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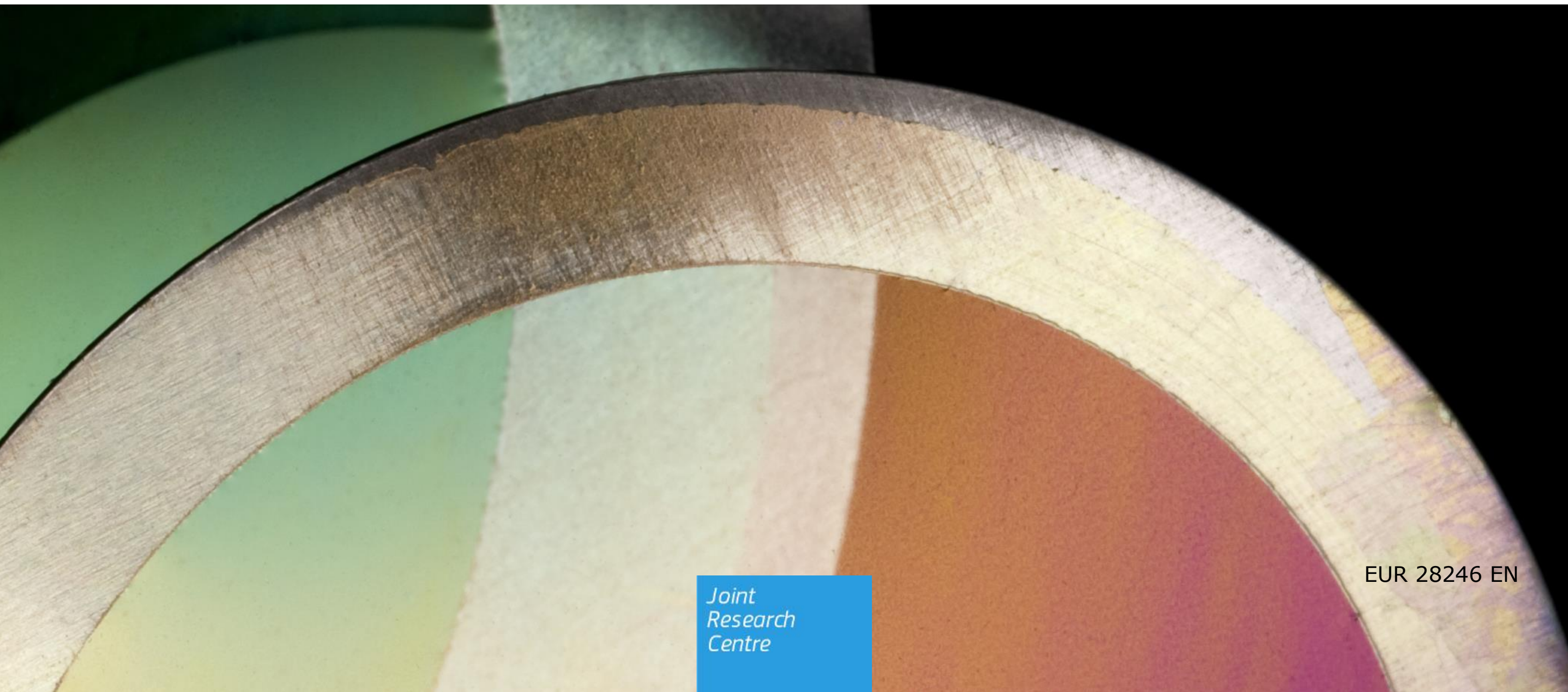

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Environmental Productivity Change in World Air Emissions: A new Malmquist-Luenberger Index Approach

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Abstract

Over the last twenty years an increasing number of studies have relied on the standard definition of the Malmquist-Luenberger index proposed by Chung et al. (1997) [J. Environ. Manage., 51, 229-240], to assess environmental sensitive productivity change. While recent contributions have shown that it suffers from relevant drawbacks related to inconsistencies and infeasibilities, no one has studied systematically the performance of the original model, and to what extent the existing results are unreliable. We introduce the optimization techniques that implement the model by Aparicio et al. (2013) [Eur. J. Oper. Res., 229(3), 738-742] solving these problems, and using a country level database on air pollutants systematically compare the results obtained with both approaches. Over the 1995-2007 period environmental productivity stagnation prevails across developed and developing countries, and while increasing technical progress takes place in the later years, it is offset by declining efficiency. Results show also that inconsistencies and infeasibilities in the original model are increasing in the number of undesirable outputs included, reaching remarkable values that seriously question the reliability of results, and compromise any environmental policy recommendation based on them.

Keywords: Malmquist-Luenberger Index, Technical Change, Data Envelopment Analysis, Computational Analysis.

JEL Classification: C61; D24; O47; Q53.

1. Introduction

The study of environmentally sensitive productivity change accounting for undesirable outputs such as those considered in environmental studies have grown exponentially in recent years. The asymmetric modelling of outputs when measuring efficiency and productivity change depending on their nature, increasing those that are market oriented while reducing those that are detrimental to the environment—resulting in negative externalities, was initiated in the Malmquist-Luenberger productivity index context by Chung et al. (1997) —hereafter denoted *CFG*. Mirroring the definition of the Malmquist index proposed by Färe et al. (1994) based on Shephard's (1953) input or output distance functions, these authors introduced the Malmquist–Luenberger index—hereafter denoted *ML*—exploiting the flexibility of the directional distance function—Chambers et al. (1996). They also followed its traditional Malmquist counterpart so as to identify the sources of productivity change, by decomposing the *ML* index into two mutually exclusive components interpreted in terms of efficiency change and technical change.

The *ML* literature draws from previous contributions on how to model undesirable (or bad) outputs when calculating efficiency. Most particularly, if the axioms underlying the production technology and their Data Envelopment Analysis (*DEA*) approximations should reflect their strong or weak disposability, and eventually, if they should be modeled as outputs or as if they were inputs. But in this latter case an infinite amount of undesirable outputs could be produced with limited inputs, which is an untenable assumption as discussed in the following methodological section. For many years there has been an ongoing debate on this issue in the framework of radial environmental efficiency measurement, although it seamlessly extends to other non-radial measures such as the directional distance function making up the *ML* index. While this debate revolves around technological axioms and is mainly theoretical, the alternative models had been ultimately put to the test in empirical studies.[1]

With respect to the definition of the standard *ML* index, Aparicio et al (2013)—hereafter *APZ*—have shown that the original technological postulates underlying the definition of the directional distance function by Chambers et al. (1996), result in inconsistencies related to the numerical interpretation of its technical change component, which eventually plagues the *ML* index itself. Specifically, these authors show that this component may not measure the actual shift in the production possibility set properly. For example, environmentally friendly technical progress by which the same amount of desirable outputs is produced with less undesirable outputs, is measured numerically with an index lower than one, indicating technological regress—and *viz.*. Consequently the numerical value of the technical change index in empirical applications will yield erroneous results, which in turn support misguided policy recommendations. Ultimately, Aparicio et al.'s (2013) findings question the validity of the standard approach as an empirical tool for environmental productivity measurement. A suspicion that is corroborated in this study by the existence of a remarkable number of inconsistencies that result in wrong interpretations. To overcome this theoretical drawback, these authors redefine the technological axioms by assuming a new postulate that ensures that production possibility sets are nested over time, while limiting the amount of undesirable outputs that can be produced by a—finite amount—of observed inputs.

Although the new approach solves the inconsistency issue, the *ML* index still suffers from one more weakness related to the infeasibility of the cross period directional distance functions conforming the technical change component. Nevertheless, as shown

[1] See, for example, the exchange between Hailu and Veeman (2001), Färe and Grosskopf (2003) and Hailu (2003) in the *Am. J. Agric. Econ*., and Seiford and Zhu (2002), Färe and Grosskopf (2004) and Seiford and Zhu (2005) in the *Eur. J. Oper. Res.*.

in our empirical application, while infeasibilities are pervasive in the standard approach, they diminish by several orders of magnitude in the *APZ* model, becoming negligible and showing one more advantage of the new theoretical framework. Indeed, Aparicio et al.'s (2013) objective was to mend the original approach in the most parsimonious manner, thereby preventing the existence of inconsistencies and reducing infeasibility issues.

As the popularity of the standard approach is unquestionable given the number of empirical applications that rely on this methodology, it is mandatory to assess the reliability of the results through systematic numerical simulations and model definitions, and compare its performance with respect to the new approach. Since it was introduced, many empirical studies have adopted the Chung's et al. (1997) theoretical framework—hereafter *CFG*, while relying on Data Envelopment Analysis techniques to approximate the production technology. Among these, and focusing on the fields of energy, industrial and environmental economics, we can highlight Färe et al. (2001) and Weber and Domazlicky (2001) in manufacturing industries, Kumar (2006) and Yörük and Zaim (2005) for OECD countries, Kumar and Managi (2010b) for electric generating plants, etc. Table 1 summarizes the most relevant contributions to leading journals in the field of environmental economics and management that use the standard *ML* index, including the number of observations (countries, firms,…) and period studied; the included desirable outputs, undesirable outputs and inputs; as well as their main findings regarding environmentally friendly or detrimental productivity change, as well as its efficiency and technical sources.[2]

[2] The list of studies was elaborated following these steps: First, using the ISI Web of Knowledge, we searched for contributions citing the *CFG* approach, finding 458 hits; secondly, among these we identified 32 papers actually using it (but excluding those that proposed some extension of standard *ML* index). Thirdly, we selected those studies that have been published in relevant journals in the field of environmental economics and management, reaching the 19 contributions summarized in Table 1.

Table 1. Review of selected environmental economics and management literature studies applying the standard *CFG* approach.

