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ABSTRACT

A spent cartridge case exhibits characteristic markings (firearm fingerprint) that can be used to identify the type and possibly make of weapon in which the cartridge was fired. This report details research into the use of discriminant analysis for the purpose of matching spent rim-fire cartridge cases to specific make and model firearms. The discrimination and classification are based on several scalar shape parameters for the two-dimensional silhouette of the firing pin (FP) impression— shape factor calculated from the second order moment of inertia, G factor calculated from the distance transform, and the P2A factor— as well as the distance between the centre of the cartridge case and the centroid of the FP impression, and the orientation of the principal centroidal axes associated with the FP impression. Classification results for two case studies are detailed: (i) 3 different make/model weapons producing different shaped FP impressions, and (ii) 5 different make/model weapons each producing a rectangular FP impression.

Keywords* - *Ballistics, Class characteristics, Discriminant analysis, Image matching, Shape descriptors

1. INTRODUCTION

"The Federal Bureau of Investigation (FBI) defines firearms identification as 'the study by which a bullet[1], cartridge case or shotshell casing may be identified as having been fired by a particular weapon to the exclusion of all other weapons.' " (Giannelli, 1991, p. 196). Firearms identification is more commonly known as *ballistics* — a misnomer because ballistics is the study of projectile motion. There are typically three types of weapons encountered in ballistics: rifles, handguns, and shotguns. The evidence associated with a firearm used to commit a felony includes: powder residues, fingerprints, blood, trajectory (true ballistics), bullets (projectiles), and cartridge cases. "Typically, the most definitive evidence obtained from material associated with firearms is not intrinsic to the firearm per se. It arises rather from the interaction of the firearm with components of the cartridge" (Heye & Thornton, 1994, p. 83). Figure 1 is a sketch of an unfired cartridge case. "Cartridge cases are generally made of brass. Several different shapes—straight, tapered, or bottleneck—are manufactured. The type of rim may also differ—e.g., rimmed, semirimmed, rimless, belted, or rebated (rim diameter is less than case diameter)" (Giannelli, 1991, p. 199). Rifles and handguns are generally classified according to their calibre[2]. The cartridges used in these weapons comprise the case, bullet (inserted into the mouth of the case), propellant (powder), and percussion priming mixture. They are generally of two types: rim-fire and centre-fire.

> **The rim-fire cartridge is simply a short tube of copper [nowadays cartridges are typically made from brass], closed at one end, having a charge of powder in it, and carrying a bullet in the open end. The closed end is formed into a flat head, with a hollow rim, and inside this rim there is a layer of percussion priming mixture. When the gun is fired, the firing pin, or the hammer nose, as the case may be, strikes this hollow rim, and mashes a small portion of it, crushing the mixture at this point, and causing it to ignite and fire the charge. (Hatcher, 1946, p. 67). . . .While the rim-fire cartridge still enjoys enormous popularity in the .22 caliber, it has been superseded in practically every other caliber by the later type known as the center-fire cartridge. This is a brass cartridge with a thick, heavy head, containing a percussion cap, or *primer*, pressed into a circular recess in the center. (p. 69).**

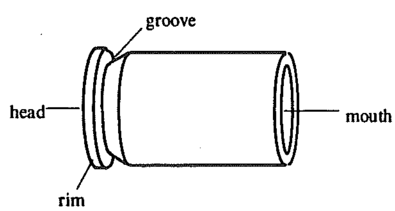

Figure 1. Sketch showing the parts of an unfired cartridge case.

Using a comparison microscope a forensic scientist can compare evidence bullets or cartridge cases with those obtained from test firings either from the evidence firearm or from an array of test weapons. When there is no evidence firearm the expert can only infer the make or type of weapon from which the projectiles or cases came. If the firearm is available then the expert may be able to infer whether or not they came from that particular weapon. "It may frequently happen that in a crime of violence with firearms the bullet may not be recovered; it may pass entirely through the victim and be lost" (Hatcher, 1946, p. 259). Even when a bullet is recovered its condition often precludes the positive identification of the type or specific firearm from which it was fired – the bullet may be too mutilated, or too little of the bullet may have been recovered. If the firearm used to commit a crime is an automatic weapon then investigators are likely to find the empty cartridges (cases) expelled by the gun. "In many cases, also, the repeating . . . rifle may be fired two or more times in the commission of a crime, or if it is fired only once, the chances are that the criminal will reload at once, unconsciously causing the gun to eject the

[1] The term *bullet* is sometimes used in place of the term *cartridge*. Herein, *bullet* means *projectile*.
[2] The diameter of the bore (inside surface of the barrel) expressed in either hundredths or thousandths of an inch (e.g. .22 and .243), or in millimetres (e.g. 9 mm).

empty cartridge" (Hatcher, 1946, p. 259). A cartridge case recovered from the scene of the crime is then most valuable.

2. CARTRIDGE CASE IDENTIFICATION

Figure 2 is a sketch of a revolver showing a cartridge in position ready for firing. A recovered cartridge case has several prominent *class characteristics* including: the initials of the manufacturer stamped on its head, its type and calibre, extractor[3] hook marks (particularly in the case of automatic arms), ejector[4] mark (which has a fixed relation to the position of the extractor mark), and the shape of the firing pin impression (in the case of rim-fire cartridges). "It is a well-known fact that at the moment when a cartridge is fired, the empty shell is hurled violently against the breech face, as a result of the recoil. The primer and shell [cartridge case] head receive certain imprints from the breech face . . . . These impressions vary considerably in their nature, and may be of great value in determining the type or even the particular make of weapon used" (Mathews, 1962, p. 311). Breech faces can to some degree be classified according to the characteristic patterned markings they possess. This is "because each manufacturer has a certain procedure for the production of a certain model of arm [which produces the characteristic markings], which may differ from that used by some or all other manufacturers, and because he follows this procedure fairly consistently, possibly in his different models" (Mathews, 1962, p. 28). In the case of centre-fire cartridges the primer is made of a softer material, e.g. copper, than the brass of the shell itself. The primer therefore is able to take a better impression of the breech face than the rest of the cartridge case head. The rim-fire cartridge case on the other hand

> **takes a good impression showing the shape of the firing pin, but it does not often take a clear impression of the fine file marks and other irregular scratches on the breech block, which form the "finger-prints" of the gun; hence when an empty rim-fire cartridge is found at the scene of a shooting, it is often easy to say what type of arm was used; but it is seldom possible to identify a rim-fire cartridge to a definite individual gun by the impression of the file marks it left on the head, as is so often done in the case of a center-fire cartridge. (Hatcher, 1946, p. 68).**

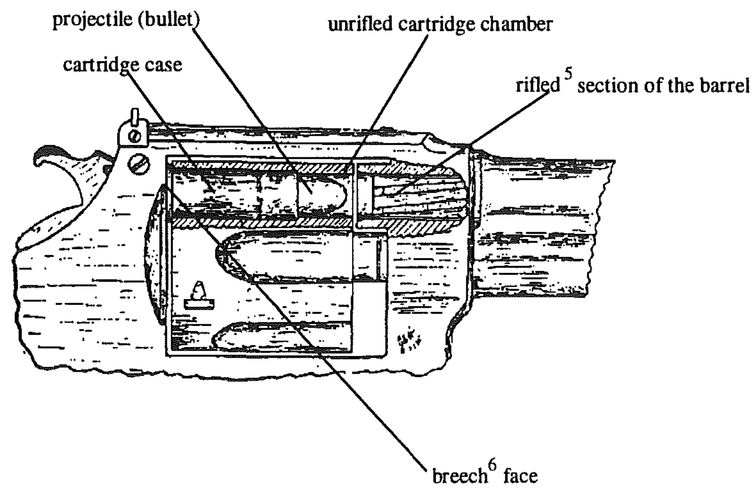

Figure 2. Cut-away of a revolver (Smith & Wesson .38/44) showing a cartridge in position ready for firing (Adapted from Hatcher, 1946, p. 49).

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[3] "The extractor is the mechanism that withdraws the cartridge case from the chamber after the firearm has been fired" (Giannelli, 1991, p. 204)

[4] "The ejector is the mechanism that throws or 'kicks out' the cartridge case from the firearm after it has been fired" (Giannelli, 1991, p.204). In some weapons the firing pin acts as the ejector.

[5] *Rifling* are the parallel spiral grooves cut into the bore of modern rifles and hand guns and designed to impart a spin on the projectile as it passes through.

[6] The *breech* is the end of the bore into which the cartridge is inserted.

To establish that a recovered cartridge case was fired in a specific firearm the unique markings (striations and impressed marks) imparted on the case head by the firing pin and the breech face, as well as the impressed chamber marks on the side of the case, need to be examined microscopically. These markings are those unique to that particular weapon; they are not even exhibited on cartridge cases fired in exactly the same type of weapon produced by the same manufacturer. "By utilizing these individual characteristics, 'causal identity' is established, that is, the markings on both evidence and test case were caused by the same event, namely, the interaction of that firearm with the cartridge case" (Heye & Thornton, 1994, p. 84). Collectively the class characteristics and the individual features on a fired cartridge case constitute a firearm fingerprint.

Forensic laboratories around the world still use comparison microscopes to examine and compare markings on cartridge cases. This is true even of the FBI; they began using comparison microscopes in 1925 (Kaplan, 1993, p. 54). Consequently cartridge case comparisons are very labour intensive. In Police Ballistics Units around Australia, this very fact prohibits the routine checking of catalogued exhibits (e.g. from unsolved shootings) against those obtained from weapons that come into the possession of the Police (Lawrence, 1993, p. 1). Moreover these exhibits are not routinely circulated around the country. There is clearly a need for a national computerised image storage and comparison system.

3. EXISTING COMPUTER SYSTEMS AND ONGOING RESEARCH

A Canadian company, Walsh Automation, has developed a commercial hardware/software system, called *Bulletproof*, that can acquire and store images of projectiles and brass cartridge cases, and automatically search the image database for particular striations on projectiles (but not impressed markings or striations on cartridge cases). Lawrence (1993, p. 4) stated that the system would cost around $540,000 US per participating Australian state. The cost and inherent limitations (particularly with respect to cartridge cases) of the system prohibit its use in Australia. The Australian Institute of Security and Applied Technology (AISAT), in conjunction with the Western Australian Police, have developed a prototype database called *FIREBALL* (Smith, Cross, & Variyan, 1995). It is modelled after the FBI's *general rifling characteristics* file (GRC) but with the added capability of storing and retrieving images of cartridge case heads, and of interactively obtaining position metrics for the firing-pin impression, ejector mark, and extractor mark. However, like Bulletproof, the system has no facility to perform image matching of breech face marks, firing-pin impressions, or the like.

An inherent problem with digital images of cartridge cases obtained using light stereo microscopy is that depth information of features, such as the firing pin impression, must be inferred from the shadows induced by the light source. Moreover the position of the source relative to the case dramatically influences the amount of detail that can be seen. This has led research teams in other countries to investigate alternative methods for determining an accurate description of the morphology of the head of a cartridge case. In Germany ultrasound is being investigated as a means of obtaining depth information at points on the head of a cartridge case (Senior Constable P. Lawrence, personal communication, June, 1994). In the USA the Air Gage Company and the Industrial Technology Institute have been contracted by the FBI to integrate their product CADEYES into the *Drugfire*[7] database system. CADEYES uses *moiré interferometry* to obtain "depth information at each point in the image of the object [cartridge case] being measured" (Kaplan, 1993, p. 54). Other researchers have reported on more novel and specialised techniques for cartridge case identification. For example Fatuzzo and Puglishi (1992) made a study of the characteristic markings produced on cartridge cases fired in weapons with delayed blowback action (the advantages of the delayed blowback mechanism include: simplicity of the bolt assembly, fewer parts, fast automatic cycle, lack of violent jerking, and economy and strength). Another example is the research reported by Heye and Thornton (1994) into compositional matching of cartridge cases using atomic absorption spectrometry. The method is used to determine the concentrations of nickel, iron, and lead in brass cartridge cases. However not only is the technique very specialised but it also interferes with the evidence cartridge case(s).

[7] *Drugfire* is "a database driven multimedia image-analysis management system" (Kaplan, 1993, p. 56).

4. AISAT RESEARCH

The aim of this research is to utilise existing comparison microscopes (fitted with adaptors for both still and video cameras) in conjunction with a computer and custom imaging software to perform automated/semi-automated matching of rim-fire cartridge case images stored in a database. Whilst images acquired in this manner are dependent on the orientation of the light source this need not be an issue when dealing with gross features such as the shape of the firing pin impression and the relative positions of the ejector and extractor marks. For fine detail such as breech face marks, chamber marks, and feed marks however, orientation is significant. In the former case a video camera and frame-grabber are sufficient to obtain a digital image of the head of a cartridge case. In the latter case, high resolution imaging and several captures per case are necessary to obtain adequate detail.

In this report we describe a pilot study that focused on several scalar parameters associated with the firing pin impression on spent rim-fire[8] cartridge cases, for the purpose of identifying the make/model firearms in which each was fired. "The size, shape, and location of the firing pin impression is of value in determining the make of arm used" (Mathews, 1962, p. 22). For the study, images of rim-fire cartridge case heads were captured using a conventional stereo light microscope and a video camera and frame-grabber. These images provided sufficient detail so that the shape of the firing pin impression and on occasion its location relative to the ejector mark were discernible. Using custom software, incorporating a graphics user interface (GUI), the outlines of the firing pin impressions were traced and quantified (using scalar feature parameters). Discriminant analysis was then used to discriminate and classify the cartridge case images according to the specific make/model weapon in which each cartridge was fired. We specifically report on the details and results for two case studies: (i) 3 different make/model weapons producing different shaped FP impressions, and (ii) 5 different make/model weapons each producing a rectangular FP impression.

4.1. Characterising shape

"Shape is an elusive property, difficult to define without being vague" (Danielsson, 1978, p. 292). In the field of image processing there exist a wide variety of *shape descriptors;* these are either single, dimensionless, and scale independent parameters called *shape factors*, or coding schemes that in some way characterise the boundary of an object. Reviews of shape coding techniques and shape descriptors can be found in Pavlidis (1978) and more recently in Marshall (1989). Shape descriptors fall into two categories:

(i) *internal* – based on the area within the boundary of an object, and
(ii) *external* – based on the boundary itself.

Furthermore, they can be classified as either *scalar transform* or *space domain*. A well known shape factor is *P2A* defined:

$$P2A = \frac{P^2}{4\pi A}, \text{where } P = \text{perimeter, and } A = \text{area},$$

which is equal to 1 for a circle. It is an example of an internal scalar transform descriptor (to calculate the perimeter it is not necessary to determine or track the boundary – e.g. Crofton's formula (see Mehnert, 1994a, p. 89)). Examples of internal space domain transforms are the *medial axis transform* and the *morphological skeleton* (see Serra, 1982, pp. 382-387). *Chain coding* is an example of an external space domain technique. Chain coding techniques (see Gonzalez & Wintz, 1987, chapter 8) involve tracking the boundary of an object, pixel by pixel. For images digitised on a square grid, either 4- or 8-connectivity can be used to code the boundary. For a given pixel, and proceeding in either a clockwise or anticlockwise direction, the next 8-connected (resp. 4-connected) boundary pixel must necessarily be in one of 8 (resp. 4) adjacent positions. Thus if each direction is given a unique code, then as the boundary is traversed, a chain of codes is established. *Fourier shape descriptors* (Krzyzak,

[8] It is interesting to note that in Western Australia, at least, the majority of firearms involved in shootings are .22 rifles and shotguns.

Leung, & Suen, 1988) are examples of external scalar transforms. They are derived from a normalisation of the Fourier coefficients obtained from the Fourier transformation of a 1D or 2D representation of the object boundary (Marshall, 1989, p. 284). Scale change and change in orientation of shapes manifest themselves as simple transformations of the Fourier coefficients. Figure 3 is a compendium of shape descriptors, most of which are reviewed in Marshall (1989) and Pavlidis (1978).

