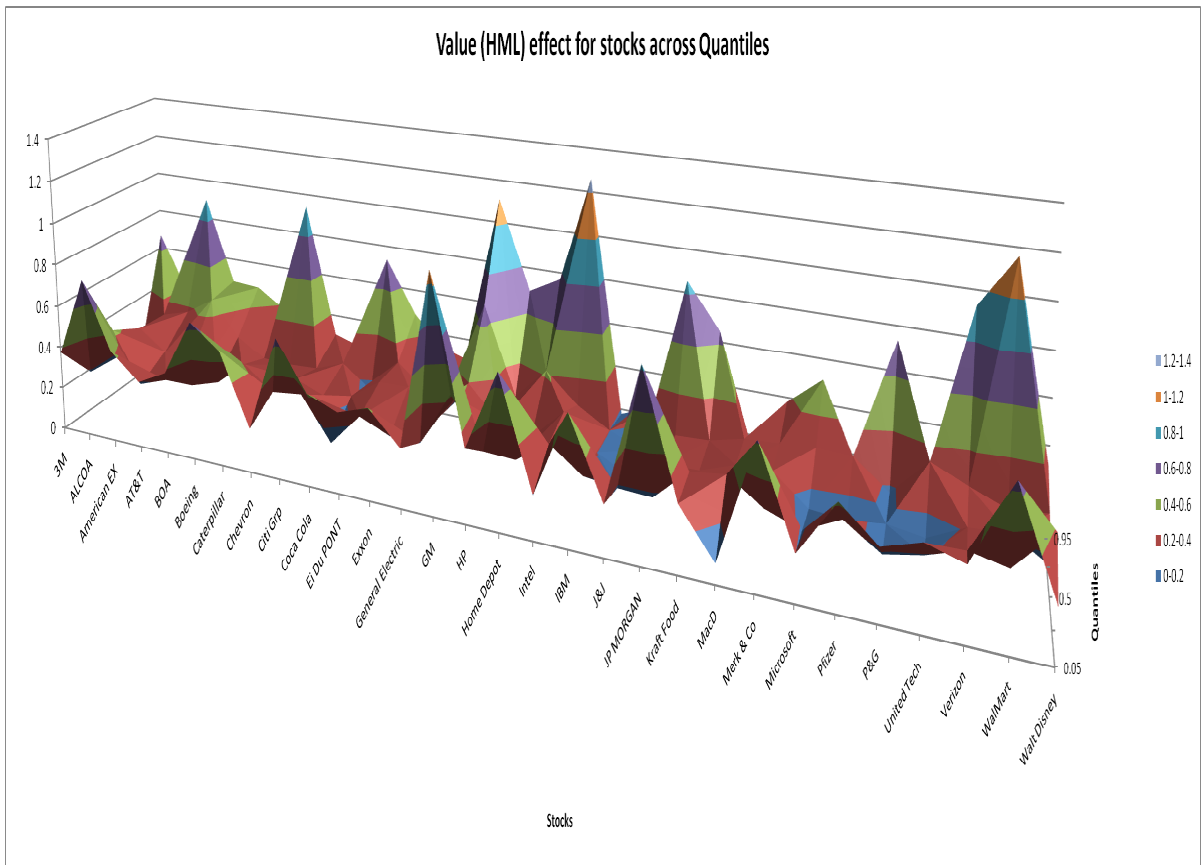vantage of quantile regression whereby its estimated coefficients can be combined by certain weighting schemes to yield more robust measurements of sensitivity to the factors across the quantiles, as opposed to OLS estimates around the mean. This approach was originally proposed by Chan and Lakonishok (1992) in a paper which featured simulations to establish the facility of quantile regressions in equity beta estimations. Their results show that the weighted average of quantile beta coefficients is more robust than the OLS beta estimates. We will test two weighting schemes for robust measurement of size and book to market effects based on the quantile regression coefficients. The resulting estimators have weights which are the linear combination of quantile regression coefficients.