Publication	Sector and/or Country (Time period)	Desirable outputs	Undesirable outputs	Inputs	Main results		
					MLEFFCH	***MLTECH***	***ML***
Boyd et al. (2002)	US firms in container glass industry (1987-1990)	1.Value of shipment	1. NO_x	1.Capital 2.Stock 3.Labour 4.Cost of energy 5.Cost of materials	1987/1988: increase 1988/1989: decrease 1989/1990: increase	1987-1990: increase	1987-1990: increase
Chen and Golley (2014)	38 China industrial sectors (1980–2010)	1.Value added	1.Energy-induced emissions	1.Capital 2.Labor 3.Energy	1.0036*	1.0158*	1.0146*
Domazlicky and Weber (2004)	48 US states chemical industry (1988-1993)	1.Value added	1.Toxic air emissions 2.Toxic water emissions 3.Toxic land emissions 4.Toxic underground emissions	1.Labor 2.Capital	1.0481*	1.0499*	1.0351*
He et al. (2013)	50 China firms in iron and steel industry (2006-2008)	1.Value added	1.Waste water 2.Waste gas 3.Solid waste	1.Net fixed assets 2.Employees 3.Energy	0.8930	1.3420	1.1980
Krautzberger and Wetzel (2012)	17 European countries commercial transport sector (1995-2006)	1.GDP	1.CO_2	1.Intermediate inputs 2.Capital stock 3.Employees	0.9349	1.0564	0.9872
Kumar (2006)	41 countries (1971–1992)	1.GDP	1.CO_2	1.Labor, 2.Capital, 3.Energy consumption	0.9997	1.0006	1.0002
Kumar and Khanna (2009)	38 countries (1971-1992)	1.GDP	1.CO_2	1.Labor 2.Capital 3.Energy consumption	0.9680*	1.0889*	1.0534*
Kumar and Managi (2010a)	51 countries (1971-2000)	1.GDP 2.Income per capita	1.CO_2 2.SO_2	1.Capital 2.Labor 3.Energy use	0.9939*	1.0185*	1.0052*
Kumar and Managi (2010b)	50 US electric generating plants (1995-2007)	1.Electricity output	1.SO_2 1.NO_x	1.Heat 2.Labor 3.Capital	1.0217*	1.0822*	1.0931*
Li and Lin (2016a)	28 China manufacturing sectors (2006-2010)	1.Gross industrial output value	1.CO_2	1.Capital stock 2.Labor 3.Energy consumption	1.0002	1.0270	1.0272
Li and Lin (2016b)	30 China provinces (1997-2012)	1.Gross region product	1.CO_2	1.Capital 2.Labor 3.Energy	1.0093	1.0253	1.0340

Managi et al. (2005)	406 oil and gas production fields in Gulf Mexico in the USA (1968-1995)	1.Oil production 2.Gas production	1.Water pollution 2. Oil spill	1.No of platforms 2.Avg platform size 3(4).No of exploration (development) wells 5(6).Avg drilling distance for exploratory (development) wells 7.Produced water 8.Environmental compliance cost	-	1.4800	1.6500
Oh and Heshmati (2010)	26 OECD countries (1970–2003)	1.GDP	1.CO_2	1.Labor 2.Capital	1.0005	0.9938	0.9941
Oh (2010)	46 countries (1993 and 2003)	1.GDP	1.CO_2	1.Labor 2.Capital 3.Energy consumption	0.9992	1.0053	1.0043
Piot-Lepetit and Le Moing (2007)	320 French pig farms (1996-2001)	1.Gross output	1.Nitrogen surplus	1.Land 2.Livestock population 3.No of workers 4.Variable expenses	1996/1997: increase 1997/1998: decrease 1998/1999: increase 1999/2000: increase 2000/2001: decrease	1996/1997: decrease 1997/1998: decrease 1998/1999: increase 1999/2000: increase 2000/2001: increase	1996/1997: decrease 1997/1998: decrease 1998/1999: increase 1999/2000: increase 2000/2001: increase
Wang at al. (2013)	28 China provinces (2005-2010)	1.Provincial GDP	1.CO_2	1.Capital stock 2.Labor 3.Energy consumption	0.9600	1.0444	1.0027
Yu et al. (2016)	16 China provinces pulp and paper industry (2010 and 2013)	1.Total industrial output value	1.Wastewater emissions 2.Ammonia nitrogen	1.Water consumption	1.2930	0.9410	1.2170
Zhang (2015)	8 China cities (2001-2009)	1.GDP	1.CO_2	1.Capital 2.Labor 3.Energy	0.9980	0.9910	0.9900
Zhang et al. (2011)	30 China provinces (1989-2008)	1.GDP	1.SO_2	1.Labor 2.Capital	0.9976	1.0270	1.0246

Notes: *MLEFFCH* = Efficiency change; *MLTECH* = Technical change; *ML* = Productivity change.
*Average calculated from reported results.
Although Oh and Heshmati (2010), and Oh (2010) propose an extension of M-L index, we include these studies in the table as they report also the results of the standard M-L index. Wang at al. (2013) and Mangi et al. (2005) additionally report also the results of other models, but we restrict ourselves to the model that takes into account both desirable and undesirable outputs. Managi et al. (2005) do not report the results for efficiency change. Boyd et al. (2002) and Piot-Lepetit and Le Moing (2007) do not report the exact values for indices.
Source: own elaboration

In general, most of the studies consider only few individual pollutants in the analysis. Besides data reliability and availability, it is well known that as the number of decision variables increases with respect to the number of observations, the discriminatory power of *DEA* in terms of efficiency decreases. In terms of the *ML* index, and given the limited number of observations—particularly at the country level—studies tend to select the most relevant undesirable outputs based on the damage they cause to the environment. It turns out nevertheless that the number of inconsistencies and infeasibilities associated to the standard approach increase with the number of variables—undesirable outputs in particular. Therefore, we find the contradiction that limiting the number of available variables in the calculation of the *ML* index so as to increase *DEA*'s discriminatory power yields biased results with respect to the real figures. But if these variables were to be included in the model, results would be unreliable given the increasing number of inconstancies and infeasibilities.

Acknowledging the possibility to incorporate more pollutants into the analysis to better represent environmental productivity change and study the previous trade-off, in this study we solve successive models with increasing number of undesirable outputs under the standard *CFG* and new *APZ* approaches for a sample of 39 developed and developing countries committed to environmentally friendly policies. In total we solve up to 16.380 linear programs per round by exhausting all feasible combinations of undesirable outputs. We choose environmental performance with respect to air pollution mainly because of its relevance and the volume of empirical research exploring this issue. It is undoubtedly one of the most pressing environmental concerns, drawing increasing attention given the ongoing debate around its effects on global warming, soil acidification, and ozone depletion, as well as the existing international agreements on its limits and abatement programs; e.g., from the 1997 Kyoto Protocol extending the 1992

United Nations Framework Convention on Climate Change (UNFCCC), to the most recent December 2015 Paris agreement between 195 countries adopting the first-ever universal, legally binding global climate deal.

The paper unfolds as follows. In the next section we discuss the standard and new approaches recalling the axioms underlying the production technology, the definition of the directional distance function as a measure of environmental efficiency, and the inconsistency issue that affects the original *ML* definition. We also show how these two approaches can be operationalized by approximating both technologies through Data Envelopment Analysis, introduce the mathematical programs corresponding to the new one. Section 3 starts out presenting the dataset on domestic production, air pollutants and inputs that have been collected for a comprehensive set of developed and developing countries. Afterwards we report and compare the results that are obtained using the standard and new approaches. For this purpose a model with two undesirable outputs is initially chosen as benchmark for comparison purposes. Also, a systematic discussion of the inconsistency and infeasibility issues regarding the *ML* index and its components is presented. Subsequently we perform sensitivity and robustness checks by increasing the number of undesirable outputs and solving the corresponding linear programs. Section 4 draws relevant methodological and computational conclusions.

2. The Malmquist-Luenberger productivity index

2.1 The standard approach: CFG

In this section we briefly introduce the definition and main features of the Malmquist-Luenberger productivity index introduced by Chung et al. (1997) constituting the standard *CFG* approach. To this end, we first need to introduce some concepts and notation.

Formally, let us denote the desirable (good) outputs by $y \in \mathbb{R}_{+}^{M}$, the undesirable (bad) outputs by $b \in \mathbb{R}_{+}^{I}$, while inputs are denoted by $x \in \mathbb{R}_{+}^{N}$. Then, the production technology can be represented by way of the following output correspondence P: $\mathbb{R}_{+}^{N} \rightarrow P(x) \subseteq \mathbb{R}_{+}^{M+I}$, $P(x) = \{(y,b) : x \text{ can produce } (y,b)\}$.

Given $x \in R_{+}^{N}$, we assume the usual technological axioms, that is, (A1): $0_{M+I} \in P(x)$; (A2): $P(x)$ is compact; (A3) if $x' \geq x$, then $P(x) \subseteq P(x')$; (A4) $(y,b) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta y, \theta b) \in P(x)$; (A5) if $(y,b) \in P(x)$ and $b = 0_I$, then $y = 0_M$; and (A6) $(y,b) \in P(x)$ and $y' \leq y$ imply $(y',b) \in P(x)$ (see Färe et al., 2007).

Axiom A2 is particularly important since it implies that the undesirable outputs are treated as real outputs and not as inputs. Compactness implies boundedness and, consequently, in words, A2 says that finite inputs can only produce finite (good and bad) outputs. As anticipated in the introduction, this is in contrast to the strand of literature that adheres to the input interpretation of undesirable outputs for empirical convenience and simplicity. Without further axiomatic qualifications this implies that, for example, a ton of coal could be used to produce a finite quantity of electricity and an infinite quantity of CO_2 (e.g., Hailu and Veeman, 2000, 2001).

The *ML* index used to measure productivity change is based on the directional distance function (Chambers et al. 1996, 1998)[3], which seeks the largest feasible increase in desirable outputs compatible with a simultaneous reduction in undesirable outputs (see Chung et al., 1997):

[3] Luenberger (1992, 1995) introduced the concept of benefit function as a representation of the amount that an individual is willing to trade, in terms of a specific reference commodity bundle *g*, for the opportunity to move from a consumption bundle to a utility threshold. Luenberger also defined a so-called shortage function (Luenberger, 1992, p. 242, Definition 4.1), which basically measures the distance in the direction of a vector *g* of a production plan from the boundary of the production possibility set. In recent times, Chambers et al. (1996, 1998) redefined the benefit function and the shortage function as efficiency measures, introducing to this end the so-called directional distance function.

$$\overrightarrow{D}_o(x,y,b;g) = \sup\left\{\beta : (y,b) + \beta g \in P(x)\right\}, \qquad (1)$$

where g is the directional vector setting the particular orientation in which outputs are scaled. A standard choice of orientation corresponds to the observed values of the desirable and undesirable outputs: $g = (y, -b)$, with the latter expressed in negative values, thereby allowing for their reduction.[4]

In the context of Data Envelopment Analysis (*DEA*), the directional distance function (1) can be determined from the mathematical formulation of a linear output production set $P(x)$ that satisfies axioms A1-A6, and that is defined in terms of K observations. In this respect, we assume that for each period of time t there are $k = 1,...,K$ observations of inputs and (good and bad) outputs, denoted as $\left(x_k^t, y_k^t, b_k^t\right)$. From this sample, it is possible to construct the output production set $P^t(x)$ (see Chung et al., 1997):

$$P^t(x^t) = \left\{ \left(y,b\right) \in R_+^M \times R_+^I : \quad \begin{array}{c} \sum_{k=1}^{K} z_k y_{km}^t \geq y_m, \;\; m = 1,...,M \\ \sum_{k=1}^{K} z_k b_{ki}^t = b_i^t, \;\; i = 1,...,I \\ \sum_{k=1}^{K} z_k x_{kn}^t \leq x_n^t, \;\; n = 1,...,N \\ z_n \geq 0, \;\; k = 1,...,K \end{array} \right\}. \qquad (2)$$

From (2), the directional output distance function can be computed as follows:

[4] See Figure 1 in Chung et al. (1997) for a graphical illustration of the directional distance function in a setting with good and bad outputs.

$$\overrightarrow{D}_o(x_0^t, y_0^t, b_0^t; y_0^t, -b_0^t) = \quad Max \quad \beta \tag{3.1}$$

$$s.t.$$

$$\sum_{k=1}^{K} z_k y_{km}^t \geq y_{0m}^t + \beta y_{0m}^t, \quad m = 1,\dots,M \tag{3.2}$$

$$\sum_{k=1}^{K} z_k b_{ki}^t = b_{0i}^t - \beta b_{0i}^t, \quad i = 1,\dots,I \tag{3.3}$$

$$\sum_{k=1}^{K} z_k x_{kn}^t \leq x_{0n}^t, \quad n = 1,\dots,N \tag{3.4}$$

$$z_n \geq 0, \quad k = 1,\dots,K \tag{3.5}$$

(3)

We now turn to the definition of the *ML* index and its decomposition. Following Färe et al. (2001), the index based on period *s* technology is:

$$ML^s = \frac{1 + \vec{D}_o^s\left(x^t, y^t, b^t; y^t, -b^t\right)}{1 + \vec{D}_o^s\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}, \quad s = t, t+1 \tag{4}$$

Note that the definition of the Malmquist–Luenberger index is such that when the direction g is (y, b) rather than (y, $-b$), it coincides with the standard Malmquist index. However, since the direction (y, b) is not suitable for dealing with the production of bad outputs, the direction (y, $-b$) must be used instead and, consequently, the values of the *ML* index will differ from those of the standard Malmquist index.