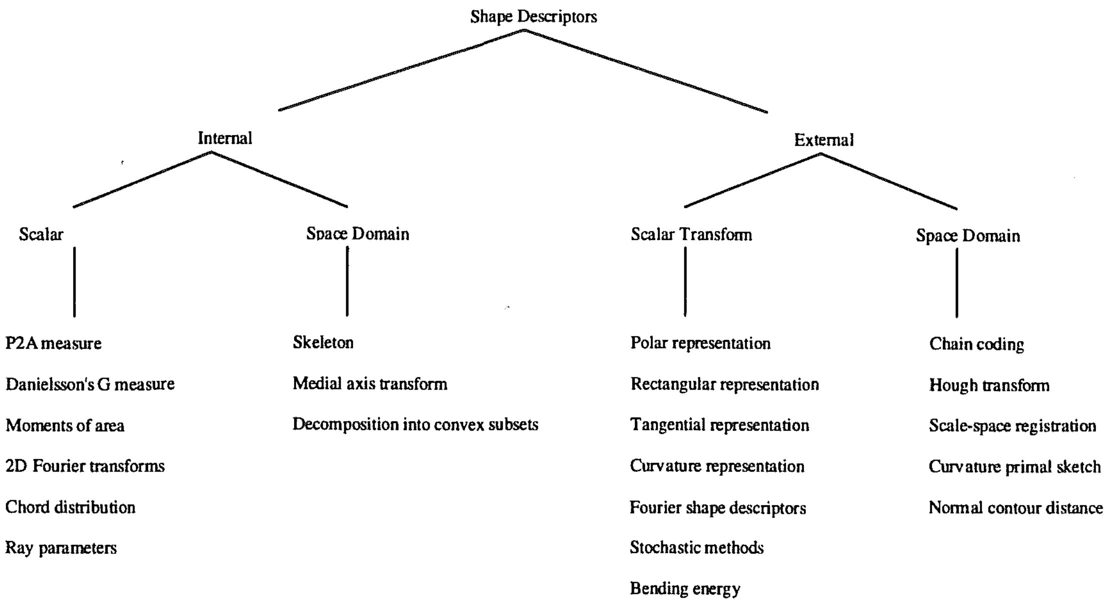


Figure 3. A compendium of shape descriptors.

4.2. Discriminant analysis and classification

Discriminant analysis and classification are techniques of multivariate statistics "concerned with *separating* distinct sets of objects (or observations) and with *allocating* new objects (observations) to previously defined groups" (Johnson & Wichern, 1988, p. 470). R. A. Fisher provided the first modern treatment of separatory problems. In the discussion that follows Fisher's original methods for separation and classification are described. This is followed by a discussion of the *minimum total probability of misclassification rule for normal populations* — a generalisation of Fisher's methods when specifically dealing with multivariate normal populations.

Fisher's original approach to dealing with two populations is as follows. Let π_1 and π_2 denote the two populations; e.g. Winchester 9422 XTR repeating rifles and Ruger 10/22 self loading rifles. Separation and classification is done on the basis of measurements on several random variables $\mathbf{X}' = [X_1, X_2, \cdots, X_p]$. A single realisation (observation) of values is denoted $\mathbf{x}' = [x_1, x_2, \cdots, x_p]$. The two populations can be described by their respective probability density functions $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$. Fisher's method involves taking a linear combination of the multivariate observations $\mathbf{x}$ so that they are transformed into univariate observations y. The chosen linear combination is the one that best separates the y values derived from each population. This is achieved by determining the linear combination that maximises the squared distance between the mean of the Y values for π_1 and the mean of the Y values for π_2 relative to the variability of the Y values. The linear combination that satisfies this requirement, assuming that both populations have the same covariance matrix Σ, is called Fisher's linear discriminant function:

$$\underset{(1\times1)}{Y} = \underset{(1\times p)}{\ell'}\ \underset{(p\times1)}{\mathbf{X}} = (\mu_1 - \mu_2)'\,\Sigma^{-1}\mathbf{X}, \qquad (1)$$

where $\mu_1 = E(\mathbf{X}|\pi_1)$, $\mu_2 = E(\mathbf{X}|\pi_2)$, and $\Sigma = E(\mathbf{X}-\mu_1)(\mathbf{X}-\mu_1)' = E(\mathbf{X}-\mu_2)(\mathbf{X}-\mu_2)'$. In practice one has n_1 observations of the random variable $\mathbf{X}$ for π_1 and n_2 observations for π_2 from which the sample mean vectors $\bar{\mathbf{x}}_1, \bar{\mathbf{x}}_2$, and the sample (pooled) covariance matrix $\mathbf{S}_{\text{pooled}}$ are calculated (see Johnson & Wichern, 1988, p. 474). These are then substituted for μ_1, μ_2, and Σ, respectively, in (1) to give Fisher's sample linear discriminant function:

$$y = \hat{\ell}'\mathbf{x} = (\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)' \, \mathbf{S}_{\text{pooled}}^{-1}\mathbf{x}.$$

The midpoint between the two univariate sample means $\bar{y}_1 = \hat{\ell}'\bar{\mathbf{x}}_1$ and $\bar{y}_2 = \hat{\ell}'\bar{\mathbf{x}}_2$ is given by:

$$\hat{m} = \tfrac{1}{2}(\bar{y}_1 + \bar{y}_2)$$

and leads to the following classification rule for a new observation $\mathbf{x}_0$:

allocate to π_1 *if* $y_0 = \hat{\ell}'\mathbf{x}_0 \geq \hat{m}$ *otherwise allocate to* π_2.

Fisher also devised a generalisation of his two population discriminant method for dealing with several populations. The method once again requires that all the populations have a common covariance matrix. Moreover this covariance matrix must be of full rank[9]; i.e. it must be invertible. As before no assumption about the distributions of the populations needs to be made. The method involves finding the linear combinations that maximise "the variability *between* the groups of Y—values relative to the common variability *within* groups" (Johnson & Wichern, 1988, p. 515) subject to certain constraints. The first discriminant is the maximising linear combination for which there are no constraints. The second discriminant is the maximising linear combination subject to the constraint $\text{cov}(\ell_1'\mathbf{X}, \ell_2'\mathbf{X}) = 0$. This continues such that $\ell_k'\mathbf{X}$ maximises the variability ratio subject to $\text{cov}(\ell_k'\mathbf{X}, \ell_i'\mathbf{X}) = 0$, for all $i < k$. In practice one calculates the sample discriminants $\hat{\ell}_i'\mathbf{X}$ which are then used as the basis for a classification rule (see Johnson & Wichern ,1988, p. 524).

When the populations do not have the same covariance matrix Fisher's method is not valid. However if the populations are multivariate normal then the *total minimum probability of misclassification rule* can be used for discrimination and classification. This rule is based on the calculation of *quadratic discriminant scores*:

$$d_i^Q(\mathbf{x}) = -\tfrac{1}{2}\ln|\Sigma_i| - \tfrac{1}{2}(\mathbf{x}-\mu_i)'\Sigma_i^{-1}(\mathbf{x}-\mu_i) + \ln p_i, \qquad (2)$$

where Σ_i is the covariance matrix for the i-th population, $|\Sigma_i|$ is its determinant, and p_i is the prior probability for population i (these probabilities can be used to reflect relative occurrences for each population; e.g. that certain weapons are more frequently used to commit crimes). The allocation rule for g populations is then:

allocate $\mathbf{x}$ *to* π_k *if the quadratic discriminant score* $d_k^Q(\mathbf{x}) = \max\{d_1^Q(\mathbf{x}), d_2^Q(\mathbf{x}), \ldots, d_g^Q(\mathbf{x})\}$ *for* $i = 1, 2, \ldots, g$.

[9] "If not, we let $\mathbf{P} = [\mathbf{e}_1, \ldots, \mathbf{e}_q]$ be the eigenvectors of Σ corresponding to nonzero eigenvalues $[\lambda_1, \ldots, \lambda_q]$. Then we replace $\mathbf{X}$ by $\mathbf{P}'\mathbf{X}$, which has a full rank covariance matrix $\mathbf{P}'\Sigma\mathbf{P}$" (Johnson & Wichern, 1988, p. 514).

In practice the estimates $\hat{d}_i^Q(\mathbf{x}) = -\frac{1}{2}\ln|\mathbf{S}_i| - \frac{1}{2}(\mathbf{x}-\overline{\mathbf{x}}_i)'\mathbf{S}_i^{-1}(\mathbf{x}-\overline{\mathbf{x}}_i) + \ln p_i$, where the $\mathbf{S}_i$ are the sample covariance matrices, are used. If the Σ_i are all equal the term $-\frac{1}{2}\ln|\Sigma_i|$ in (2) is a constant and the expression simplifies (for allocatory purposes) to:

$$-\tfrac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)'\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) + \ln p_i. \qquad (3)$$

In fact (3) can be simplified further, because allocation is based on relative magnitudes, to give the expression:

$$d_i(\mathbf{x}) = \boldsymbol{\mu}_i'\Sigma^{-1}\mathbf{x} - \tfrac{1}{2}\boldsymbol{\mu}_i'\Sigma^{-1}\boldsymbol{\mu}_i + \ln p_i, \qquad (4)$$

called the *linear discriminant score*. In practice one uses the estimates $\hat{d}_i(\mathbf{x}) = \overline{\mathbf{x}}_i'\mathbf{S}_{\text{pooled}}^{-1}\mathbf{x} - \frac{1}{2}\overline{\mathbf{x}}_i'\mathbf{S}_{\text{pooled}}^{-1}\overline{\mathbf{x}}_i + \ln p_i$. When the prior probabilities are all equal, i.e. $p_1 = p_2 = \ldots = p_g = \frac{1}{g}$, the term $\ln p_i$ in (4) is a constant and allocation based on the linear discriminant scores is equivalent to allocation based on Fisher's linear discriminant function. Moreover an examination of expression (3) shows that the only remaining variable term (ignoring the coefficient $-\frac{1}{2}$) is $(\mathbf{x}-\boldsymbol{\mu}_i)'\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu}_i)$. This is the expression for the squared *Mahalanobis distance* between $\mathbf{x}$ and the i-th population mean $\boldsymbol{\mu}_i$. Thus Fisher's procedure equates to assigning an observation $\mathbf{x}$ to the *closest* population. More generally classification based on linear discriminant scores equates to assigning an observation $\mathbf{x}$ to the *closest* group taking into account a distance penalty $\ln p_i$.

To summarise, Fisher's linear discriminants can be used for classification when all of the populations have the same covariance matrix, regardless of their respective distributions. However studies have shown "that there are nonnormal cases where Fisher's linear classification function performs poorly even though the population covariance matrices are the same" (Johnson & Wichern, 1988, p. 493). If in addition to covariance homogeneity the populations are also multivariate normal then classification can be performed using linear discriminant scores. If additionally the prior probabilities are all equal then classification based on Fisher's linear discriminants is equivalent to classification based on linear discriminant scores. Finally if the populations are multivariate normal but have different covariance matrices then classification can be performed using quadratic discriminant scores. It is important to keep in mind though that "classification with quadratic functions is rather awkward in more than two dimensions and can lead to some strange results. This is particularly true when the data are not (essentially) multivariate normal" (Johnson & Wichern, 1988, p. 493).

4.3. Case study 1: three different shaped FP impressions

To be able to evaluate the effectiveness of a particular set of feature parameters (variables) in discriminating between different FP impressions it is necessary to

(i) obtain images of multiple cartridge cases fired from the same weapon, and
(ii) obtain images of multiple cartridge cases fired from different weapons but of the same make and model.

In this regard, with the cooperation of the Forensic Ballistics Unit of the Western Australian Police, a total of 36 fired 0.22 calibre rim-fire cartridge cases were obtained for study. Table 1 lists the numbers of cartridges fired in each of three different make/model weapons. In the case of the Ruger 10/22, 10 rounds were recovered from one weapon and another 3 rounds each from two other Ruger 10/22 rifles. The data set comprises three different shapes of FP impression (see Appendix A).

Table 1. *Data set comprising three different shaped FP impressions.*

Make	Model	Type	Calibre	Single Weapon	Multiple Weapons
Ruger	10/22	SLR	22LR	10 rounds	2 × 3 rounds
Winchester	9422 XTR	RR	22LR	10 rounds	N/A
Erma-Werke	E M1.22	SLR	22LR	10 rounds	N/A

Note: SLR = self loading rifle, RR = repeating rifle, and LR=long rifle.

4.3.1. Image acquisition, tracing, and measurement

Images of the cartridge case heads were captured using a PC and frame-grabber[10], in conjunction with a Citoval stereo trinocular microscope fitted with a monochrome video camera[11] and a fibre optic ring light source (see Figure 4). Images were acquired at a spatial resolution of 768H × 512V pixels and grey-scale resolution of 8-bits per pixel (see Figure 5). Each cartridge case was orientated so that the FP impression was approximately located at 12 o'clock.

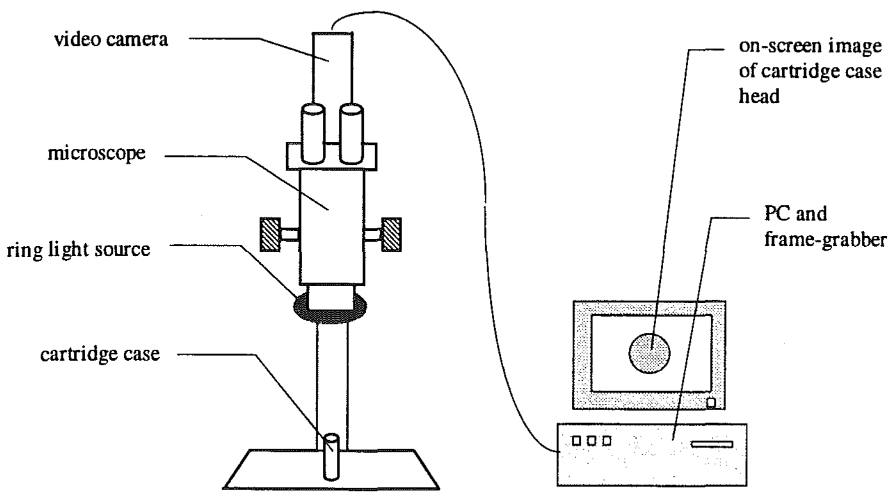


Figure 4. Imaging system.

A custom program—hereinafter referred to as *TREASURE*[12] (from the words: trace and measure) — was used to obtain several metrics pertaining to each cartridge case and in particular the FP impression. TREASURE displays images of cartridge case heads (normally or as a photographic negative) on-screen and permits the user to trace the outline of the firing pin impression. Tracing is initiated by double-clicking the left mouse button. This produces an anchor point from which a *rubber-banded* line is drawn to the current mouse position. A single click of the left button produces a new anchor point. Thus the boundary of the impression is approximated by a number of straight line segments. A final double-click of the left button closes the traced contour and initiates measurement. All measurements are logged to a ASCII file and the traced boundary is also saved (see Figure 6). TREASURE automatically locates the centroid of the FP impression when a new image is loaded. The underlying algorithm is similar to that developed for the GUI in FIREBALL. The algorithm comprises a calculation of the maximum value of the morphological gradient, thresholding, binary erosion, and calculation of the centroid. Details of the dilation and erosion operations can be found in Mehnert, Cross, Smith, and Chia (1995, p. 34). The algorithm is as follows:

[10] Model DT2855 QuickCapture, Data Translation Incorporated, Marlboro, MA, USA.

[11] Model TC354X, Burle Industries Incorporated, Lancaster, PA, USA.

[12] Written in ANSI *C* for IBM's OS/2 2.x operating system. Written by Andrew Mehnert, 1995. All rights reserved.

(1) Let $f(x, y)$ be the function describing the grey-level surface of the image; i.e. the value of f at (x, y) represents a brightness value in the range [0,255]. Find the maximum value, t, of the morphological gradient of f viz.

$$t = \max_{(x,y)}\left(\frac{f \oplus \mathrm{B} - f \ominus \mathrm{B}}{2}\right),$$

where B is a 3×3 cross-shaped structuring element.

