УДК 004.93'1; 004.932

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КЛИФФОРДОВСКИЕ МОДЕЛИ МНОГОКАНАЛЬНЫХ ИЗОБРАЖЕНИЙ, ВОЗНИКАЮЩИХ В VISUAL CORTEX

CLIFFORDEAN MODELS OF MULTICHANNEL IMAGES IN VISUAL CORTEX

  

Ключевые слова: *цветные и мультиканальные изображения, гиперкомплексные числа, алгебры Клиффорда, обработка изображений.*

Одна из главных целей работы состоит в том, чтобы доказать, что аппарат гиперкомплексных алгебр и алгебр Клиффорда более адекватно описывает процессы обработки и распознавания цветных и многоспектральных 2D-, 3D- и nD- изображений, чем векторно-матричный математический аппарат. Можно утверждать, что визуальные системы животных с различной эволюционной историей используют различные коммутативные гиперкомплексные алгебры и алгебры Клиффорда для обработки и распознавания цветных и мультиканальных изображений. Поэтому отдел VC головного мозга вероятно имеет способность оперировать как устройство, работающее в алгебре Клиффорда.

Keywords: *color, multicolor, hyperspectral images, algebraic model, hypercomplex numbers, Clifford algebra, image processing.*

The main goal of the paper is to show that commutative hypercomplex algebras and non-commutative Clifford algebras can be used to solve problems of color, multicolor and hypercomplex 2D-, 3D- and nD- images in a natural and effective manner. One can argue that nature has, through evolution, also learned to utilize properties of hypercomplex numbers. Thus, the visual cortex of a brain might have the ability to operate as a Clifford algebra computing device.

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Introduction

"The LORD created the integers, the rest is the work of man"
Leopold Kronecker

We develop a conceptual framework and design methodologies for multichannel image processing systems with assessment capability. The term multichannel image is used for an image with more than one component. They are composed of a series of images $f_{\lambda_1}(\mathbf{x}), f_{\lambda_2}(\mathbf{x}),..., f_{\lambda_K}(\mathbf{x})$ in different optical bands at wavelengths $\lambda_1,\lambda_2,...,\lambda_K$, called spectral channels, where K is the number of different optical channels. A multichannel images (MCI) can be considered as a n–D K-component (vector–valued) functions

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),..., f_K(\mathbf{x}) \right) : \mathbf{R}^n \to \mathbf{V}^K \quad (1)$$

with values into K-D perceptual space $\mathbf{V}^K$ (dichromatic $\mathbf{V}^2$, color $\mathbf{V}^3_{rgb}$, multichannel $\mathbf{V}^K$), where $\mathbf{x} = (x_1, x_2,..., x_n) \in \mathbf{R}^n$, $n = 2,3,....$.

For processing and recognition of 2-D, 3-D and n-D images we turn the perceptual spaces into corresponding hypercomplex algebras (and call them perceptual algebras). In this work our approach to multichannel processing based on noncommutative Clifford algebras. In the algebraic-geometrical approach, each multichannel pixel is considered not as a K-D vector, but as a K-D hypercomplex number. We will interpret multichannel images as Cliffordean-valued signals

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),\ldots, f_K(\mathbf{x}) \right) = f_1(\mathbf{x})J_1 + f_2(\mathbf{x})J_2 + ... + f_K(\mathbf{x})J_K, \quad (2)$$

which take values in so called Clifford algebra $Alg^{Vis(u,v,w)}_{2^K}\left(\mathbf{R}|1, J_1, J_2,..., J_{K-1}\right) = Alg^{Vis(u,v,w)}_{2^K}$ $= Alg^{Vis(u,v,w)}_{2^K}$, where $J_1, J_2,..., J_K$ are hyperimaginary units with the following non-commutative multiplication rule $J_r J_s = -J_s J_r$, $J_s^2 = -1$ for $s,r = 1,2,...,K$.

Our hypotheses are (*Labunets-Rundblad E.V., Labunets V.G.,2001)*:

1. Brain (Visual cortex) of primates operates with Cliffordean numbers during image recognition. In the algebraic approach, each pixel is considered not as a multi–dimensional vector, but as a multi–dimensional Cliffordean number. For this reason, we assume that the human retina and human visual cortex use triplet numbers and 8-D Clifford numbers to process and recognition of color (RGB)–images, respectively.

2. Brain uses different algebras for retina and for Visual cortex (VC) levels. Multichannel images appear on the retina as functions with values in a multiplet K-D algebra (Greaves, 1847) (in particular, in K–cycle algebra), where K is the number of spectral channels. For example, RGB–color images as they appear on the human retina are represented as triplet–valued functions. But multichannel images in an human Visual cortex are functions (2) with values in a 8-D Clifford algebra.

3. Visual systems of animals with different evolutionary history use different hypercomplex algebras for color and multichannel image processing. Thus, the Visual Cortex might have the ability to operate as a Clifford algebra computing device.

We don't agree with L.Kronecker in that that *"the LORD created the integers, the rest is the work of man"*. We assume that the LORD was the first engineer who knew hypercomplex algebras and used them for designing the visual systems of animals.

Clifford algebras for physical and visual spaces

As we see in (1), a n–D K-component images

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_K(\mathbf{x}) \right) : \mathbf{R}^n \to \mathbf{V}^K$$

have two attributes: n-D physical $\mathbf{R}^n$ and K-D perceptual space $\mathbf{V}^K$ spaces. We suppose (*Labunets V.G.,2000)* that a brain operates with hypercomplex numbers when processing image and calculates hypercomplex–valued invariants of an image recognizing it. In order to operate with n-*D* vectors $\mathbf{x} = (x_1, x_2, ..., x_n) \in \mathbf{R}^n$ and K–D vectors $\mathbf{f} = (f_1, f_2, ..., f_K) \in \mathbf{V}^K$ as with numbers, we embed $\mathbf{R}^n$ and $\mathbf{V}^K$ into spatialn $Alg_{2^n}^{Sp}(\mathbf{R}|1, I_1, \ldots, I_K)$ and visual $Alg_{2^K}^{Vis}(\mathbf{R}|1, J_1, \ldots, J_K)$ hypercomplex Clifford algebras:

$$\mathbf{R}^n \to Alg_{2^n}^{Sp}(\mathbf{R}|1, I_1, \ldots, I_n), \quad \mathbf{V}^K \to Alg_{2^K}^{Vis}(\mathbf{R}|1, J_1, \ldots, J_K).$$

