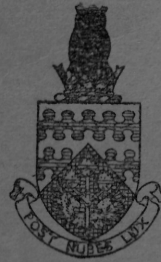


CoA/N-83

CoA Note No. 83

THE COLLEGE OF AERONAUTICS
CRANFIELD

ST. NO. R 18,301/B
U.D.C.
AUTH.



THERMAL STRESSES IN THIN CYLINDRICAL SHELLS
STIFFENED BY PLANE BULKHEADS FOR ARBITRARY
TEMPERATURE DISTRIBUTIONS

by

D. J. JOHNS

R 18,301/B



NOTE NO. 83

July, 1958

THE COLLEGE OF AERONAUTICS

C R A N F I E L D



Thermal Stresses in Thin Cylindrical Shells, Stiffened by
Plane Bulkheads, for Arbitrary Temperature Distributions

- by -

D. J. Johns, B.Sc., M.I.A.S.

SUMMARY

A study has been made of the thermal stresses resulting near the joint of a cylinder and internal bulkhead due to arbitrary temperature distributions in the configuration and to the consequent compatibility forces and moments at the joint. The method is general enough to permit the inclusion of joint thermal resistance but certain limitations are placed on the form of the axial temperature distribution in the cylinder.

An approximate method to determine the transient temperatures for completely general heating programmes is also proposed.

CONTENTS

1. Introduction
 2. Theory
 3. Thermal Stresses for Arbitrary Temperature Distributions
 4. Assumed Temperature Distributions
 5. Possible Modifications to the Theory
 6. Conclusions
 7. References
 8. Notation.
- Appendix
1. Prediction of Transient Temperature Distributions in Cylinder - Bulkhead Configurations

1. Introduction

The purpose of this note is to develop formulae for the thermal stress distributions in thin cylindrical shells and internal stiffening bulkheads which arise as a consequence of the restraint forces at the cylinder-bulkhead joint due to differential expansion of the two components. The physical nature of the problem can easily be visualised and one obvious application occurs in the kinetic heating of any typical missile or aircraft fuselage.

Because the thin shell attains a higher temperature quicker than the thicker, internal bulkhead, differential expansion will result. A system of self-equilibrating forces and moments must therefore be applied to the two components at the joint in order to make the resultant deformations of shell and bulkhead compatible.

Przemieniecki (Ref.1) has already considered this problem and presented some very useful results. The approach in Ref. 1 was to assume a rather restricted flight programme i.e. constant height, instantaneous acceleration and then to derive exactly expressions for the transient temperature distributions in the bulkhead diaphragm. The main assumptions made in the structural analysis were that bulkhead spacing was so large that restraint effects at any one joint did not influence conditions at other joints, and that the cylinder temperature was constant axially at any time.

The former assumption regarding bulkhead spacing should be valid for most practical configurations, since the discontinuity stresses introduced at the joints are extremely localised; and this assumption will also be made in the present note. The latter assumption cannot be easily justified since it is known that heavy internal members attached to the shell act as heat sinks thereby lowering the local shell temperature. To remedy this therefore the thermal stress problem will now be solved for arbitrary temperature distributions in both shell and bulkhead. Suitable approximations to realistic temperature distributions will then be made yielding convenient thermal stress formulae.

2. Theory

For the configuration shown in Fig. 1 it will be assumed that the temperatures in shell and bulkhead are constant through their thickness but vary axially and radially respectively.

The variation along the cylinder will be $T_c = F(x)$ and in the bulkhead $T_B = f(r)$.

2.1. Thermal expansion effects in the cylinder

Consider the cylinder separated from the bulkhead and consider a small ring element, dx , separated from the rest of the cylinder. The radial expansion of the ring due to a temperature change T_c is equal to $\alpha_c R T_c$.

It produces no stress in the ring.

Imagine now an external pressure p which is applied to the ring to restore it to its original diameter. The contraction due to p must equal the expansion due to T_c . Hence,

$$\frac{pR^2}{E_c t} = \alpha_c R T_c,$$

and therefore
$$p = \alpha_c E_c T_c \cdot \frac{t}{R} \quad (1)$$

Since $T_c = F(x)$, pressure p is also a function of x and produces a hoop stress in the ring. The remaining analysis follows that of Ref. 2 (p.423) exactly. Since there is no actual applied pressure on the complete cylinder a pressure of $-p$ must be applied to the now-undisturbed cylinder, which will cancel the hoop pressure due to p but introduce longitudinal bending stresses as the sole equivalent of the axial temperature distribution.