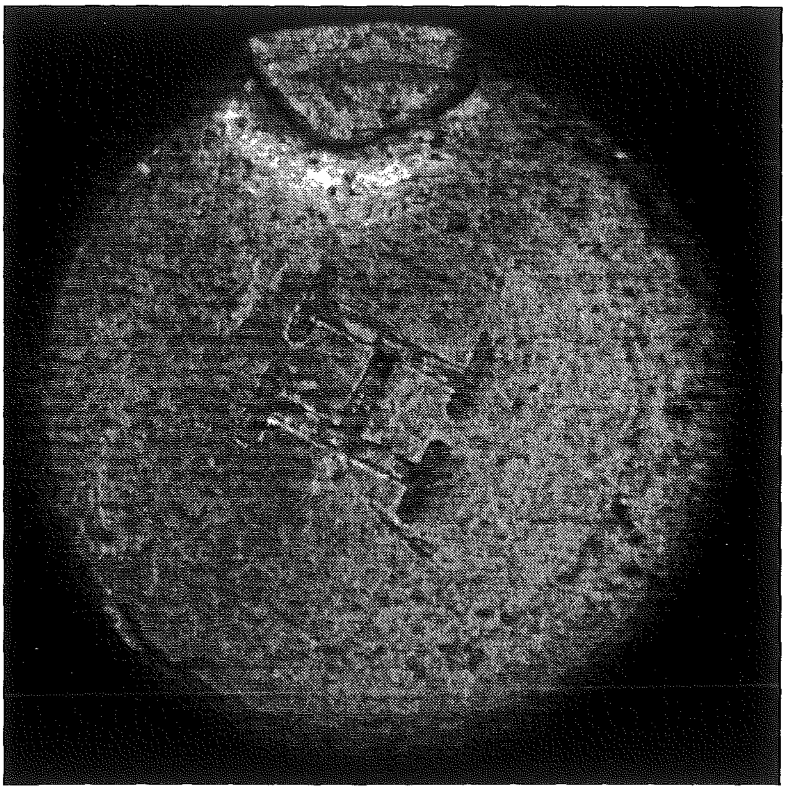
(2) Obtain the threshold set $\mathrm{T} = \{(x, y) | f(x, y) \geq t\}$.

(3) Erode the threshold set: $\mathrm{T}' = \mathrm{T} \ominus \mathrm{B}$.

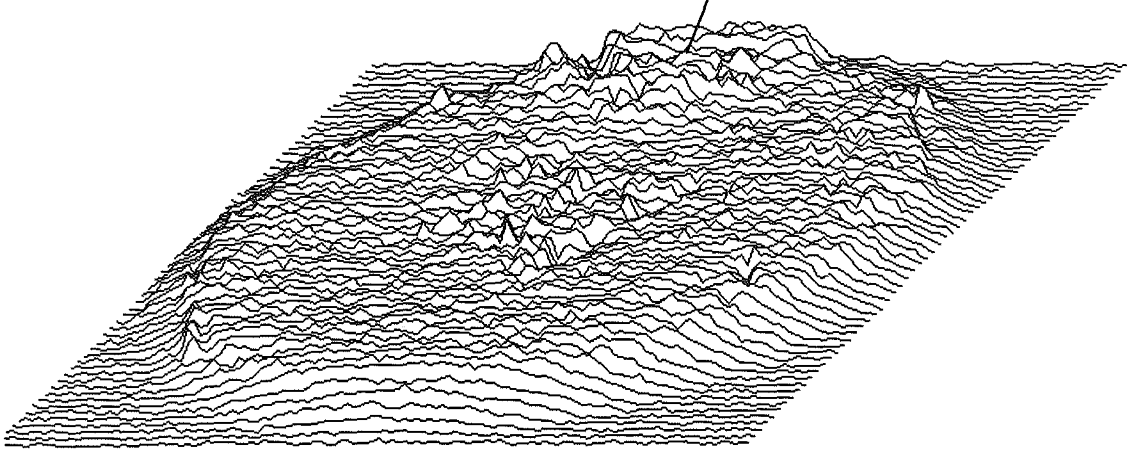
(4) Determine the mean of the x coordinates and the mean of the y coordinates for T' thus obtaining the centroid $(\bar{x}, \bar{y})$.

The images in Appendix A and Appendix C reveal that the boundary of a FP impression is not always well defined. Often the boundaries are pitted, blurred, broken, or are ambiguous because of *double strike* or brightness flare. The following procedure was adopted for all tracing:

(i) view the image as both a (photographic) positive and negative and decide which is clearer to trace;
(ii) using the centre of the impression as a reference point, the boundary constitutes the first enclosing border;
(iii) in the case of double strike, trace the most distinct single impression—do not merge boundaries.



(a)



(b)

Figure 5. Head of a cartridge case fired in a Ruger 10/22 self loading rifle: (a) digitised light micrograph; (b) 3D perspective view — DIMPAL (Mehnert, 1994b) command: *surface(byte(sqrt(image)*4),8)*

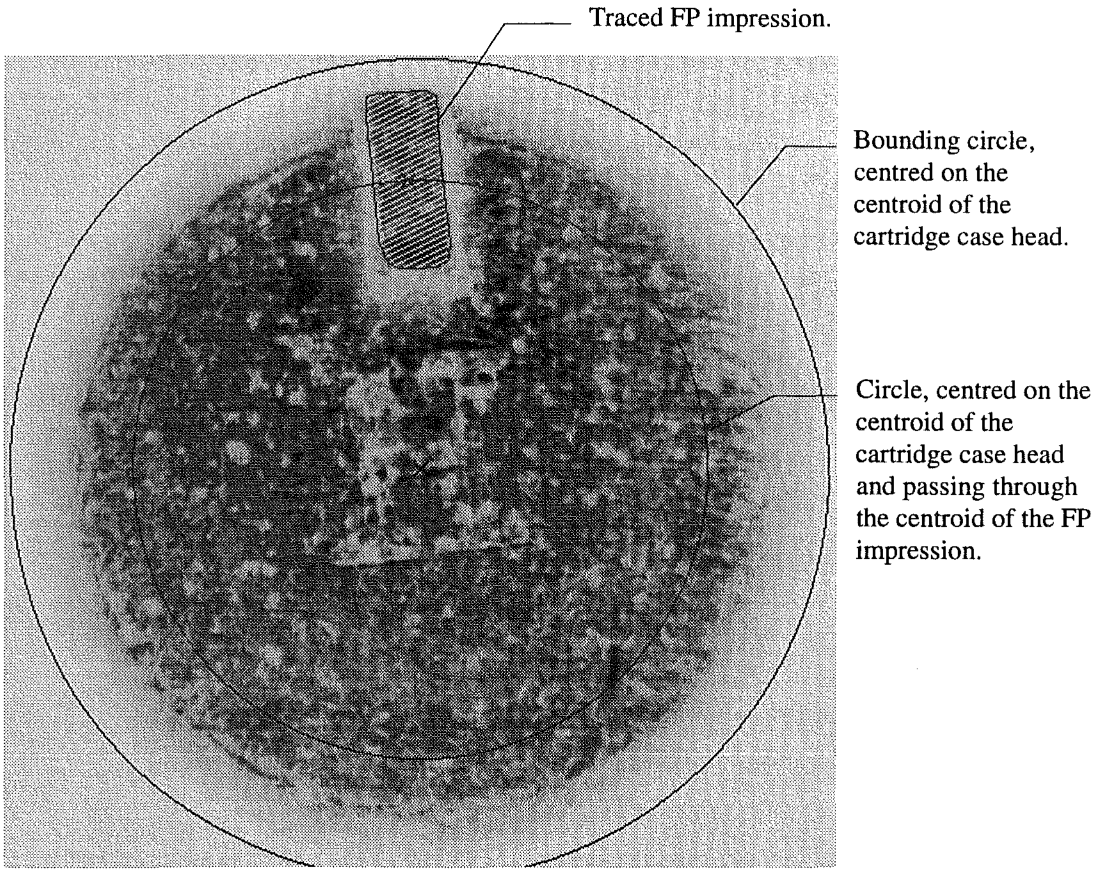
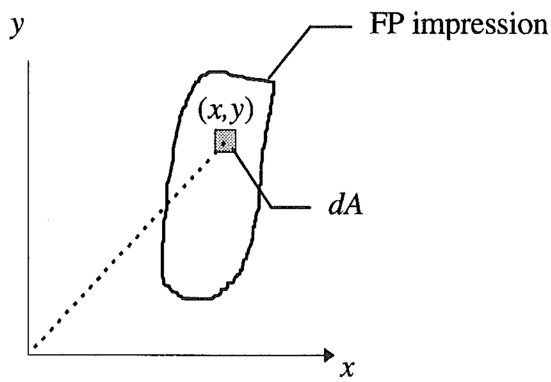


Figure 6. Image (negative) of the head of a rim-fire cartridge case traced using TREASURE.

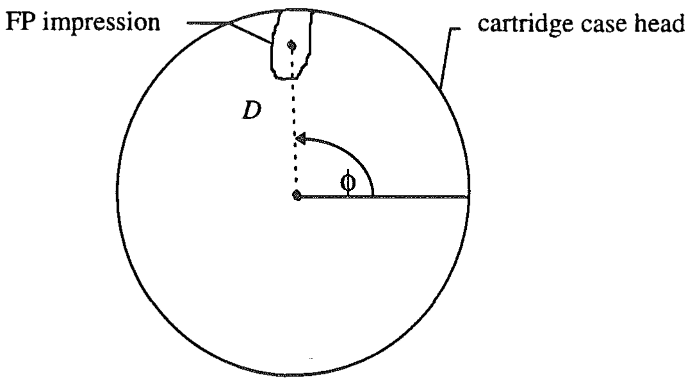
After tracing, TREASURE calculates the following:

(i) centroid of the FP impression $(\bar{x}, \bar{y})$ where $\bar{x} = \frac{1}{A}\iint_R x\,dA$ and $\bar{y} = \frac{1}{A}\iint_R y\,dA$;



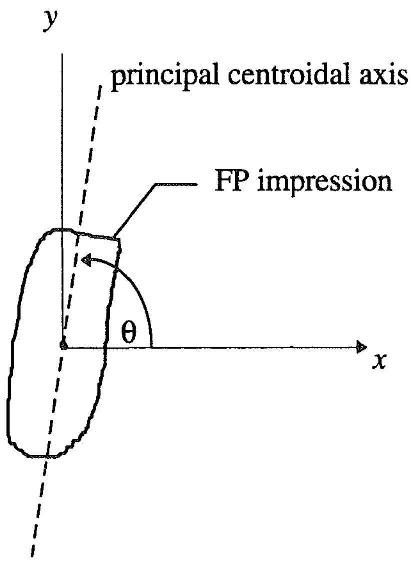
(ii) distance, D, from the centroid of the FP impression to the centroid of the cartridge case head;

(iii) angle of orientation, ϕ, of the centroid-to-centroid line;

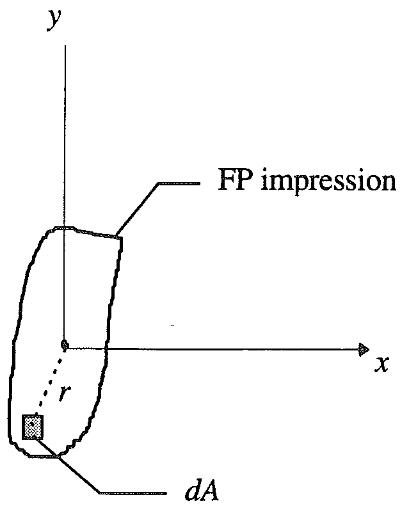


(iv) the moments of inertia $I_x = \iint_R y^2 dA$, $I_y = \iint_R x^2 dA$, and product of inertia $P_{xy} = \iint_R xy\, dA$ about the centroid of the FP impression;

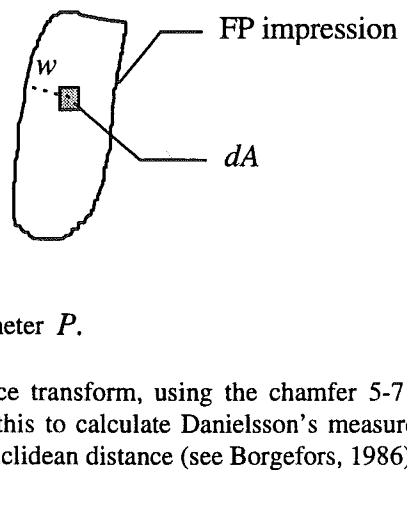
(v) the angle of orientation of the principal centroidal axes viz. $\tan 2\theta = -\dfrac{2P_{xy}}{I_x - I_y}$;



(vi)the first and second order moments of inertia $J = \iint_R r\,dA$ and $I_0 = \iint_R r^2\,dA$, where $r^2 = x^2 + y^2$, about the centroid of the FP impression ;



(vii)Danielsson (1978) measure $\iint_R w\,dA$ where w is the shortest distance to the boundary;



(viii)Area A and perimeter P.

TREASURE generates a distance transform, using the chamfer 5-7 metric, for the silhouette of the traced FP impression and uses this to calculate Danielsson's measure. The chamfer 5-7 metric is an approximation to 5 times true Euclidean distance (see Borgefors, 1986).

4.3.2. Selection of variables for discrimination and classification

From the set of TREASURE measurements for a single cartridge case the following set of shape factors (normalised to be unity for a disk) can be calculated:

$C_{P2A} = \dfrac{P^2}{4\pi A}$ (P2A shape factor), $C_{mom1} = \dfrac{9\pi J^2}{4A^3}$ (based on first order moment),

$C_{mom2} = \frac{2\pi I_0}{A^2}$ (based on second order moment), and $C_G = \frac{A}{9\pi(\bar{d})^2}$ (Danielsson's G shape factor) where $\bar{d} = \frac{1}{A}\iint_R w\, dA$.

These factors are rotation and size invariant. Another such parameter is the acute angle of intersection between the centroid-to-centroid line and the principal centroidal axis given by

$$\tan\alpha = \frac{\tan\phi - \tan\theta}{1 + \tan\phi\tan\theta}.$$

Unfortunately the function *tan* is not continuous so that it is not possible to attach a sign to α. As a consequence α denotes only the *closeness* of the principal axis to the centroid-to-centroid line and not its relative direction. Given that all of the images captured for this study were captured under exactly the same conditions, the length of the centroid-to-centroid line can also be considered to be a rotationally and scale invariant parameter. More generally, to guarantee scale invariance, the ratio of centroid-to-centroid length to the radius of the cartridge case head should be used.

4.3.3. Results

The MINITAB (1993) statistical package was used to analyse the data collected for the exhibits (3 weapons × 10 rounds) in Table 1 — hereinafter referred to as *data set 1*. MINITAB implements discrimination and classification based on either linear discriminant scores (covariance homogeneity) or quadratic discriminant scores (lack of covariance homogeneity). Appendix B is an edited log file of a MINITAB session for data set 1 Each feature parameter corresponds to a MINITAB worksheet column as shown in Table 2.

Table 2. *MINITAB column names.*

MINITAB COLUMN NAME	FEATURE PARAMETER
CENTDIST	D
CENTANG	ϕ
PRINCANG	θ
MOMENT1	J
MOMENT2	I_0
MEANDIST	$\bar{d}$
AREA	A
PERIM	P
MOM1SHAP	C_{mom1}
MOM2SHAP	C_{mom2}
P2A	C_{P2A}
G	C_G
ANGDIFF	α

The column entitled FIREARM contains the integers 1, 2, and 3 representing respectively the Ruger, Winchester, and Erma-Werke rifles. The correlation matrix for the shape factors indicates that they are highly correlated and indeed the factors C_{mom1} and C_{mom2} are very highly correlated (1.00 to two decimal places). Dotplots for the variables CENTDIST, G, and ANGDIFF for each firearm give an indication of the separatory characteristics of these variables. In addition the dotplots indicate that the variables are approximately normally distributed. The DISCRIM command was used to perform linear discriminant analysis — under the assumptions of multivariate normality and covariance homogeneity. The subcommand XVAL invokes cross-validation (Lachenbruch's *holdout* procedure):

Denote the set of observations belonging to group (firearm) one π_1, to group two π_2, and to group three π_3.

1. Start with the π_1 group of observations. Omit a single observation and derive a classification rule based on the remaining π_1 observations, and the π_2 and π_3 observations.
2. Classify the *holdout* observation using this classification rule.
3. Repeat steps 1 and 2 until all π_1 observations have been classified.
4. Repeat steps 1 to 3 for the π_2 and then the π_3 observations.