When one speaks about both algebras simultaneously, then they are denoted by $Alg_{2^t}(\mathbf{R}|1, B_1, \ldots, B_t)$ or Alg_{2^t}. Obviously,

$$Alg_{2^t}(\mathbf{R}|1, B_1, \ldots, B_t) = \begin{cases} Alg_{2^n}^{Sp}(\mathbf{R}|1, I_1, \ldots, I_K), & \text{if } t = n \text{ and } B_1 = I_1, \ldots, B_K = I_K, \\ Alg_{2^K}^{Vis}(\mathbf{R}|1, J_1, \ldots, J_K), & \text{if } t = K \text{ and } B_1 = J_1, \ldots, B_K = J_K. \end{cases}$$

Let «small» t-D space $\mathbf{R}^t$ be spanned on the orthonormal basis of t *hyperimaginary units* B_i, $i = 1, 2, \ldots, t$ (basic imagineries). We assume

$$B_i^2 = \begin{cases} +1, & \text{for } i = 1, 2, \ldots, u, \\ -1, & \text{for } i = u+1, u+2, \ldots, u+v, \\ 0, & \text{for } i = u+v+1, u+v+2, \ldots, u+v+w = t, \end{cases} \tag{2}$$

and $B_i B_j = -B_j B_i$. Now, we construct the «big» multicolor 2^t-D hypercomplex space $\mathbf{R}^{2^k}$ as a direct sum of subspaces of dimensions $C_t^0, C_t^1, C_t^2, ..., C_t^t$:

$$\mathbf{R}^{2^t} = \mathbf{R}^{C_t^0} \oplus \mathbf{R}^{C_t^1} \oplus \mathbf{R}^{C_t^2} \oplus ... \oplus \mathbf{R}^{C_t^S} \oplus ... \oplus \mathbf{R}^{C_t^{t-1}} \oplus \mathbf{R}^{C_t^t}, \tag{3}$$

where subspaces $\mathbf{R}^{C_t^S}$, $s = 0, 1, 2, ..., t$ are spanned on the S–products of units $B_{m_1} B_{m_2} \cdots B_{m_s}$ $(m_1 < m_2 < ... < m_s)$. By definition, we suppose that $B_0 \equiv 1$ is the classical real units 1. So

$$
\begin{aligned}
&\mathbf{R}^{C_t^0} = \{a_0 B_0 \mid a_0 \in \mathbf{R}\},\\
&\mathbf{R}^{C_t^1} = \{a_1 B_1 + a_2 B_2 + \ldots + a_t B_t \mid a_1, a_2, \ldots, a_t \in \mathbf{R}\},\\
&\mathbf{R}^{C_t^2} = \{a_{1,2} B_1 B_2 + a_{1,3} B_1 B_3 + \ldots + a_{t-1,t} B_{t-1} B_t \mid a_{1,2}, a_{1,3}, \ldots, a_{t-1,t} \in \mathbf{R}\},\\
&\mathbf{R}^{C_t^3} = \{a_{1,2,3} B_1 B_2 B_3 + a_{1,2,4} B_1 B_2 B_4 + \ldots + a_{t-2,t-1,t} B_{t-2} B_{t-1} B_t \mid a_{123}, a_{124}, \ldots, a_{t-2,t-1,t} \in \mathbf{R}\},\\
&\ldots\ldots\ldots\ldots\ldots\ldots,\\
&\mathbf{R}^{C_t^t} = \{a_{1,2,3,\ldots,t} B_1 B_2 \cdots B_t \mid a_{1,2,3,\ldots,t} \in \mathbf{R}\}.
\end{aligned} \tag{5}
$$

Example 1. Let us consider 1-,2-,3-D small spaces $\mathbf{R}^t = \mathbf{R}^1, \mathbf{R}^2, \mathbf{R}^3$. The corresponding "big" spaces are

$$
\begin{aligned}
&\mathbf{R}^{2^1} = \mathbf{R}^2 = \mathbf{R}^1 \oplus \mathbf{R}^1 = \mathbf{R}\cdot B_0 + \mathbf{R}\cdot B_1,\\
&\mathbf{R}^{2^2} = \mathbf{R}^4 = \mathbf{R}^1 \oplus \mathbf{R}^2 \oplus \mathbf{R}^1 = \mathbf{R}\cdot B_0 + \underbrace{[\mathbf{R}\cdot B_1 + \mathbf{R}\cdot B_2]}_{\mathbf{R}^2} + \mathbf{R}\cdot B_1 B_2,\\
&\mathbf{R}^{2^3} = \mathbf{R}^8 = \mathbf{R}^1 \oplus \mathbf{R}^3 \oplus \mathbf{R}^3 \oplus \mathbf{R}^1 =\\
&\quad = \mathbf{R}\cdot B_0 + \underbrace{[\mathbf{R}\cdot B_1 + \mathbf{R}\cdot B_2 + \mathbf{R}\cdot B_3]}_{\mathbf{R}^3} \oplus \underbrace{[\mathbf{R}\cdot B_1 B_2 + \mathbf{R}\cdot B_1 B_3 + \mathbf{R}\cdot B_2 B_3]}_{\mathbf{R}^3} \oplus \mathbf{R}\cdot B_1 B_2 B_3.
\end{aligned} \tag{6}
$$

As we will see complex numbers live in $\mathbf{R}^{2^1}$, quaternions - in $\mathbf{R}^{2^2}$, and biquaternions in $\mathbf{R}^{2^3}$.

