COLLISION OF TWO LIQUID WEDGES

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1. INTRODUCTION

Splashing following the collisions of liquids or granular materials is often observed in a variety of natural and engineering problems, such as plunging or breaking water waves, liquid droplets impacting on a free surface or a thin film of the same liquid, etc. Splashing leads to fluid fragmentation and the generation of drops and spray as well as fluid aeration. These phenomena involve the momentum and energy exchanges processes at the air-liquid interface, evaporation and air entrainment, cavity, bubbles, secondary drops, erosion, noise and sprays. A review of this of kind problem was presented by Yarin [1], which focused on drop impacts on thin liquid layers and dry surfaces. Kiger & Duncan [2] described the mechanism of air-entrainment, and Thoroddsen [3] centred the discussions on the initial stage of drop impact when liquid masses come into contact and coalesce. The current mathematical models may not be able to cover all the problems mentioned above. Nevertheless the formation of the splash jet and its effects on the main flow plays a major role in these phenomena. The present work aims to shed some light along this direction.

Much of the work on splashing jet has mostly been based on experimental observation. Although this has greatly improved our understanding, it is still far from giving a thorough insight into this phenomenon, especially at the initial stage of impacts when physical parameters change rapidly. .

Direct numerical simulations of splashing during droplets impacting on liquid layers were performed by Weiss [4] and Davidson [5] based on velocity potential theory with the boundary integral method and the surface tension was included. Their result showed that a splash jet might be formed, which moved close to the film, leading to the possibility of bubble entrapment. Incompressible Navier-Stokes equations with surface tension were solved numerically by Josserand & Zaleski [6]. The initial stage of a high speed droplet impact on a shallow water layer was investigated by Howison et al. [7] using the method of matched asymptotic expansions, with special attention given to the splash jet mechanics. A numerical investigation of splashing and wave breaking processes using the SPH method was performed by Landrini et al. [8]. Their numerical results reflected the experimental observations of breaking waves.

In this study, we investigate impact between two liquid wedges of the same density. It is assumed that the liquid is inviscid and incompressible, the flow is irrotational. When gravity and surface tension forces are neglected, the flow is self-similar. Such a formulation with fully nonlinear boundary conditions may be applicable to two sharp cornered wave crests colliding with each other at initial stage, and to other similar problems. The integral hodograph method [9, 10] is employed to derive analytical expressions for the complex-velocity potential, the complex-conjugate velocity, and the mapping function the physical plan and a parameter plan. The problem is reduced to a system of integro-differential equations in terms of the velocity magnitude and the velocity angle with the tangential direction of the liquid boundary. The results are presented as streamline patterns and the pressure distributions along the symmetry line of the wedge and near the root of the splash jet. It is found that the secondary impact could exist especially if the difference between the wedge angles is sufficiently large. The implications of such events are discussed.

2. THEORETICAL ANALYSIS AND NUMERICAL RSULTS

Two liquid wedges of half-angles α_+ and α_- move in the opposite directions with velocity V_O and V_D, respectively, and their apexes meet at point A right before the impact at time $t=0$, where the origin of the Cartesian coordinate system xy is chosen. Dynamically, the problem depends on only the relative velocity V_O+V_D. Here we consider the case in which we fix V_O and adjust V_D to ensure that A is the stagnation point. A sketch of the problem and the definitions of the geometric parameters are shown in figure 1*a*. The liquid wedges are assumed to be symmetric about the $y-$axis. For time $t>0$, a splash jet with the tip at point C appears, which is assumed to have evolved form A.