Cross-validation compensates for the bias introduced by using the same observations to construct and evaluate the classification rule. Appendix B shows the results of discrimination and classification using just a single shape factor: first C_G, then C_{mom2}, and finally C_{P2A}. Clearly these shape factors alone can discriminate between the three different shaped FP impressions. Next Appendix B shows the results of discrimination and classification using just α. This time only 90% of the observations are correctly classified. Three of the Erma-Werke observations are misclassified. This is not surprising as the shape of the FP impression is circular and for a perfect circle there are infinitely many principal centroidal axes. The results of discrimination and classification using just D follow. Once again there are misclassifications. Next Appendix B shows the results of discrimination and classification using both α and D. This time there are no misclassifications. Thus for data set 1 classification and discrimination on the basis of shape alone or on both location and orientation is sufficient. Figure 7 is a plot of the ordered triples (D, α, C_G) for all of the observations. Three clusters of points are clearly seen. Next Appendix B shows the results of discrimination and classification using C_G, C_{mom2}, C_{P2A}, α, D. A plot of the first three principal components[13], which account for 99.8% of the variability exhibited by all of the variables, is shown in Figure 8. An examination of the squared distances between groups output for each DISCRIM command in Appendix B shows that this last combination of variables provides the best separation.

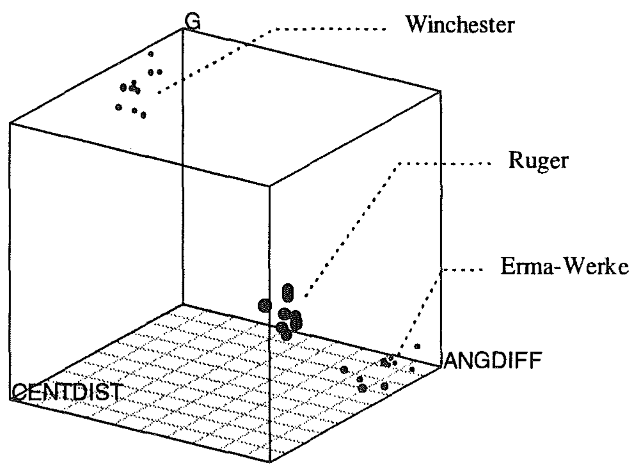


Figure 7. Plot of the triples (D, α, C_G) for data set 1.

[13] Principal components analysis is a multivariate technique primarily concerned with data reduction. Several variables are replaced by a smaller number of principal components—each of which is a particular linear combination of the original variables—that account for most of the original variablity.

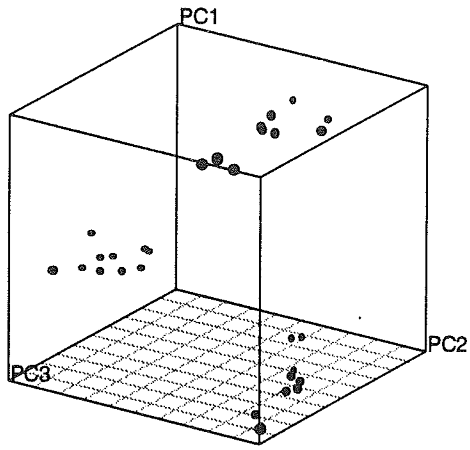


Figure 8. Plot of the first three principal components for data set 1.

Finally Appendix B shows the classification results for the 6 rounds obtained from the two additional Ruger 10/22 SLRs. These rounds, though not used in the construction of the classification function, were unambiguously classified as having been fired in a Ruger 10/22 SLR.

4.4. Case study 2: five different make/model rifles; all rectangular FPs

For the previous case study each make/model weapon produced a distinctly different shaped FP impression. Thus discrimination and classification based solely on shape was adequate. For this case study 150 spent 0.22 calibre cartridge cases, 30 rounds from each of five different make/model rifles, were obtained for study (see Table 3). The rifles were chosen because they all produce rectangular shaped FP impressions. As before TREASURE was used to trace and measure the captured images of the cartridge case heads. Unfortunately the quality of two of the images, one of a Fast Deer case and the other of a Glenfield case, was so poor that they could not be traced (see Appendix C). It was decided to use 29 cases from each weapon to construct a classification rule and to use the remaining 3 images—AR7, JW-20, and Armi Jaeger—to test it.

Table 3. *Data set comprising five rectangular shaped FP impressions.*

Make/Model	Calibre	Serial Number	No. Cases
Colt AR7	22LR	A28092	29 rounds
JW-20 (China)	22LR	230492	29 rounds
Armi Jaeger (Italy)	22LR	19690	29 rounds
Fast Deer (China)	22LR	840158	29 rounds
Glenfield Model 60 (Marlin)	22LR	22433098	29 rounds

4.4.1. Selection of variables for discrimination and classification

Hereinafter the measurements obtained for the exhibits listed in Table 3 are referred to as data set 2. Appendix D is an edited log file of a MINITAB session for data set 2. Each worksheet column name has the same meaning as for the previous case study with the exception of the ANGDIFF and FIREARM columns. Here the ANGDIFF column is defined to be the values $\alpha = \phi - \theta$. As each weapon produces a rectangular FP impression that is taller than wide the major principal centroidal axis is always vertically aligned. Consequently the signed angular difference between the orientation of the centroid-to-centroid line and the major principal centroidal axis conveys more information than the magnitude of the acute angle of intersection alone (used for data set 1). The FIREARM column contains the integers 1, 2, 3, 4, and 5 representing respectively the Colt AR7, JW-20, Armi Jaeger, Fast Deer, and Glenfield Model 60 rifles. Given the reasonably large size of data set 2 it was feasible to assess multivariate normality. To begin with each variable was assessed for univariate normality. For each weapon and each variable this involved ordering the observations, calculating their standard normal quantiles, and calculating the Pearson product moment correlation coefficient between the ordered scores and standard normal quantiles. Using MINITAB this procedure reduces to

```
MTB > NSCORES C1 C100
MTB > CORR C1 C100
```

where C1 contains the observations on a single variable for a single weapon and C100 is used to hold the standard normal quantiles. Table 4 lists the correlation coefficients for a selection of variables. Each correlation coefficient r is used in the following test of normality:

H_0: Observations are from a normal distribution.

H_A: Observations are not from a normal distribution.

If $r > r_c$ then do not reject H_0 and conclude that the observations are from a normal distribution, otherwise reject H_0 and conclude that the observations are not from a normal distribution.

For a test of normality at the 5% level of significance and for samples of size n=29 the critical value is $r_c = 0.964$ (see Johnson & Wichern, 1988, p. 151). The shaded entries in Table 4 represent observations on a variable for a specific rifle that failed this test of normality.

Table 4. *Correlation between ordered observations and standard normal quantiles for each variable and each rifle.*

	Colt AR7	JW-20	Armi Jaeger	Fast Deer	Glenfield 60
CENTDIST	0.997	0.991	0.987	0.733	0.946
MOMENT2	0.984	0.937	0.968	0.924	0.979
MEANDIST	0.975	0.955	0.956	0.959	0.904
AREA	0.986	0.941	0.965	0.936	0.972
PERIM	0.987	0.930	0.994	0.958	0.979
MOM2SHAP	0.956	0.990	0.899	0.986	0.990
P2A	0.987	0.984	0.988	0.983	0.990
G	0.969	0.974	0.985	0.988	0.989
ANGDIFF	0.978	0.988	0.995	0.984	0.988

Note: shaded cells are values below the critical value of 0.964 for α=0.05 and n=29.

Thus it is only the variables P2A, G, and ANGDIFF that exhibit univariate normality for each weapon. Next these variables were examined pairwise for bivariate normality. For each pair in turn and for each weapon a chi-square plot (Johnson & Wichern, 1988, p. 153) was constructed. This is a plot of the ordered squared generalised distances $d_j^2 = (\mathbf{x}_j - \bar{\mathbf{x}})' \mathbf{S}^{-1} (\mathbf{x}_j - \bar{\mathbf{x}})$, $j = 1,2,\ldots,n$ against percentiles of the χ_2^2 distribution, where $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ are the bivariate observations. "Although these distances are *not* independent or exactly chi-square distributed, it is helpful to plot them as if they were . . . .The plot should resemble a straight line. A systematic curved pattern suggests lack of normality. One or two points far to the right of the line indicate large distances, or outlying observations, that merit further attention" (Johnson & Wichern, 1988, p. 152). The 15 chi-square plots (5 weapons and 3 different pairings of variables) are shown in Appendix E. Appendix F contains the listings of the MINITAB macros written to generate these plots. Some of the plots exhibit non-linear behaviour at their extremes and there is evidence of outliers. However there is sufficient linearity to conclude that the triple (C_{P2A}, C_G, α) is approximately multivariate normal. Though it is possible to extend the chi-square plot to check for higher order multivariate normality "for practical work it is usually sufficient to investigate the univariate and bivariate distributions" (Johnson & Wichern, 1988, p. 151).

4.4.2. Results

Appendix D shows the dotplots for the variables P2A, G, ANGDIFF, and AREA and then the output from the DISCRIM command using cross-validation (Lachenbruch's holdout procedure) and quadratic discriminant scores. Johnson and Wichern (1988, p. 513) state that "if doubt exists as to the appropriateness of a linear or quadratic rule, both rules can be constructed and their error rates examined using Lachenbruch's holdout procedure". Indeed for data set 2 the quadratic rule gave better results than the linear rule — for the triple (C_{P2A}, C_G, α) 89% of all observations are classified correctly for the quadratic rule as opposed to 85.5% for the linear rule (not shown in Appendix D). These variables describe only the shape and orientation of the FP impressions. Intuitively one would expect that by including a variable that measures the relative size of the FP impressions the quality of classification could be improved. Referring again to Table 4 candidate variables are MOMENT2, MEANDIST, and AREA. MEANDIST is most definitely non-normal. This leaves only MOMENT2 and AREA. Both variables failed the normality test for the JW-20 and the Fast Deer. AREA did however achieve higher correlations for these two weapons than did MOMENT2. The correlations are still very high and so it is perhaps reasonable to assume normality for AREA, concluding that the apparent lack of normality is a consequence of bias introduced during tracing. Appendix D shows the results of quadratic discrimination using the variables $C_{P2A}, C_G, \alpha,$ and A. Indeed the quality of classification has been improved, now 92.4% of the observations are correctly classified. Figure 9 is a plot of the first three principal components for the variables $C_{P2A}, C_G, \alpha,$ and A.

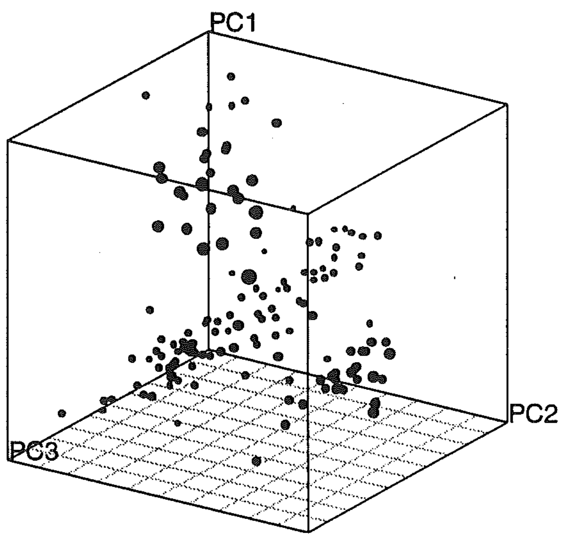


Figure 9. Plot of the first three principal components for the variables $C_{P2A}, C_G, \alpha,$ and A.

Finally Appendix D shows the classification results for the three leftover rounds: AR7, JW-20, and Armi Jaeger. Using the quadratic classification rule based on the triple (C_{P2A}, C_G, α) the three rounds were correctly classified.

4.5. Discussion

The two case studies demonstrate that discriminant analysis using a set of feature parameters that describe the size, shape, and orientation of the FP impression can be used to classify an arbitrary spent cartridge case to a specific make and model of weapon. The traditional P2A shape factor and Danielsson's G factor appear to have good separatory properties even when dealing with FP impressions all of the same shape. The orientation of the major principal centroidal axis relative to the centroid-to-centroid line also has good separatory properties. Area, or more generally the area of the FP impression relative to the area of the head of the cartridge case, also appears to be a useful separatory variable. However, in general, it may be of little value because "the size, and to some extent the shape, of a firing pin impression will depend on the depth of the penetration of the pin. Frequently a cartridge at the moment of firing is not seated firmly against the shoulder of the chamber, and 'normal' depth of penetration will not be achieved" (Mathews, 1962, p. 24). In poorly manufactured or well used firearms the firing pin may not always strike in exactly the same place. Thus it is not surprising that the centroid-to-centroid distance turned out to be a rather poor separatory variable; in particular it exhibited complete lack of normality for the Fast Deer. Some other shape factors that are worthy of investigation include: (i) aspect ratio: ratio of the lengths of the major and minor principal centroidal axes, and (ii) the mean, standard deviation, etc. of a chosen set of normalised ray parameters (see Barker, Vuori, Hegedus, & Myers, 1992). The shape factors C_G, C_{mom1}, C_{mom2}, and C_{P2A} are based on the 2D planar shape (silhouette) of the FP impression and do not take into account the grey-level dimension. It is possible to extend the general silhouette moments $M_{pq} = \iint x^p y^q \, dx \, dy$ (refer to section 4.3.1. for which it can be seen that $I_x = M_{02}$, $I_y = M_{20}$, and $P_{xy} = M_{11}$) to grey level moments $G_{pqr} = \iint x^p y^q [f(x,y)]^r \, dx \, dy$ (Savini, 1988, p. 147) where $f(x,y)$ is the grey level surface of the FP impression . This leads to the possibility of defining shape descriptors based on the moment set $\{G_{pqr}\}$ that embody the 3D morphology of the FP impression.

Several further case studies are warranted. The results for case study 1 suggest that the within group (make/model) variability is negligible compared to the between group variability. However data set 1 is far too small to make any statistically significant conclusion. Mathews (1973) stated in relation to the photographic compilations in Mathews (1962, 1973) "that there was frequently a considerable variation in the shape, dimensions, and character of firing pin impressions made by different specimens of hand guns of the same make and model" (p. 613). Thus a new data set needs to be compiled comprising measurements on spent cartridge cases from each of several different make/model rifles and such that for each make/model several physically different weapons are used. For example data set 2 could be enlarged to include four independent rifles of each make/model. This would lead to a sample of 5x4x30=600 cartridge cases. A classification rule would then be constructed and using Lachenbruch's holdout procedure the proportion of correct classifications determined. Alternatively the data set could be divided into two separate sets, one to be used to construct the classification rule, and the other to evaluate its performance. Another case study is required to examine the variability introduced by using ammunition from different manufacturers. "Copper from different sources may have different degrees of hardness due to impurities. Furthermore, the depth of the penetration [of the firing pin] depends on the thickness of the metal in the head of the shell, and this may vary from manufacturer to manufacturer" (Mathews, 1962, p. 25).

It was mentioned earlier that the positions of the ejector and extractor marks, relative to the FP impression, can be of value in determining the make/model of firearm. Indeed it would be desirable to use these additional variables for discriminant analysis. Unfortunately, though, extractors and ejectors seldom leave identifiable marks on rim-fire cases (Mathews, 1973, p. 614); this means of course that one is even less likely to see such marks in a digitised (sampled) micrograph of the head of a cartridge case.

In any case "these marks are of secondary significance because unfired cartridges are frequently removed by means of the extractor and ejector, thus possibly producing an initial set of marks which have no relation to those put on later at the time of firing" (Mathews, 1973, p. 615).

For the two case studies *continuous* variables were used for discriminant analysis. However it might be useful to use some *qualitative* (e.g. aspect: taller than wide, or wider than tall) or *categorical* (e.g. shape: circular, rectangular, . . .) variables. Unfortunately multivariate normality is then no longer a sensible assumption. If it is reasonable to assume covariance matrix homogeneity for the populations then Fisher's procedure can be used; because there is no requirement of multivariate normality. "Computer simulation experiments . . . indicate that Fisher's linear discriminant function can perform poorly or satisfactorily depending upon the correlations between the qualitative and continuous variables" (Johnson & Wichern, 1988, p. 527). Ultimately though, as with any proposed classification method, performance should be evaluated using test data.

5. SUMMARY AND CONCLUSION

The case studies described in this report demonstrate that scale and rotation invariant feature parameters relating to the shape, size, and orientation of the FP impression can be used to identify a fired 0.22 rim-fire cartridge case to a specific make/model weapon. Further research is needed to assess the performance of discriminant analysis, using the aforementioned feature parameters, when dealing with more than one firearm of a specific make and model. It must be acknowledged that "manufacturers frequently deliberately change the shape, size, and even the type of firing pin used on a particular model" (Mathews, 1973, p. 614). Effectively this introduces subgroups for make/model firearms according to the serial number (reflecting the various production lots). For large numbers of make/model firearms it is likely that the observations for some will cluster together in discriminant space. The consequence of this is that the classification of a new observation then equates to identifying a *hit list* of possible weapons in which the evidence cartridge case may have been fired.