The stresses produced by $-p$ can be found if the deformation mode of the cylinder shell is known. The skin deflection, w , (positive inwards towards the axis) is given by the differential equation (Ref. 2 p.392 eq.230).

$$\frac{d^4 w}{dx^4} + 4 \beta^4 w = -\frac{p}{D} \quad (2)$$

where
$$\beta^4 = \frac{3(1 - \nu_c^2)}{R^2 t^2}, \quad D = \frac{E_c t^3}{12(1 - \nu_c^2)}$$

and
$$4 \beta^4 D = \frac{E_c t}{R^2}.$$

Substituting from eq.(1),

$$\frac{d^4 w}{dx^4} + 4 \beta^4 w = - \frac{E_c \alpha_c T_c t}{D R} \quad (3)$$

The general solution of this equation is,

$$w = \frac{1}{2} e^{\beta x} [K_1 \cos \beta x + K_2 \sin \beta x] + \frac{1}{2} e^{-\beta x} [K_3 \cos \beta x + K_4 \sin \beta x] - K_5 T_c \quad (4)$$

where $K_1 \dots K_4$ are unknown constants depending on the boundary conditions at the ends of the cylinders, and

$$K_5 = \frac{E_c \alpha_c t}{4 \beta^4 D R} = \alpha_c R.$$

It should be noted that in assuming $w = -K_5 T_c$ to be the particular integral of eq.(3) it is implicitly assumed that the function $T_c = F(x) = \sum_{m=0}^3 F_m x^m$. In other words if F is a polynomial in x it cannot be of higher order than a cubic.

For a cylinder with moderately high bulkhead spacing the boundary conditions at bulkheads distant from $x = 0$ will not influence the conditions existing at $x = 0$. In other words $K_1 = K_2 = 0$ and

$$w = \frac{1}{2} e^{-\beta x} [K_3 \cos \beta x + K_4 \sin \beta x] - K_5 T_c \quad (5)$$

To determine K_3, K_4 it is convenient to assume for the moment that the cylinder has a free edge at $x = 0$ and therefore

$$\frac{d^2 w}{dx^2} = \frac{d^3 w}{dx^3} = 0 \text{ at } x = 0.$$

From eq. (5)

$$\frac{dw}{dx} = \frac{1}{2} \beta e^{-\beta x} \left[(K_4 - K_3) \cos \beta x - (K_4 + K_3) \sin \beta x \right] - K_5 T'_0$$

$$\frac{d^2 w}{dx^2} = -\frac{1}{2} \beta^2 e^{-\beta x} \left[2K_4 \cos \beta x - 2K_3 \sin \beta x \right] - K_5 T''_0$$

$$\frac{d^3 w}{dx^3} = \frac{1}{2} \beta^3 e^{-\beta x} \left[2(K_4 + K_3) \cos \beta x + 2(K_4 - K_3) \sin \beta x \right] - K_5 T'''_0$$

When $x = 0$ $\frac{d^2 w}{dx^2} = -K_4 \beta^2 - K_5 T''_{co} = 0$

$$\frac{d^3 w}{dx^3} = (K_4 + K_3) \beta^3 - K_5 T'''_{co} = 0$$

Therefore,

$$\left. \begin{aligned} K_4 &= -\frac{K_5}{\beta^2} T''_{co} \\ K_3 &= K_5 \left(\frac{T'''_{co}}{\beta^3} + \frac{T''_{co}}{\beta^2} \right) \end{aligned} \right\} \quad (6)$$

Where T''_{co} is value of T'' at $x = 0$ etc.

Therefore eq.(5) becomes

$$w = \frac{1}{2} e^{-\beta x} K_5 \left[\cos \beta x \left(\frac{T''_{co}}{\beta^2} + \frac{T'''_{co}}{\beta^3} \right) - \sin \beta x \frac{T''_{co}}{\beta^2} \right] - K_5 T'_0 \quad (7)$$

and at $x = 0$,

$$\left. \begin{aligned} w_{co} &= K_5 \left[\frac{1}{2} \left(\frac{T''_{co}}{\beta^2} + \frac{T'''_{co}}{\beta^3} \right) - T'_{co} \right] \\ \left(\frac{dw}{dx} \right)_{co} &= -K_5 \beta \left[\frac{1}{2} \left(\frac{2T''_{co}}{\beta^2} + \frac{T'''_{co}}{\beta^3} \right) + \frac{T'_{co}}{\beta} \right] \end{aligned} \right\} \quad (8)$$

2.2. Thermal Expansion Effects in the Bulkhead

If the temperature distribution in the bulkhead is axi-symmetric and given by the function $T_B = f(r)$ it can be shown that the radial expansion of the bulkhead at the outer circumference is given by

$$w_B = -\frac{2\alpha_B}{R} \int_0^R T_B r \, dr \quad (9)$$

and because it has been assumed that the temperature is constant through the thickness of the bulkhead no curvature of the bulkhead will result, and hence no change in slope at the circumference.

2.3. Compatibility of Deformations

Having now considered the deformations in both shell(2.1) and bulkhead (2.2) due to thermal expansion it is necessary to determine the system of internal equilibrating forces and moments that will make the overall deformations of shell and bulkhead compatible.

To this end we assume that conditions in the cylinder on both sides of the bulkhead in Fig. 1 are identical. Hence if the bulkheads are spaced far enough apart, the results of Ref. 2 (p.393 eq.232) can be applied directly.

Therefore,

$$w = \frac{e^{-\beta x}}{2\beta^3 D} \left[\beta M_0 (\sin \beta x - \cos \beta x) - Q_0 \cos \beta x \right] \quad (10)$$

expresses the deflection mode for the right-hand portion of Fig. 1 due to applied bending moments, M_0 , and shearing forces Q_0 distributed uniformly along the circumferential edge $x = 0$. (For sign convention see Ref. 2 p.392).