We will use Tukey's trimean as our first estimator:

$$\beta_t = 0.25\beta_{0.25,t} + 0.5\beta_{0.5,t} + 0.25\beta_{0.75,t}, \tag{8}$$

$$s_t = 0.25s_{0.25,t} + 0.5s_{0.5,t} + 0.25s_{0.75,t}, \tag{9}$$

$$h_t = 0.25h_{0.25,t} + 0.5h_{0.5,t} + 0.25h_{0.75,t}, \tag{10}$$

$$\alpha_t = 0.25\alpha_{0.25,t} + 0.5\alpha_{0.5,t} + 0.25\alpha_{0.75,t}, \tag{11}$$

These are the weighted average of the three quantile estimates. We will test this along with another robust estimator with symmetric weights covering all the quantile estimates, i.e. 0.05, 0.25, 0.5, 0.75, 0.95.

$$\beta_t = 0.05\beta_{0.05,t} + 0.2\beta_{0.25,t} + 0.5\beta_{0.5,t} + 0.2\beta_{0.75,t} + 0.05\beta_{0.95,t} \tag{12}$$

$$s_t = 0.05s_{0.05,t} + 0.2s_{0.25,t} + 0.5s_{0.5,t} + 0.2s_{0.75,t} + 0.05s_{0.95,t} \tag{13}$$

$$h_t = 0.05h_{0.05,t} + 0.2h_{0.25,t} + 0.5h_{0.5,t} + 0.2h_{0.75,t} + 0.05h_{0.95,t} \tag{14}$$

$$\alpha_t = 0.05\alpha_{0.05,t} + 0.2\alpha_{0.25,t} + 0.5\alpha_{0.5,t} + 0.2\alpha_{0.75,t} + 0.05\alpha_{0.95,t} \tag{15}$$

The portfolio problem using the Fama-French three factor model, requires a solution for minimum risk and maximum return. The return and risk of the portfolio is as presented in equations 16 and equation 17.

$$Return_p = \sum_{i=1}^{n} \beta_i Avg(r_M - r_F)w_i + s_i Avg(SMB)w_i + h_i(HML)w_i + \alpha_i w_i \quad (16)$$

$$Risk_p = \sum_{i=1}^{n} \beta^2{}_i Var(r_M - r_F)w^2{}_i + s^2{}_i Var(SMB)w^2{}_i + h^2{}_i Var(HML)w^2{}_i \quad (17)$$

This forms a classical portfolio optimization problem of minimizing risk (equation 17) and maximising the return (equation 16). We apply here sequential quadratic programming which is also referred to as recursive quadratic programming and is used for solving general non linear programming problems. (For details refer to *Robust portfolio optimization and management*, Frank J. Fabozzi, Petter N. Kolm, Dessislava Pachamanova, page 284-285). We leave the mathematical details of the algorithm for the sake of brevity. MATLAB's optimization toolbox is used to execute the algorithm, with, additional constrain of maximum 10% weight per asset for well diversified portfolio and minimum of 0% daily return on the portfolio formed (to prevent optimization from generating optimized weights for negative portfolio returns).

We generate portfolios using historical data for three consecutive years, 2005, 2006 and 2007 with a following hold out period of one year in each case. We use OLS and quantile risk measures with Tukey's trimean and symmetric weights to generate three different portfolios. We then roll over the weights as calculated by these respective routines to the next year and calculate the realized return and risk for the next year for each of the three portfolios. Risk for the rolled over period used as a hold-out sample is the actual diversifiable risk calculated using the covariance of the daily returns of the stocks and the weights of the selected portfolios.

We then compare the realized return and risk for the next year obtained from maintaining the portfolio through the hold out period using the Sharpe Index so as to analyse which portfolio performs better in times of severe financial distress.

Table 2, Table 3, and Table 4, give the weights generated from the historical data of the years 2005, 2006 and 2007 respectively. W1, W2, W3 represent the weights for quantile regression coefficients using Tukey's trimean, the quantile regression coefficients with symmetric weights and the OLS coefficients respectively.