On completion of the additional studies, and experiments with other shape factors, a final set of feature parameters will have been determined. The next step will be to implement the classifier in custom software. The software will need to be able to acquire images from a frame-grabber, permit the user to trace the FP impression, and to classify a new observation based on a classification rule. In practice new make/model/lot firearms that come into the possession of a Forensic Ballistics Unit would be test fired and the exhibit cases traced and their measurements forwarded to a central body responsible for periodically updating the classifier. When investigators recover a spent case at the scene of a crime the classifier would be used to determine the possible make/model/lot of firearm from which it originated. It would even be possible to expand the classifier to provide on-screen viewing of evidence and hit-list cartridge cases.

Classification based on discriminant scores involves assigning a new observation to the closest group (make/model) based on either a linear or quadratic distance function. Alternatives to discriminant analysis for classification include neural networks and expert systems. Neural networks offer the ability to classify or recognise an input pattern even when that pattern is slightly distorted; i.e. they possess a certain *noise* immunity (Kung, 1993, p. 8). A neural network processing/retrieval system comprises two subsystems: feature extraction, and a neural network. In the case of FP impressions a geometric representation such as chain coding, Fourier coefficients, or a set of feature parameters constitutes the output from the feature extractor. For classification applications neural networks based on either a supervised or unsupervised learning model are appropriate. In the former case the network is *trained* using many pairs of input/output training patterns, and in the latter the network is trained using only the set of input patterns (the network must adapt based on previous patterns). The specific neural network model chosen and its specific design are application dependent. One of the disadvantages of using a neural network is that "most neural network algorithms are computationally intensive and iterative in nature" (Kung, 1993, p. 14). In addition problems can arise in relation to the training of the network (Phillips, Millar, & Smith, 1993, p. 1):

(i) for non-linear networks the *back-propagation algorithm* is often used to train the network; unfortunately the algorithm is liable to become trapped in local minima in the *energy function* (analogous to the discriminant function/rule);
(ii) training may be difficult to control; overtraining and redundancy are possible;
(iii) non-linear and multi-layer networks are generally slow to *learn*.

An expert system could be developed that classifies a new observation based on a classification tree and a set of intelligent rules (hierarchical classification). At the top of the tree all observations are considered as one group. This group is then split into several subgroups on the basis of some rule. For example the classifier might classify the shape of the FP impression to one of the basic types shown in Figure 10. Then for each of these new groups another rule or set of rules are used to further subdivide; e.g. aspect ratio. A hypothetical classification tree is shown in Figure 11.

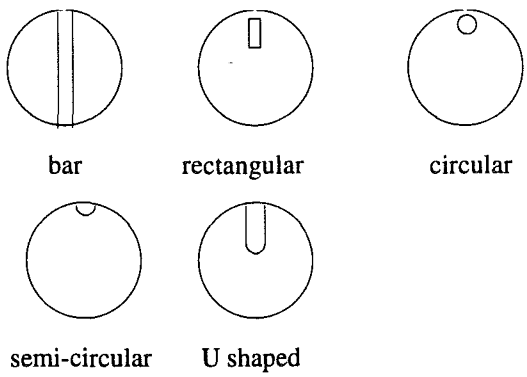


Figure 10. A simple shape classification for the most common shapes of FP impression.

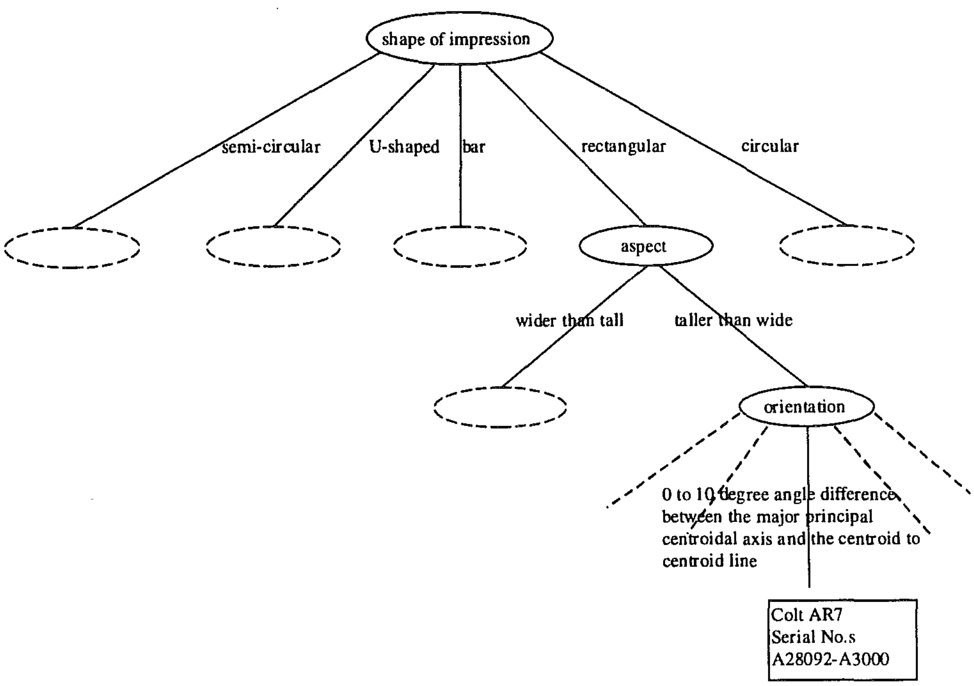


Figure 11. Hypothetical classification tree.

In spite of the difficulties and uncertainties associated with the study of different types of rim-fire FP impressions the fact remains that the morphology, size, and orientation of the FP impression is of great aid in determining the make and model of firearm in which the evidence cartridge was fired. The inherent variability of FP impressions made by several specimens of the same make and model weapon renders identification based on precision measurements unsatisfactory. Thus scale and rotation invariant feature parameters must be used. Discriminant analysis using a set of such feature parameters offers the potential for classifying an evidence cartridge case to a specific make and model weapon.

6. ACKNOWLEDGMENTS

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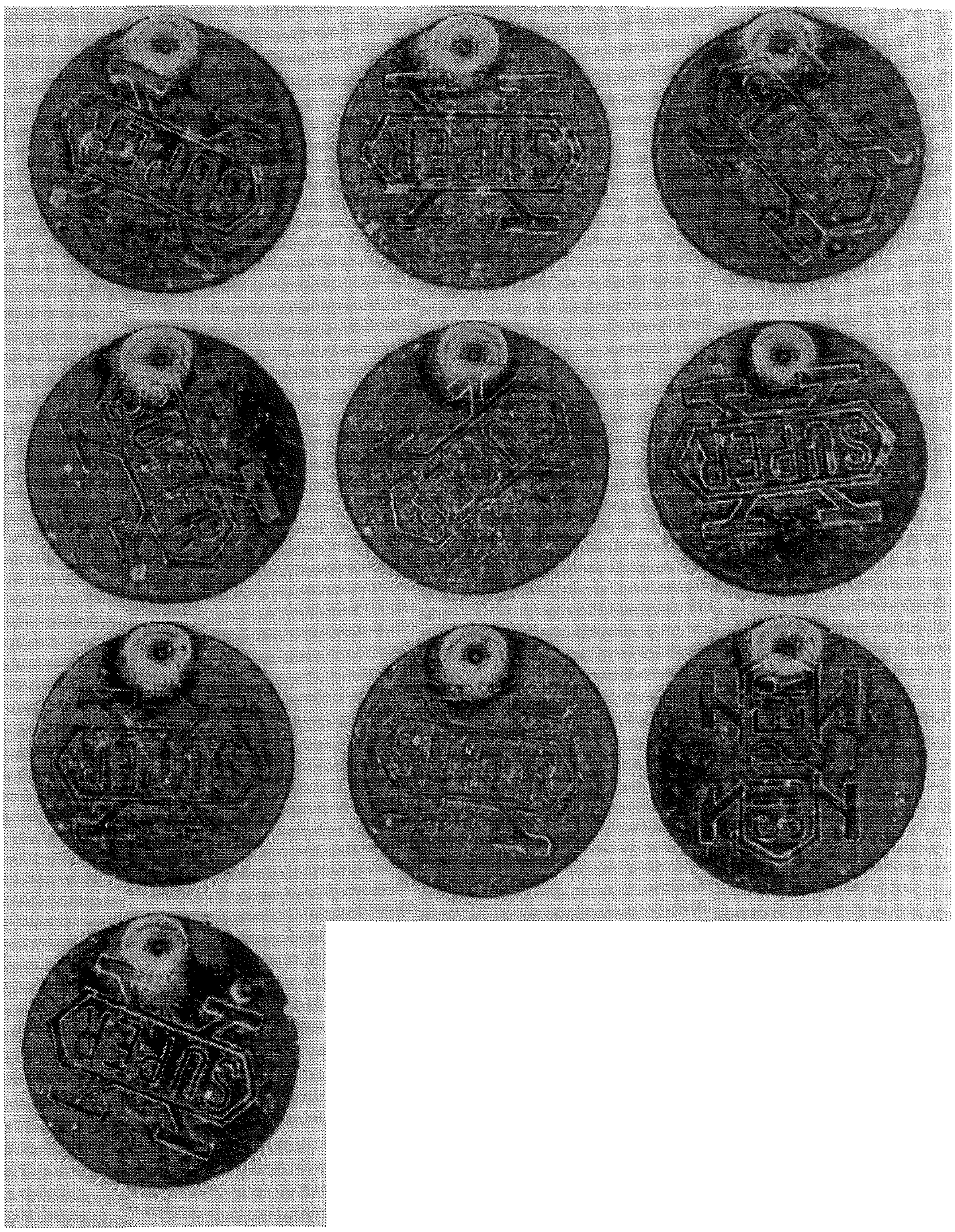
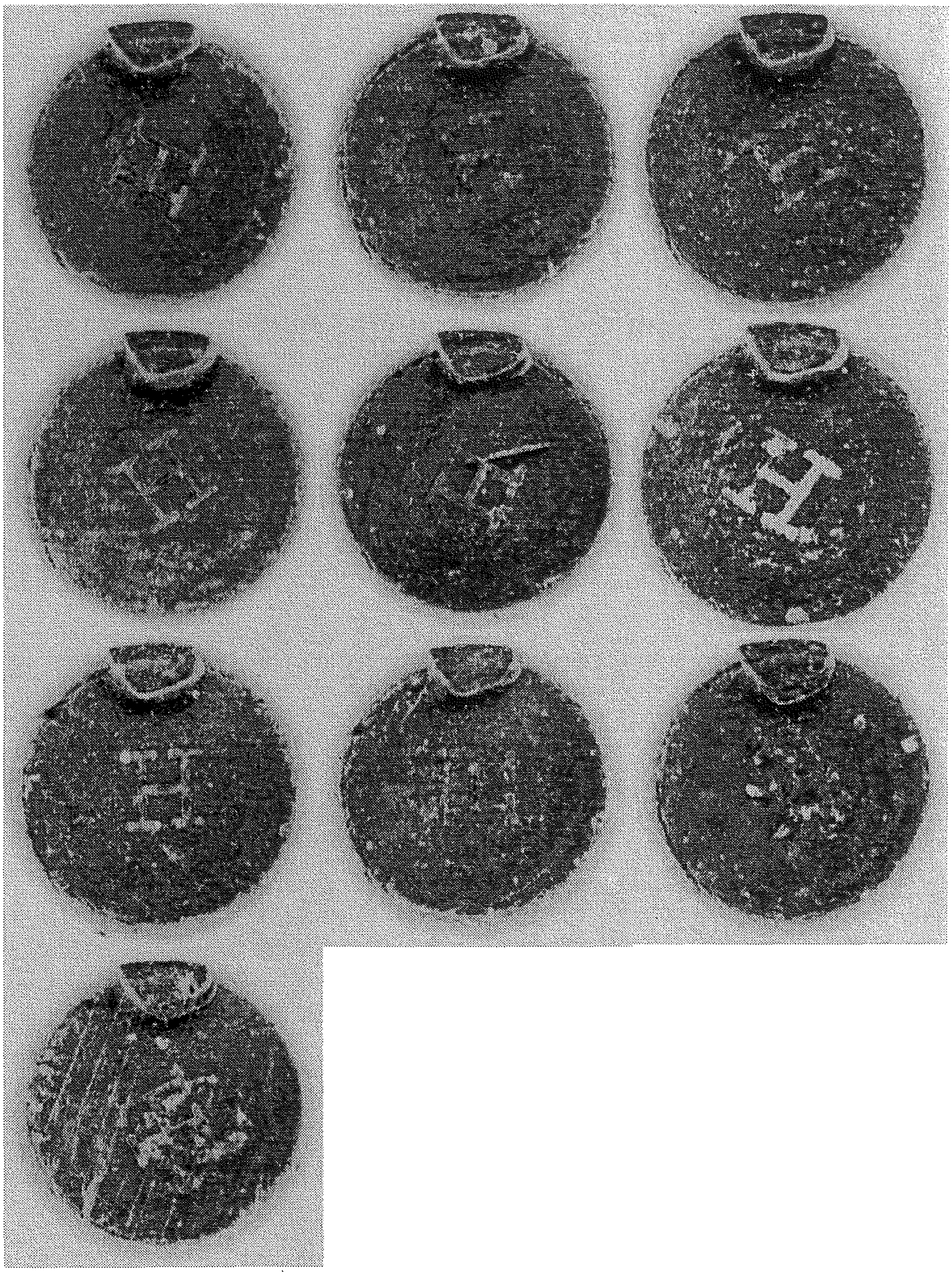
APPENDIX A