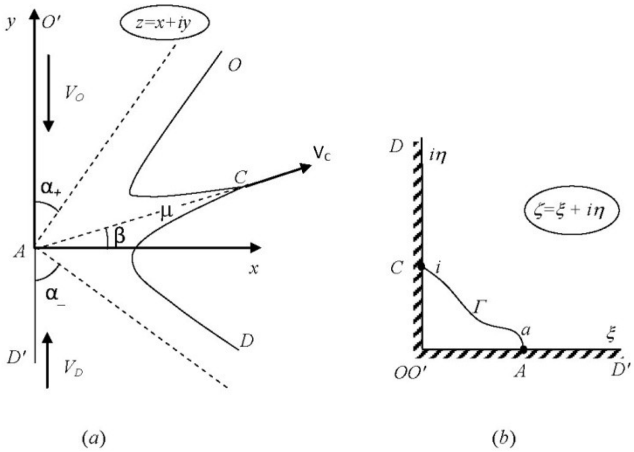

Figure 1. Sketch of the collision of two liquid wedges: (*a*) the stationary plane; (*b*) the parameter plane.

For a constant impact velocity of each liquid wedge, the time-dependent problem in the physical plane can be written in the stationary plane in terms of the self-similar variables $x = X/(V_O t)$, $y = Y/(V_O t)$. The complex velocity potential $W(Z,t) = \Phi(X,Y,t) + i\Psi(X,Y,t)$ for self-similar flows is written in the form

$$W(Z,t) = V_O^2 t w(z). \quad (1)$$

The problem is to determine the function $w(z)$ which conformally maps the stationary plane z onto the complex-velocity potential region w. We choose the first quadrant of the $\zeta-$plane as the parameter region to derive expressions for the nondimensional complex velocity, dw/dz, and for the derivative of the complex potential, $dw/d\zeta$, both as functions of the variable ζ. Once these functions are found, the velocity field and the mapping function $z = z(\zeta)$ can be determined [9].

Conformal mapping allows us to fix three arbitrary points in the parameter region, which are O, C, and D as shown in figure 1*b*. In this plane, the interval of the imaginary axis ($0<\eta<1, \xi=0$) corresponds to the free surface OC, and the interval ($1<\eta<\infty$, $\xi=0$) corresponds to the free surface CD. The positive real axis ($0<\xi<\infty$, $\eta=0$) corresponds to the symmetry line $O'D'$. The point $\zeta=a$ is the image of the stagnation point A in the stationary plane z. The parameter a is unknown and is to be determined as part of the solution.

The boundary-value problems for the complex-velocity function, dw/dz, and for the derivative of the complex potential, $dw/d\zeta$, can be formulated in the parameter plane. Then, applying the integral formulae determining an analytical function from its modulus and argument, and from its argument on the boundary of the first quadrant [9, 10], respectively, we obtain the following expression for the complex velocity and for the derivative of the complex potential:

$$\frac{dw}{dz} = v_0\left(\frac{\zeta-a}{\zeta+a}\right)\exp\left[-\frac{i}{\pi}\int_0^\infty \frac{d\ln v}{d\eta}\ln\left(\frac{i\eta-\zeta}{i\eta+\zeta}\right)d\eta + i\frac{\pi}{2}\right], \quad (2)$$

$$\frac{dw}{d\zeta} = K\zeta^{-1-2\alpha_+/\pi}\frac{\zeta^2-a^2}{(\zeta^2+1)^{1-\mu/\pi}}\exp\left[\frac{1}{\pi}\int_0^\infty \frac{d\lambda}{d\eta}\ln(\zeta^2+\eta^2)d\eta\right], \quad (3)$$

where K is a real scale factor, $v_0 = v(0) = 1$ is the nondimensionalized magnitude of the velocity at point O, $\lambda(\eta) = \theta(\eta) - \Delta_i$ is a continuous function in which $\theta(\eta)$ is the angle between the tangential direction of the free surface and the velocity vector on the free surface, and Δ_i are the jumps of the function $\theta(\eta)$ caused by corners of the flow boundary in the similarity plane.

Dividing Eq. (3) by Eq. (2), we can obtain the derivative of the mapping function

$$\frac{dz}{d\zeta} = \frac{dw}{d\zeta} / \frac{dw}{dz} \quad (4)$$

whose integration along the imaginary axis in the parameter region provides the free boundaries OC and CD in the $z-$plane.

