Redundancy in Block Coded Modulations for Channel Equalization Based on Spatial and Temporal Diversity

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ABSTRACT

Linear block codes in the complex field can be applied in spatial and/or temporal diversity receivers in order to develop high performance schemes for (almost-) blind equalization in mobile communications. The proposed technique uses the structure of the encoded transmitted information (with redundancy) to achieve equalization schemes based on a deterministic criterion. Simulations show that the proposed technique is more efficient than other schemes that follow similar equalizer structures. The result is an algorithm that provides the design of channel equalizers in low EbNo scenarios.

I. INTRODUCTION

The issue of developing equalization techniques in mobile communications has received considerable attention recently. The time-variant nature of the channel behavior suggests the definition of deterministic cost functions instead of the use of the conventional stochastic ones. The goal of the deterministic methods is to make use of the signal structure and to avoid the use of the received signal statistics. For instance, in a TDMA transmission, the channel is almost stationary into a burst but non-stationary between bursts. Notice that in deterministic algorithms, the performance of the equalizer will be dependent on the information data realization, the channel realization and the noise realization in a burst. The proposed approach relies on the availability of redundancy in block-coded modulations for *temporal* and/or *spatial diversity* digital receivers. The paper shows that the knowledge of coded signal structure allows the information recovery from a deterministic design criterion.

The proposed method can be classified as an *almost-blind* (or *semi-blind*) algorithm or, in other words, it does not introduce a *training* sequence (or *time reference*) in the transmitted signal but some kind of information dependent redundancy (*code diversity*). In some equalization methods ([4], among others), the transmitter introduces redundancy that the receiver uses to identify or equalize the channel. Although some additional redundancy is also added in our method, the proposed approach differs in the sense that the redundancy is not introduced to equalize but used to correct detection errors, too [3].

It is known that if the channel output is oversampled in the time domain (*temporal diversity*) and/or in the spatial domain (*spatial diversity*), channel compensation can be performed based on the received signal second-order statistics only. A recent study [1] used the Bezout equation to introduce a new blind equalization criterion. Basically, it proposed an algorithm that maximizes the signal-to-ISI-plus-noise ratio (SINR) at the equalizer output. Unfortunately, this algorithm offered high reliability in moderate to high SNR's scenarios but showed stability problems at lower SNR's. Following the same strategy than the method presented in [1] and [9], the paper shows that the equalizer performance and stability can be improved by introducing a systematic block-coding allowing the channel equalization at low EbNo scenarios.

The main idea is to combine the information supplied by the received signal redundancy introduced by a systematic linear block coding technique to improve the statistical stability of the equalization technique in presence of noise. The result is a robust scheme that can be applied to TDMA, DS-CDMA systems in frequency selective mobile channels, and OFDM systems in frequency flat fading (F^3) mobile channels ([9]). In a practical system, the method presented in this paper would be complemented by the receiver decoder in order to exploit the coding redundancy, not only for the channel compensation (in a first information recovery step) but also for symbol error correction (decoding step) following the complex field sequence error correction methods ([5] and [6]).

The next section illustrates the scheme and establishes the problem. Section 3 describes the linear block coding characteristics and shows how the structure of the transmitted encoded data can be used in the design of the equalizer. Section 4 applies these results to any linear block-code. Finally section 5 presents some simulation results, where it is possible to see the improvement of the proposed solution.

II. PROBLEM STATEMENT

Lets consider the following two receiving front-ends (Figures 1a and 1b) corresponding to a discrete-time model for a temporal and a spatial diversity receiver, respectively. In the time diversity receiver, the information signal $T[k]$ is transmitted through a mobile channel response $C[k]$, which distorts the signal and degraded by an AWGN term $W[k]$. The received signal is oversampled at B samples per symbol, and introduced in B different branches. In the other hand, for the spatial diversity receiver, the same information signal $T[k]$ is transmitted through B diversity branches. It is distorted by B different channel responses $C^i[k]$ and finally degraded by B AWGN terms $W^i[k]$.