Table 2: Portfolio Weights from Year 2005 Data

Stocks	W1	W2	W3
3m	0.06229200	0.05610500	0.04195300
ALCOA	0.01290900	0.01543800	0.02106300
American EX	0.02474200	0.02767300	0.02846100
AT&T	0.07121300	0.05595500	0.04779500
BOA	0.04663300	0.04683900	0.03813200
Boeing	0.06837600	0.06724900	0.05689900
Caterpillar	0.03617300	0.03536900	0.02395100
Chevron	0.02718700	0.02274000	0.01319100
Citi Grp	0.07690000	0.06416700	0.04120500
Coca Cola	0.05264500	0.05184300	0.06206800
Ei Du PONT	0.00240340	0.00237950	0.02023100
Exxon	0.01976700	0.01873900	0.01069700
General Electric	0.01643300	0.02068200	0.03077100
GM	0.00000000	0.00000000	0.00000000
HP	0.07385500	0.06914300	0.07248900
Home Depot	0.01057300	0.00832410	0.01841600
Intel	0.03736100	0.03732700	0.02088100
IBM	0.02116300	0.02294500	0.02155600
J&J	0.02146600	0.03809600	0.05243100
JP MORGAN	0.02578700	0.02795800	0.03209500
Kraft Food	0.03597500	0.05065400	0.06278900
MacD	0.02907300	0.03247500	0.02674400
Merk & Co	0.07557900	0.06570100	0.04616000
Microsoft	0.03363300	0.03609500	0.03442900
Pfizer	0.00000000	0.00000000	0.01821700
P&G	0.06333700	0.06208600	0.03596300
United Tech	0.03444800	0.03508400	0.02665100
Verizon	0.00000000	0.00000000	0.01833700
WalMart	0.01631000	0.02151200	0.04983900
Walt Disney	0.00376770	0.00742090	0.02658600

Table 3: Portfolio Weights from Year 2006 Data

Stocks	W1	W2	W3
3m	0.03284900	0.03362400	0.04019300
ALCOA	0.01351000	0.01440800	0.01544900
American EX	0.02335900	0.02438800	0.02633800
AT&T	0.03976500	0.03841900	0.03662300
BOA	0.02918900	0.02966100	0.02955800
Boeing	0.02010400	0.02069000	0.02527500
Caterpillar	0.01649400	0.01700400	0.01883900
Chevron	0.01055100	0.01124700	0.01266900
Citi Grp	0.02870300	0.02892000	0.02403100
Coca Cola	0.04776700	0.04909600	0.05077000
Ei Du PONT	0.03455100	0.03553700	0.03538400
Exxon	0.01253800	0.01309600	0.01420500
General Electric	0.04406400	0.04298500	0.03480700
GM	0.02521600	0.02807200	0.03376900
HP	0.02096300	0.01968800	0.01996100
Home Depot	0.02944500	0.03056700	0.02740400
Intel	0.01605000	0.01569300	0.01296800
IBM	0.03349600	0.03418700	0.03365800
J&J	0.04796600	0.04595400	0.04034600
JP MORGAN	0.01543100	0.01546500	0.01687000
Kraft Food	0.10000000	0.10000000	0.10000000
MacD	0.04972700	0.04953500	0.05555300
Merk & Co	0.02373600	0.02367400	0.02729900
Microsoft	0.03001600	0.02829300	0.02066000
Pfizer	0.02322900	0.02316100	0.02371600
P&G	0.07478600	0.07337400	0.06627500
United Tech	0.04301200	0.04222300	0.04210000
Verizon	0.02917800	0.02927200	0.03017800
WalMart	0.04295900	0.04418600	0.04350900
Walt Disney	0.04134800	0.03758400	0.04159400