Case Study 1: Micrographs

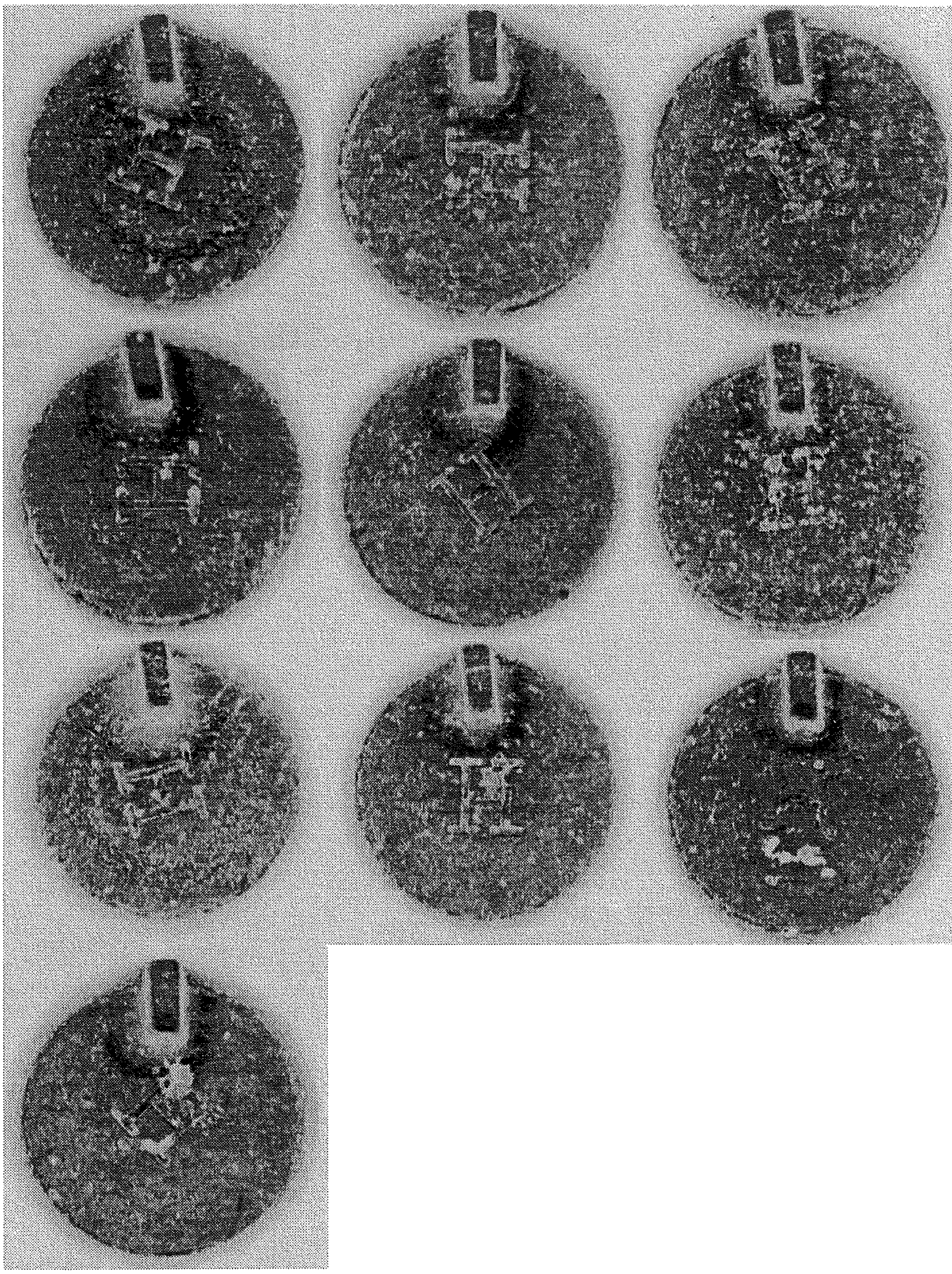
Erma-Werke EM1.22



Ruger 10/22



Winchester 9422 XTR



APPENDIX B

Case Study 1 Results

```
MTB > INFO

Column    Name          Count
C1        CENTDIST         30
C2        CENTANG          30
C3        PRINCANG         30
C4        MOMENT1          30
C5        MOMENT2          30
C6        MEANDIST         30
C7        AREA             30
C8        PERIM            30
C9        MOM1SHAP         30
C10       MOM2SHAP         30
C11       P2A              30
C12       G                30
C13       ANGDIFF          30
C14       FIREARM          30

MTB > LET C13=ABS(ATAN((TAN(C2)-TAN(C3))/(1+TAN(C2)*TAN(C3))))

MTB > CORR C9-C12

       MOM1SHAP MOM2SHAP      P2A
MOM2SHAP  1.000
P2A       0.991    0.992
G         0.991    0.995    0.993

MTB > DOTPLOT 'CENTDIST';
SUBC> BY 'FIREARM'.

 FIREARM                                              .  :
 1                                                    :: :    .
          -------+---------+---------+---------+---------+---------CENTDIST
 FIREARM                    :
 2                    . ..  :.  .  .
          -------+---------+---------+---------+---------+---------CENTDIST
 FIREARM
 3

             .    ::  .    . .  .  .
          -------+---------+---------+---------+---------+---------CENTDIST
               174.0     180.0     186.0     192.0     198.0     204.0

MTB > DOTPLOT 'G';
SUBC> BY 'FIREARM'.

 FIREARM
 1
                                 .:: : ...
         +---------+---------+---------+---------+---------+-------G
 FIREARM                                                   .
 2                                                   :   :::  .
         +---------+---------+---------+---------+---------+-------G
 FIREARM       .
 3            .:
             :::
         +---------+---------+---------+---------+---------+-------G
      0.90      1.05      1.20      1.35      1.50      1.65

MTB > DOTPLOT 'ANGDIFF';
SUBC> BY 'FIREARM'.

                                                             :
                                                             :
 FIREARM                                                     :
 1                                                           :
                                                             :
         +---------+---------+---------+---------+---------+-------ANGDIFF
 FIREARM  ::
 2        :::
         +---------+---------+---------+---------+---------+-------ANGDIFF
 FIREARM
 3
                         .                           .  :.:.:.
         +---------+---------+---------+---------+---------+-------ANGDIFF
      0.00      0.30      0.60      0.90      1.20      1.50
```

```
MTB > DISCRIM 'FIREARM' USING 'G';
SUBC> XVAL.

Linear Discriminant Analysis for FIREARM

Group         1         2         3
Count        10        10        10

Summary of Classification with Cross-validation

Put into      ....True Group....
Group            1         2         3
1               10         0         0
2                0        10         0
3                0         0        10
Total N         10        10        10
N Correct       10        10        10
Proport.      1.00      1.00      1.00

N =    30      N Correct =    30      Prop. Correct = 1.000

Squared Distance Between Groups
                 1         2         3
1            0.000    84.942    96.117
2           84.942     0.000   361.773
3           96.117   361.773     0.000

         Linear Discriminant Function for Group
                 1         2         3
Constant    -740.1   -1137.1    -410.9
G           1127.0    1397.0     839.8

MTB > DISCRIM 'FIREARM' USING 'MOM2SHAP';
SUBC> XVAL.

Linear Discriminant Analysis for FIREARM

Group         1         2         3
Count        10        10        10

Summary of Classification with Cross-validation

Put into      ....True Group....
Group            1         2         3
1               10         0         0
2                0        10         0
3                0         0        10
Total N         10        10        10
N Correct       10        10        10
Proport.      1.00      1.00      1.00

N =    30      N Correct =    30      Prop. Correct = 1.000


Squared Distance Between Groups
                 1         2         3
1            0.000   112.567    63.353
2          112.567     0.000   344.816
3           63.353   344.816     0.000

         Linear Discriminant Function for Group
                 1         2         3
Constant   -1168.5   -1737.7    -815.4
MOM2SHAP    1917.4    2338.2    1601.7

MTB > DISCRIM 'FIREARM' USING 'P2A';
SUBC> XVAL.

Linear Discriminant Analysis for FIREARM

Group         1         2         3
Count        10        10        10

Summary of Classification with Cross-validation

Put into      ....True Group....
Group            1         2         3
1               10         0         0
2                0        10         0
3                0         0        10
Total N         10        10        10
N Correct       10        10        10
Proport.      1.00      1.00      1.00

N =    30      N Correct =    30      Prop. Correct = 1.000
```

```
Squared Distance Between Groups
              1         2         3
1         0.000    82.193    58.856
2        82.193     0.000   280.154
3        58.856   280.154     0.000

        Linear Discriminant Function for Group
              1         2         3
Constant   -611.7    -969.9    -372.8
P2A        1140.0    1435.5     889.9
```

MTB > DISCRIM 'FIREARM' USING 'ANGDIFF';
SUBC> XVAL.

```
Linear Discriminant Analysis for FIREARM

Group          1        2        3
Count         10       10       10

Summary of Classification with Cross-validation

Put into      ....True Group....
Group            1        2        3
1               10        0        3
2                0       10        0
3                0        0        7
Total N         10       10       10
N Correct       10       10        7
Proport.      1.00     1.00     0.70

N =   30      N Correct =    27      Prop. Correct = 0.900

Squared Distance Between Groups
              1         2         3
1          0.00   1269.47      4.67
2       1269.47      0.00   1120.18
3          4.67   1120.18      0.00

        Linear Discriminant Function for Group
              1         2         3
Constant  -680.30     -0.79   -602.95
ANGDIFF    874.36     29.79    823.15

Summary of Misclassified Observations

Observtn      True      Pred      X-val      Group     Squared Distance      Probability
              Group     Group     Group                  Pred       X-val     Pred X-val
   21 **        3         1         1          1         0.06        0.07     0.86  0.93
                                               2      1252.37     1421.56     0.00  0.00
                                               3         3.69        5.16     0.14  0.07
   27 **        3         1         1          1         0.13        0.15     0.82  0.89
                                               2      1243.72     1381.28     0.00  0.00
                                               3         3.23        4.43     0.18  0.11
   30 **        3         1         1          1         0.08        0.09     0.85  0.92
                                               2      1248.99     1405.40     0.00  0.00
                                               3         3.50        4.87     0.15  0.08
```

MTB > DISCRIM 'FIREARM' USING 'CENTDIST';
SUBC> XVAL.

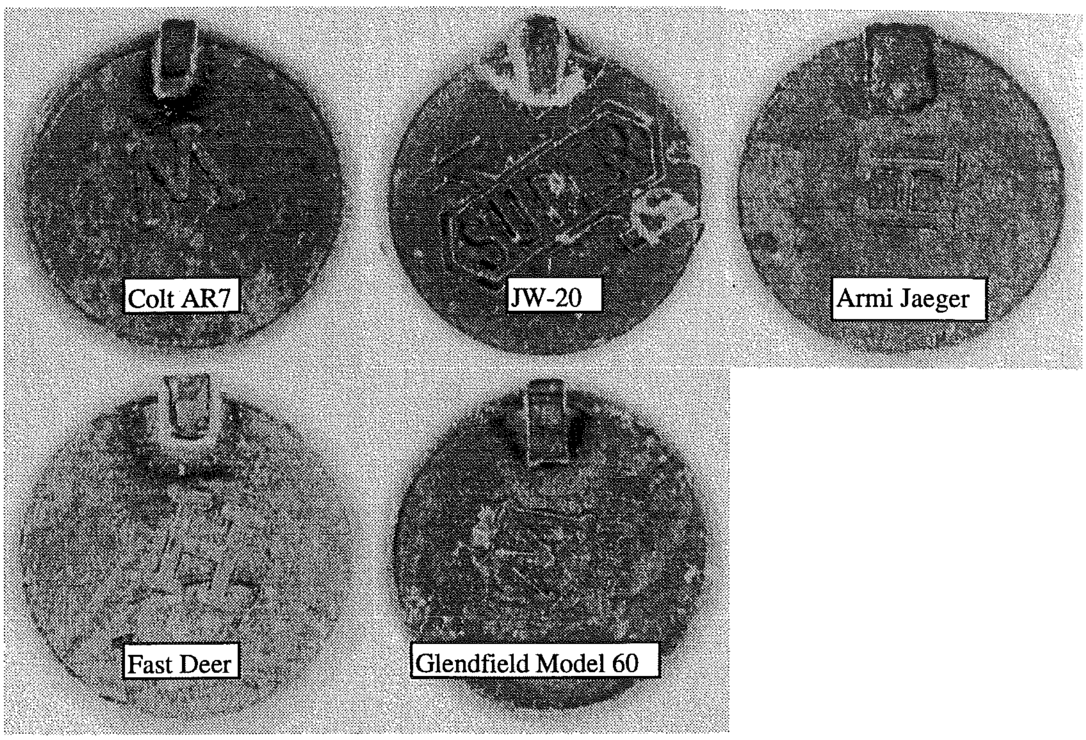
```
Linear Discriminant Analysis for FIREARM

Group          1        2        3
Count         10       10       10

Summary of Classification with Cross-validation

Put into      ....True Group....
Group            1        2        3
1               10        0        0
2                0        7        4
3                0        3        6
Total N         10       10       10
N Correct       10        7        6
Proport.      1.00     0.70     0.60

N =   30      N Correct =    23      Prop. Correct = 0.767

Squared Distance Between Groups
              1         2         3
1        0.0000   36.3770   50.1078
2       36.3770    0.0000    1.0970
3       50.1078    1.0970    0.0000

        Linear Discriminant Function for Group
              1         2         3
Constant  -2543.9   -2131.9   -2064.0
```

```
CENTDIST      25.8      23.6      23.2

MTB > DISCRIM 'FIREARM' USING 'CENTDIST' 'ANGDIFF';
SUBC> XVAL.

Linear Discriminant Analysis for FIREARM

Group          1         2         3
Count         10        10        10

Summary of Classification with Cross-validation

Put into       ....True Group....
Group              1         2         3
1                 10         0         0
2                  0        10         0
3                  0         0        10
Total N           10        10        10
N Correct         10        10        10
Proport.        1.00      1.00      1.00

N =    30      N Correct =    30      Prop. Correct = 1.000

Squared Distance Between Groups
                  1         2         3
1              0.00   1269.54     50.96
2           1269.54      0.00   1170.19
3             50.96   1170.19      0.00

        Linear Discriminant Function for Group
                  1         2         3
Constant    -2848.3   -2186.4   -2346.0
CENTDIST       24.2      24.3      21.7
ANGDIFF       594.2    -251.5     571.9

MTB > DISCRIM 'FIREARM' USING 'G' 'P2A' 'MOM2SHAP' 'CENTDIST' 'ANGDIFF';
SUBC> XVAL.

Linear Discriminant Analysis for FIREARM

Group          1         2         3
Count         10        10        10

Summary of Classification with Cross-validation

Put into       ....True Group....
Group              1         2         3
1                 10         0         0
2                  0        10         0
3                  0         0        10
Total N           10        10        10
N Correct         10        10        10
Proport.        1.00      1.00      1.00

N =    30      N Correct =    30      Prop. Correct = 1.000

Squared Distance Between Groups
                  1         2         3
1              0.00   1713.11    205.42
2           1713.11      0.00   1508.63
3            205.42   1508.63      0.00

        Linear Discriminant Function for Group
                  1         2         3
Constant    -4343.3   -4712.3   -3748.3
G           -3461.1   -6268.9   -4733.5
P2A           919.2    1542.9    1052.5
MOM2SHAP     5758.6    9193.0    6990.7
CENTDIST       22.1      21.6      19.8
ANGDIFF       548.9    -445.8     416.3

MTB > PCA 'G' 'P2A' 'MOM2SHAP' 'CENTDIST' 'ANGDIFF';
SUBC> SCORES C20-C24.

Eigenanalysis of the Correlation Matrix

Eigenvalue      3.7692     1.2023     0.0198     0.0081     0.0006
Proportion       0.754      0.240      0.004      0.002      0.000
Cumulative       0.754      0.994      0.998      1.000      1.000

Variable           PC1        PC2        PC3        PC4        PC5
G                0.506     -0.166      0.337     -0.286      0.722
P2A              0.510     -0.111      0.171      0.825     -0.135
MOM2SHAP         0.512     -0.080      0.219     -0.483     -0.671
CENTDIST        -0.016     -0.910     -0.413     -0.030     -0.017
ANGDIFF         -0.471     -0.354      0.799      0.061     -0.101
```

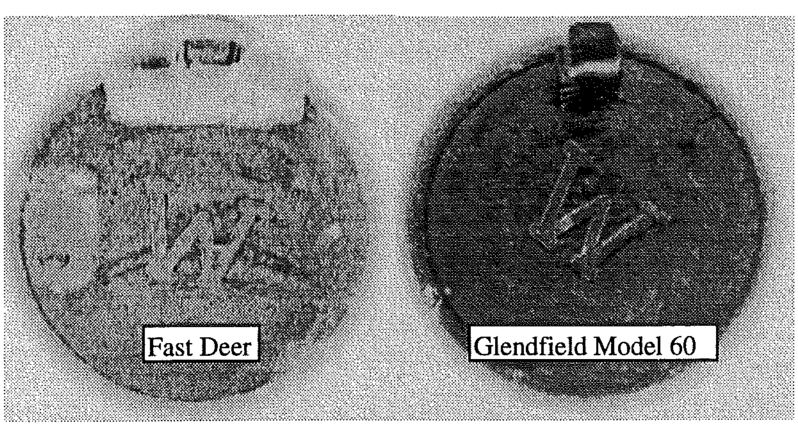
```
MTB > DISCRIM 'FIREARM' USING 'G' 'P2A' 'MOM2SHAP' 'CENTDIST' 'ANGDIFF';
SUBC> PREDICT 1.25060   0.99325   1.11699        190    1.51576;
SUBC> PREDICT 1.31604   1.07971   1.20991        204    1.56262;
SUBC> PREDICT 1.22935   1.03063   1.14456        198    1.55416;
SUBC> PREDICT 1.48998   1.08413   1.33957        210    1.53726;
SUBC> PREDICT 1.40464   1.09985   1.28092        203    1.54185;
SUBC> PREDICT 1.42976   1.10056   1.30827        206    1.53122.

Prediction for Test Observations

Set number  1

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1      110.739       1.000
                                   2     2182.722       0.000
                                   3      382.890       0.000
Set number  2

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1        9.220       1.000
                                   2     1811.061       0.000
                                   3      274.253       0.000
Set number  3

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1       10.977       1.000
                                   2     1812.747       0.000
                                   3      199.024       0.000
Set number  4

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1       81.754       1.000
                                   2     1920.299       0.000
                                   3      490.263       0.000
Set number  5

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1       14.948       1.000
                                   2     1759.022       0.000
                                   3      314.867       0.000
Set number  6

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1       26.260       1.000
                                   2     1704.769       0.000
                                   3      333.833       0.000
```