The parameters a and K are determined from the following physical consideration that the tip of the splash jet, point C, is evolved from point A at which the origin of the coordinate system ($Z=0$) is chosen. Denoting the coordinate of point C, $Z=Z_C$, the magnitude of the velocity V_C and the angle β with the $x-$axis, we can write the following equation in the similarity plane

$$z_C = v_C e^{i\beta}, \quad (5)$$

where $v_C = V_C / V_O = v(\eta)_{\eta=1}$, $\beta = -\arg(dw/dz)_{\zeta=i}$. $z_C = Z_C / (V_O t)$ can be obtained by integration of the mapping function (4) from point A to point C along an arbitrary contour Γ in the parameter region shown in figure 1*b*. This equation makes it possible to determine the parameters K and a , while the functions $v(\eta)$ and $\lambda(\eta)$ are determined from dynamic and kinematic boundary conditions which for the self-similar flow take the form [9]:

$$\frac{d\lambda}{d\eta} = \frac{v + s\cos\theta}{s\sin\theta}\frac{d\ln v}{d\eta}, \tag{6}$$

$$\frac{1}{\tan\theta}\frac{d\ln v}{d\eta} = \frac{d}{d\eta}\left[\arg\left(\frac{dw}{dz}\right)\right], \tag{7}$$

where $s = S/(V_O t)$ and $s = s(\eta)$ is the spatial length coordinate along the free surface in the similarity plane obtaining by integration of $ds/d\eta = \left|dz/d\zeta\right|_{\zeta=i\eta}$ with using Eq.(4).

By choosing in the Bernoulli equation the location of the reference point at the stagnation point A , putting there $S=0$ and $W(Z_A,t)=0$, and taking advantage of the self-similarity of the flow, we can determine the pressure at any point of the liquid region

$$c_p^* = \frac{2(P-P_A)}{\rho V_O^2} = \mathrm{Re}\left(-2w + 2z\frac{dw}{dz}\right) - \left|\frac{dw}{dz}\right|^2. \tag{8}$$

The method of successive approximations is used for solving the system of integro-differential equations.

In figure 2, streamline patterns are shown for an upper liquid wedge of $\alpha_+ = 10^\circ$ and different angles α_- of the lower liquid wedge. Here, the pressure coefficient along the line of symmetry $x=0$ and the "zero" streamline passing through point A are also shown by dashed and dot-dashed lines, respectively. For the case shown in figure 2*a* the symmetry of the flow about the *x* axis, is clearly seen. The obtained value of the tip angle μ at point C shown in figure 1*a* for two identical wedges is $\mu = 9.47^\circ$ which is close to the value $\mu = 9.50^\circ$ obtained by Semenov & Wu [11] as the double contact angle for a liquid wedge impacting a solid wall.

The streamline patterns for an upper liquid wedge of $\alpha_+ = 10^\circ$ colliding with a lower wedge of $\alpha_- = 30^\circ, 70^\circ, 90^\circ$ respectively, are shown in figures 3*b* – 3*d*, respectively. For the case $\alpha_- = 30^\circ$, it can be seen that the splash jet is directed into the half-plane of the liquid wedge of smaller angle due to the larger momentum of the liquid wedge of larger angle.