Table 4: Portfolio Weights from Year 2007 Data

Stocks	W1	W2	W3
3m	0.054393	0.100000	0.040137
ALCOA	0.013114	0.000000	0.011691
American EX	0.010286	0.000000	0.010973
AT&T	0.021150	0.004994	0.023461
BOA	0.019031	0.000000	0.019723
Boeing	0.036850	0.000000	0.037031
Caterpillar	0.023927	0.037533	0.023411
Chevron	0.017666	0.069037	0.017536
Citi Grp	0.010951	0.000000	0.011810
Coca Cola	0.071481	0.100000	0.078598
Ei Du PONT	0.027253	0.000000	0.029683
Exxon	0.015905	0.039803	0.015837
General Electric	0.026498	0.014420	0.027831
GM	0.009273	0.000000	0.012645
HP	0.024137	0.066279	0.022169
Home Depot	0.020272	0.000000	0.024160
Intel	0.020009	0.067342	0.017283
IBM	0.037055	0.100000	0.033876
J&J	0.100000	0.027851	0.100000
JP MORGAN	0.011823	0.000000	0.012573
Kraft Food	0.041194	0.000000	0.046818
MacD	0.060709	0.100000	0.076052
Merk & Co	0.042629	0.000000	0.043153
Microsoft	0.025494	0.026575	0.024309
Pfizer	0.033774	0.000000	0.035079
P&G	0.100000	0.100000	0.078537
United Tech	0.032082	0.072465	0.031047
Verizon	0.034559	0.073701	0.033326
WalMart	0.031722	0.000000	0.034201
Walt Disney	0.026762	0.000000	0.027047

Table 5: Final Risk and Return for all the three types of weights after a roll over period of a year

Stocks	Ret (2006)	Ret (2007)	Ret(2007)
3m	0.00553305	0.07880810	-0.38213873
ALCOA	0.01477033	0.19715053	-1.17742456
American EX	0.16464454	-0.15382108	-1.03115847
AT&T	0.37828540	0.15058790	-0.37723408
BOA	0.14572650	-0.25772995	-1.07513816
Boeing	0.23491905	-0.01565545	-0.71768545
Caterpillar	0.05979951	0.16814469	-0.48511167
Chevron	0.25868547	0.23844811	-0.23248199
Citi Grp	0.13779798	-0.63762585	-1.47875534
Coca Cola	0.17979625	0.24052529	-0.30427655
Ei Du PONT	0.13638027	-0.09965135	-0.5554286
Exxon	0.31060584	0.20100281	-0.16009209
General Electric	0.05980193	-0.00376953	-0.82779677
GM	0.45861045	-0.21044779	-2.05131531
HP	0.36374039	0.20338171	-0.330035
Home Depot	-0.00793655	-0.39925930	-0.15724877
Intel	-0.20911975	0.27500952	-0.59804162
IBM	0.16710087	0.10680055	-0.25033698
J&J	0.09394788	0.01024723	-0.10869775
JP MORGAN	0.19633229	-0.10122828	-0.32526381
Kraft Food	0.23689311	-0.08991858	-0.19496629
MacD	0.27357052	0.28434921	0.05418336
Merk & Co	0.31527644	0.28728062	-0.64789516
Microsoft	0.13267060	0.17582585	-0.60501284
Pfizer	0.10493161	-0.13055733	-0.24955619
P&G	0.10472106	0.13310342	-0.17196944
United Tech	0.11174325	0.20232697	-0.35626441
Verizon	0.24929073	0.15973593	-0.24901024
WalMart	-0.01333640	0.02881429	0.165061455
Walt Disney	0.35746867	-0.04643890	-0.35252354

2006					
Quantile Regression (Trimean)		Quantile Regression (Symmetric Weights)		OLS	
Return	Risk	Return	Risk	Return	Risk
0.17707517	0.00623073	0.17178290	0.00613572	0.17189773	0.00603117
Sharpe Ratio	25.20975199	24.73760491		25.18545442	