The *ML* index may be decomposed into efficiency change and technical change in periods t and t+1 as follows:

$$ML^t = \underbrace{\frac{1 + \vec{D}_o^t\left(x^t, y^t, b^t; y^t, -b^t\right)}{1 + \vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}}_{MLEFFCH^t} \cdot \underbrace{\frac{1 + \vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}{1 + \vec{D}_o^t\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}}_{MLTECH^t}, \tag{5}$$

$$ML^{t+1} = \underbrace{\frac{1+\vec{D}_o^t\left(x^t, y^t, b^t; y^t, -b^t\right)}{1+\vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}}_{MLEFFCH^{t+1}} \cdot \underbrace{\frac{1+\vec{D}_o^{t+1}\left(x^t, y^t, b^t; y^t, -b^t\right)}{1+\vec{D}_o^t\left(x^t, y^t, b^t; y^t, -b^t\right)}}_{MLTECH^{t+1}}. \tag{6}$$

To avoid the use of an arbitrary reference technology, the geometric mean of the two based period indices is considered, thereby defining $ML_t^{t+1} = \left(ML^t \cdot ML^{t+1}\right)^{1/2}$. ML_t^{t+1} credits producers for simultaneously increasing good outputs and reducing the production of bad outputs. Also, from (5) and (6), ML_t^{t+1} can be decomposed into the same two components, accounting for efficiency change and technical change. Noting that $MLEFFCH^t = MLEFFCH^{t+1}$, one obtains the following breakdown:

$$ML_t^{t+1} = \underbrace{\frac{1+\vec{D}_o^t\left(x^t, y^t, b^t; y^t, -b^t\right)}{1+\vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}}_{MLEFFCH_t^{t+1}} \cdot \underbrace{\left[MLTECH^t \cdot MLTECH^{t+1}\right]^{1/2}}_{MLTECH_t^{t+1}}. \tag{7}$$

Following the literature, any improvement in productivity, efficiency and technical change corresponds to values greater than one. On the contrary, values less than one indicate regress. In particular, Färe et al. (2001; 391) interpret the values of the technical change component of the *ML* index as: “Shifts of the production possibilities frontier in the direction of ‘more goods and fewer bads’ results in the value of the $MLTECH_t^{t+1}$ index exceeding unity. If the $MLTECH_t^{t+1}$ index equals unity, this indicates that there was no shift in the production possibilities frontier. Finally, an $MLTECH_t^{t+1}$ index value of less than unity indicates a shift of the production possibilities frontier in the direction of ‘fewer goods and more bads’”. Additionally, Kumar (2006; 284-285) states that “If technical change enables more production of good and less production of bad output, then

$MLTECH_t^{t+1} > 1$, whereas if $MLTECH_t^{t+1} < 1$, there has been a shift in the frontier in the direction of fewer good outputs and more bad outputs”.

2.2 The inconsistency of the standard ML index

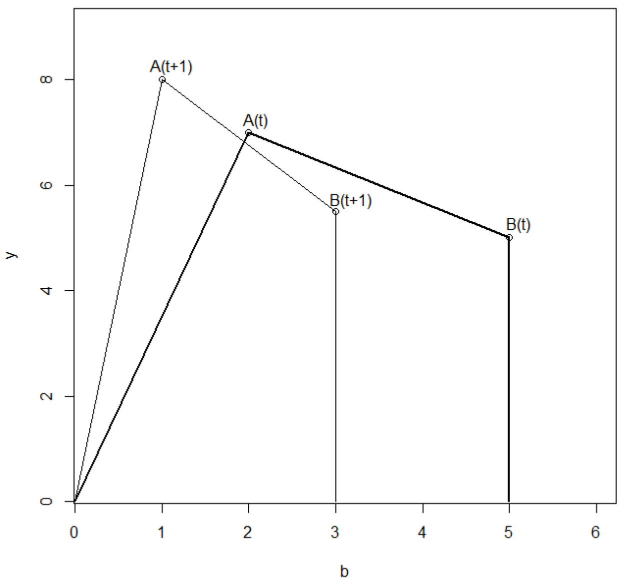
In this subsection we briefly revise the drawback of the *ML* index related to the existence of inconsistent results for the technical change term $MLTECH_t^{t+1}$, which seriously compromise the reliability of the analyses based on the standard approach. We also discuss a second weakness related to the existence of infeasible solutions when solving for the cross period distance functions conforming the same term.

With respect to the first shortcoming, Aparicio et al. (2013) showed that the interpretation of the technical change component in terms of production frontier shifts can be inconsistent with its numerical value. These authors illustrated this problem through a numerical example, showing that this measure does not correctly measure the actual shift in the production possibility set. Environmentally friendly technical progress was found in the example since the observed shift was in the direction of ‘more goods and fewer bads’. However, this progress was mistakenly associated with a value of *MLTECH* < 1, indicating unreal technological regress. Let us briefly reproduce at this point the simple numerical example used by these authors.

Consider two observations, *A* and *B* in *t* and *t*+1 time periods, which use an equal amount of a single input (x) to produce one good output (y) and one bad output (b): A^t=(1,7,2), B^t=(1,5,5), A^{t+1}=(1,8,1) and B^{t+1}=(1,5.5,3). The corresponding output production sets are illustrated in Figure 1.

Focusing the analysis on unit *B*, we see that this observation is efficient in periods *t* and *t*+1, and therefore $\vec{D}_o^t\left(x_B^t, y_B^t, b_B^t; y_B^t, -b_B^t\right) = \vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right) = 0$, resulting in $MLEFFCH_t^{t+1}$ = 1, and any improvement or decrease in productivity must be exclusively consequence of technological progress or regress.

Figure 1. The inconsistency of the standard approach.



Continuing with the example, we calculate the technical change component for the *ML* index based on period *t* as the reference technology. In this way, we obtain $MLTECH^t = \dfrac{1+\vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right)}{1+\vec{D}_o^{t}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right)} < 1$, since $\vec{D}_o^{t+1}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right) = 0$ and $\vec{D}_o^{t}\left(x_B^{t+1}, y_B^{t+1}, b_B^{t+1}; y_B^{t+1}, -b_B^{t+1}\right) > 0$. This value suggests that *B* has experienced technological regress, i.e., a shift in the direction of 'fewer goods and more bads' following, for example, Färe et al. (2001) and Kumar (2006). However, the actual change is exactly in the opposite direction, i.e., 'more goods and fewer bads', for both unit B and for the overall technology in general. This is the inconsistency that we wanted to show and that allows to claim that the *ML* index can yield wrong results.

As for the infeasibility weakness, it is relatively well-known since it is inherited from the directional distance function. It is recognized that 'mixed period' directional distance functions, which reflect the distance of a data point in time period t relative to the technology of period t+1 or *vice versa*, may yield infeasible results (Briec and Kerstens, 2009).[5] Additionally, Briec and Kerstens (2009) showed that infeasibilities can also occur even in single period (contemporaneous) calculations when the output directional vector is non-zero and the number of inputs is larger than or equal to two, or the directional input vector is not of full dimension whenever the output direction is null. In empirical studies, it is normally observed that a small fraction of the linear programs calculating the distance functions are unfeasible. However, how serious is this weakness deserves to be studied in terms of the frequency of this result. In our context, in terms of the number of outputs that are considered in the model, as we explore in the empirical application.

2.3 Overcoming inconsistencies and infeasibilities: The APZ approach

Abiding by the principle of parsimony, Aparicio et al. (2013) searched for a new definition of the technology that would solve the inconsistency issue while reducing the likelihood of infeasible solutions in the *DEA* approach. Ideally, and given the popularity of the standard approach, such solution would preserve from a theoretical perspective the analytical framework of the *ML* index based on the directional distance function, while from an empirical perspective should not increase the complexity of the mathematical programming, or result in additional computational burdens. They finally proposed a solution based on a new postulate on the environmental technology that complements those usually accepted in the related literature (i.e., axioms A1-A6 in Section 2.1).

[5] Considering the example, note that if one attempts to calculate the technical change component resorting to the period t+1 technology as reference, the numerator of $MLTECH^{t+1}$ is undetermined since no $\beta \in R$ exists for B such that $\left(y_B^t, b_B^t\right) + \beta\left(y_B^t, -b_B^t\right) \in P^{t+1}(x)$. Therefore, this simple example also illustrates the possibility of infeasible results.

Therefore, it builds upon the existing axioms by qualifying the production technology, while preventing the inconsistency and infeasibility issues.

Given $x \in \mathbb{R}_{+}^{N}$, let $\bar{b}(x): \mathbb{R}_{+}^{N} \to \mathbb{R}_{++}^{I}$ be a correspondence representing the upper bound for the generation of each considered bad output from the input vector x. In this way, given x, if the vector (y,b) is feasible, then $b \le \bar{b}(x)$. The new postulate states that if x can produce outputs (y,b), then it is feasible to produce more contaminants up to a certain limit, $\bar{b}(x)$:

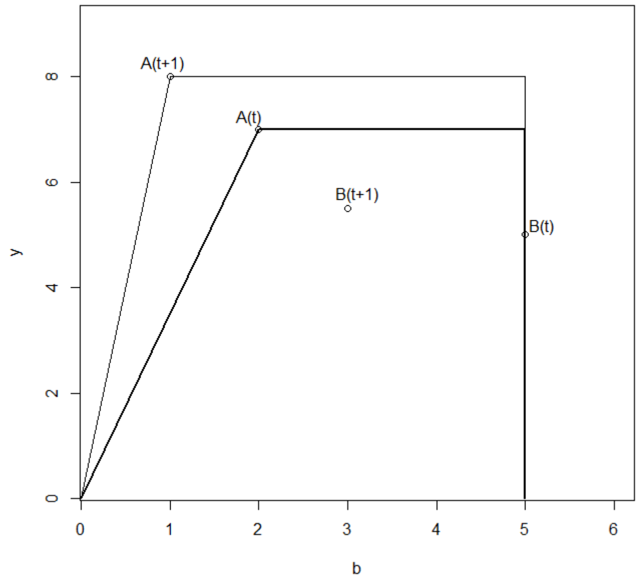
(A7) If $(y,b) \in P(x)$ and $b \le b' \le \bar{b}(x)$, then $(y,b') \in P(x)$.

