Ellipse-based principal component analysis for self-intersecting curve reconstruction from noisy point sets

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Abstract

Surface reconstruction from cross cuts usually requires curve reconstruction from planar noisy point samples. The output curves must form a possibly disconnected 1-manifold for the surface reconstruction to proceed. This article describes an implemented algorithm for the reconstruction of planar curves (1-manifolds) out of noisy point samples of a self-intersecting or nearly self-intersecting planar curve C. $C:[a,b]\subset R\rightarrow R^2$ is self-intersecting if $C(u)=C(v)$, $u\neq v$, $u,v\in(a,b)$ ($C(u)$ is the self-intersection point). We consider only *transversal* self-intersections, i.e. those for which the tangents of the intersecting branches at the intersection point do not coincide ($C'(u)\neq C'(v)$). In the presence of noise, curves which self-intersect cannot be distinguished from curves which nearly self-intersect. Existing algorithms for curve reconstruction out of either noisy point samples or pixel data, do not produce a (possibly disconnected) Piecewise Linear 1-manifold approaching the whole point sample. The algorithm implemented in this work uses Principal Component Analysis (PCA) with elliptic support regions near the self-intersections. The

algorithm was successful in recovering contours out of noisy slice samples of a surface, for the *Hand*, *Pelvis* and *Skull* data sets. As a test for the correctness of the obtained curves in the slice levels, they were input into an algorithm of surface reconstruction, leading to a reconstructed surface which reproduces the topological and geometrical properties of the original object. The algorithm robustly reacts not only to statistical non-correlation at the self-intersections (non-manifold neighborhoods) but also to occasional high noise at the non-self-intersecting (1-manifold) neighborhoods.

Keywords

Self-intersecting curve reconstruction Elliptic support region Principal component analysis Noisy samples

Glossary

PL: — Piecewise Linear.
C: — Planar open or closed, possibly self-intersecting or nearly self-intersecting, curve.
$S=\{p_0,p_1,\ldots,p_n\}$: — An unorganized noisy point sample of *C*.
ε: — Stochastic component of the point sample.
$B(p,r)$: — The disk of radius *r* centered at point *p*.
$L(\lambda) = p + \lambda * \hat{v}$: — Parametric form of the straight line passing through *p*, directed by the unit vector $\hat{v}$ with signed distance parameter λ.
f_1,f_2: — Foci of an ellipse in R^2.
$E(f_1,f_2,a)$: — Ellipse $\{p \in R^2 : d(p,f_1)+d(p,f_2)=2a\}$.
$\rho_{X,Y}$: — Linear regression correlation coefficient between variables *Y* and *X*.
$[\rho, p, \hat{v}] = pca(S_E)$: — Principal Component Analysis of the point set S_E, rendering as a result the linear trend $L(\lambda) = p + \lambda * \hat{v}$ with correlation coefficient ρ.
Q: — Queue whose elements are pairs $[p,v]$ formed by a vector *v* anchored at point *p*.
$PL\_Curve\_Set=\{c_1,c_2,\ldots,c_m\}$: — Set of PL pairwise disjoint curves $c_1,c_2,\ldots,c_m$.

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