2007					
Quantile Regression (Trimean)		Quantile Regression (Symmetric Weights)		OLS	
Return	Risk	Return	Risk	Return	Risk
0.02816570	0.00880364	0.02651735	0.00883844	0.03048237	0.00886899

Sharpe Ratio	1.72266282		1.52938198		1.97117930
2008					
Quantile Regression (Trimean)		Quantile Regression (Symmetric Weights)		OLS	
Return	Risk	Return	Risk	Return	Risk
-0.35877338	0.02189977	-0.28532662	0.02118938	-0.35993622	0.02202203
Sharpe Ratio	-16.56516722	**-13.65431927**		-16.52600367	

Table 5, provides the final risk and returns after a hold out period of a year. The risk (standard deviation), is the total portfolio risk calculated using the covariance of daily returns of the stocks and the relevant weights. Return is calculated using the first and the last day's prices for the stocks for the particular year; the annualized rate of return. The Sharpe ratio values indicate the efficiency of the portfolios formed through the three different regression estimates. We can quickly analyse the effectiveness of the portfolios based on the Sharpe ratio, which is the excess return of a portfolio divided by its risk.

We analyse the return and risk profiles of the portfolios based on the Sharpe Index and also on the basis of their risk. For the years 2006, and 2007 we can see that the portfolios formed using OLS do well, as these periods coincide with at time when market was stable and there were no major losses of the scale that occurred in the year 2008 as a result of the GFC, yet even so, during these periods the portfolios formed using quantile regressions performed reasonably well.

Figure 6 shows the returns for all three portfolios for the three observation years, (the return lines for portfolio 1 and portfolio 3 are almost overlapping due to similar returns). These years range in period from pre GFC to the onset and establishment of the GFC. The returns of the portfolios present a rational picture consistent with these varying circumstances. We can see from Figure 6 that the three test portfolios performed almost equally well in the year 2006, as the distribution of the returns in the prior historical analysis period, in which the weights were formed, i.e. the year 2005 were less skewed towards the lower tails; as they were in years prior to the financial crisis period. We can further conclude from Figure 6 that as we approach closer to the financial crisis period, our symmetrically weighted quantile regression coefficient portfolio begins to perform better than the other two methods; given that during the time of financial distress the return distributions are more skewed towards the lower tails and portfolio selection methods