APPENDIX C

Case Study 2: Sample Micrographs



Case Study 2: Discarded Micrographs



APPENDIX D

Case Study 2 Results

```
MTB > INFO

Column    Name        Count
C1        CENTDIST      145
C2        CENTANG       145
C3        PRINCANG      145
C4        MOMENT1       145
C5        MOMENT2       145
C6        MEANDIST      145
C7        AREA          145
C8        PERIM         145
C9        MOM1SHAP      145
C10       MOM2SHAP      145
C11       P2A           145
C12       G             145
C13       ANGDIFF       145
C14       FIREARM       145

MTB > LET C13=C2-C3

MTB > DOTPLOT 'P2A';
SUBC> BY 'FIREARM'.

FIREARM                                 .
1                                     . :.:
                                .  .:.:.::: ..:. ...
          -----+---------+---------+---------+---------+---------+-P2A
                                                 :
FIREARM                                          :      .: :
2                                          .     :. :::.::.: .   .
          -----+---------+---------+---------+---------+---------+-P2A
FIREARM                     .
3                         ..:.:.
                    .....:::::::  .
          -----+---------+---------+---------+---------+---------+-P2A
FIREARM                                          .
4                                          .  .::
                               . .   ..:::: :::  :.  .
          -----+---------+---------+---------+---------+---------+-P2A
FIREARM                                                     . .
5                                          .     . ...: ::::.:. :.: : .
          -----+---------+---------+---------+---------+---------+-P2A
             1.00      1.10      1.20      1.30      1.40      1.50

MTB > DOTPLOT 'G';
SUBC> BY 'FIREARM'.

FIREARM                               .. : .:
1                                  :..:: :.::.. ....        .
          -+---------+---------+---------+---------+---------+-----G
FIREARM                                                    .:.
2                                     . .  ..  ...:: :.::: ::        .
          -+---------+---------+---------+---------+---------+-----G
                          :
FIREARM                ::. :.
3              .. .:::: ::. . ..
          -+---------+---------+---------+---------+---------+-----G
FIREARM                             . .         :
4                         . : . :.::: :.:. :.: .
          -+---------+---------+---------+---------+---------+-----G
FIREARM                                                    ..
5                                . .     .  .: .:.. ::... . :... .  . .
          -+---------+---------+---------+---------+---------+-----G
         1.08      1.20      1.32      1.44      1.56      1.68

MTB > DOTPLOT 'ANGDIFF';
SUBC> BY 'FIREARM'.

FIREARM                                                       ..
1                                                             ::::
                                                         .: ..::::.:.  ..
          -------+---------+---------+---------+---------+---------ANGDIFF
FIREARM                                          .  .
2                                               .:: :
                                            .: ::::::: : .
          -------+---------+---------+---------+---------+---------ANGDIFF

FIREARM                                         . .
3                  .  .  .:    ....:...... : :..   .:        .
          -------+---------+---------+---------+---------+---------ANGDIFF
```

```
 FIREARM                                  .
  4                                       ::. .
                                    ..:.. :::::::   . .
         -------+---------+---------+---------+---------+---------ANGDIFF
                                          .
 FIREARM                                  :  .
  5                                       ::: :
                                        :.::::::..
         -------+---------+---------+---------+---------+---------ANGDIFF
             -0.24     -0.12      0.00      0.12      0.24      0.36

MTB > DOTPLOT 'AREA';
SUBC> BY 'FIREARM'.

                ..
 FIREARM        ::..
 1             . ::::
               :::::
         -+---------+---------+---------+---------+---------+-----AREA
                                  .
 FIREARM                          :
 2                  .  ..      .::
                   .: ::..  ...:::
         -+---------+---------+---------+---------+---------+-----AREA
                                                     .
 FIREARM                                             :
 3                                                   :...
                                             .   ..::::::. .. : .
         -+---------+---------+---------+---------+---------+-----AREA
                    .
 FIREARM            ::
  4                 :: :  :
                    :: :.:::..    .
         -+---------+---------+---------+---------+---------+-----AREA
                         :
 FIREARM                 :
  5                      :..
                   .:  :.:.:::.....   .
         -+---------+---------+---------+---------+---------+-----AREA
        3200      4800      6400      8000      9600     11200

MTB > DISCRIM 'FIREARM' USING 'P2A' 'G' 'ANGDIFF';
SUBC> QUADRATIC;
SUBC> XVAL.

Quadratic Discriminant Analysis for FIREARM

Group        1         2         3         4         5
Count       29        29        29        29        29

Summary of Classification with Cross-validation

Put into      ....True Group....
Group            1         2         3         4         5
1               29         1         0         1         0
2                0        22         0         1         3
3                0         0        28         2         0
4                0         3         1        25         1
5                0         3         0         0        25
Total N         29        29        29        29        29
N Correct       29        22        28        25        25
Proport.      1.00      0.76      0.97      0.86      0.86

N =   145      N Correct =   129       Prop. Correct = 0.890

From     Generalized Squared Distance to Group
Group            1         2         3         4         5
1           -19.30      5.48     16.11      1.17     93.73
2             0.51    -19.19     45.50    -11.13     -9.21
3            41.88     25.40    -18.94     -3.51     15.84
4            15.04     -8.22      3.23    -18.75    -11.75
5            17.57    -12.83     68.37     -7.06    -19.21

Summary of Misclassified Observations

Observtn     True      Pred     X-val     Group     Squared Distance     Probability
            Group     Group     Group                Pred      X-val    Pred X-val
   33 **        2         4         4         1      5.18       5.18    0.00  0.00
                                              2    -15.23     -14.44    0.23  0.17
                                              3     14.98      14.98    0.00  0.00
                                              4    -17.37     -17.37    0.68  0.74
                                              5    -13.16     -13.16    0.08  0.09
   37 **        2         5         5         1     12.86      12.86    0.00  0.00
                                              2    -15.90     -15.33    0.21  0.17
                                              3     62.50      62.50    0.00  0.00
```

				4	-8.74	-8.74	0.01	0.01
				5	-18.53	-18.53	0.78	0.83
44 **	2	5	5	1	8.21	8.21	0.00	0.00
				2	-16.50	-16.09	0.38	0.33
				3	49.32	49.32	0.00	0.00
				4	-10.87	-10.87	0.02	0.02
				5	-17.43	-17.43	0.60	0.65
47 **	2	5	5	1	11.40	11.40	0.00	0.00
				2	-16.09	-15.59	0.31	0.26
				3	71.62	71.62	0.00	0.00
				4	-5.96	-5.96	0.00	0.00
				5	-17.71	-17.71	0.69	0.74
51 **	2	4	4	1	5.43	5.43	0.00	0.00
				2	-13.51	-11.88	0.20	0.10
				3	14.45	14.45	0.00	0.00
				4	-15.98	-15.98	0.68	0.76
				5	-12.58	-12.58	0.12	0.14
57 **	2	4	4	1	-8.89	-8.89	0.05	0.06
				2	-13.34	-11.60	0.44	0.24
				3	2.57	2.57	0.00	0.00
				4	-13.68	-13.68	0.52	0.69
				5	11.46	11.46	0.00	0.00
58 **	2	1	1	1	-13.28	-13.28	0.54	0.74
				2	-12.83	-10.75	0.43	0.21
				3	19.50	19.50	0.00	0.00
				4	-7.80	-7.80	0.03	0.05
				5	21.64	21.64	0.00	0.00
87 **	3	3	4	1	20.90	20.90	0.00	0.00
				2	3.20	3.20	0.00	0.00
				3	-12.41	-10.20	0.56	0.29
				4	-11.97	-11.97	0.44	0.71
				5	4.29	4.29	0.00	0.00
99 **	4	3	3	1	10.16	10.16	0.00	0.00
				2	2.84	2.84	0.00	0.00
				3	-17.21	-17.21	0.92	0.97
				4	-12.27	-10.10	0.08	0.03
				5	13.83	13.83	0.00	0.00
101 **	4	1	1	1	-13.45	-13.45	0.84	0.96
				2	-6.34	-6.34	0.02	0.03
				3	10.21	10.21	0.00	0.00
				4	-9.74	-5.07	0.13	0.01
				5	45.91	45.91	0.00	0.00
103 **	4	2	2	1	-6.25	-6.25	0.01	0.01
				2	-16.62	-16.62	0.95	0.99
				3	27.55	27.55	0.00	0.00
				4	-10.63	-6.99	0.05	0.01
				5	-1.02	-1.02	0.00	0.00
104 **	4	3	3	1	12.23	12.23	0.00	0.00
				2	0.46	0.46	0.00	0.00
				3	-15.71	-15.71	0.64	0.74
				4	-14.52	-13.63	0.36	0.26
				5	5.09	5.09	0.00	0.00
120 **	5	2	2	1	7.20	7.20	0.00	0.00
				2	-16.67	-16.67	0.50	0.55
				3	56.76	56.76	0.00	0.00
				4	-8.66	-8.66	0.01	0.01
				5	-16.63	-16.25	0.49	0.44
122 **	5	2	2	1	10.14	10.14	0.00	0.00
				2	-16.06	-16.06	0.37	0.40
				3	31.11	31.11	0.00	0.00
				4	-15.46	-15.46	0.27	0.30
				5	-15.98	-15.44	0.36	0.30
123 **	5	4	4	1	20.04	20.04	0.00	0.00
				2	-2.86	-2.86	0.00	0.01
				3	16.74	16.74	0.00	0.00
				4	-12.59	-12.59	0.52	0.79
				5	-12.38	-9.94	0.47	0.21
125 **	5	2	2	1	16.55	16.55	0.00	0.00
				2	-16.75	-16.75	0.67	0.75
				3	80.86	80.86	0.00	0.00
				4	-3.83	-3.83	0.00	0.00
				5	-15.32	-14.56	0.33	0.25

MTB > DISCRIM 'FIREARM' USING 'P2A' 'G' 'ANGDIFF' 'AREA';
SUBC> QUADRATIC;
SUBC> XVAL.

Quadratic Discriminant Analysis for FIREARM

Group	1	2	3	4	5
Count	29	29	29	29	29

Summary of Classification with Cross-validation

Put into Group	....True Group.... 1	2	3	4	5
1	29	1	0	0	0
2	0	23	0	2	2
3	0	0	29	0	0
4	0	3	0	27	1
5	0	2	0	0	26
Total N	29	29	29	29	29
N Correct	29	23	29	27	26
Proport.	1.00	0.79	1.00	0.93	0.90

N = 145 N Correct = 134 Prop. Correct = 0.924

From Group	Generalized Squared Distance to Group 1	2	3	4	5
1	-9.49	23.48	152.20	37.78	113.13
2	154.00	-6.17	132.75	1.32	3.09
3	2374.15	89.51	-6.13	122.24	160.70
4	144.90	4.80	90.44	-6.51	0.54
5	192.78	0.46	153.88	5.79	-7.23

Summary of Misclassified Observations

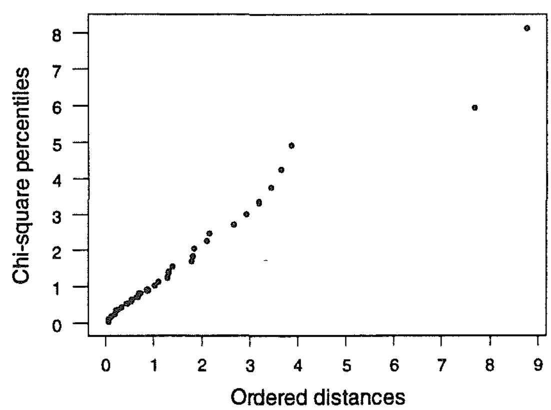
Observtn	True Group	Pred Group	X-val Group	Group	Squared Distance Pred	Squared Distance X-val	Probability Pred	Probability X-val
32 **	2	2	1	1	27.02	27.02	0.00	1.00
				2	10.72	39.55	1.00	0.00
				3	289.84	289.84	0.00	0.00
				4	48.17	48.17	0.00	0.00
				5	41.59	41.59	0.00	0.00
33 **	2	4	4	1	39.712	39.712	0.00	0.00
				2	-0.799	0.688	0.43	0.26
				3	128.752	128.752	0.00	0.00
				4	-1.283	-1.283	0.54	0.70
				5	4.681	4.681	0.03	0.04
37 **	2	5	5	1	258.602	258.602	0.00	0.00
				2	-2.092	-1.221	0.12	0.08
				3	138.605	138.605	0.00	0.00
				4	4.970	4.970	0.00	0.00
				5	-6.149	-6.149	0.88	0.92
47 **	2	5	5	1	121.015	121.015	0.00	0.00
				2	-3.034	-2.478	0.33	0.28
				3	168.881	168.881	0.00	0.00
				4	6.521	6.521	0.00	0.00
				5	-4.398	-4.398	0.66	0.72
51 **	2	4	4	1	112.927	112.927	0.00	0.00
				2	-0.490	1.178	0.15	0.07
				3	106.844	106.844	0.00	0.00
				4	-3.650	-3.650	0.71	0.77
				5	-0.512	-0.512	0.15	0.16
57 **	2	4	4	1	103.772	103.772	0.00	0.00
				2	-0.310	1.470	0.46	0.26
				3	92.836	92.836	0.00	0.00
				4	-0.640	-0.640	0.54	0.74
				5	23.459	23.459	0.00	0.00
101 **	4	4	2	1	560.761	560.761	0.00	0.00
				2	11.752	11.752	0.09	0.99
				3	55.892	55.892	0.00	0.00
				4	7.121	21.369	0.91	0.01
				5	75.563	75.563	0.00	0.00
103 **	4	2	2	1	271.267	271.267	0.00	0.00
				2	-2.123	-2.123	0.93	0.99
				3	97.633	97.633	0.00	0.00
				4	2.969	8.301	0.07	0.01
				5	15.039	15.039	0.00	0.00
123 **	5	4	4	1	84.752	84.752	0.00	0.00
				2	10.255	10.255	0.00	0.00
				3	117.186	117.186	0.00	0.00
				4	-0.132	-0.132	0.51	0.81
				5	-0.017	2.780	0.48	0.19
125 **	5	2	2	1	236.040	236.040	0.00	0.00
				2	-3.586	-3.586	0.56	0.67
				3	160.957	160.957	0.00	0.00
				4	8.568	8.568	0.00	0.00
				5	-3.097	-2.204	0.44	0.33
140 **	5	2	2	1	689.277	689.277	0.00	0.00
				2	8.947	8.947	0.64	0.99
				3	85.835	85.835	0.00	0.00
				4	17.568	17.568	0.01	0.01
				5	10.185	42.497	0.35	0.00

```
MTB > DISCRIM 'FIREARM' USING 'P2A' 'G' 'ANGDIFF';
SUBC> QUADRATIC;
SUBC> XVAL;
SUBC> PREDICT 1.18646    1.39296    0.176795;
SUBC> PREDICT 1.26710    1.44252    0.030980;
SUBC> PREDICT 1.03272    1.11863   -0.123141.

Prediction for Test Observations

Set number  1

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           1
                                   1     -18.695       1.000
                                   2       5.256       0.000
                                   3      25.301       0.000
                                   4       0.160       0.000
                                   5      93.141       0.000
Set number  2

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           2
                                   1      -1.834       0.000
                                   2     -18.827       0.933
                                   3      32.795       0.000
                                   4     -13.360       0.061
                                   5      -8.627       0.006
Set number  3

 Observation Pred. Group  From Group Sqrd Distnc Probability
          1           3
                                   1      66.272       0.000
                                   2      47.233       0.000
                                   3     -17.583       1.000
                                   4       5.371       0.000
                                   5      29.685       0.000
```