If the radial shear force reaction on the bulkhead in Fig. 1 is P , then by symmetry $Q_0 = -P/2$ and the value of $\left(\frac{dw}{dx}\right)_0$ is given by,

$$\left(\frac{dw}{dx}\right)_{00} = \frac{1}{2\beta^2 D} \left[2\beta M_0 - P/2 \right] \quad (11)$$

To calculate the moment M_0 which appears in eq. (11) the boundary condition for the shell at the joint is used viz. that the net slope at $x = 0$ must be zero.

Therefore from eqs. (8) and (11),

$$\frac{1}{2\beta^2 D} \left[2\beta M_0 - \frac{P}{2} \right] = + \left[\frac{1}{2} \left(\frac{2 T''_{co}}{\beta^2} + \frac{T'''_{co}}{\beta^3} \right) + \frac{T'_{co}}{\beta} \right] K_5 \beta, \quad (12)$$

$$\therefore M_0 = \frac{P}{4\beta} + \beta^2 DK_5 \left[\frac{T'_{co}}{\beta} + \frac{T''_{co}}{\beta^2} + \frac{T'''_{co}}{2\beta^3} \right] \quad (13)$$

Substituting eq. (13) into eq.(10) gives

$$w = \frac{e^{-\beta x}}{2\beta^3 D} \left[(\sin\beta x - \cos\beta x) \left\{ \frac{P}{4} + \beta^3 DK_5 \left[\frac{T'_{co}}{\beta} + \frac{T''_{co}}{\beta^2} + \frac{T'''_{co}}{2\beta^3} \right] \right\} + \frac{P}{2} \cos\beta x \right] \quad (14)$$

which added to eq.(7) gives the overall deformation mode of the shell. Hence,

$$w = \frac{e^{-\beta x}}{2\beta^3 D} \left[\frac{P}{4} (\sin\beta x + \cos\beta x) + \beta^3 DK_5 \left\{ \sin\beta x \left(\frac{T'_{co}}{\beta} + \frac{T'''_{co}}{2\beta^3} \right) - \cos\beta x \left(\frac{T'_{co}}{\beta} - \frac{T'''_{co}}{2\beta^3} \right) \right\} \right] - K_5 T_c \quad (15)$$

Therefore at $x = 0$ the shell deformation is

$$w_{co} = \frac{1}{2\beta^3 D} \left[\frac{P}{4} - \beta^3 DK_5 \left\{ 2 T_{co} + \frac{T'_{co}}{\beta} - \frac{T'''_{co}}{2\beta^3} \right\} \right] \quad (16)$$

Similarly, for the bulkhead, the resultant radial displacement due to the applied force P and the thermal expansion is,

$$w_B = - \left\{ \frac{P(1 - \nu_B)R}{E_B d} + \frac{2\alpha_B}{R} \int_0^R T_B r dr \right\} \quad (17)$$

Equating eqs. (16) and (17) the value of P can be determined, which on rearrangement yield the result

$$P = \frac{2\alpha_c \left[T_{co} + \frac{T'_{co}}{2\beta} - \frac{T'''_{co}}{4\beta^3} \right] - \frac{4\alpha_B}{R^2} \int_0^R T_B r dr}{\left[\frac{\beta R}{E_c t} + \frac{2(1 - \nu_B)}{E_B d} \right]} \quad (18)$$

2.4. Thermal Stresses in Cylindrical Shells

From eqs. (14) and (15) it is now possible to determine the values of the various stress resultants and stress couples in the shell. Eq. (15) is the sum of the two equations, eq. (7) and eq. (14). The former represents the deformation mode of a free ended cylinder and is a function of axial temperature distribution only. The latter derives from the compatibility relationships between cylinder and bulkhead and is a function therefore of the external forces and moments applied to the end of the cylinder ($x = 0$).

Since thermal expansion effects in the free cylinder do not introduce hoop stresses (see Sect. 2.1), the eq. (7) is not considered in calculating the circumferential stress resultant $N\phi$. Hence eq. (14) gives $N\phi$ directly. To determine the remaining stress resultants and stress couples eq. (15) is used. Hence,

$$N_x = 0 \quad (19)$$

$$N\phi = -\frac{e^{-\beta x}}{2} \beta R \left[P (\sin \beta x + \cos \beta x) + K_6 \left(\frac{T'_{co}}{\beta} + \frac{T''_{co}}{\beta^2} + \frac{T'''_{co}}{2\beta^3} \right) (\sin \beta x - \cos \beta x) \right] \quad (20)$$

$$Q_x = -\frac{e^{-\beta x}}{2} \left[P \cos \beta x + K_6 \left\{ \sin \beta x \left(\frac{T'_{co}}{\beta} \right) + \cos \beta x \left(\frac{T''_{co}}{2\beta^3} \right) \right\} \right] + \frac{K_6}{4} \left(\frac{T'''_{co}}{\beta^3} \right) \quad (21)$$

$$M_x = -\frac{e^{-\beta x}}{4\beta} \left[P (\sin \beta x - \cos \beta x) - K_6 \left\{ \sin \beta x \left(\frac{T'_{co}}{\beta} - \frac{T''_{co}}{2\beta^3} \right) + \cos \beta x \left(\frac{T'_{co}}{\beta} + \frac{T''_{co}}{2\beta^3} \right) \right\} \right] + \frac{K_6}{4\beta} \left(\frac{T'''_{co}}{\beta^2} \right) \quad (22)$$

$$M\phi = \nu_c M_x \quad (23)$$

where $K_6 = \frac{E_c \alpha_c t}{R \beta}$ and P is given by eq. (18)