Every element of $\mathbf{R}^{2^t}$ has the following representation. Let $\mathbf{b} = (b_1, b_2, \ldots, b_t) \in \mathbf{B}_2^t$ be an arbitrary t-bit binary vector, where $b_i \in \mathbf{B}_2 = \{0,1\}$ and $\mathbf{B}_2^t$ is the t-D Boolean. Let us introduce 2^t elements $\mathbf{B}^{\mathbf{b}} := B_1^{b_1} B_2^{b_2} \cdots B_t^{b_t}$, where $\mathbf{b} = (b_1, b_2, \ldots, b_t) \in \mathbf{B}_2^t$. Let $w(\mathbf{b}) = b_1 + b_2 + \cdots + b_t$ be the weight of $\mathbf{b} = (b_1, b_2, \ldots, b_K) \in \mathbf{B}_2^K$. Elements $B^{\mathbf{b}} = B_1^{b_1} B_2^{b_2} \cdots B_t^{b_t}$ form a basis of 2^t-D space (full set of imaginaries – FSoI). For all $\mathcal{C} \in \mathbf{R}^{2^t}$ we have the following hypercomplex representations

$$
\begin{aligned}
\mathcal{C} &:= \sum_{\mathbf{b}\in\mathbf{B}_2^k} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} = \sum_{s=0}^{K} \sum_{w(\mathbf{b})=s} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} = \sum_{w(\mathbf{b})=0} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} + \left(\sum_{w(\mathbf{b})=1} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} + \sum_{w(\mathbf{b})=2} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} + \ldots + \sum_{w(\mathbf{b})=s} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} + \ldots + \sum_{w(\mathbf{b})=t} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} \right) =\\
&= \mathbf{Sc}(\mathcal{C}) + \left(\mathbf{Vec}^1(\mathcal{C}) + \mathbf{Vec}^2(\mathcal{C}) + \ldots + \mathbf{Vec}^s(\mathcal{C}) + \ldots + \mathbf{Vec}^t(\mathcal{C}) \right),
\end{aligned} \tag{7}
$$

where

- $\mathbf{Sc}(\mathcal{C}) = \sum_{w(\mathbf{b})=0} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} = a_0 B_0 \in \mathbf{R}^{C_t^0}$ is the *scalar part* of the Clifford number $\mathcal{C}$,
- $\mathbf{Vec}^1(\mathcal{C}) = \sum_{w(\mathbf{b})=1} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} \in \mathbf{R}^{C_t^1}$ is its the pure vector part,
- $\mathbf{Vec}^2(\mathcal{C}) = \sum_{w(\mathbf{b})=2} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} \in \mathbf{R}^{C_t^2}$ is its the bivector part, …,
- $\mathbf{Vec}^s(\mathcal{C}) = \sum_{w(\mathbf{b})=s} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} \in \mathbf{R}^{C_t^s}$ is its the s-vector part,…, and
- $\mathbf{Vec}^t(\mathcal{C}) = \sum_{w(\mathbf{b})=t} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} \in \mathbf{R}^{C_t^t}$ is its the t-vector part.

Among the FSoI imaginary units $\mathbf{B}^{\mathbf{b}} := B_1^{b_1} B_2^{b_2} \cdots B_t^{b_t}$ with $w(\mathbf{b}) = 1$ (i.e. $B_1, \ldots, B_t$) are called the basic imaginaries and units $\mathbf{B}^{\mathbf{b}} := B_1^{b_1} B_2^{b_2} \cdots B_t^{b_t}$ with $w(\mathbf{b}) > 1$ are called the derivative imaginaries.

Binary vector $\mathbf{b} = (b_1, b_2, \ldots, b_t) \in \mathbf{B}_2^t$ can be considered as binary code of an integer $\mathbf{b} = (b_1, b_2, \ldots, b_t) \in [0, 2^t - 1]$, hence $\mathbf{B}^{\mathbf{b}} \in \{\mathbf{B}^{\mathbf{0}}, \mathbf{B}^{\mathbf{1}}, \ldots, \mathbf{B}^{\mathbf{2t-1}}\}$ and

$$B_0 = \mathbf{B}^{\mathbf{0}} = \mathbf{B}^{(0,0,0,0\ldots,0)}, B_1 = \mathbf{B}^{\mathbf{1}} = \mathbf{B}^{(1,0,0,0\ldots,0)},$$
$$B_2 = \mathbf{B}^{\mathbf{2}} = \mathbf{B}^{(0,1,0,0\ldots,0)}, B_3 = \mathbf{B}^{\mathbf{4}} = \mathbf{B}^{(0,0,1,0\ldots,0)},$$
$$\ldots\ldots\ldots\ldots\ldots\ldots, B_t = \mathbf{B}^{\mathbf{2^{t-1}}} = \mathbf{B}^{(0,0,0,0\ldots,1)}.$$

Definition 1. *Elements of the form (7) are called the Clifford physical (if $t = n$) or hyperspectral numbers (if $t = K$).*

If $\mathrm{A} = \sum_{\mathbf{b} \in \mathbf{B}_2^t} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}}$, $\mathrm{B} = \sum_{\mathbf{c} \in \mathbf{B}_2^t} b_{\mathbf{c}} \mathbf{B}^{\mathbf{c}} \in \mathbf{R}^{2^t}$ are two Clifford numbers then their product is