As far as the spatial diversity receiver corresponds to a polyphase representation of the temporal diversity receiver, only the spatial diversity scheme with the following z-transform associated equations will be considered further on:

$$Y^i(z) = T(z)C^i(z) + W^i(z) \quad i = 1,\dots,B \tag{1}$$

Similar equations can be derived for OFDM signals through frequency-flat fading channels in the time domain (see [1], [9]).

As shown in [1] and [9], the equalization process can be designed following a blind criterion. The multiple temporal or spatial diversity branches are combined by means of FIR filters $E^i[k]$ to generate an output $R[k]$:

$$R(z) = \sum_{i=1}^{B} Y^i(z)E^i(z) = T(z)\sum_{i=1}^{B} C^i(z)E^i(z) + \sum_{i=1}^{B} W^i(z)E^i(z) \tag{2}$$

Under noise-free conditions, the perfect equalization criterion requires $R(z)=T(z)$, and therefore:

$$\sum_{i=1}^{B} C^i(z)E^i(z) = 1 \tag{3}$$

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The Bezout equation [8] guarantees that the previous equation has solution if and only if the B channel responses have no common zeros, or in other words $T(z) = g.c.d.\{Y^{i}(z)\}$ [9].

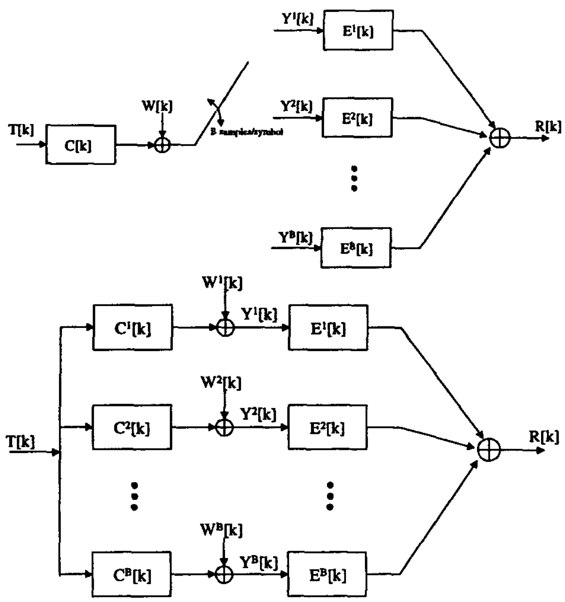

Figure 1a and 1b. Temporal and spatial diversity schemes.

A matrix formulation for the method can be found in [9] and is briefly summarized here in order to approach the problem. As shown in [9], equation (2) can be written in matrix notation as:

$$\mathbf{r} = \mathbf{Y}\mathbf{e} \tag{4}$$

where $\mathbf{r}$ is the equalizer output vector, $\mathbf{Y}$ is a generalized Sylvester matrix with the received data and $\mathbf{e}$ is the equalizer weight vector. The perfect equalization noise-free case can be written as:

$$\mathbf{r} = \alpha\begin{bmatrix}\mathbf{t}\\ \mathbf{0}\end{bmatrix} = \begin{bmatrix}\mathbf{Y}_t\\ \mathbf{Y}_0\end{bmatrix}\mathbf{e} \tag{5}$$

where the received data matrix $\mathbf{Y}$ has been split in two parts, $\mathbf{t}$ is the transmitted data vector and α is an arbitrary multiplicative constant.

In [1] a blind scheme was suggested such that the signal-to-ISI-plus-noise-ratio (SINR) at the equalizer output is maximized, that is:

$$SINR = \frac{\mathbf{e}^H \mathbf{Y}_t^{\,H} \mathbf{Y}_t \mathbf{e}}{\mathbf{e}^H \mathbf{Y}_0^{\,H} \mathbf{Y}_0 \mathbf{e}} \tag{6}$$

In noisy environments, many stability problems with the $\mathbf{Y}_0$ matrix appeared. The goal of the next section is to introduce the use of the redundancy of linear block-codes in the complex field in order to improve the algorithm robustness. Both, systematic and non-systematic codes will be considered.