2.5. Thermal Stresses in Circular Bulkhead

The stress distribution in a circular bulkhead of uniform thickness subjected to arbitrary axi-symmetrical distribution of temperature is given by (Ref. 3, p.366)

$$\sigma_{r_T} = \alpha_B E_B \left\{ \frac{1}{R^2} \int_0^R T_B r^2 dr - \frac{1}{r^2} \int_0^r T_B r^2 dr \right\} \quad (24)$$

$$\sigma_{\theta_T} = \alpha_B E_B \left\{ -T_B + \frac{1}{R^2} \int_0^R T_B r^2 dr + \frac{1}{r^2} \int_0^r T_B r^2 dr \right\} \quad (25)$$

where σ_{r_T} and σ_{θ_T} are the radial and circumferential stresses respectively

due to temperature alone. In addition to these stresses there is a uniform stress field throughout the bulkhead due to the transverse shear in the shell. This uniform stress is given by $\frac{P}{d}$ and is added to both equations (24) and

(25) to give total values of σ_r and σ_θ .

3. Thermal Stresses for Arbitrary Temperature Distributions

In section 2 formulae have been developed giving the thermal stress distributions in a bulkhead and cylinder combination in terms of arbitrary temperature distributions radially in the bulkhead and axially in the cylinder.

It is possible therefore to use the formulae with either experimental or theoretical temperature distributions. In either case the axial temperature distribution in the cylinder should be expressed as a polynomial in x up to the third power, even if the true variation is of a higher order (see Sect. 2.1). To minimise any errors in approximating the exact, arbitrary distribution to a polynomial of the form

$$T_c = F(x) = F_0 + F_1 x + F_2 x^2 + F_3 x^3, \quad (26)$$

the choice of constants F_0, \dots, F_3 should be such as to satisfy conditions at the joint with the true temperature distribution.

$$\begin{aligned} \text{i.e.} \quad F_0 &= T_{co} \\ F_1 &= T'_{co} \\ 2F_2 &= T''_{co} \\ 6F_3 &= T'''_{co} \end{aligned} \quad (27)$$

In this way no errors will be introduced into eqs. (13), (16) and (18) and in Sect. 2.4 the only errors introduced into the various stress resultants and stress couples will be those in the function T_c away from the joint. Since the greatest restraint thermal stresses are expected at the joint with a rapid diminution as x increases, the errors should therefore be small.

4. Assumed Temperature Distribution

In a recent paper (Ref.4) Biot showed that by assuming arbitrary, but realistic, temperature distribution modes in skin-web combinations, fairly simple analyses enabled the complete temperature time histories of such combinations to be obtained. He assumed that the temperature distribution in the web should be parabolic and the skin temperature would be constant except in the vicinity of the internal member which caused a local parabolic variation.

Without any justification such distributions will be assumed here for the analogous case of a cylinder with a bulkhead.

There are two phases in the heating of the configuration shown in Fig. 1. Initially the heat penetrates radially into the bulkhead and the temperature at the centre has not yet begun to rise. The penetration depth at any time is denoted by q and the corresponding assumed temperature distributions are shown in Fig. 2. It will be noticed that a temperature drop is considered over the cylinder bulkhead joint.

The corresponding temperature distributions during the second phase of heating, i.e. after the temperature at the centre has begun to rise, are shown in Fig. 3. This phase begins after a time t_1 known as the "transit" time.

Biot assumed that for the first heating phase $l = q$ and he did not consider the second heating phase in detail. In this analysis the parameters l, q will retain their separate identities during both heating phases.

The temperature distributions shown in Figs. 2 and 3 can be expressed thus,

$$\left. \begin{aligned} T_C &= T_0 - (T_0 - T_1) \left(1 - \frac{x}{l}\right)^2 & \text{for } x < l \\ &= T_0 & \text{for } x > l \end{aligned} \right\} \quad (28)$$

$$\left. \begin{aligned} T_B &= T_2 \left[\frac{R-q-r}{q} \right]^2 & \text{for } r > R - q \\ &= 0 & \text{for } r < R - q \end{aligned} \right\} \text{for } t_0 < t_1 \quad \left. \begin{aligned} & & \\ & & \\ & = T_3 + (T_2 - T_3) \left(\frac{r}{R}\right)^2 & \text{for } t_0 > t_1 \end{aligned} \right\} \quad (29)$$