For the simple numerical example utilized above, where $A^{t+1} > A^{t}$ and $B^{t+1} > B^{t}$, i.e. unit A uses the same quantity of inputs to produce more good outputs and less bad outputs in period $t+1$ than unit A in period t, and the same for unit B, the effects of taking into account the new postulate are depicted in Figure 2. Note that, in contrast to Figure 1, in Figure 2 the environmental technologies are nested[6]. Note also that the maximum limit permitted for polluting was defined as $\bar{b}^{t}(x) = \bar{b}^{t+1}(x) = \max\limits_{\substack{1 \le k \le K \\ s=t,t+1}} \{b_k^s\}$.

In the context of nested technologies, like in Figure 2, we now analyze what happens with respect to $MLTECH^{t} = \dfrac{1+\vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}{1+\vec{D}_o^{t}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)}$. The relationship $P^{t}(x) \subseteq P^{t+1}(x)$ implies that $\vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right) \ge \vec{D}_o^{t}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)$ and, consequently, $MLTECH^{t} \ge 1$. The same can be shown for $MLTECH^{t+1}$. In this way, by (6), we finally have that $MLTECH_{t}^{t+1} \ge 1$, as desired.

[6] Two papers that support the use of nested technologies in the measurement of productivity change are Tulkens and Vanden Eeckaut (1995) and Shestalova (2003) but, in this case, in the traditional context of the Malmquist index.

Figure 2. The new approach solving the inconsistency of the *ML* index



As regards the infeasibility problem, we show in this paper that assuming the new postulate minimizes, although not avoids, this weakness of the *ML* index. In particular, in the simple numerical example, the technical change component can be determined for unit B without problems of this type, since in Figure 2 $\exists\beta \in R$ for B such that $\left(y_B^t, b_B^t\right) + \beta\left(y_B^t, -b_B^t\right) \in P^{t+1}(x)$. This contrasts to what happened with the same unit when $MLTECH^{t+1}$ is computed under the standard approach as illustrated in Figure 1.

2.4 The Data Envelopment Analysis formulation

The new methodology introduced by Aparicio et al. (2013) remains to be operationalized as these authors did not show how it can be mathematically implemented, nor applied to real data. In this subsection, in the framework of *DEA*, we present the expression of the production possibility set under axioms A1-A7 and introduce the optimization program that must be solved to determine the directional distance function defined on this set.

The output production set $P^t(x)$ in (2) is modified as follows to satisfy additionally postulate A7:

$$\bar{P}^t\left(x^t\right)=\left\{\left(y,b\right)\in R_+^M\times R_+^I: \begin{array}{l} \sum_{k=1}^{K} z_k y_{km}^t \geq y_m,\ m=1,\dots,M \\ \sum_{k=1}^{K} z_k b_{ki}^t \leq b_i,\ i=1,\dots,I \\ \sum_{k=1}^{K} z_k x_{kn}^t \leq x_n^t,\ n=1,\dots,N \\ b_i \leq \bar{b}_i^t\left(x^t\right),\ i=1,\dots,I \\ z_n \geq 0,\ k=1,\dots,K \end{array}\right\}. \qquad (8)$$

Proposition 1. Let $x^t \in R_{++}^N$, $b_k^t \in R_{++}^I$ for all $k=1,\dots,K$ and $\bar{b}_i^t\left(x^t\right):=\max_{1\leq k\leq K}\left\{b_{ki}^t\right\}$, $i=1,\dots,I$. Then $\bar{P}^t\left(x^t\right)$ meets A1-A7.

Proof. (A1) Defining $z_n=0,\ k=1,\dots,K$, we have that $0_{M+I}\in\bar{P}^t\left(x^t\right)$ for any $x^t\in R_+^N$. (A2) From $\sum_{k=1}^{K} z_k x_{kn}^t \leq x_n^t$ we have that $z_k \leq \min_{1\leq n\leq N}\left\{\frac{x_n^t}{x_{kn}^t}\right\}$ for all $k=1,\dots,K$. Then, $y_m \leq \sum_{k=1}^{K} z_k y_{km}^t \leq \sum_{k=1}^{K}\left(\min_{1\leq n\leq N}\left\{\frac{x_n^t}{x_{kn}^t}\right\} y_{km}^t\right),\ m=1,\dots,M$. Additionally, $b_i \leq \bar{b}^t\left(x^t\right),\ i=1,\dots,I$. Consequently, $\bar{P}^t\left(x^t\right)$ is bounded. Moreover, $\bar{P}^t\left(x^t\right)$ is a polyhedral set and, hence, it is closed. As a result, $\bar{P}^t\left(x^t\right)$ is compact. (A3) Let $\tilde{x}^t \geq x^t$ and let $\left(y,b\right)\in\bar{P}^t\left(x^t\right)$. Then $\exists z_k \geq 0,\ k=1,\dots,K$, such that $\left(y,b,z\right)$ satisfies the constraints in (8). So, we have that $\sum_{k=1}^{K} z_k x_{kn}^t \leq x_n^t \leq \tilde{x}_n^t,\ n=1,\dots,N$. Consequently, $\left(y,b\right)\in\bar{P}^t\left(\tilde{x}^t\right)$, which implies that $\bar{P}^t\left(x^t\right)\subseteq\bar{P}^t\left(\tilde{x}^t\right)$. (A4) Let $\left(y,b\right)\in\bar{P}^t\left(x^t\right)$ and $0\leq\theta\leq 1$. Then $\exists z_k \geq 0,\ k=1,\dots,K$, such that $\left(y,b,z\right)$ satisfies the constraints in (8). In this way, $\left(\theta y,\theta b,\theta z\right)$ also meets the constraints in (8) and, consequently, $(\theta y, \theta b) \in \bar{P}^t(x^t)$. (A5) Let $(y,b) \in \bar{P}^t(x^t)$ with $b = 0_I$. By the constraints $\sum_{k=1}^{K} z_k b_{ki}^t \le b_i$, $i = 1,...,I$, we have that $z_n = 0,\ k = 1,...,K$, since by hypothesis $b_k^t \in R_{++}^I$ for all $k = 1,...,K$. Finally, from $\sum_{k=1}^{K} z_k y_{km}^t \ge y_m,\ m = 1,...,M$, and $y \in R_+^M$ we have that $y = 0_M$. (A6) Let $\tilde{y} \le y$ and $(y,b) \in \bar{P}^t(x^t)$. Then $\exists z_k \ge 0$, $k = 1,...,K$, such that (y,b,z) satisfies the constraints in (8). It is easy to prove that $(\tilde{y},b,z)$ also satisfies the same constraints since $\sum_{k=1}^{K} z_k y_{km}^t \ge y_m \ge \tilde{y}_m,\ m = 1,...,M$, and, therefore, $(\tilde{y},b) \in \bar{P}^t(x^t)$. (A7) Given $z_k \ge 0$, $k = 1,...,K$, such that (y,b,z) satisfies the constraints in (8), we have that $(y,b') \in \bar{P}^t(x^t)$ with $b \le b' \le \bar{b}(x^t)$ since $\sum_{k=1}^{K} z_k b_{ki}^t \le b_i \le b_i',\ i = 1,...,I$, and $b_i' \le \bar{b}_i^t(x^t),\ i = 1,...,I$. ■

From (8), it is possible to define the directional output distance function in (1) under the satisfaction of the new postulate. In particular, we show how this distance can be calculated for observations of period *h*, *h=t*, *t+1*, with respect to the frontier of technology $\bar{P}^s$, *s=t,t+1*.

$$\begin{array}{llll}
\overrightarrow{D}_o^s(x_0^h, y_0^h, b_0^h; y_0^h, -b_0^h) = & Max\ \beta & & (9.1) \\
& s.t. & & \\
& \sum_{k=1}^{K} z_k y_{km}^s \ge y_{0m}^h + \beta y_{0m}^h, & m = 1,...,M & (9.2) \\
& \sum_{k=1}^{K} z_k b_{ki}^s \le b_{0i}^h - \beta b_{0i}^h, & i = 1,...,I & (9.3) \quad (9) \\
& \sum_{k=1}^{K} z_k x_{kn}^s \le x_{0n}^h, & n = 1,...,N & (9.4) \\
& b_{0i}^h - \beta b_{0i}^h \le \bar{b}_i^s(x_0^h), & i = 1,...,I & (9.5) \\
& z_n \ge 0, & k = 1,...,K & (9.6)
\end{array}$$

In contrast to model (3), constraint (3.3) is transformed into an inequality and (9.5) is added to the model in order to bound the maximum pollution associated with the potential projection benchmark. Specifically, the inequality related to (9.3) denotes that this constraint is really an input-type restriction. Therefore, model (9) can be seen as a bridge between the two previously mentioned approaches in the literature for dealing with good and bad outputs. Indeed model (9) forces the undesirable outputs projection to be greater or equal than the benchmark frontier combination—adopting the rationale underlying input modeling, but upper bounding the feasible values. This bound prevents that from finite input it is possible to produce infinite pollutants, which is the situation if the bad outputs are dealt with as usual inputs.

3 Data and results

3.1. Data and sources

The data used in this study comes from World Input-Output Database (WIOD) (Timmer et al., 2015). This database is a result of the project financed by the European Union (EU) that aims to develop databases, accounting frameworks and models in order to explain some of the tradeoffs between worldwide socioeconomic and environmental factors. WIOD contains annual time series of input-output and environmental variables for 40 countries covering the period from 1995 to 2011. Because of the lack of input-output data for some countries and years, the final database used in this study contains a balanced panel of 39 countries for the period 1995-2007. [7] The countries analyzed include 27 EU countries: Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain,

[7] In particular, the data on capital was very limited after 2007. We also needed to exclude Taiwan from the dataset because of the lack of data on Purchasing Power Parity for this country.

Sweden and the UK, and 12 other major countries in the world: Australia, Brazil, Canada, China, Indonesia, India, Japan, Korea, Mexico, Russia, Turkey and United States.

The *DEA* model used in this study includes one desirable output, two inputs and seven undesirable outputs. The desirable output corresponds to gross value added. The two inputs are number of employees (further referred to as labor) and gross capital stock. The undesirable outputs are the main air pollutants emissions that cause three environmental hazards related to global warming, acidification, and tropospheric ozone formation. These main air pollutants emissions are formed by two groups: (1) main greenhouse gases: carbon dioxide CO_2, methane CH_4, nitrous oxide N_2O, and (2) other main air pollutants that are not greenhouse gases: nitrogen oxides NO_X, sulphur oxides SO_X, ammonia NH_3, and non-methane volatile organic compound NMVOC. While in the empirical analysis we explore all combinations of these pollutants, we choose two of them for our reference model: CO_2 and NO_X. CO_2 is the main greenhouse gas that causes global warming, while NO_X is responsible for smog, acid rain and tropospheric ozone, being particularly dangerous to humans.