$$\mathrm{C} = \mathrm{AB} := \left(\sum_{\mathbf{b} \in \mathbf{B}_2^t} a_{\mathbf{b}} \mathbf{B}^{\mathbf{b}} \right) \cdot \left(\sum_{\mathbf{c} \in \mathbf{B}_2^t} b_{\mathbf{c}} \mathbf{B}^{\mathbf{c}} \right) = \left(\sum_{\mathbf{b} \in \mathbf{B}_2^t} \sum_{\mathbf{c} \in \mathbf{B}_2^t} a_{\mathbf{b}} b_{\mathbf{c}} \mathbf{B}^{\mathbf{b}} \mathbf{B}^{\mathbf{c}} \right) = \left(\sum_{\mathbf{b} \in \mathbf{B}_2^t} \sum_{\mathbf{c} \in \mathbf{B}_2^t} (-1)^{\langle \mathbf{b} | R | \mathbf{c} \rangle} a_{\mathbf{b}} b_{\mathbf{c}} \mathbf{B}^{\mathbf{b} \oplus \mathbf{c}} \right) =$$
$$= \left(\sum_{\mathbf{d} \in \mathbf{B}_2^t} \sum_{\mathbf{b} \in \mathbf{B}_2^t} (-1)^{\langle \mathbf{b} | R | \mathbf{d} \oplus \mathbf{b} \rangle} a_{\mathbf{b}} b_{\mathbf{d} \oplus \mathbf{b}} \mathbf{B}^{\mathbf{d}} \right) = \left(\sum_{\mathbf{d} \in \mathbf{B}_2^t} \sum_{\mathbf{b} \in \mathbf{B}_2^t} (-1)^{\langle \mathbf{b} | R | \mathbf{d} \oplus \mathbf{b} \rangle} a_{\mathbf{b}} b_{\mathbf{d} \oplus \mathbf{b}} I^{\mathbf{d}} \right) = \sum_{\mathbf{d} \in \mathbf{B}_2^K} c_{\mathbf{d}} \mathbf{B}^{\mathbf{d}}, \quad (8)$$

where

$$c_{\mathbf{d}} = \sum_{\mathbf{b} \in \mathbf{B}_2^t} (-1)^{\langle \mathbf{b} | R | \mathbf{d} \oplus \mathbf{b} \rangle} a_{\mathbf{b}} b_{\mathbf{d} \oplus \mathbf{b}}, \quad R := \begin{bmatrix} 1 & & & & \\ 1 & 1 & & & \\ 1 & 1 & 1 & & \\ . & . & . & . & . \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}. \quad (9)$$

There are 3^t possibilities for $B_s^2 = +1, 0, -1, \ \forall s = 1, 2, \ldots, t$. Every possibility generates algebra. Therefore, the space $\mathbf{R}^{2^t}$ with 3^t rules of the multiplication forms 3^t different 2^t-D algebras, which are called the Clifford algebras. We denote these algebras by $Alg_{2^t}^{(u,v,w)}(\mathbf{R}|1, B_1, \ldots, B_t)$, or Alg_{2^t} if $B_1, \ldots, B_K$ and u, v, w are fixed. If $t = n$ then we have 3^n different 2^n-D spatial Clifford algebras $Alg_{2^n}^{Sp(p,q,r)}(\mathbf{R}|1, I_1, \ldots, I_n)$. For $t = K$ we have 3^K different 2^K-D visual Clifford algebras $Alg_{2^K}^{Sp(u,v,w)}(\mathbf{R}|1, J_1, \ldots, J_K)$. Spatial and visual Clifford algebras can have different signatures (3): $(u, v, w) \neq (p, q, r)$. In Alg_{2^t} we introduce the conjugation operation which maps every Clifford number $\mathrm{C} := c_0 J_0 + \sum_{\mathbf{b} \neq 0} c_{\mathbf{b}} B^{\mathbf{b}}$ to the number $\bar{\mathrm{C}} := c_0 J_0 - \sum_{\mathbf{b} \neq 0} c_{\mathbf{b}} B^{\mathbf{b}}$.

The algebras Alg_{2^t} are transformed into 2^t-D pseudometric spaces designed as $\mathbf{Geo}_{2^t}$, if the pseudodistance between two Clifford numbers A and B is defined by

$$\rho(\mathrm{A}, \mathrm{B}) = |\mathrm{A} - \mathrm{B}| = \sqrt{(\mathrm{A} - \mathrm{B})\overline{(\mathrm{A} - \mathrm{B})}} = |\mathrm{U}| = \mathrm{U}\bar{\mathrm{U}} = \sqrt{\sum_{\mathbf{b} \in \mathbf{B}_2^t} (-1)^{w(\mathbf{b})} u_{\mathbf{b}} \left(B^{\mathbf{b}} \right)^2}, \quad (10)$$

where $\mathrm{A} - \mathrm{B} = \mathrm{U} = \sum_{\mathbf{b} \in \mathbf{B}_2^t} u_{\mathbf{b}} B^{\mathbf{b}}$.

Subspaces of pure vector Clifford numbers $\mathbf{R}^t = \{\mathbf{x} \mid \mathbf{x} = \sum_{w(\mathbf{b})=1} x_{\mathbf{b}}\mathbf{B}^{\mathbf{b}} = x_1B_1 + \ldots + x_tB_t\}$ in this case is transformed into different t-D pseudometric spaces $\mathbf{R}^t \to \mathbf{Geo}_t = \langle\langle \mathbf{R}^t, \rho(\mathrm{A},\mathrm{B})\rangle\rangle$, since pseudometrics $\rho(\mathrm{A},\mathrm{B})$ constructed in $\mathbf{R}^{2^t}$ induce corresponding pseudometrics in $\mathbf{R}^t$. The pseudometric spaces $\mathbf{Geo}_t$ are *the Cayley-Klein geometries* (Labunets-Rundblad et al., 2001; Labunets et al., 2001a,b). Obviously,

$$\mathbf{R}^n \to \mathbf{Geo}_n^{Sp(p,q,r)} = \langle\langle \mathbf{R}^n, \rho^{Sp(p,q,r)}\rangle\rangle, \quad \mathbf{V}^K \to \mathbf{Geo}_K^{Vis(u,v,w)} = \langle\langle \mathbf{V}^K, \rho^{Vis(u,v,w)}\rangle\rangle,$$

where $\mathbf{Geo}_n^{Sp(p,q,r)}$ is a spatial geometry for the physical space $\mathbf{R}^n$ with a metric $\rho^{Sp(p,q,r)}$ and $\mathbf{Geo}_K^{Vis(u,v,w)}$ is a geometry for the visual space $\mathbf{V}^K$ (in the Visual Cortex) with a metric $\rho^{Vis(u,v,w)}$.