The boundary condition at the bulkhead cylinder joint, where a drop in temperature of $T_1 - T_2$ is assumed, can be written,

$$k_B \frac{\partial T_B}{\partial r} = H_j(T_1 - T_2), \quad r = R. \quad (30)$$

where H_j is the thermal conductance of the joint and k_B is the thermal conductivity of the bulkhead material. Using eq. (29) the boundary condition gives,

$$\left. \begin{aligned} T_2 &= T_1 / \left(1 + \frac{2k_b}{H_j q} \right) \quad \text{for } t < t_1 \\ &= T_1 / \left(1 + \frac{2k_b}{H_j R} \right) + T_3 / \left(1 + \frac{RH_j}{2k_b} \right) \quad \text{for } t > t_1. \end{aligned} \right\} \quad (31)$$

Equation (31) can be generalised to give

$$T_2 = \frac{T_1}{\left(1 + \frac{\theta}{n} \right)} + \frac{T_3 \theta}{(1 + \theta)}, \quad (32)$$

where $n = \frac{q}{R}$ and θ is the non-dimensional parameter known as the relative thermal resistance $\left(= \frac{2k_b}{H_j R} \right)$, and in the first phase $n < 1$, $T_3 = 0$; whilst in the second phase, $n = 1$, $T_3 \neq 0$.

It is seen that two extreme values of the parameter θ can be considered, corresponding to,

- (a) Zero joint thermal resistance or $\theta = 0$
- (b) Infinite joint thermal resistance or $\theta = \infty$.

Using the foregoing formulae it can be shown that the values of the parameters necessary to determine P (eq.(18)) and hence solve the thermal stress problem are :-

$$\int_0^R T_b r \, dr = \left(\frac{4n - n^2}{12} \right) R (T_2 + T_3) \quad (33)$$

$$\left. \begin{aligned} T_{co} &= T_1 \\ T_{co}' &= \frac{2}{1} (T_o - T_1) \\ T_{co}'' &= \frac{-2}{1^2} (T_o - T_1) \\ T_{co}''' &= 0 \end{aligned} \right\} \quad (34)$$

Hence the thermal stress problem can be solved if the values of the temperatures T_0, \dots, T_3 , and the parameter q are known as functions of time. Biot's analysis has been developed for this purpose for the analogous case of the skin-web combination (Ref. 5) and will be similarly developed here in Appendix 1.

5. Possible Modifications to The Theory

5.1. Assumption of Simply Supported Edge Conditions on the Shell at the Joint

In the analysis of section 2.3 it was assumed that clamped edge conditions existed at the cylinder - bulkhead joint. Such conditions could result from the method of attachment and/or the fact that the cylinder is continuous over the joint. If however the cylinder is discontinuous at the joint and depending on the method of attachment, simply-supported edge conditions may pertain.

The effect of this modification on the analysis of Sect. 2.3 is to neglect the compatibility of slopes at the joint and to remove the distributed moment M_0 . Hence eq.(1) becomes

$$w = - \frac{e^{-\beta x}}{2\beta^3 D} Q_0 \cos \beta x. \quad (35)$$

Using the fact that $Q_0 = -\frac{P}{2}$ the addition of eqs. (8) and (35) gives the following equation for the resultant shell deformation, at $x = 0$

$$w_{00} = \frac{1}{2\beta^3 D} \left[\frac{P}{2} - \beta^3 D K_5 \left\{ 2 T_{c0} = \frac{T''_{c0}}{\beta^2} - \frac{T'''_{c0}}{\beta^3} \right\} \right] \quad (36)$$

On equating the shell deformations and bulkhead deformations at $x = 0$ the following value of P is obtained.

$$P = \frac{2\alpha_c \left[T_{c0} - \frac{T''_{c0}}{2\beta^2} - \frac{T'''_{c0}}{2\beta^3} \right] - \frac{4\alpha_B}{R^2} \int_0^R T_B r dr}{\left[\frac{2\beta^3 R}{E_c t} + \frac{2(1-\nu_B)}{E_B d} \right]} \quad (37)$$

The values of the stress-couples, stress resultants in the shell and stresses in the bulkhead follow from equations (37) and (35) in the same manner as in Sects. 2.4 and 2.5.

Comparing eqs. (37) and (18) it can be seen that the former equation gives the lower value of P and hence the lower thermal stresses.

5.2. Use of Circular Frames (See Ref. 1)

If circular frames are used as internal stiffening members instead of circular bulkheads the solution is obtained by comparing radial displacements of the joint between the frame and the shell. Assuming A_F to be the cross-sectional area of the frame with a mean frame temperature of T_F and a mean radius of R_F then the frame radial displacement at the joint is

$$w = - \left[R_F \alpha_F T_F + P R_F^2 / A_F E_F \right] . \quad (38)$$

Equating eqs. (38) and (16) yields the result

$$P = \frac{2\alpha_c \left[T_{co} + \frac{T'_{co}}{2\beta} - \frac{T''_{co}}{4\beta^3} \right] - \frac{2\alpha_F T_F R_F}{R}}{\left[\frac{\beta R}{E_c t} + \frac{2 R_F^2}{R A_F E_F} \right]} \quad (39)$$

Since, in general, $\frac{2 R_F^2}{R A_F E_F} \ll \frac{2(1 - \nu_B)}{E_B d}$ (cf. eq.(18))

and, $\alpha_c T_{co} \ll \frac{\alpha_F T_F R_F}{R}$, the value of P in eq.(39)

should be small and severe thermal stresses should not result from frame stiffening.