All variables in monetary units (that is gross value added and capital stock) are compiled from WIOD in local currencies and in current prices. On one hand, to facilitate cross-country comparisons these variables are adjusted by the Purchasing Power Parity (PPP) of the local currency to the US dollar, obtained from the World Bank. On the other, to enable comparisons across periods, these variables are deflated to constant prices of the year 1995 using country-specific price indices as reported by WIOD.

Table 2 shows the descriptive statistics for the input-output data, aggregated across countries. While in the empirical results we analyze all years in the 1995-2007 period, we choose the initial years (1995 and 1996), middle years (2000 and 2001) and final years (2006 and 2007) as reference for detailed analyses. Therefore, the data on descriptive statistics in Table 2 is presented for these years. From the table, it is clear that average gross value added systematically increased along the period. Regarding inputs, on average, the labor increased, while capital initially decreased between 1995 and 1996 and then increased between 2000 and 2001, and 2006 and 2007, with an overall increasing trend observed between 1995 and 2007. The average values for air pollutants follow diverging trends. The emissions of CO_2, CH_4, and NH_3 systematically increased over the years. NO_X and N_2O emissions increased between 1995 and 1996, then between 2000 and 2001; N_2O continued to increase while NO_X decreased, and finally between 2006 and 2007 N_2O decreased while NO_X increased. SO_X emissions were decreasing between 1995 and 1996, and 2000 and 2001, while they finally increased between 2006 and 2007. Overall, the trend for emissions of NO_X, N_2O and SO_X was increasing between 1995 and 2007. The only variable that systematically decreased, on average, are emissions of NMVOC. The data in Table 2 also shows the large values of standard deviations relatively to their respective means, hence a relative variation in the sample.

Table 2. Descriptive statistics for input-output data.

Year	Variable	Mean	Std. dev.	Min.	Max.
1995	Gross value added (millions of PPP)	686,253.53	1,279,484.99	5,058.14	7,421,307.33
	CO_2 (kilotonnes)	460,636.57	930,467.07	2,188.13	4,953,562.45
	NO_X (tonnes)	1,978,321.32	4,041,603.20	12,846.73	22,831,722.76
	SO_X (tonnes)	2,003,458.16	4,539,153.12	7,922.61	23,556,746.58
	CH_4 (tonnes)	4,700,829.14	9,651,495.17	9,273.47	45,286,442.20
	N_2O (tonnes)	174,426.62	323,105.54	143.42	1,567,921.37
	NH_3 (tonnes)	485,048.18	967,269.11	854.35	5,442,662.04
	NMVOC (tonnes)	2,176,422.67	4,108,955.91	7,968.99	19,586,083.47
	Labour (thousand)	46,022.54	122,778.93	138.87	680,650.00
	Capital (millions of PPP)	2,203,607.98	3,598,642.36	16,431.69	18,820,738.92
1996	Gross value added	686,286.66	1,335,601.21	4,866.75	7,730,078.42
	CO_2	471,576.46	951,895.59	2,240.78	5,083,435.38
	NO_X	1,992,477.21	4,063,781.13	12,637.13	22,897,280.36
	SO_X	1,984,641.12	4,546,819.59	6,708.82	23,679,088.94
	CH_4	4,708,135.86	9,714,490.04	9,509.88	46,016,614.24
	N_2O	177,468.82	333,245.97	131.11	1,632,111.86
	NH_3	493,864.75	1,004,855.15	882.18	5,689,107.52
	NMVOC	2,125,670.68	3,932,284.81	7,968.99	17,251,889.20
	Labour	46,671.24	124,913.67	139.49	689,500.00
	Capital	2,181,879.51	3,712,337.20	15,777.09	19,498,613.65
2000	Gross value added	756,656.52	1,608,892.39	1,461.88	9,215,202.31
	CO_2	492,175.58	1,005,168.82	2,320.39	5,514,270.26
	NO_X	1,951,070.60	3,866,781.83	8,374.46	21,059,372.34
	SO_X	1,787,754.71	3,984,392.41	1,535.43	20,239,383.54
	CH_4	4,564,497.76	9,356,762.74	4,834.22	43,620,314.31
	N_2O	170,495.54	327,719.02	140.39	1,618,908.95
	NH_3	496,430.69	1,006,818.29	1,816.02	5,594,203.70
	NMVOC	2,084,779.51	3,827,823.53	3,096.31	16,788,208.81
	Labour	49,138.15	131,072.63	145.53	720,850.00
	Capital	2,346,676.97	4,387,850.01	7,326.73	23,387,726.47
2001	Gross value added	776,671.60	1,652,629.91	1,463.77	9,388,564.60
	CO_2	495,623.09	1,002,302.13	2,437.62	5,466,773.01
	NO_X	1,928,598.40	3,730,947.74	9,105.28	19,917,643.37
	SO_X	1,774,729.74	3,940,051.08	1,549.01	20,141,167.74
	CH_4	4,589,226.10	9,441,084.38	5,057.59	43,978,120.89
	N_2O	171,421.25	333,492.77	136.48	1,638,270.08
	NH_3	502,356.52	1,026,454.66	1,823.23	5,686,233.82
	NMVOC	2,082,803.09	3,843,465.41	3,134.55	16,946,994.51
	Labour	49,861.03	133,470.15	148.52	730,250.00
	Capital	4,561,993.48	2,417,593.70	7,176.93	24,337,515.19
2006	Gross value added	945,564.39	2,018,858.10	1,686.56	10,800,042.32
	CO_2	577,106.55	1,219,914.21	2,634.57	5,524,517.08
	NO_X	2,078,251.77	4,089,787.65	11,696.16	19,353,454.94
	SO_X	1,987,082.35	5,605,053.22	1,545.55	32,981,245.81
	CH_4	5,111,910.63	11,429,000.42	5,426.04	58,888,765.24
	N_2O	180,032.54	371,686.69	128.75	1,946,832.83
	NH_3	547,716.45	1,208,896.26	1,596.46	6,857,957.67
	NMVOC	2,140,119.45	4,184,685.79	3,517.95	20,568,570.76
	Labour	52,800.66	141,138.14	154.19	764,000.00
	Capital	2,830,231.08	5,457,198.48	7,797.18	29,027,701.09
2007	Gross value added	987,118.81	2,098,971.74	1,690.66	11,033,197.31
	CO_2	597,620.24	1,283,481.75	2,693.49	5,962,552.39
	NO_X	2,101,036.66	4,194,824.52	11,566.63	20,589,660.91
	SO_X	2,037,660.41	5,920,675.42	1,312.46	35,194,456.64
	CH_4	5,169,666.63	11,714,306.90	5,143.63	61,036,665.45
	N_2O	179,524.49	377,934.53	133.81	1,991,630.72
	NH_3	552,819.37	1,234,504.54	1,692.26	7,028,573.20
	NMVOC	2,133,170.72	4,249,305.41	3,269.60	21,162,541.46
	Labour	53,034.90	141,092.65	159.11	769,900.00
	Capital	2,928,664.64	5,659,777.43	7,837.54	30,191,130.26

Source: World Input-Output Database (WIOD) and own elaboration.

3.2. Environmental productivity change: Comparing the standard and new approaches.

In this section we study the main trends in environmental productivity change of developed and developing countries, and discuss the consequences of adopting the standard approach by Chung, et al. (1997), *CFG*, in terms of the emerging inconsistencies and infeasibilities that do not only cast doubts on its reliability, but also greatly reduces the set of results. These results are confronted with those attained relying on the new approach by Aparicio, et al. (2013), *APZ*, solving both problems (3) and (9). As anticipated, while we systematically explore all existing combinations of the seven undesirable outputs included in our database, we initially choose a reference model with two relevant air pollutants: CO_2 and NO_X, and for illustration purposes focus on the interannual productivity change of the initial years 1996/1995, the middle years corresponding to 2001/2000, and the last years 2007/2006. Table 3 displays the Malmquist-Luenberger index (*ML*)—eq. (7), as well as its decomposition into its technical efficiency change (*MLEFFCH*), and technical change (*MLTECH*) components computed using both the *CFG* and *APZ* approaches.

The productivity indices' calculations have been performed using the *DEA Toolbox* developed by Álvarez et al. (2016) in the MATLAB environment.[8] The linear optimization problems are solved using the dual-simplex algorithm with the optimality tolerance and constraint tolerance set to 10^{-10} and 10^{-7}, respectively. Infeasibilities correspond to those cases in which the optimization program returns 'No feasible point was found' as exit flag. Infeasibilities plaguing the *CFG* approach are reported with a dash punctuation mark, with relevant frequencies in the technical change and productivity indices. As we report in the following section, a complete computational analysis reveals

[8] Data Envelopment Analysis Toolbox is available as free software, under the GNU General Public License version 3, and can be downloaded from http://www.deatoolbox.com, with all the supplementary material: Manual, source code, examples and data.

an increasing and monotonic relationship between the frequency of infeasibilities in the *CFG* approach and the number of undesirable outputs.

Taking as reference feasible results only—particularly in the *CFG* where about one third of the calculations go unsolved, and leaving aside countries whose unitary values report unchanging indices, the values of the *ML* index resulting from applying both approaches show that in the initial 1996/1995 period the majority of countries in the sample experience a decline in environmental productivity (18 and 22, respectively), due to efficiency losses as well as technical regress (exceptions are Greece, Korea, Portugal, etc.)—Table 3. However, the results reveal that the *APZ* approach reports more countries experiencing technical regress compared to the *CFG* approach (24 vs 16). These findings change in the middle 2001/2000 period with regard to the environmental technical change index *MLTECH*, as most countries exhibit technical progress according to *APZ* (22), while *CFG* shows that there is an even number of countries experiencing technical progress and regress (14). As a result, and taking into account that in this period there are similar patterns for environmental efficiency change *MLEFFCH* in both approaches (i.e., there are more countries with efficiency decline than regress), the previous technical change patterns translate in to the *ML* index. Turning to the last period 2007/2006 results, we observe that an overwhelming majority of countries experience technical progress *MLTECH* (37 and 27), while many of them exhibit declining efficiency (29 and 18). As technical change increases, while efficiency sharply declines for most countries, the gap between leading and lagging countries widens; i.e., the catching up speed of most of countries is slower than that of the frontier technology advancement. As a result, while in 2007/2006 the majority of countries experience improving environmental productivity, it is driven by technical progress for most of countries. To sum up, although many of the patterns of environmental productivity change and its components are similar in the *CFG* and *APZ* approaches, in some periods we find relevant dissimilarities with regard to the technical change index *MLTECH*. Dissimilarities that are further confirmed by the inconsistency results discussed thereafter.

Indeed, inconsistencies reflecting conflicting results with the *CFG* approach, wrongly measuring either decreasing or increasing productivity and technical change, with the *APZ* approach yields opposite trends (i.e., < 1 *vs.* > 1 and *viz.*), are highlighted in bold. Results include several inconsistencies where technical change *MLTECH* has decreased or increased when computed using the *CFG* approach, and the opposite when the *APZ* model is considered. It is worth remarking that the inconsistencies detected in the technical change component carry over to the *ML* index itself, but since the technical efficiency change term *MLEFFCH* may also differ between the two approaches due to the alternative definitions of the production possibilities sets, such difference in their values may compensate the technical change inconsistencies. An example of the former case is India in the initial 1997/1996 period. Its inconsistent *MLTECH* index under the *CFG* approach is 0.9462 reflecting technical regress, while its value is 1.0074 under *APZ*. Reinforcing the difference in the final *ML* index, its efficiency change *MLEFFCH* values are also opposite to each other: 0.9837 and 1.0139. As a result the *CFG* approach reflects productivity decline to the tune of 0.9308, while the *APZ* model accurately reflects productivity growth: 1.0214. An example of the latter case with the technical change inconsistency of the *CFG* model not passing to the *ML* is Poland, whose efficiency change components counterbalance the conflicting technical change differential, with both *ML* indices finally reflecting productivity growth.