Every algebra $Alg_{2^K}^{Vis(u,v,w)}$ can be decomposed as

$$Alg_{2^t} = {}^0Alg_{2^t} + {}^1Alg_{2^t} = \sum_{s=0}^{]t/2[} \mathbf{Vec}^{2s} + \sum_{s=0}^{]t/2[} \mathbf{Vec}^{2s+1}, \tag{11}$$

where ${}^0Alg_{2^t} = \sum_{s=0}^{]t/2[} \mathbf{Vec}^{2s}$ and ${}^1Alg_{2^t} = \sum_{s=0}^{]t/2[} \mathbf{Vec}^{2s+1}$ are even and odd parts of Alg_{2^t}. We will see that all orthogonal transforms of «small» perceptual t-D space $\mathbf{R}^t$ live in ${}^0Alg_{2^t}$. Clifford numbers $\mathrm{E} \in {}^0Alg_{2^t}$ of unit modulus represent the rotation group for the corresponding space $\mathbf{R}^t$ which is called *spinor group* and is denoted by $\mathbf{Spin}(Alg_{2^t})$.

We know that complex numbers and quaternions of unit modulus have the following forms:

$$\mathrm{e}_0 = e^{i\varphi} = \cos\varphi + i\sin\varphi, \quad Q_0 = e^{\mathbf{u}_0\varphi} = \cos\varphi + \mathbf{u}_0\sin\varphi, \tag{12}$$

where $\cos\varphi$ and $\sin\varphi$ are trigonometric functions in the corresponding 2-D geometries, respectively, φ is a rotation angle around vector-valued quaternion $\mathbf{u}_0$ of unit modulus ($|\mathbf{u}_0| = 1$, $\mathbf{u}_0 = -\bar{\mathbf{u}}_0$). In general case Clifford spinors $\mathrm{E}_0 \in \mathbf{Spin}(Alg_{2^t})$ with unit modulus have the same form

$$\mathrm{E}_0 = e^{\mathrm{U}\varphi} = \cos\varphi + \mathrm{U}\cdot\sin\varphi \in \mathbf{Spin}(Alg_{2^t}), \tag{13}$$

where $\mathrm{U} = \sum_{k_1=1}^{t}\sum_{k_2=1}^{t} u_{k_1k_2}B_{k_1}B_{k_2} = \sum_{w(\mathbf{b})=2} u_{\mathbf{b}}\mathbf{B}^{\mathbf{b}} \in \mathbf{Vec}^2$ is a unit bivector $(\mathrm{U}^2 = -1)$, and φ is a rotation angle.

Theorem 1 (Labunets-Rundblad et al., 2001; Labunets et al., 2001a). *The transforms*

$$\mathrm{Q}' = e^{\mathrm{U}_1\varphi_1/2}\mathrm{Q}, \quad \mathrm{Q}'' = \mathrm{Q}e^{-\mathrm{U}_2\varphi_2/2}, \quad \mathrm{Q}''' = e^{\mathrm{U}_1\varphi_1/2}\mathrm{Q}e^{-\mathrm{U}_2\varphi_2/2} \tag{14}$$

are the rotations of the "big" space $\mathbf{Geo}_{2^t}$, *where* $\mathrm{Q}, \mathrm{Q}', \mathrm{Q}'', \mathrm{Q}''' \in \mathbf{Geo}_{2^t}$ *and* $e^{\mathrm{U}_1\varphi_1/2}, e^{-\mathrm{U}_2\varphi_2/2} \in \mathbf{Spin}(Alg_{2^t})$. *They form groups* $\mathbf{Rot}_L(\mathbf{Geo}_{2^t})$, $\mathbf{Rot}_R(\mathbf{Geo}_{2^t})$, $\mathbf{Rot}_{LR}(\mathbf{Geo}_{2^t})$ *of left, right and double-side rotations of "big" space* $\mathbf{Geo}_{2^t}$ *and transforms*

$$\mathbf{x}' = e^{\mathrm{U}_1\varphi_1/2}\mathbf{x}e^{-\mathrm{U}_2\varphi_2/2}, \tag{15}$$

where $\mathbf{x}, \mathbf{x}' \in \mathbf{Geo}_t$ *and* $e^{\mathrm{U}_1\varphi_1/2}, e^{-\mathrm{U}_2\varphi_2/2} \in \mathbf{Spin}(Alg_{2^t})$ *are rotations of "small" space* $\mathbf{Geo}_t$. *They form group of rotations* $\mathbf{Rot}_{LR}(\mathbf{Geo}_t)$.

Theorem 2 (Labunets-Rundblad et al., 2001; Labunets et al., 2001a). *The transforms*

$$\mathrm{Q}' = e^{\mathrm{U}_1\varphi_1/2}\mathrm{Q} + \mathrm{P},\ \ \mathrm{Q}'' = \mathrm{Q}e^{-\mathrm{U}_2\varphi_2/2} + \mathrm{P},\ \ \mathrm{Q}''' = e^{\mathrm{U}_1\varphi_1/2}\mathrm{Q}e^{-\mathrm{U}_2\varphi_2/2} + \mathrm{P} \qquad (16)$$

form three groups of left, right and double-side multicolor motions $\mathbf{Mov}_L\left(\mathbf{Geo}_{2^t}\right)$, $\mathbf{Mov}_R\left(\mathbf{Geo}_{2^t}\right)$, $\mathbf{Mov}_{LR}\left(\mathbf{Geo}_{2^t}\right)$ *of "big" space* $\mathbf{Geo}_{2^t}$

Theorem 3. *Every motion of "small"* $\mathbf{Geo}_t$ *-space is represented in the following form*

$$\mathbf{z}' = e^{\mathrm{U}\varphi/2}\cdot\mathbf{z}\cdot e^{-\mathrm{U}\varphi/2} + \mathbf{w},\quad \mathbf{z},\mathbf{z}',\mathbf{w}\in\mathbf{Geo}_t. \qquad (17)$$

Cliffordean models of multichannel images

In classical approach multichannel images $\mathbf{f}(\mathbf{x})$ are considered as a n–D K-component (vector–valued) functions