5.3. Use of a Cylindrical Shell Stiffened by Longitudinal Stiffeners (See Ref.1)

The preceding analyses which have been developed for thin isotropic cylindrical shells is also applicable to shells with longitudinal stiffeners. The main effect is to neglect flexural rigidity in the circumferential direction and to let $D = E I/b$ where I is the moment of inertia of the stringer-skin combination and b is the stringer pitch. With this modification the previous analyses are identical except that $M\phi = 0$, and β is replaced by γ , where $\gamma^4 = \frac{b t}{c}$.

$$\frac{c}{4 I R^2}$$

6. Conclusions

The general problem of the thermal stresses resulting in a cylinder bulkhead configuration due to arbitrary temperature distributions has been considered and suitable formulae derived. The method is restricted in that the axial temperature variation in the shell must be a polynomial of positive order n less than or equal to three; but it is general enough to permit the inclusion of joint resistance. An approach has been suggested for minimising errors incurred in approximating any arbitrary temperature distribution to satisfy the above restriction.

The method ideally awaits the development of a concise theory to predict the transient temperature distributions in both cylinder and bulkhead. One approximate method based on the assumption of parabolic temperature distributions is presented in the Appendix giving formulae sufficient to determine the complete transient temperature distributions. The accuracy of this method has not been assessed by comparison with either experimental or more precise theoretical results. It is considered that the results of the Appendix can be safely applied in project design studies at least.

7. References

1. Przemieniecki, J.S. Transient Temperature Distributions
and Thermal Stresses in Fuselage Shells
with Bulkheads or Frames.
Jour. Royal Aero. Soc. December 1956.
2. Timoshenko, S. Theory of Plates and Shells.
McGraw-Hill Book Co., First Edition (1940).
3. Timoshenko, S. Theory of Elasticity.
McGraw-Hill Book Co. First Edition (1934).
4. Biot, M.A. New Methods in Heat Flow Analysis with
Application to Flight Structures.
Jour. Inst. Aero. Sci. Vol.24.
December 1957. pp.857-873.
5. Johns, D.J. Approximate Formulae for Thermal Stress
Analyses.
Jour.Inst.Aero.Sci. Vol.25 Aug. 1958.
pp. 524-525.
6. Carslaw, H.S.,
Jaeger, J.C. Conduction of Heat in Solids.
Oxford University Press 1947, p.174.
7. Ingersoll, A.
Zobel, O. Heat Conduction.
McGraw-Hill Book Co. 1948, p.176.

8. Notation

| | |
|---------------|---|
| A_F | Frame cross-sectional area |
| b | Stringer pitch in stiffened shell |
| B | Heat Parameter = $2R^2 C_B / k_B$ |
| c | Specific heat/unit volume |
| d | Bulkhead thickness |
| D | Shell flexural rigidity |
| E | Youngs modulus |
| $f(r)$ | Radial temperature distribution function in bulkhead |
| $F(x), F_m$ | Axial temperature distribution functions in shell |
| G_1 | Heat flow per unit length into the shell, from the boundary layer, at the joint (eq. A.5) |
| h | Convective heat transfer coefficient |
| H_j | joint thermal conductance |
| I | Moment of inertia of the skin-stringer combination |
| k | thermal conductivity |
| K_n | Parameters defined in Text (eq.(4); eqs.(20)-(22)) |
| l | Temperature distribution parameter |
| m | Power of x in polynomial expression for $F(x)$ |
| M_x, M_ϕ | Bending moments per unit length about ϕ and x axes respectively |
| n | (= q/R) |
| N_x, N_ϕ | Stress - resultants in x and ϕ directions respectively |
| p | hypothetical pressure (eq. (1)) |
| P | Radial reaction between the shell and bulkhead or frame |
| q | Penetration depth of temperature rise in bulkhead |
| Q_x | Transverse shear force in shell per unit length |

| | |
|---------------------|---|
| r | Radial co-ordinate in bulkhead |
| R | Radius of cylinder |
| R_F | Mean radius of Frame |
| t | Shell thickness; t_0 =time; t_1 =transit time |
| T | Temperature rise above the initial value |
| T_s | Adiabatic wall temperature |
| w | Radial displacement |
| x | Co-ordinate measured along shell axis |
| α | Coefficient of expansion |
| β | Shell parameter = $\left[\frac{3(1-\nu_c^2)}{R^2 t^2} \right]^{\frac{1}{4}}$ |
| β_s | Stefan-Boltzman Radiation constant |
| ν | Poissons ratio |
| θ | Relative thermal resistance ($= \frac{2k_B}{H_j R}$) |
| ϕ | Angle denoting position on shell penphery |
| ψ | Heat sink parameter due to bulkhead $\left(= 1 + \frac{dc_B(T_2+T_3)(4-n)}{8 t c_c T_1} \right)$ |
| $\sigma_{r,\theta}$ | Radial and tangential stresses on the bulkhead respectively |
| ϵ | Emmissivity |
| Suffioes | |
| c | Cylinder |
| B | Bulkhead |
| F | Frame |
| co | at $x = 0$ in cylinder |
| 0,1,2,3 | refer to Temperatures in configuration (Figs. 2,3) |
| \dot{T} | refers to differentiation with respect to time |
| T' | refers to differentiation with respect to x |

APPENDIX 1.