based on OLS and the Tukey's trimean quantiles are unable to capture these extreme characteristics of the return distributions, and hence unable to give a proper measure of the risks involved. Our portfolio analyses show the useful applicability of quantile regression analysis, as a tool for the quantification of the tail risks involved with the return distributions of financial assets.
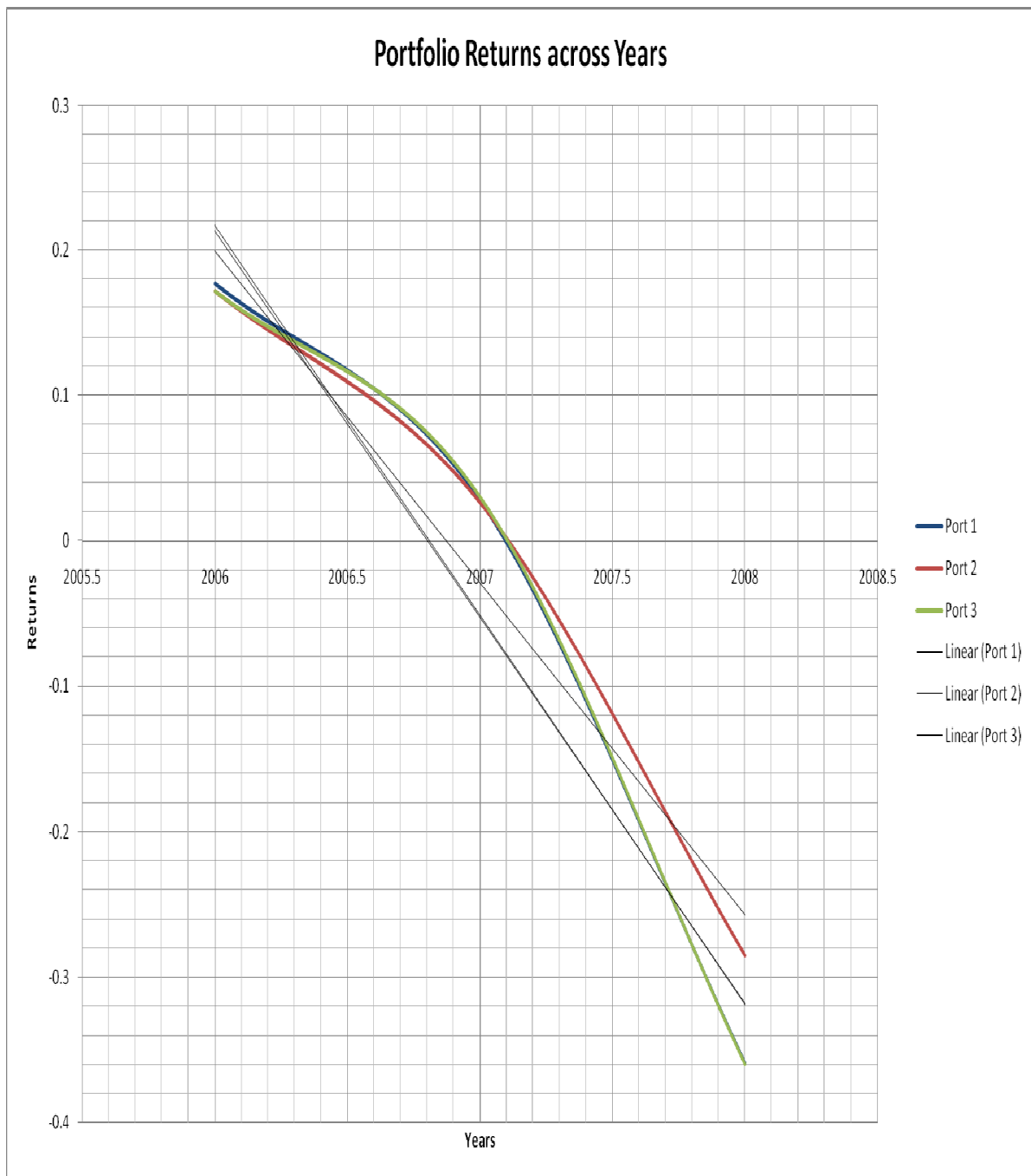
Figure 6: Portfolio Returns across Years

Our main focus is the period of immense financial distress and downturn in equity markets. We are testing here, whether quantile regression was able to predict the heavy risks and whether its application helps to reduce the losses that occurred during this particularly extreme hold out period. The analysis of portfolios held during the year 2008 clearly shows that the portfolio formed with symmetric weights from the quantile regression coefficients, which automatically covered both the extreme lower and upper bounds of the return distributions performed better than the other two methods. This portfolio saved around 2% of the relative potential losses to the investor.

The analysis shows that a well distributed quantile regression analysis of historical returns can give better estimates of the inherent risks than standard OLS analysis. We also show that the weighting scheme tested here proves more effective in capturing information from the extreme quantile coefficients that receive more emphasis than that given in the other two methods considered.

7. CONCLUSION

In this paper we have introduced quantile regression as a tool for investment analysis and portfolio management. Our study shows that quantile regression can provide more effective use of information in the entire distribution than is the case with estimates from the customarily used OLS. We can achieve more efficient risk measures using this robust regression technique. The technique becomes particularly useful when we want to analyse the behaviour in the tails of the distributions of returns or to capture a more complete picture of the risk of a financial instrument. Our analysis suggests that further research using quantile regression in the context of the application of linear asset pricing models and their empirical effectiveness in extreme market conditions for portfolio formation is likely to be fruitful.

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