Geodesic-based manifold learning for parameterization of triangular meshes

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Authors Authors and affiliations

Diego A. Acosta, Oscar E. Ruiz, Santiago Arroyave, Roberto Ebratt, Carlos Cadavid ,
Juan J. Londono

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Abstract

Reverse Engineering (RE) requires representing with free forms (NURBS, Spline, Bézier) a real surface S_0 which has been point-sampled. To serve this purpose, we have implemented an algorithm that minimizes the accumulated distance between the free form and the (noisy) point sample. We use a dual-distance calculation point to / from surfaces, which discourages the forming of outliers and artifacts. This algorithm seeks a minimum in a function f that represents the fitting error, by using as tuning variable the control polyhedron for the free form. The topology (rows, columns) and geometry of the control polyhedron are determined by alternative geodesic-based dimensionality reduction methods: (a) graph-approximated geodesics (Isomap), or (b) PL orthogonal geodesic grids. We assume the existence of a triangular mesh of the point sample (a reasonable expectation in current RE). A bijective composition mapping $S_0 \subset \mathbb{R}^3 \longleftrightarrow \mathbb{R}^2$ allows to estimate a size of the control polyhedrons favorable to uniform-speed parameterizations. Our results show that orthogonal geodesic grids is a direct and intuitive parameterization method, which requires more exploration for irregular triangle meshes. Isomap gives a usable initial parameterization whenever the graph approximation of geodesics on S_0 be faithful. These initial guesses, in turn, produce efficient free form optimization processes with minimal errors. Future work is required in further exploiting the usual triangular mesh underlying the point sample for (a) enhancing the segmentation of the point set into faces, and (b) using a more accurate approximation of the geodesic distances within S_0, which would benefit its dimensionality reduction.

Keywords

Computational geometry Parametric surfaces Surface reconstruction Reverse engineering

Abbreviations

PL
Piecewise linear

B
Solid object in $\mathbb{R}^3$. $B \subset \mathbb{R}^3$ is the closure of a bounded and connected open set, whose border ∂B is a 2-dimensional manifold.

S_0
Freeform parametric surface on which a Face of ∂B is mounted

$\mathbf{P}$
$\{p_0, p_1, \ldots\}$ Unordered point sample of S_0

$S(u, v)$
Parametric surface, which fits the set $\mathbf{P}$,so $S \approx S_0$

u, v
Surface parameters

$N_{i,p}, N_{j,q}$
B-spline base functions $\mathbb{R} \to \mathbb{R}$,

n, m
Number of control points of S in u, v directions respectively

$\mathbf{Cp}$
Control polyhedron for S

k
Norm degree. $|(x_1, x_2, \ldots, x_n)|_k = \sqrt[k]{\sum_{i=1}^{i=n} |x_i|^k}$

f
Function minimized when fitting S to $\mathbf{P}$

d_i
Minimum distance between the i-th point p_i of $\mathbf{P}$ and S

LM
Levenberg-Marquardt

RE
Reverse engineering

Gr

Regular, axis-aligned vertex grid in $\mathbb{R}^2$

G

Graph $(\mathbf{P}, E)$ with vertex set $\mathbf{P}$ and edge set E, nearly embedded in S_0

D

Square matrix in which $D(i,j) = dist(p_i, p_j)$, with *dist()* approaching the geodesic distance on S_0 between sample points p_i and p_j

T

$\{t_1, t_2, \ldots\}$ Triangular mesh of triangles t_i with vertices in $\mathbf{P}$

B_{UV}

Parametric rectangular connected subset of $\mathbb{R}^2$

c_G

PL geodesic curve on T

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