Prediction of Transient Temperature Distributions in Cylinder-Bulkhead Configurations

In Ref. 1 Przemieniecki analysed exactly the transient temperature distributions in a cylinder-bulkhead configuration for the case of convective heating of the cylinder for constant values of the heat transfer coefficient, stagnation temperature and axial temperature distribution in the cylinder. Since these assumptions do not allow of a completely general study the analysis of Ref. 1 will not be adopted here, nor will it be extended to make it more general since the results of such a study would be tedious to obtain and inconvenient to incorporate into the main analysis of this note.

Recourse will therefore be had to an approximate method of heat flow analysis based on an extension of Ref. 4. The basic method of Ref. 4 will not be discussed here and the main extension introduced will be in allowing the shell temperature T_0 to be a completely arbitrary function of time. This extension has already been applied to the analogous skin-web problem of an aircraft wing in Ref. 5.

The results of the analysis were as follows.

A.1. First Heating Phase in Bulkhead

The parameter q can be expressed as a function of the temperature T_2 as follows (see Fig. 2).

$$n \dot{m} T_2 (13 - 3n) + \frac{5n^2 \dot{T}_2 (6 - n)}{4} = 21 \frac{T_2}{B} (7 + n) \quad \text{A.1.}$$

where $n = \frac{q}{R}$, $B = \frac{2 R^2 C_B}{k_B}$, and where C_B , k_B are specific heat and thermal conductivity bulkhead material respectively

The above equation can be solved for q using the boundary condition that $q = n = 0$ at $t_0 = 0$.

Hence it is possible to find the value of $t_0 = t_1$, which makes $n = 1$.

Equation A.1 has not in fact been solved and it is proposed to use the result quoted by Timoshenko, (Ref. 3 p.370) for the problem with $T_2 = \text{const.}$, by exact analysis which gave

$$t_1 = .025 \frac{R^2 C_B}{k_b} \quad \text{or} \quad t_1 = .0125 B. \quad \text{A.2.}$$

See also Refs. 6, 7.

Biot showed in Ref. 4 that $q \propto t_0^{\frac{1}{2}}$ therefore if it is assumed that $q = 6.34 \sqrt{\frac{k_B t_0}{c_B}}$, A.3.

Ingersoll's⁽⁷⁾ exact result is obtained for $t_0 = t_1$ when $q = R$.

Although this does not follow from eq. A.1 it is considered to be accurate enough for most practical problems.

A.2. Second Heating Phase in Bulkhead

In this phase, if the generalised co-ordinate is taken as T_3 , the temperature at the bulkhead centre, the following differential equation is obtained, using the temperature distribution of Fig. 3.

$$B \left(T_3 + \frac{3 T_2}{2} \right) = 42 T_2 - 12 T_3 \quad A.4$$

A.3. First Heating Phase in The Cylinder

In this analysis, the local heat-sink effect of the bulkhead is considered and the temperature distributions of Fig. 2 are assumed except that l is assumed equal to q , the penetration depth into the bulkhead. The subsequent analysis yields the result

$$G_1 = t c_c T_1 + \frac{1}{2} d c_B T_2 \left(1 - \frac{n}{4} \right) \quad A.5$$

where G_1 is the heat per unit length to have flowed through the boundary layer into the skin at the joint station.

$$\begin{aligned} \text{If } G_1 &= t c_c T_1 \psi, \\ \psi &= 1 + \frac{d c_B T_2}{8 t c_c T_1} (4-n) \end{aligned} \quad A.6$$

A.4. Second Heating Phase in the Cylinder

The result is obtained.

$$\begin{aligned} G_1 &= t c_c T_1 + \frac{3}{8} d c_B (T_2 + T_3) \\ \psi &= 1 + \frac{3 d c_B (T_2 + T_3)}{8 t c_c T_1} \end{aligned} \quad A.7$$

A.5. Shell Temperatures

For the shell away from the joint ($x > 1$) the temperature T_o obeys the differential equation

$$t c_c \dot{T}_o = h (T_s - T_o) - \epsilon \beta_s T_o^4 \quad \text{A.8.}$$

where ϵ is emissivity

β_s is Stefan-Boltzman Constant

h is convective heat transfer coefficient

T_s is Stagnation temperature.

The corresponding value of H_o is $t c_c T_o$.