III. SYSTEMATIC LINEAR BLOCK STRUCTURES

Equation (7) depicts the construction of linear block codes using matrix notation. It describes how to encode a k-symbols data information vector $\mathbf{t}$ using the code generator nxk matrix $\mathbf{G}_c$ to obtain the n-symbols *code-word* $\mathbf{t}'$:

$$\mathbf{t}' = \mathbf{G}_c \mathbf{t} \tag{7}$$

For the sake of simplicity and without loss of generality, we will consider a *systematic code* for which the encoder matrix (or transform [2] and [7]) $\mathbf{G}_c$ becomes:

$$\mathbf{G}_c = \begin{bmatrix}\mathbf{I}_{kxk}\\ \cdots\cdots\cdots\cdots\\ \mathbf{G}_{r\,(n-k)xk}\end{bmatrix} \tag{8}$$

for any full rank $(n-k)xk$ $\mathbf{G}_r$ matrix. The first k-symbols of the code-word are always identical to the information sequence to be transmitted while the other (n-k) symbols are the redundant symbols, that is:

$$\mathbf{t}' = \left[\mathbf{t}_k^H \;\vdots\; \mathbf{f}_{(n-k)}^H\right]^H \tag{9}$$

The decoder applies the $(n-k)xn$ check matrix $\mathbf{C}_c$ over the mobile channel output data and it is defined as:

$$\mathbf{C}_c = \left[-\mathbf{G}_{r(n-k)xk} \;\vdots\; \mathbf{I}_{(n-k)x(n-k)}\right] \tag{10}$$

and $\mathbf{G}_c$, $\mathbf{C}_c$ matrices are defined such that:

$$\mathbf{C}_c \mathbf{G}_c \mathbf{t} = \mathbf{C}_c \mathbf{t}' = \mathbf{0} \tag{11}$$

Thus, the check matrix can detect changes between the transmitted code and the received information. If both sequences are identical, the null vector will be obtained. On the contrary, the effect of the noise and the ISI channel introduces differences in the received sequence and the product of that sequence with the check matrix differs from the null vector (*residue* or *syndrome*). To ensure that the encoding process maintains constant the symbol energy of all the transmitted symbols we are interested on those transform such that:

$$|\mathbf{g}_1|^2 = |\mathbf{g}_2|^2 = \ldots = |\mathbf{g}_n|^2 = 1 \tag{12}$$

where $|\mathbf{g}_i|$ is the Euclidean norm of the *i-th* row vector of $\mathbf{G}_c$.

Thus, the two equalizer design equations becomes:

$$\begin{aligned}\mathbf{t}' &= \mathbf{Y}_t \mathbf{e}\\ \mathbf{C}_c \mathbf{t}' &= \mathbf{C}_c \mathbf{Y}_t \mathbf{e} = \mathbf{0}\end{aligned} \tag{13}$$

According to equations (13), the new *SNIR* estimate is formulated as:

$$SINR' = \frac{\mathbf{e}^H \mathbf{Y}_t^{\,H} \mathbf{Y}_t \mathbf{e}}{\mathbf{e}^H \mathbf{Y}_t^{\,H} \mathbf{C}_c^{\,H} \mathbf{C}_c \mathbf{Y}_t \mathbf{e}} \tag{14}$$

This new cost function to be optimized is more robust in presence of AWGN because the residual $\mathbf{Y}_0$ matrix is not needed. According to equation (14), the equalizer that maximizes the new signal to noise plus *ISI* ratio corresponds to the generalized eigenvector associated with the maximum generalized eigenvalue:

$$\mathbf{Y}_t^{\,H} \mathbf{Y}_t \mathbf{e} = \lambda_{max} \mathbf{Y}_t^{\,H} \mathbf{C}_c^{\,H} \mathbf{C}_c \mathbf{Y}_t \mathbf{e} \tag{15}$$

As usual, the equalization performance can be optimized if a delay is allowed in *R[k]* in equation (2), and the best equalizer is selected as that one which yields the greatest λ_{max} (maximum *SINR'* in equation (15)).