Table 3. Malmquist-Luenberger results: *ML*, *ML*TEC and *ML*TC. *CFG* and *APZ* models. Selected years.

Period	**1996/1995**						**2001/2000**						**2007/2006**					
Index	***ML***		***ML*TEC**		***ML*TC**		***ML***		***ML*TEC**		***ML*TC**		***ML***		***ML*TEC**		***ML*TC**	
Country	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***	***CFG***	***APZ***
Australia	−	1.0196	1.0000	1.0177	−	1.0018	−	1.0113	1.0000	1.0143	−	0.9970	0.9876	0.9781	0.9704	0.9692	1.0178	1.0091
Austria	0.9673	0.9842	1.0000	1.0000	0.9673	0.9842	0.9723	0.9690	0.9740	0.9369	**0.9982**	**1.0343**	1.0224	1.0238	0.9758	0.9763	1.0478	1.0487
Belgium	−	1.0103	1.0000	0.9973	−	1.0131	−	0.9995	1.0000	0.9997	−	0.9998	−	1.0026	**1.0000**	**0.9846**	−	1.0183
Bulgaria	0.8868	0.8808	0.8875	0.8853	0.9992	0.9950	−	0.9999	1.0000	1.0001	−	0.9998	1.0048	1.0005	**1.0018**	**0.9997**	1.0029	1.0008
Brazil	−	0.9140	1.0000	1.0000	−	0.9140	−	0.9734	1.0000	0.9834	−	0.9898	**1.0029**	**0.9925**	0.9326	0.9588	1.0754	1.0351
Canada	1.0017	1.0015	0.9849	0.9853	1.0171	1.0164	1.0063	1.0068	1.0032	1.0059	1.0031	1.0009	0.9950	0.9984	0.9837	0.9911	1.0115	1.0074
China	1.0386	1.0396	1.0726	1.0413	0.9684	0.9983	1.0437	1.0439	1.0000	1.0000	1.0437	1.0439	1.0786	−	1.0000	1.0000	1.0786	−
Cyprus	0.9863	0.9862	0.9959	0.9958	0.9903	0.9904	1.0338	1.0280	1.0208	1.0210	1.0128	1.0069	1.0134	1.0127	0.9859	0.9929	1.0279	1.0199
Czech Republic	0.9840	0.9830	0.9849	0.9885	0.9991	0.9944	0.9876	0.9963	0.9743	0.9859	1.0137	1.0105	1.0501	1.0190	**1.0037**	**0.9977**	1.0463	1.0213
Germany	−	1.0140	1.0000	1.0079	−	1.0061	−	1.0193	1.0000	0.9888	−	1.0308	−	1.0434	1.0000	0.9968	−	1.0468
Denmark	−	0.9713	0.9725	0.9705	−	1.0008	−	0.9898	1.0000	1.0015	−	0.9883	−	0.9886	1.0000	0.9725	−	1.0166
Spain	1.0187	1.0166	1.0313	1.0182	0.9877	0.9984	1.0062	1.0064	**0.9995**	**1.0017**	1.0068	1.0047	1.0045	1.0029	0.9670	0.9701	1.0388	1.0338
Estonia	−	0.9614	1.0000	0.9657	−	0.9956	−	0.9936	1.0000	0.9915	−	1.0022	−	0.9729	1.0000	0.9600	−	1.0134
Finland	**0.9981**	**1.0070**	0.9944	0.9998	1.0038	1.0072	0.9914	0.9944	1.0005	1.0014	0.9909	0.9930	1.0445	1.0389	1.0346	1.0301	1.0096	1.0085
France	**0.9961**	**1.0010**	1.0000	1.0000	**0.9961**	**1.0010**	1.0159	1.0201	1.0385	1.0291	0.9782	0.9912	1.0251	1.0262	0.9663	0.9677	1.0608	1.0604
United Kingdom	1.0124	1.0035	1.0040	1.0191	**1.0083**	**0.9847**	0.9932	0.9925	**0.9951**	**1.0032**	0.9981	0.9893	1.0118	1.0080	0.9981	0.9932	1.0137	1.0149
Greece	0.9774	0.9767	0.9519	0.9703	1.0268	1.0066	0.9915	0.9947	0.9889	0.9937	1.0026	1.0010	0.9193	0.9622	0.9009	0.9494	1.0204	1.0135
Hungary	0.9523	0.9523	0.9645	0.9645	0.9873	0.9873	1.0130	1.0064	0.9973	0.9963	1.0157	1.0102	0.9828	0.9890	0.9687	0.9736	1.0146	1.0158
Indonesia	**1.0157**	**0.9946**	1.0000	1.0000	**1.0157**	**0.9946**	**0.8806**	**1.0189**	1.0000	0.9988	**0.8806**	**1.0201**	−	1.0044	1.0000	0.9749	−	1.0303
India	**0.9308**	**1.0214**	**0.9837**	**1.0139**	**0.9462**	**1.0074**	**0.9969**	**1.0003**	1.0000	0.9871	**0.9969**	**1.0134**	0.9829	0.9907	**1.0018**	**0.9755**	**0.9811**	**1.0157**
Ireland	1.0666	1.0329	1.0000	1.0003	1.0666	1.0326	−	0.9794	1.0000	0.9978	−	0.9815	0.9835	0.9968	0.9551	0.9750	1.0297	1.0224
Italy	0.9929	0.9929	0.9945	0.9933	0.9985	0.9996	1.0130	1.0072	0.9902	0.9961	1.0231	1.0111	1.0081	1.0072	0.9872	0.9777	1.0211	1.0302
Japan	1.0259	1.0266	1.0328	1.0338	0.9934	0.9931	1.0064	1.0149	0.9801	0.9839	1.0269	1.0315	1.0562	1.0449	1.0427	1.0287	1.0129	1.0158
Korea	0.9927	0.9929	0.9889	0.9910	1.0038	1.0019	1.0037	1.0029	1.0002	1.0014	1.0035	1.0015	1.0211	1.0175	1.0096	1.0052	1.0114	1.0123
Lithuania	0.9909	0.9649	0.9861	0.9736	**1.0049**	**0.9911**	1.0102	1.0215	1.0172	1.0255	0.9931	0.9961	0.9840	0.9849	0.9617	0.9651	1.0231	1.0204
Luembourg	−	0.9958	1.0000	1.0000	−	0.9958	**0.9976**	**1.0030**	1.0000	1.0000	**0.9976**	**1.0030**	1.0274	1.0408	1.0000	1.0000	1.0274	1.0408
Latvia	0.9775	0.9759	0.9743	0.9776	**1.0033**	**0.9982**	1.0368	1.0220	1.0924	1.0363	0.9491	0.9862	0.9780	0.9791	0.9899	0.9742	**0.9879**	**1.0050**
Mexico	0.9093	0.9095	0.9337	0.9333	0.9739	0.9745	0.9800	0.9940	1.0036	0.9958	0.9764	0.9982	0.9952	0.9999	**1.0090**	**0.9832**	**0.9863**	**1.0170**
Malta	0.9469	0.9720	0.9824	0.9827	0.9639	0.9892	0.9530	0.9554	0.9436	0.9492	1.0099	1.0065	1.0254	1.0214	1.0119	1.0053	1.0133	1.0161
Netherlands	1.0116	1.0109	1.0003	1.0006	1.0113	1.0103	1.0107	1.0006	0.9927	0.9921	1.0182	1.0086	1.0050	1.0111	0.9839	0.9871	1.0215	1.0243
Poland	1.0005	1.0117	**0.9948**	**1.0231**	**1.0057**	**0.9889**	**1.1340**	**0.9825**	**1.1455**	**0.9937**	0.9900	0.9887	1.0709	1.0267	**0.9787**	**1.0178**	1.0943	1.0087
Portugal	0.9976	0.9868	1.0078	1.0020	0.9899	0.9848	0.9815	0.9801	0.9859	0.9876	0.9956	0.9924	1.0231	1.0200	0.9903	0.9917	1.0331	1.0285
Romania	**1.0122**	**0.9224**	0.9996	0.9273	**1.0126**	**0.9947**	**0.9939**	**1.0240**	0.9905	0.9937	1.0034	1.0305	−	1.0011	1.0000	0.9956	−	1.0055
Russia	−	0.9111	1.0000	0.9452	−	0.9639	−	1.0285	1.0000	1.0000	−	1.0285	−	1.0143	1.0000	0.9270	−	1.0942
Slovak Republic	1.1442	1.0806	1.1262	1.0806	1.0160	1.0000	1.0482	1.0141	1.0277	1.0088	1.0200	1.0053	1.0160	1.0500	**0.9666**	**1.0325**	1.0510	1.0169
Slovenia	0.9727	0.9729	1.0000	0.9999	0.9727	0.9730	0.9741	0.9727	0.9883	0.9864	0.9857	0.9861	**1.0425**	**0.9975**	1.0035	0.9919	1.0389	1.0056
Sweden	**0.9741**	**1.0010**	1.0000	1.0000	**0.9741**	**1.0010**	0.9814	0.9885	1.0000	1.0000	0.9814	0.9885	1.0078	1.0088	1.0000	1.0000	1.0078	1.0088
Turkey	−	0.9803	1.0000	0.9901	−	0.9901	−	0.9369	1.0000	0.9077	−	1.0322	−	0.9988	1.0000	0.9128	−	1.0942
United States	−	−	1.0000	1.0000	−	−	−	−	1.0000	1.0000	−	−	−	0.9977	1.0000	1.0000	−	0.9977

Source: Own elaboration.

3.3. Numerical results: Sensitivity and robustness of results to different number of undesirable outputs.

To study systematically how the number of undesirable outputs included in the model drives inconsistencies and infeasibilities in the technical change component of the Malmquist-Luenberger index (*MLTECH*), we perform a series of simulations using all possible combinations in the number of undesirable outputs across all time periods available in the sample. We solve the *CFG* and *APZ* models for each combination of undesirable outputs increasing the number of undesirable outputs, which totalizes 127 combinations. As for the time periods, we compute the model for each pair of years between 1995 and 2007. Table 4 shows the combinations for each number of undesirable outputs included in the model, the time periods available, the product of these two, and the total number of problems solved, which correspond to solving all combinations of air pollutants for the 39 countries across all time periods, bringing the total to 59,436 linear programs, LPs, solved.