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),..., f_K(\mathbf{x})\right) : \mathbf{R}^n \to \mathbf{V}^K \qquad (18)$$

with values into K-*D* perceptual spaces $\mathbf{V}^K$ *(dichromatic* $\mathbf{V}^2$, *color* $\mathbf{V}^3_{rgb}$, *multichannel* $\mathbf{V}^K$*)*, where $\mathbf{x}\in\mathbf{R}^n$, $n = 2,3,....$. Now we can interpret multichannel images $\mathbf{f}(\mathbf{x})$ as $Alg^{Vis(u,v,w)}_{2^K}\left(\mathbf{R}|1, J_1,\ldots,J_K\right)$–valued signal of hypercomplex variables $\mathbf{x}\in Alg^{Sp(p,q,r)}_{2^n}\left(\mathbf{R}|1, I_1,\ldots,I_n\right)$:

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),..., f_K(\mathbf{x})\right) : Alg^{Sp(p,q,r)}_{2^n}\left(\mathbf{R}|1, I_1,\ldots,I_n\right) \to Alg^{Vis(u,v,w)}_{2^K}\left(\mathbf{R}|1, J_1,\ldots,J_K\right) \qquad (19)$$

Obviously,

$$\mathbf{R}^n = \mathbf{Vec}^1\left(Alg^{Sp(p,q,r)}_{2^n}\left(\mathbf{R}|1, I_1,\ldots,I_n\right)\right) = I_1\mathbf{R} + I_2\mathbf{R} + \ldots + I_n\mathbf{R} = \mathbf{Vec}^1\left(Alg^{Sp(p,q,r)}_{2^n}\right),$$

$$\mathbf{V}^K = \mathbf{Vec}^1\left(Alg^{Vis(u,v,w)}_{2^K}\left(\mathbf{R}|1, J_1,\ldots,J_K\right)\right) = J_1\mathbf{R} + J_2\mathbf{R} + \ldots + J_K\mathbf{R} = \mathbf{Vec}^1\left(Alg^{Vis(u,v,w)}_{2^K}\right)$$

There are four algebraic models for *multichannel* images:

1) The first model

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),..., f_K(\mathbf{x})\right) : \mathbf{Vec}^1\left(Alg^{Sp(p,q,r)}_{2^n}\right) \to \mathbf{Vec}^1\left(Alg^{Vis(u,v,w)}_{2^K}\right), \qquad (20)$$

where we used basic spatial and multichannel imaginaries, i.e.

$$\mathbf{f}(I_1x_1 + I_2x_2 + \ldots + I_nx_n) = \\ = J_1\cdot f_1(I_1x_1 + I_2x_2 + \ldots + I_nx_n) + J_2\cdot f_2(I_1x_1 + I_2x_2 + \ldots + I_nx_n) + ... + J_K\cdot f_K(I_1x_1 + I_2x_2 + \ldots + I_nx_n). \qquad (21)$$

2) The second model

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),..., f_K(\mathbf{x})\right) : \mathbf{Vec}^1\left(Alg^{Sp(p,q,r)}_{2^n}\right) \to Alg^{Vis(u,v,w)}_{2^m}\left(\mathbf{R}|1, J_1,\ldots,J_m\right). \qquad (22)$$

where we used basic spatial and the full set of multichannel imaginaries, i.e.

$$\mathbf{f}(I_1x_1 + I_2x_2 + \ldots + I_nx_n) = \\ = \mathbf{J}^0\cdot f_0(I_1x_1 + I_2x_2 + \ldots + I_nx_n) + \mathbf{J}^1\cdot f_1(I_1x_1 + I_2x_2 + \ldots + I_nx_n) + ... + \mathbf{J}^{\mathbf{K}\text{-}\mathbf{1}}\cdot f_{K-1}(I_1x_1 + I_2x_2 + \ldots + I_nx_n). \qquad (23)$$

Here we suppose that $K = 2^m$ and (22) is a 2^m-channel image.

3) The third model

$$\mathbf{f}(\mathbf{x}) = \left(f_1(\mathbf{x}), f_2(\mathbf{x}),..., f_K(\mathbf{x})\right) : Alg^{Sp(p,q,r)}_{2^n}\left(\mathbf{R}|1, I_1,\ldots,I_n\right) \to \mathbf{Vec}^1\left(Alg^{Vis(u,v,w)}_{2^K}\right), \qquad (24)$$

where we used the full set of spatial and basic multichannel imaginaries, i.e.

$$\mathbf{f}(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n)=$$
$$=J_1\cdot f_1(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n)+J_2\cdot f_2(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n)+\ldots+J_K\cdot f_K(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n). \quad (25)$$

Here we suppose that $n=2^l$ and (23) is a 2^l D image.

4) The fourth model

$$\mathbf{f}(\mathbf{x})=\left(f_1(\mathbf{x}), f_2(\mathbf{x}),\ldots, f_K(\mathbf{x})\right): Alg_{2^n}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1,\ldots,I_n\right)\to Alg_{2^K}^{Vis(u,v,w)}\left(\mathbf{R}|1,J_1,\ldots,J_K\right). \quad (26)$$

where we used the full set of spatial and multichannel imaginaries, i.e.

$$\mathbf{f}(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n)=$$
$$=\mathbf{J}^0\cdot f_0(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n)+\mathbf{J}^1\cdot f_1(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n)+\ldots+\mathbf{J}^{\mathrm{K-1}}\cdot f_{K-1}(\mathbf{I}^0x_0+\mathbf{I}^1x_1+\ldots+\mathbf{I}^{\mathrm{n}}x_n). \quad (27)$$

Here we suppose that $n=2^l$, $K=2^m$ and (27) is a 2^l dimension 2^m-channel image.