Therefore, an equation of the form of eq. A.8 will define the temperature variation of T_1 also, provided the parameter $t c_c$ is factored by ψ , which may be generalised as

$$\psi = 1 + \frac{d c_B (T_2 + T_3)(4 - n)}{8 t c_c T_1} \quad \text{A.9.}$$

A.6. Procedure

The procedure necessary to determine the transient temperature distribution in the shell and bulkhead is as follows :

- (a) Using eq. A.8 determine T_o as a function of time
- (b) Using eq. A.3 or A.1 determine q as a function of time
- (c) For the two distinct phases of heating the temperatures T_1, T_2, T_3 are determined from the following three equations solved simultaneously.

$$t c_c \psi \dot{T}_1 = h (T_s - T_1) - \epsilon \beta_s T_1^4 \quad \text{A.10}$$

$$B \left(\dot{T}_3 + \frac{3\dot{T}_2}{2} \right) = 4.2 T_2 - 1.2 T_3 \quad \text{A.4}$$

$$\tau_2 = \frac{T_1}{(1 + \frac{\theta}{n})} + \frac{T_3 \theta}{(1 + \theta)} \quad (32)$$

where ψ is given by eq. A.9 and depends on the value of n .

For a completely arbitrary flight programme it should be easier to solve the equations above (possibly by numerical integration) than to solve the exact equations of heat flow in the shell and bulkhead.

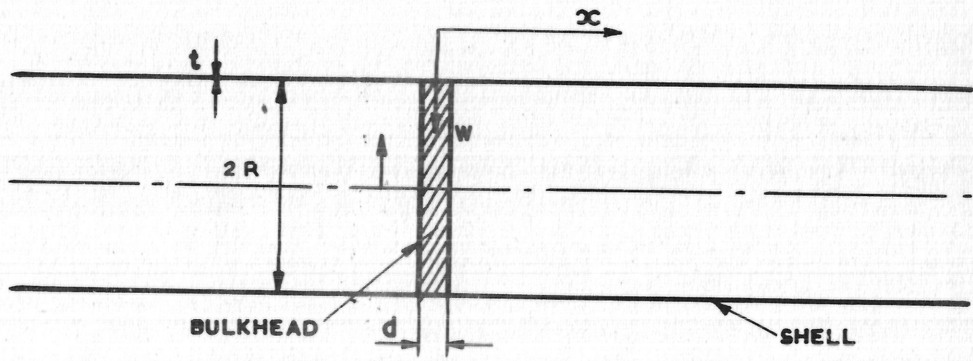


FIG 1. SHELL - BULKHEAD CONFIGURATION

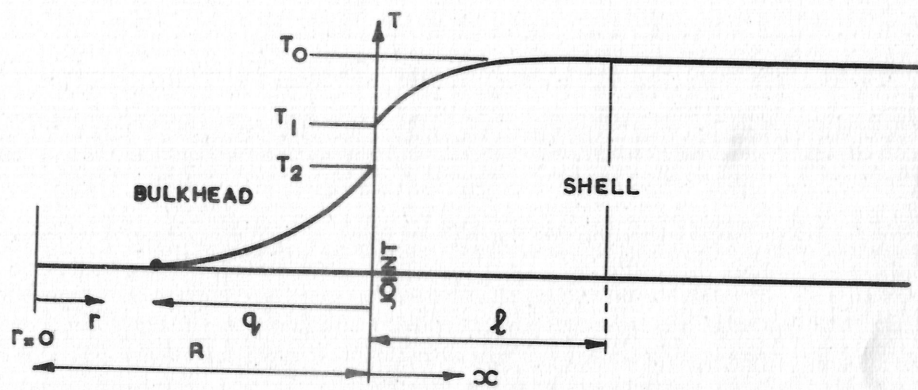


FIG. 2. TEMPERATURE DISTRIBUTIONS DURING FIRST HEATING PHASE.

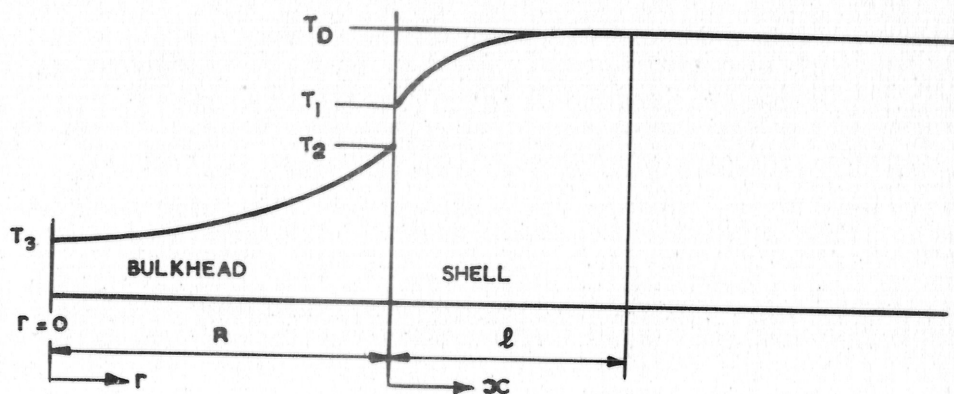


FIG. 3. TEMPERATURE DISTRIBUTIONS DURING SECOND HEATING PHASE.