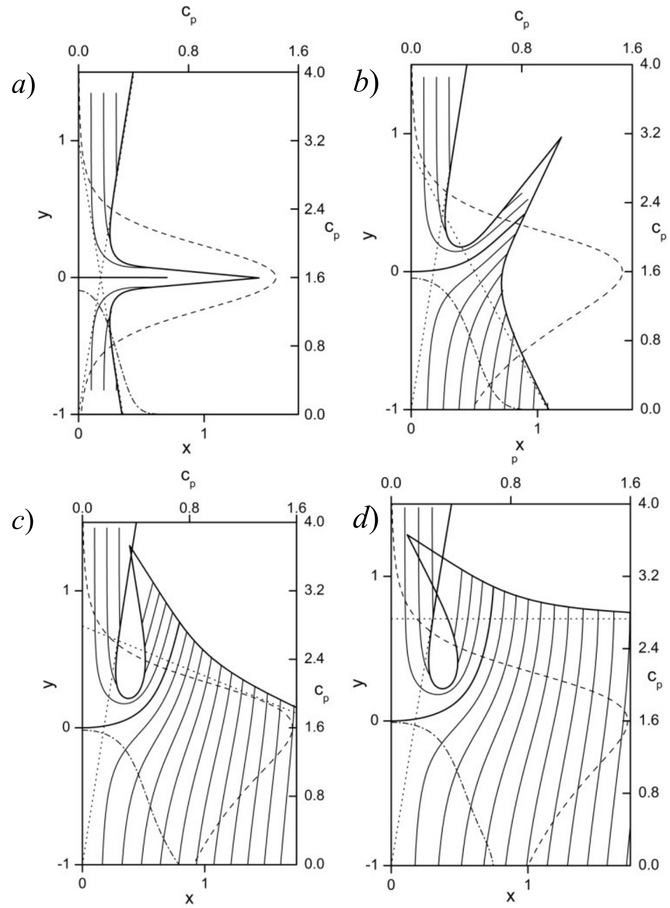

Figure 2. Streamline patterns (solid lines) and the pressure distribution along the $y-$axis (dashed line) and along the "zero" streamline starting at the origin (dot-dashed lines) for $\alpha_+ = 10^\circ$ and (*a*) $\alpha_- = 10^\circ$, (*b*) 30°, (*c*) 70° and (d) 90°. Here, $c_p = c_p^* - c_{pO}^*$, where c_{pO}^* is the pressure coefficient at point O or at any point on the free surface.

The dotted lines show the free surfaces of the undisturbed wedges at the same time point. It is seen that at some distance from the origin the dotted lines coincide with the free surfaces of the impacting liquid wedges. This distance corresponds to the region affected by the impact between the wedges. The $y-$coordinate of the tip of the undisturbed upper wedge is its incoming velocity which is obviously $v_O = -y = 1$. Similar the y - coordinate of the tip of the lower wedge is its incoming velocity. These figures show $v_D = y \le 1$ in these four cases.

The streamline pattern for $\alpha_- = 70°$ in figure 2*c* shows that the splash jet moves into the free surface of the upper liquid wedge forming a cavity. At the same time, the velocity direction of the liquid in the splash jet, which can be seen as the streamline slope, is almost parallel to the undisturbed free surface of the upper wedge. This secondary impact is not included in the present model and therefore the result does not fully reflect the real physics. Another issue is about the closed cavity. For the similarity solution to hold, the pressure of the trapped air inside the cavity has to be constant in the similarity coordinate system when the size of the cavity expands in the physical system. This is of course hardly the case in the real flow. From a mathematical point of view, the splash jet moves into the second sheet of the Riemann surface without interaction with the main flow.

The streamline pattern for $\alpha_- = 90°$ corresponding to a flat free surface is shown in figure 2*d*. The splash jet has moved further into the upper liquid wedge. In physical reality, the overlapping may lead to a bigger secondary impact between the splash jet and the wedge than in figure 2*c*. This could produce subsequent impacts and new splash jets. Such multi-impact processes with the formation of multiple cavities could produce a liquid/air mixture, liquid aeration, and the transformation of the splash jet into a spray, although it is speculative rather than conclusive at this stage. Similar situations occur for plunging breaking waves reviewed by Kiger & Duncan [2], in which the splash jet formed as a result of an impact between the wave crest and the free surface may be observed clearly in the case of oblique impacts or in the form of an air/liquid mixture in the case of nearly vertical impacts like a waterfall.

The numerical solutions are compared with asymptotic predictions based on the extended Wagner's theory in the small deadrise angle limit in which the half angle of both liquid wedges is close to $\pi/2$. In particular comparisons are made for the locations of the turning points (near the root of the splash jet); the splash jet thickness and length; and the pressure distribution on the line of symmetry of the wedge. The details will be presented in the workshop.

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