Example 2. Let

$$\mathbf{f}(\mathbf{x})=\left[f_Y(\mathbf{x}), f_R(\mathbf{x}), f_G(\mathbf{x}), f_B(\mathbf{x})\right] \quad (28)$$

be a grey-level and color retinal image. We can consider four models for this image. For this purpose we introduce the following physical space and multichannel Clifford algebras

$$Alg_{2^1}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1\right),\qquad Alg_{2^2}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1,I_2\right),$$
$$Alg_{2^2}^{Vis(u,v,w)}\left(\mathbf{R}|1,J_1,J_2\right),\ Alg_{2^4}^{Vis(u,v,w)}\left(\mathbf{R}|1,J_1,J_2,J_3,J_4\right).$$

We are going to consider two variants for $\mathbf{x}=(x,y)$:

$$\mathbf{x}=(x,y)=xI_0+yI_1\in Alg_{2^1}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1\right)=\mathbf{R}I_0+\mathbf{R}I_1,$$
$$\mathbf{x}=(x,y)=xI_1+yI_2\in Vec^1\left(Alg_{2^1}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1,I_2\right)\right)=\mathbf{R}I_1+\mathbf{R}I_2\subset\mathbf{R}I_0+\underbrace{\left(\mathbf{R}I_1+\mathbf{R}I_2\right)}_{Vec^1}+\mathbf{R}I_1I_2,$$

and two variants for $\left[f_Y, f_R, f_G, f_B\right]$:

$$\left[f_Y, f_R, f_G, f_B\right]\in Alg_{2^2}^{Sp(p,q,r)}\left(\mathbf{R}|1,J_1,J_2\right)=\mathbf{R}J_0+\mathbf{R}J_1+\mathbf{R}J_2+\mathbf{R}J_1J_2=\mathbf{R}J_Y+\mathbf{R}J_R+\mathbf{R}J_G+\mathbf{R}J_B,$$
$$\left[f_Y, f_R, f_G, f_B\right]\in Vec^1\left(Alg_{2^4}^{Sp(p,q,r)}\left(\mathbf{R}|1,J_1,J_2,J_3,J_4\right)\right)=\mathbf{R}J_1+\mathbf{R}J_2+\mathbf{R}J_3+\mathbf{R}J_4\subset$$
$$\subset\mathbf{R}I_0+\underbrace{\left(\mathbf{R}J_1+\mathbf{R}J_2+\mathbf{R}J_3+\mathbf{R}J_4\right)}_{Vec^1}+\left(\mathbf{R}J_1J_2+\ldots+\mathbf{R}J_3J_4\right)+\left(\mathbf{R}J_1J_2J_3+\ldots+\mathbf{R}J_2J_3J_4\right)+\mathbf{R}J_1J_2J_3J_4=$$
$$\mathbf{R}I_0+\underbrace{\left(\mathbf{R}J_1+\mathbf{R}J_2+\mathbf{R}J_3+\mathbf{R}J_4\right)}_{Vec^1}+\left(\mathbf{R}J_1J_2+\ldots+\mathbf{R}J_3J_4\right)+\left(\mathbf{R}J_1J_2J_3+\ldots+\mathbf{R}J_2J_3J_4\right)+\mathbf{R}J_1J_2J_3J_4,$$

where $J_Y=J_1, J_R=J_2, J_G=J_3, J_B=J_4$. Now we can consider the following four algebraic models of retinal image (28):

$$\mathbf{f}(\mathbf{x}): Vec^1\left(Alg_{2^2}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1,I_2\right)\right)\to Vec^1\left(Alg_{2^4}^{Vis(u,v,w)}\left(\mathbf{R}|1,J_1,J_2,J_3,J_4\right)\right),$$
$$\mathbf{f}(\mathbf{x})=\mathbf{f}(xI_1+yI_2)=f_Y(xI_1+yI_2)\cdot J_1+f_R(xI_1+yI_2)\cdot J_2+f_G(xI_1+yI_2)\cdot J_3+f_B(xI_1+yI_2)\cdot J_4,$$

$$\mathbf{f}(\mathbf{x}): Vec^1\left(Alg_{2^2}^{Sp(p,q,r)}\left(\mathbf{R}|1,I_1,I_2\right)\right)\to Alg_{2^2}^{Vis(u,v,w)}\left(\mathbf{R}|1,J_1,J_2\right),$$
$$\mathbf{f}(\mathbf{x})=\mathbf{f}(xI_1+yI_2)=f_Y(xI_1+yI_2)\cdot J_0+f_R(xI_1+yI_2)\cdot J_1+f_G(xI_1+yI_2)\cdot J_2+f_B(xI_1+yI_2)\cdot J_1J_2,$$

$$\mathbf{f}(\mathbf{x}): Alg_{2^1}^{Sp(p,q,r)}\left(\mathbf{R}|1, I_1\right) \to Vec^1\left(Alg_{2^4}^{Vis(u,v,w)}\left(\mathbf{R}|1, J_1, J_2, J_3, J_4\right)\right),$$

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(xI_0 + yI_1) = f_Y(xI_0 + yI_1)\cdot J_1 + f_R(xI_0 + yI_1)\cdot J_2 + f_G(xI_0 + yI_{12})\cdot J_3 + f_B(xI_0 + yI_1)\cdot J_4,$$

$$\mathbf{f}(\mathbf{x}): Alg_{2^1}^{Sp(p,q,r)}\left(\mathbf{R}|1, I_1\right) \to Alg_{2^2}^{Vis(u,v,w)}\left(\mathbf{R}|1, J_1, J_2\right),$$

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(xI_0 + yI_1) = f_Y(xI_0 + yI_1)\cdot J_0 + f_R(xI_0 + yI_1)\cdot J_1 + f_G(xI_0 + yI_{12})\cdot J_2 + f_B(xI_0 + yI_1)\cdot J_1 J_2,$$

Geometric properties of these images depend on signaturies (p,q,r) and (u,v,w).