IV. EXTENSION TO A GENERAL LINEAR ENCODER

The interest of the systematic encoding schemes is the low computational complexity in the decoding process. A more general framework is given when considering non-systematic codes and this is the main goal of the this section. As we will see, the minimum redundancy condition is also derived.

Let us consider, once again, the information vector $\mathbf{t}$ of dimension k and a nxk ($n>k$) full rank linear transform $\mathbf{G}_c$ in the data encoding process, such that the transmitted symbol vector $\mathbf{t}'$ of length n is given by equation (7), being $r=n-k$ the transmitted redundancy. As we see, vector $\mathbf{t}$ is contained in the signal subspace S spanned by the k columns of matrix $\mathbf{G}_c$.

Let's consider the orthogonal subspace $S^{\perp}$ spanned by an $r=n-k$ dimensional orthogonal basis and its associated generation matrix $\mathbf{G}_c^{\perp}$ (*check matrix*). The outputs of the marginal channels $\mathbf{C}^i$ $(i=1,2,...,B)$ to the transmitted data $\mathbf{t}'$ and the channel noise contributions $\mathbf{w}^i$ will force the received data to be contained in the $S \oplus S^{\perp}$. Basically, the projection of the received data in the orthogonal subspace $S^{\perp}$ is used by the equalizer to characterize the channel response and noise distribution [1]. The main point is to establish the minimum required redundancy to ensure the channel equalization. Three conditions are necessary to ensure a correct channel compensation:

Cond. 1: The B channel responses $\mathbf{C}^i$ $(i=1,2,...,B)$ have no common zeros.

Cond. 2: For a channel response length. L, the equalizer length v has to satisfy:

$$v \geq \frac{L-1}{B-1} \qquad (16)$$

Cond. 3: The minimum required redundancy r for the correct channel compensation is given by:

$$r \geq L+v-2 \qquad (17)$$

The Bezout equation guaranties a solution for equation (3) if and only if conditions 1 and 2 are satisfied. The third condition is introduced in this paper to give the minimum redundancy requirement and its proof. If we consider the generalized Silverter matrix containing the channel response $\mathbf{C}^i$ $(i=1,2,...,B)$, the output at each branch will be given by:

$$\mathbf{y}^i = \mathbf{C}^i\mathbf{t}' \quad (i=1,2,...,B) \qquad (18)$$

Combining all branches, the equalizer output will be given by:

$$\mathbf{r} = \mathbf{Y}\mathbf{e} = \mathbf{T}\mathbf{C}\mathbf{e} = \alpha\left[\mathbf{t}'^H \quad \mathbf{0}^T\right]^H \qquad (19)$$

The *residue* or *syndrome* under noise-free conditions has to be null, or in other words:

$$\left[\mathbf{G}_c^{\perp} \quad \mathbf{0}\right]\mathbf{r} = \left[\mathbf{G}_c^{\perp} \quad \mathbf{0}\right]\mathbf{Y}\mathbf{e} = \mathbf{0} \qquad (20)$$

Substituting the received data matrix $\mathbf{Y} = \mathbf{T}\mathbf{C}$ in the previous equation:

$$\left[\mathbf{G}_c^{\perp} \quad \mathbf{0}\right]\mathbf{T} \equiv \mathbf{G}_c^{\perp}\mathbf{T}_d = \mathbf{0} \qquad (21)$$
$$\mathbf{C}\mathbf{e} \neq \mathbf{0}$$

[1] Note that the same idea is used in the blind technique described in [1] when using the information supplied by the residual matrix $\mathbf{Y}_0$ to estimate the residual *ISI* plus noise power.

where $\mathbf{T}_d = \mathbf{T}[1:n,:]$ is by definition a sub-matrix of $\mathbf{T}$ matrix composed by rows *1* to *n*, such that the first column is the transmitted vector $\mathbf{t}'$ (orthogonal to matrix $\mathbf{G}_c^{\perp}$).