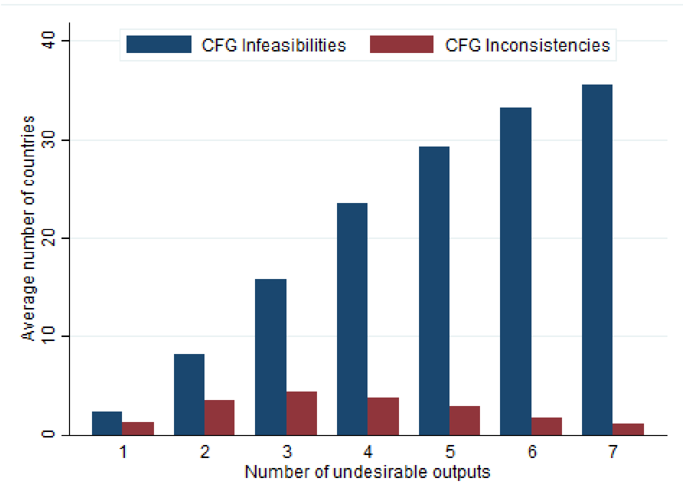
Table 4. Combinations and LPs solved

# Undesirable Outputs	**Combinations (a)**	**Periods (b)**	**Comb. x Periods (a)·(b)**	**LPs Solved**
1	7	12	84	3,276
2	21	12	252	9,828
3	35	12	420	16,380
4	35	12	420	16,380
5	21	12	252	9,828
6	7	12	84	3,276
7	1	12	12	468
TOTAL:	127	84	1,524	59,436

Source: Own elaboration

The comparison of results between both models are shown in Figure 3. Firstly, the average number of infeasibilities for the total of the 39 countries when computing the technical change component *MLTECH* with the *CFG* model is presented in the first (left) bar. Secondly, the average number of inconsistencies that emerge when comparing it to that of the *APZ* model is presented in the second (right) bar. Results are striking and challenge the conclusions obtained in previous studies, which normally include one or two undesirable outputs at most, as the average number of infeasibilities increases quite rapidly from about 2 infeasible solutions out of 39 with one undesirable output, to over 30 in the model with all available undesirable outputs. Indeed there is a monotonic causal relationship between these variables. As for the inconsistencies, their number also increases with the number of undesirable outputs, but finally falls beyond four undesirable outputs because the prevalence of infeasibilities is so high that the number of inconsistencies in the remaining solutions ought to decrease.

Figure 3. Number of *CFG* infeasibilities and inconsistencies in the technical change index.

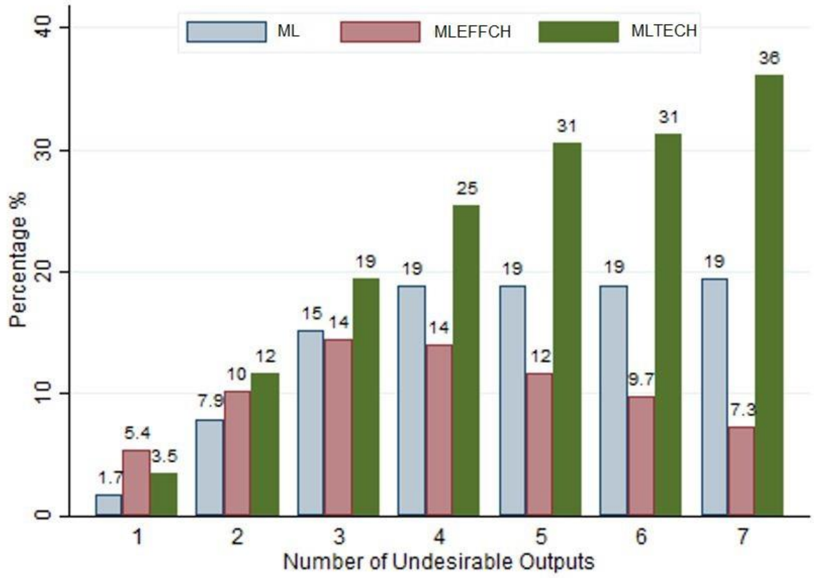


Precisely to gain better knowledge of the inconsistencies that emerge in the *ML* index and its components as the number of undesirable outputs increases, Figure 4 depicts the average percentage of inconsistencies over all feasible solutions, rather than over all possible combinations as in Figure 3; i.e., considering only solutions that do not return infeasibilities in the computation of the technical change component. For *MLTECH* results now confirm that when considering only feasible solutions, the number of inconsistencies is also monotonically increasing in the number of undesirable outputs.

Consequently, both the number of inconsistencies and infeasibilities increase with the number of undesirable outputs included in the model, and presenting non-negligible frequencies around 30% for five or more undesirable outputs. More worryingly, the combination of both infeasibilities and inconsistencies practically prevents any analysis when the number of undesirable outputs exceeds five, questioning the whole approach since those pollutants are normally available to the researcher, but are omitted in the existing empirical applications.

As for the technical efficiency change index, the frequency of inconsistencies in *MLEFFCH* exhibits a non-monotonic relationship, with an inverted-u-shape, suggesting that the differences in the own-period *DEA* production possibilities sets corresponding to the *CFG* and *APZ* models reduce with the number of facets−as opposed to the intertemporal cross-period frontiers involved in the calculation of the technical change index *MLTECH*. The combined effect of both types of inconsistencies on the productivity index *ML* is also presented in Figure 4. Its frequency ranges between both indices in which it decomposes, reflecting that they tend to counterbalance each other; e.g., as the previously referred Polish case in Table 3 for the 1996/1995 period.

Figure 4. Average percentage of inconsistencies over feasible solutions.



As the *APZ* greatly reduces the number of infeasibilities but does not rule out its existence, Table 4 compares their number when calculating the technical change component *MLTECH* for each number of undesirable outputs. While it is worth remarking that there are not infeasibilities when computing the technical efficiency change component *MLEFFCH*, those affecting *MLTECH* translate into the Malmquist-Luenberger index, so we only report results on the latter. Table 5 also displays the number of inconsistencies in the *ML* index as well as its decomposition into the *MLEFFCH* and *MLTECH* indices. Results show that the number of infeasibilities in the *APZ* approach is one order of magnitude smaller than in *CFG* approach (and two orders of magnitude smaller over four undesirable outputs). It is remarkable that the number of infeasibilities with four undesirable outputs is 9,856 out of 16,380 LPs solved. Consequently, the *APZ* model greatly reduces the number of infeasibilities making the analysis viable when the number of undesirable outputs exceeds two, while ensuring the correctness of the technical change measure *MLTECH*, and unmasking the large number of inconsistent results that are obtained with the standard approach.

Table 5. Number of infeasibilities and inconsistencies

Undesirable Ouputs	**Problems Solved**	***ML* Infeasibilities**			***CFG* Inconsistencies**		
		CFG	***APZ***	**Any**	***ML***	***MLEFFCH***	***MLTECH***
1	3,276	193	61	240	53	177	106
2	9,828	2,062	177	2,205	603	1,001	887
3	16,380	6,626	284	6,856	1,445	2,366	1,850
4	16,380	9,856	278	10,084	1,180	2,300	1,597
5	9,828	7,367	162	7,507	437	1,145	708
6	3,276	2,783	52	2,831	84	318	139
7	468	425	7	432	7	34	13
TOTAL:	59,436	29,312	1,021	30,155	3,809	7,341	5,300

Source: Own elaboration

Table 6 summarizes previous results showing the percentage of infeasibilities and inconsistencies for each number of undesirable outputs, as well as the percentage of inconsistencies over the feasible solutions. These results suggests that results obtained in environmental productivity studies using the standard approach−as those reviewed in the introduction−should be cautiously reassessed, and reinforces the need to shift to newer proposals such as the *APZ* approach that solves the inconsistency problems.

Table 6. Percentage of infeasibilities and inconsistencies

Undesirable Outputs	**% Infeasibilities *ML***			**% *CFG* Inconsistencies**			**% *CFG* Inconsistencies over feasibles**		
	CFG	***APZ***	**Any**	***ML***	***MLEFFCH***	***MLTECH***	***ML***	***MLEFFCH***	***MLTECH***
1	5.89	1.86	7.33	1.62	5.40	3.24	1.75	5.40	3.49
2	20.98	1.80	22.44	6.14	10.19	9.03	7.91	10.19	11.64
3	40.45	1.73	41.86	8.82	14.44	11.29	15.17	14.44	19.42
4	60.17	1.70	61.56	7.20	14.04	9.75	18.74	14.04	25.37
5	74.96	1.65	76.38	4.45	11.65	7.20	18.83	11.65	30.50
6	84.95	1.59	86.42	2.56	9.71	4.24	18.88	9.71	31.24
7	90.81	1.50	92.31	1.50	7.26	2.78	19.44	7.26	36.11

Source: Own elaboration

Finally, we determine whether the distributions of the productivity indices obtained with the standard and new approaches differ significantly for the whole sample by comparing their distributions. As solving each model under both approaches yields paired samples, we rely on the Wilcoxon rank-signed test and the *t*-test, thereby testing whether the medians and means of both distributions are equal, respectively. We also perform the Spearman test whose null hypothesis is the existence of correlation in the rankings of both distributions. As the distributional assumptions and degrees of freedom underlying these tests require a minimum size (e.g., normality and calculation of *p*-values), we perform the test for those models returning at least twenty feasible results, which rules out of the comparison all combinations with more than four undesirable outputs, whose high number of infeasibilities prevents reliable testing below that threshold.

Table 7 presents the results for all models and the number of combinations that have been tested. While for the Malmquist-Luenberger index itself *ML,* both models are equivalent, this is not the case for the sources of productivity change, both the efficiency change and the technical change components. Indeed, for *MLEFFCH* the maximum percentage of distributional differences is as high as 46.15% when considering the Wilcoxon test for four undesirable outputs, while it reaches 12,11% for *MLTECH* in the case of three undesirable outputs. Results are similar in the case of the *t*-test and Spearman test, with statistical disparities between distributions increasing with the number of undesirable outputs—i.e., different means for *t*-tests, while for the Spearman tests results show the percentage of the pairwise combinations whose rankings are statistically uncorrelated. These results confirm that the new characterization of the production technology preventing technical change inconsistencies, modifies the production possibility set significantly, as there are not only differences at the individual level, with countries exhibiting inconsistencies with the standard approach, but also at the sample level. More interestingly, the difference in the production possibility sets seems to affect most the efficiency change distributions. As we contend that the efficiency change values associated to the new model are reliable since the measurement of productivity and its decomposition does not suffer from the inconsistencies that plague the standard approach, these results question once again the interpretation of the sources contributing to productivity change in empirical applications. Indeed, not only individual results are into question, but also those corresponding to whole samples, whenever they are feasible.