Example 2. Binocular vision is vision in which both eyes are used together. Each eye (right and left) views the visual world from slightly different horizontal positions. Each eye's image differs from the other (Fig. 1), *i.e.*,

$$\mathbf{f}^r(\mathbf{x}) = \left(f_1^r(\mathbf{x}), f_2^r(\mathbf{x}), ..., f_K^r(\mathbf{x})\right), \quad \mathbf{f}^l(\mathbf{x}) = \left(f_1^l(\mathbf{x}), f_2^l(\mathbf{x}), ..., f_K^l(\mathbf{x})\right). \quad (29)$$

Objects at different distances from the eyes project images in the two eyes that differ in their horizontal positions, giving the depth cue of horizontal disparity Depth perception is commonly referred to as stereopsis. Stereopsis appears to be processed in the visual cortex in binocular cells having receptive fields in different horizontal positions in the two eyes. Such a cell is active only when its preferred stimulus is in the correct position in the left eye and in the correct position in the right eye, making it a disparity detector. The two eyes can influence each other. We take to account this influence using a new hyperimaginary binocular unit B and construct images on VC as signal

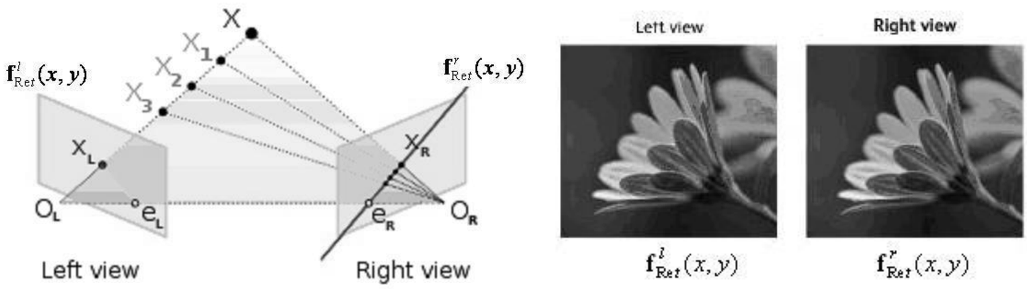

Fig. 1. When two eyes (or cameras) view a 3D scene from two distinct positions, there are a number of geometric relations between the 3D points and their projections onto the 2D images that lead to constraints between the image points.

$$\mathbf{f}^{Bin}(\mathbf{x}) = \mathbf{f}^r(\mathbf{x}) + B\mathbf{f}^l(\mathbf{x}), \quad (30)$$

i.e.,

$$\mathbf{f}^{Bin}(\mathbf{x}) = \left(f_1^r(\mathbf{x})\cdot J_1 + f_2^r(\mathbf{x})\cdot J_2 + ... + f_K^r(\mathbf{x})\cdot J_K\right) + B\left(f_1^l(\mathbf{x})\cdot J_1 + f_2^l(\mathbf{x})\cdot J_2 + ... + f_K^l(\mathbf{x})\cdot J_K\right). \quad (31)$$

for the first model and

$$\mathbf{f}^{Bin}(\mathbf{x}) = \left(\sum_{s=0}^{m}\sum_{w(\mathbf{b})=s} f_{\mathbf{b}}^l(\mathbf{x}) J^{\mathbf{b}}\right) + B\left(\sum_{s=0}^{m}\sum_{w(\mathbf{b})=s} f_{\mathbf{b}}^r(\mathbf{x}) J^{\mathbf{b}}\right) \quad (32)$$

for the second one, which take values in the Clifford binocular bialgebra $Alg_{2^{m+1}}^{Vis(u,v,w)}\left(\mathbf{R}|1, J_1, J_2, ..., J_{m-1}; B, J_1B, J_2B, ..., J_{m-1}B\right)$, with $B^2 = \delta_3 = +1, 0, -1$.

Obviously,

$$\mathbf{f}^{Bin}(\mathbf{x}) = \left(f_1^r(\mathbf{x}) + Bf_1^l(\mathbf{x})\right)\cdot J_1 + \left(f_2^r(\mathbf{x}) + Bf_2^l(\mathbf{x})\right)\cdot J_2 + \ldots + \left(f_K^r(\mathbf{x}) + Bf_K^l(\mathbf{x})\right)\cdot J_K$$
$$= f_1^{Bin}(\mathbf{x})\cdot J_1 + f_2^{Bin}(\mathbf{x})\cdot J_2 + \ldots + f_K^{Bin}(\mathbf{x})\cdot J_K, \qquad (33)$$

and

$$\mathbf{f}^{Bin}(\mathbf{x}) = \sum_{s=0}^{m}\sum_{w(\mathbf{b})=s}\left[f_{\mathbf{b}}^l(\mathbf{x}) + Bf_{\mathbf{b}}^r(\mathbf{x})\right]J^{\mathbf{b}} = \sum_{s=0}^{m}\sum_{w(\mathbf{b})=s} f_{\mathbf{b}}^{Bin}(\mathbf{x})J^{\mathbf{b}} \qquad (34)$$

where $f_k^{Bin}(\mathbf{x}) = f_k^r(\mathbf{x}) + Bf_k^l(\mathbf{x})$ and $f_{\mathbf{b}}^{Bin}(\mathbf{x}) = f_{\mathbf{b}}^l(\mathbf{x}) + Bf_{\mathbf{b}}^r(\mathbf{x})$ are channel binocular images. In this form, these binocular images are complex-valued images, where B binocular complex unit.

Conclusion

We developed a novel algebraic approach based on hypercomplex algebras to algebraic models of color, multicolor and hyperspectral images. It is our aim to show that the use of hypercomplex algebras fits more naturally to the tasks of recognition of multicolor patterns than does the use of color vector spaces. One can argue that Nature has, through evolution, also learned to utilize properties of hypercomplex numbers. Thus, a brain might have the ability to operate as a Clifford algebra computing device. We don't agree with L. Kronecker in that that *"the LORD created the integers, the rest is the work of man"*. We assume that the LORD was the first engineer who knew hypercomplex algebras and used them for designing the visual systems of animals.

Acknowledgment

This work was supported by grants the RFBR № 17-07-00886, № 17-29-03369 and by Ural State Forest Engineering's Center of Excellence in "Quantum and Classical Information Technologies for Remote Sensing Systems".

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