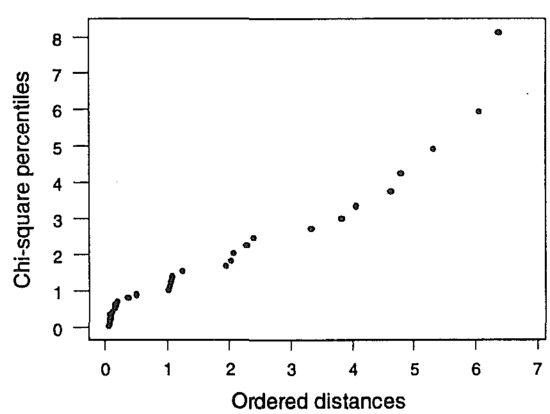
APPENDIX E

Colt AR7

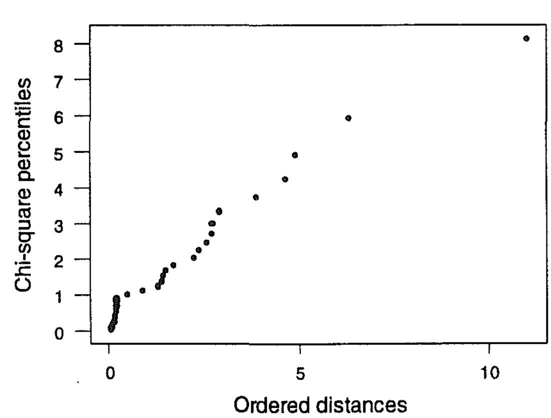
Chi-square Plot for P2A and G



Chi-square Plot for P2A and ANGDIFF

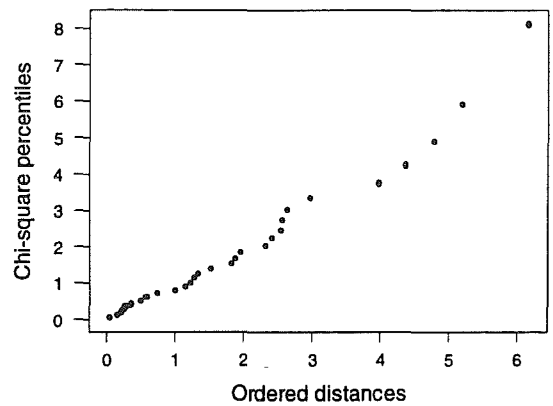


Chi-square Plot for G and ANGDIFF

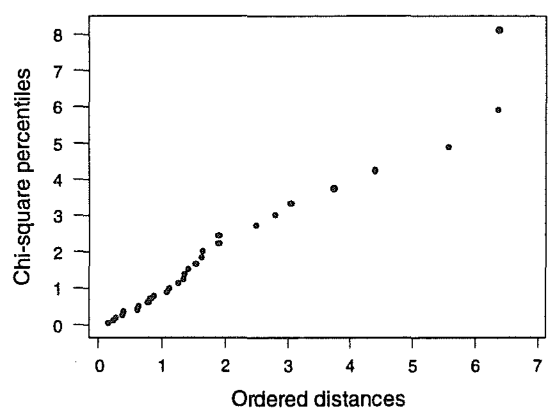


JW-20

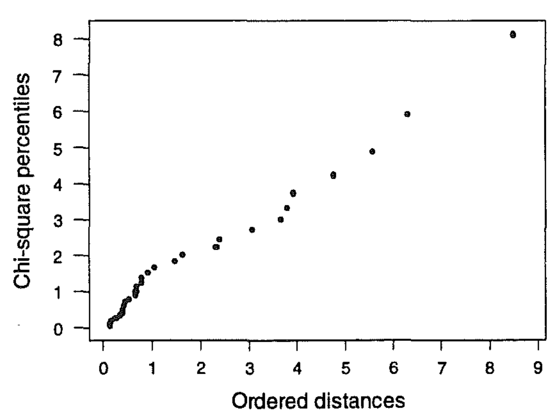
Chi-square Plot for P2A and G



Chi-square Plot for P2A and ANGDIFF

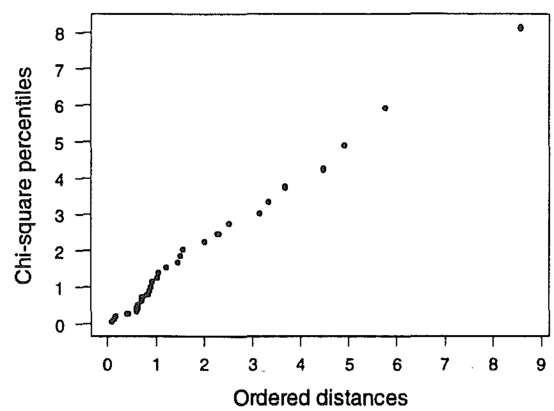


Chi-square Plot for G and ANGDIFF

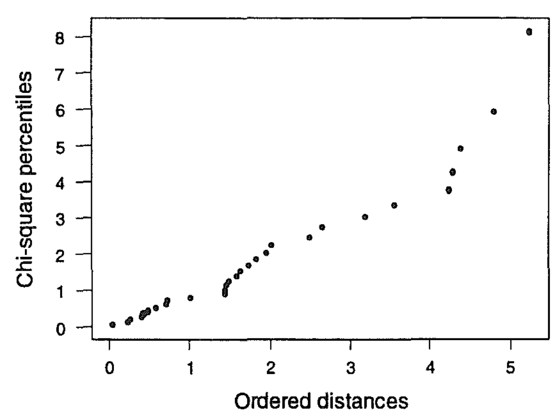


Armi Jaeger

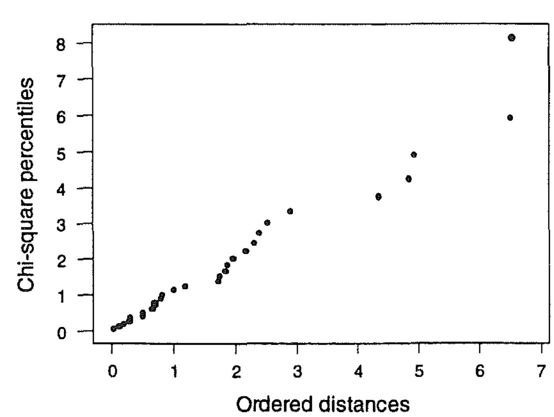
Chi-square Plot for P2A and G



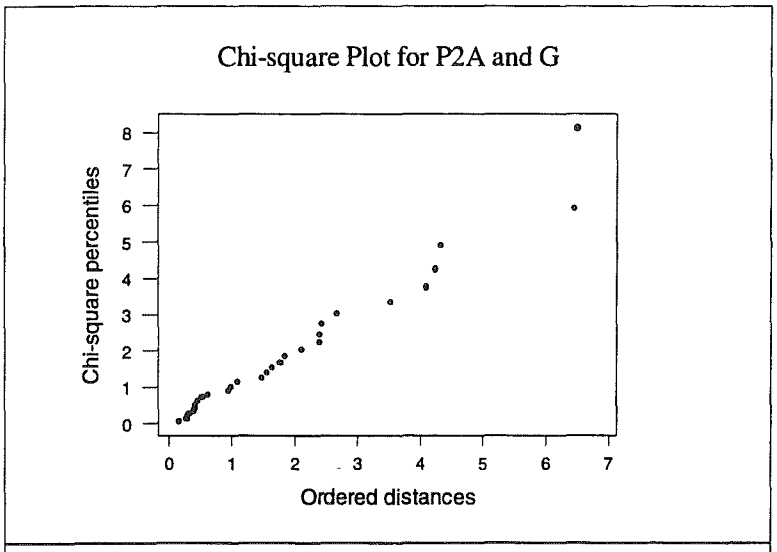
Chi-square Plot for P2A and ANGDIFF

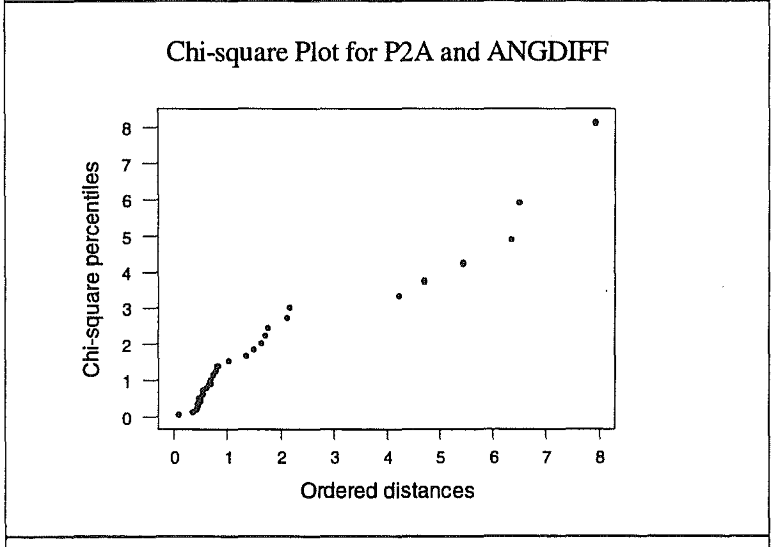


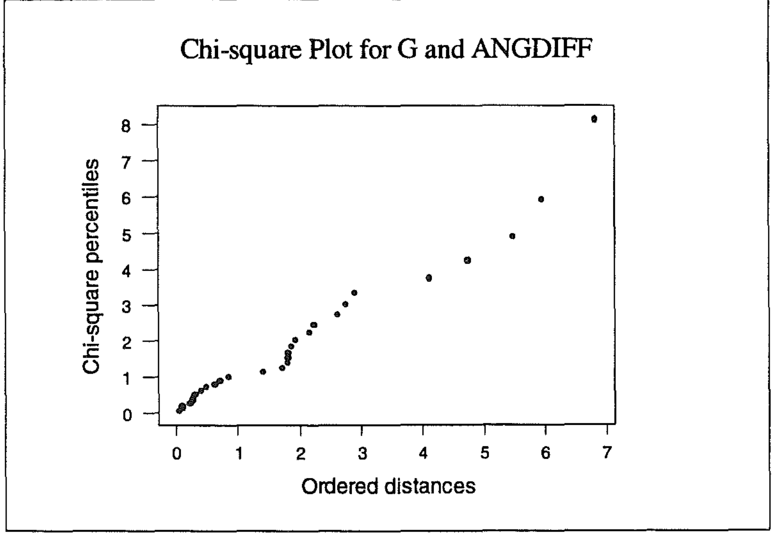
Chi-square Plot for G and ANGDIFF



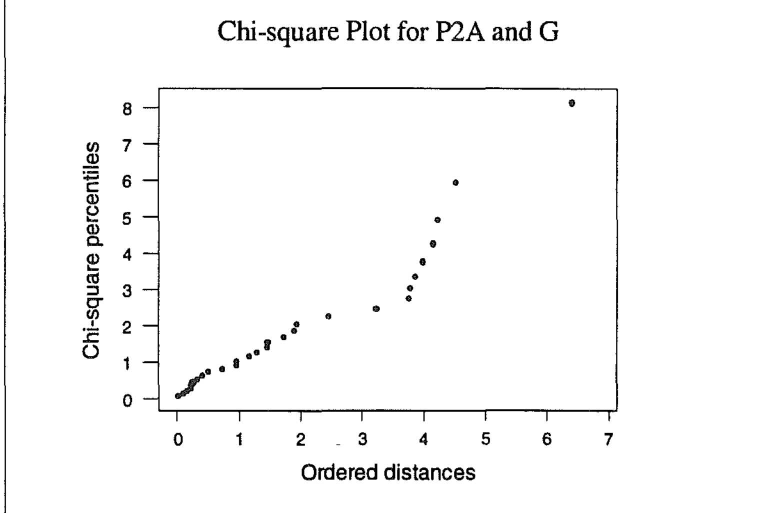
Fast Deer

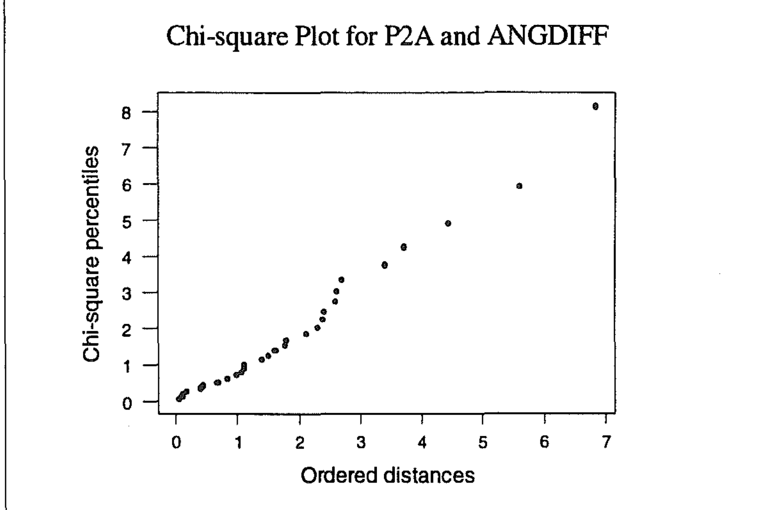

Chi-square Plot for P2A and G
Chi-square percentiles
Ordered distances

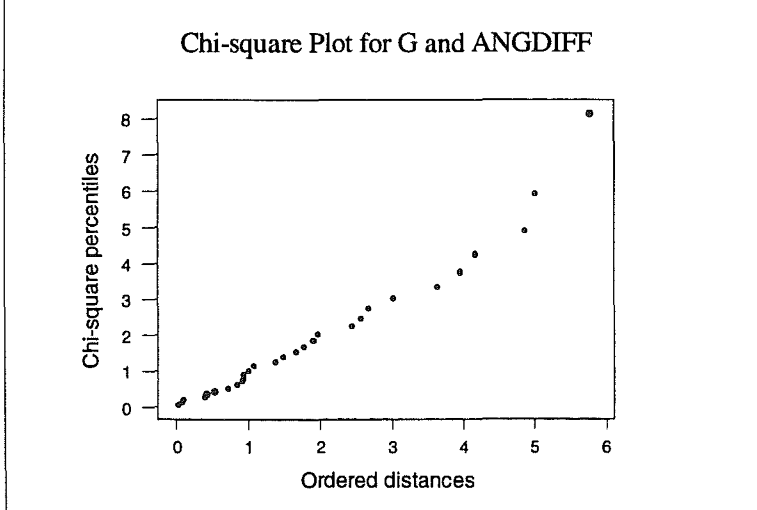

Chi-square Plot for P2A and ANGDIFF
Chi-square percentiles
Ordered distances


Chi-square Plot for G and ANGDIFF
Chi-square percentiles
Ordered distances

Marlin


Chi-square Plot for P2A and G
Chi-square percentiles
Ordered distances
0
1
2
3
4
5
6
7
8


Chi-square Plot for P2A and ANGDIFF
Chi-square percentiles
Ordered distances
0
1
2
3
4
5
6
7
8


Chi-square Plot for G and ANGDIFF
Chi-square percentiles
Ordered distances
0
1
2
3
4
5
6
7
8

APPENDIX F

MINITAB macros for the chi-square plots
(written by Andrew Mehnert)

MACRO: chiplot.mtb
USAGE:
- K1 is the column number for the observations on the first variable
- K2 is the column number for the observations on the second variable
- K3 is the column number of the column used to store the square distances d_j^2
- column (K3 + 1) is used to store the χ_2^2 percentiles
- column (K3 + 2) is used to store the ordered distances $d_{(j)}^2$

```
noecho
note Calculating the squared generalised distances
name ck3 'd-sq'
let k4=count(ck1)
let k5=1
let k6=mean(ck1)
let k7=mean(ck2)
covariance ck1 ck2 m1
invert m1 m1
execute 'chidist' k4
note Calculating the chi-square percentiles
execute 'chiperc'
note Generating the chi-square plot
execute 'gamplot'
end
```

MACRO: chidist.mtb
NOTES:
- called by macro *chiplot.mtb*
- C100 is used as work area
- K5 is used as a counter that counts from 1 to n (number of observations)

```
let c100(1)=ck1(k5)-k6
let c100(2)=ck2(k5)-k7
copy c100 m2
transpose m2 m3
multiply m3 m1 m4
multiply m4 m2 c100
let ck3(k5)=c100(1)
let k5=k5+1
end
```

MACRO: chiperc.mtb
NOTES:
- called by macro *chiplot.mtb*
- calculates the χ_2^2 percentiles; i.e. $\chi_2^2\left(\frac{j-\frac{1}{2}}{n}\right)$ (n of them)

```
let k3=k3+1
name ck3 'Chi-sq%'
set ck3
1:k4
let ck3=(ck3-0.5)/k4
invcdf ck3 ck3;
chisquare 2.
end
```

MACRO: chiperc.mtb

NOTES:
- called by macro *chiplot.mtb*
- sorts the the d_j^2 into ascending order to give $d_{(j)}^2$
- plots the χ_2^2 percentiles against the ordered distances

```
let k3=k3+1
name ck3 'd-sq ord'
sort 'd-sq' 'd-sq ord'
plot 'Chi-sq%'*'d-sq ord';
symbol;
type 6;
size 0.5;
Title "Chi-square Plot";
TFont 1;
Axis 1;
Label "Ordered distances";
Axis 2;
Label "Chi-square percentiles".
end
```