Table 7. Comparing distributions: Wilcoxon, *t*-tests and Spearman

Nº Undesirable Outputs	Comb.	*ML*	*MLEFFCH*	*MLTECH*	*ML* (%)	*MLEFFCH* (%)	*MLTECH* (%)
Wilcoxon							
1	84	0	0	0	0.00	0.00	0.00
2	252	0	23	14	0.00	9.13	5.56
3	380	0	88	46	0.00	23.16	12.11
4	26	0	12	2	0.00	46.15	7.69
T-tests							
1	84	0	0	0	0.00	0.00	0.00
2	252	0	14	11	0.00	5.56	4.37
3	380	0	43	30	0.00	11.32	7.89
4	26	0	5	2	0.00	19.23	7.69
Spearman							
1	84	0	0	0	0.00	0.00	0.00
2	252	0	0	3	0.00	0.00	1.19
3	380	5	44	65	1.32	11.58	17.11
4	26	0	7	10	0.00	26.92	38.46

Notes: 5% confidence level.
Source: Own elaboration

4. Conclusions

The standard definition of the Malmquist-Luenberger index introduced by Chung et al. (1997) is prone to inconsistencies that severely challenge the validity of empirical results, and may result in misleading industrial, energy, or transportation policies aimed at reducing undesirable outputs production though investments in environmentally friendly technological change—e.g., it may induce inefficient overinvestment levels when the technical change index signals technical regress, while the opposite is actually happening.

While the inconsistency of the standard *ML* index has been known since 2013, practitioners are still using it as the pervasiveness of the inconsistencies is generally unknown, and regardless the number of infeasibilities that prevent obtaining results for many observations. Indeed, recent publications and ongoing contributions suggest that both authors and reviewers seem to be generally unaware of the problem, or simply disregard it under the impression that the presence of inconsistencies is very unlikely, affecting only a few observations.

This paper shows quite the opposite. Relying on the new approach proposed by Aparicio et al. (2013), who solve the inconsistency problem by changing the technology axioms the minimum necessary (thereby retaining the directional distance function definition, nature, and interpretability of the Malmquist-Luenberger index), we show how to render it operational using Data Envelopment Analysis techniques, and subsequently study the severity of these problems in a systematic way through computational analyses.

Using data for 39 countries over a thirteen years period—from 1995 to 2007—on gross value added, labor, capital, and 7 air pollutants, the research strategy is as follow. First a benchmark model including two relevant pollutants: CO_2 and NO_x, is solved under the standard and new approaches. General productivity trends associated to each approach are presented, as well as the relevant frequency of both inconsistencies and infeasibilities. We show how these inconsistencies in the technical change component *MLTECH* may result in opposite productivity trends, as they carry on to the *ML* indices themselves. Also, an unexpected result emerges. As the production possibility sets in both approaches differ due to the new axiom limiting undesirable outputs' production, technical efficiency indices *MLEFFCH* can also exhibit opposite trends depending on the approach.

Subsequently, an analysis of the pervasiveness and sensitivity of these results to different number of undesirable outputs is performed. Increasing the number of undesirable outputs in the model reveals the limits of the standard approach, with the number of infeasibilities and inconsistencies increasing rapidly. In the model including all 7 undesirable outputs, one third of the runs are infeasible, seriously hampering the representativeness and robustness of the results, while the number of inconsistencies over the feasible solutions also increases to a similar value.

We therefore make a precautionary call to researchers to avoid the use of the standard approach and adopt the new model—or devise one of their own—that does not suffer from these drawbacks. To this end, and since the linear programs associated to the new model are now available in a *DEA* package for the MATLAB environment—Álvarez et al. (2016), which can be readily accessed and adapted by practitioners, we believe the present contribution allows them to avoid the problems discussed here.

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References

Álvarez, I.C., Barbero, J. and Zofío, J. L. (2016), A Data Envelopment Analysis Toolbox for MATLAB, Working Papers in Economic Theory 2016/3, Universidad Autónoma de Madrid (Spain), Department of Economic Analysis (https://ideas.repec.org/p/uam/wpaper/201603.html)

Aparicio, J., Pastor, J.T. and Zofío, J.L. (2013). On the inconsistency of the Malmquist-Luenberger index. European Journal of Operational Research, 229(3): 738-742.

Boyd, G.A., Tolley, G. and Pang, J. (2002). Plant level productivity, efficiency and environmental performance of the container glass industry. Environmental and Resource Economics, 23, 29-43.

Briec, W. and Kerstens, K. (2009). Infeasibility and directional distance functions with application of determinateness of the Luenberger productivity indicator. Journal of Optimization Theory and Applications, 141, 55-73.

Chambers, R.G., Chung, Y. and Färe, R. (1996). Benefit and distance functions. Journal of Economic Theory, 70(2): 407-419.

Chambers, R. G., Chung, Y. and Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. Journal of Optimization Theory and Applications, 98: 351-364.

Chen, S. and Golley, J. (2014). 'Green' productivity growth in China's industrial economy. Energy Economics, 44, 89-98.

Chung, Y., Färe, R. and Grosskopf, S. (1997). Productivity and undesirable outputs: a directional distance function approach. Journal of Environmental Management, 51, 229-240.

Domazlicky, B.R. and Weber, W.L. (2004). Does environmental protection lead to slower productivity growth in the chemical industry? Environmental and Resource Economics, 28, 301-324.

Färe, R. and Grosskopf, S. (2003). Nonparametric productivity analysis with undesirable outputs: Comment. American Journal of Agricultural Economics, 85(4), 1070-74.

Färe, R. and Grosskopf, S. (2004). Modeling undesirable factors in efficiency evaluation: Comment. European Journal of Operational Research, 157(1), 242-245.

Färe, R., Grosskopf, S. and Pasurka Jr., C.A. (2001). Accounting for air pollution emissions in measures of state manufacturing productivity growth. Journal of Regional Science, 41(3), 381-409.

Färe, R., Grosskopf, S. and Pasurka, C.A. (2007). Environmental production functions and environmental directional distance functions. Energy, 32, 1055-1066.

Färe, R., Grosskopf, S., Lindgren, P. and Ross, P. (1994). Productivity developments in Swedish hospitals: A Malmquist output index approach. In A. Charnes, A., Cooper, W.W., Lewin, A.Y. and Seiford, L.M. (eds.), Data Envelopment Analysis: Theory, Methodology and Applications. Norwell: Kluwer Academic Publishers.

Hailu, A. (2003). Nonparametric productivity analysis with undesirable outputs: Reply. American Journal of Agricultural Economics, 85(4), 1075-77.

Hailu, A. and Veeman, T.S. (2000), Environmentally Sensitive Productivity Analysis of the Canadian Pulp and Paper Industry, 1959-1994: An Input Distance Function Approach. Journal of Environmental Economics and Management, 40, 251-274.

Hailu, A. and Veeman, T.S. (2001) Nonparametric productivity analysis with undesirable outputs: An application to the Canadian pulp and paper industry. American Journal of Agricultural Economics, 83, 605-616.

He, F., Zhang, Q., Lei, J., Fu, W. and Xu, X. (2013). Energy efficiency and productivity change of China's iron and steel industry: Accounting for undesirable outputs. Energy Policy, 54, 204-213.

Krautzberger, L. and Wetzel, H. (2012). Transport and CO2: Productivity growth and carbon dioxide emissions in the European commercial transport industry. Environmental and Resource Economics, 53, 435-454.

Kumar, S. (2006). Environmentally sensitive productivity growth: A global analysis using Malmquist-Luenberger index. Ecological Economics, 56, 280-293.

Kumar, S. and Khanna, M. (2009). Measurement of environmental efficiency and productivity: a cross country analysis. Environment and Development Economics, 14, 473-495.

Kumar, S. and Managi, S. (2010a). Environment and productivities in developed and developing countries: The case of carbon dioxide and sulfur dioxide. Journal of Environmental Management, 91, 1580-1592.

Kumar, S. and Managi, S. (2010b). Sulfur dioxide allowances: Trading and technological progress. Ecological Economics, 69, 623-631.

Li, K. and Lin, B. (2016a). Impact of energy conservation policies on the green productivity in China's manufacturing sector: Evidence from a three-stage DEA model. Applied Energy, 168, 351-363.

Li, K. and Lin, B. (2016b). Heterogeneity analysis of the effects of technology progress on carbon intensity in China. International Journal of Climate Change Strategies and Management, 8, 129-152.

Luenberger, D.G. (1992). New optimality principles for economic efficiency and equilibrium. Journal of Optimization Theory and Applications, 75(2), 221-264.

Luenberger, D.G. (1995). Microeconomic Theory. New York: McGraw-Hill.

Managi, S., Opaluch, J.J., Jin, D. and Grigalunas, T.A. (2005). Environmental regulations and technological change in the offshore oil and gas industry. Land Economics, 81, 303-319.

Oh, D.-H. (2010). A metafrontier approach for measuring an environmentally sensitive productivity growth index. Energy Economics, 32, 146-157.

Oh, D.-H. and Heshmati, A. (2010). A sequential Malmquist–Luenberger productivity index: Environmentally sensitive productivity growth considering the progressive nature of technology. Energy Economics, 32, 1345-1355.

Piot-Lepetit, I. and Le Moing, M. (2007). Productivity and environmental regulation: the effect of the nitrates directive in the French pig sector. Environmental and Resource Economics, 38, 433-446.

Seiford, L.M. and Zhu, J. (2005). A response to comments on modeling undesirable factors in efficiency evaluation. European Journal of Operational Research, 161(2), 579-581.

Seiford, L.M. and Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation. European Journal of Operational Research, 142(1), 16-20.

Seiford, L.M. and Zhu, J. (2005). A response to comments on modeling undesirable factors in efficiency evaluation. European Journal of Operational Research, 161(2), 579-581.

Shephard, R.W. (1953). Cost and Production Functions. Princeton: Princeton University Press.

Shestalova, V. (2003). Sequential Malmquist indices of productivity growth: An application to OECD industrial activities. Journal of Productivity Analysis, 19, 211-226.

Timmer, M.P., Dietzenbacher, E. Los, B., Stehrer, R. and De Vries, G.J. (2015). An illustrated user guide to the World Input–Output Database: The case of global automotive production. Review of International Economics, 23(3), 575-605.

Tulkens, H. and Vanden Eeckaut, P. (1995). Non-parametric efficiency, progress and regress measure for panel data: Methodological aspects. European Journal of Operational Research, 80, 474-499.

Wang, H., Zhou, P. and Zhou, D.Q. (2013). Scenario-based energy efficiency and productivity in China: A non-radial directional distance function analysis. Energy Economics, 40, 795-803.

Weber, W.L. and Domazlicky, B. (2001). Productivity growth and pollution in state manufacturing. Review of Economics and Statistics, 83(1), 195-199.

Yu, Ch., Shi, L., Wang, Y., Chang, Y. and Cheng, B. (2016). The eco-efficiency of pulp and paper industry in China: An assessment based on slacks-based measure and Malmquist-Luenberger index. Journal of Cleaner Production, 127, 511-521.

Yörük, B.K. and Zaim, O. (2005). Productivity growth in OECD countries: A comparison with Malmquist indices. Journal of Comparative Economics, 33(2), 401-420.

Zhang, S. (2015). Evaluating the method of total factor productivity growth and analysis of its influencing factors during the economic transitional period in China. Journal of Cleaner Production, 107, 438-444.

Zhang, Ch., Liu, H., Bressers, H.T.A. and Buchanan, K.S. (2011). Productivity growth and environmental regulations - accounting for undesirable outputs: Analysis of China's thirty provincial regions using the Malmquist–Luenberger index. Ecological Economics, 70, 2369-2379.

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