Defining:

$$\mathbf{P} \equiv \mathbf{G}_c^{\perp}\mathbf{T}_d = \left[\mathbf{0} \quad \mathbf{p}_2 \quad \cdots \quad \mathbf{p}_{L+v-1}\right] \qquad (22)$$

we have that:

$$\mathbf{G}_c^{\perp}\mathbf{T}_d\mathbf{C}\mathbf{e} = \left[\mathbf{0} \quad \mathbf{p}_2 \quad \cdots \quad \mathbf{p}_{L+v-1}\right]\mathbf{C}\mathbf{e} = \mathbf{0} \qquad (23)$$

If the column vectors $\{\mathbf{p}_i\}$ $i=1,2,...,L+v\text{-}1; i \neq 1$ are linearly independent, the only solution for equation (23) is equivalent to the perfect equalization criterion given in equation (3):

$$\mathbf{C}\mathbf{e} = \alpha\begin{bmatrix}1\\ \mathbf{0}\end{bmatrix} \qquad (24)$$

or, in other words, the minimum required redundancy for a perfect channel compensation becomes:

$$r = rank(\mathbf{G}_c^{\perp}\mathbf{T}_d) \geq L+v-2 \qquad (25)$$

V. SIMULATION RESULTS

The simulations display the percentage of realizations over 1000 realizations for which the equalizer output EbNo was higher than the value indicated in the x-axis. In all cases the transmitted *TDMA* information consisted of 118 *QPSK* data symbols plus 10 redundancy symbols generated with a sub-Hadamard matrix $\mathbf{G}_r$. For both cases, equalization with temporal diversity and spatial diversity, each branch of the equalizer had four coefficients and four (B=4, v=16).

For the spatial diversity scenario, the four channel responses were:

$$C^1(z) = (1+j)+(-0.1-0.2j)z^{-1}+0.4z^{-2}+z^{-3}+0.5z^{-4}$$
$$C^2(z) = 0.1+2z^{-1}+4jz^{-2}+0.2z^{-3}+z^{-4}$$
$$C^3(z) = 0.1j+z^{-1}-0.4jz^{-2}+0.2z^{-3}-0.5z^{-4}$$
$$C^4(z) = (1+0.8j)-2jz^{-1}-0.4jz^{-2}+0.2z^{-3}+(1-0.5j)z^{-4}$$

Notice that the four channels have no common zeros. In order to consider a relatively difficult scenario, the four channels have been selected such that some of them exhibited a high attenuation in certain frequencies (in special channels 1 and 3), with some close zeros and with a non-minimum phase behavior.

Two temporal diversity scenarios have been simulated by taking 4 samples per symbol at the output of channel 1 and 3. The considered pulse was 0.5 roll-off Nyquist's shaping.

In Figure 2, the channel *EbNo* was 12dB. In both plots, the performance of the new method (I) was much better than the performance of the algorithm without linear block codes (II). Notice that the new method guarantees that with only an *EbNo* penalty of 0.35 dB for transmitting the redundant symbols, a recovered EbNo gain always higher than 2 dB for channel 3 and more than 4 dB for channel 1 in the temporal diversity scheme and a significant gain in the spatial diversity receiver.

An interesting point is to compare the performance of the proposed method with the time reference minimum mean square equalizer (Wiener). In these simulations (Figure 3), the performance of the proposed method (doted line) is compared with the behavior of the m.m.s.e. solution (solid line) obtained assuming that all the redundant symbols are *a priori* known for training. Only the spatial diversity receiver has been simulated for EbNo of 12dB and 15dB. In order to establish an upper bound for the performance, the EbNo obtained for the unreal case of considering that all the burst symbols are known during training in a m.m.s.e. equalizer has been also plotted (solid line with TrSeq=All label).

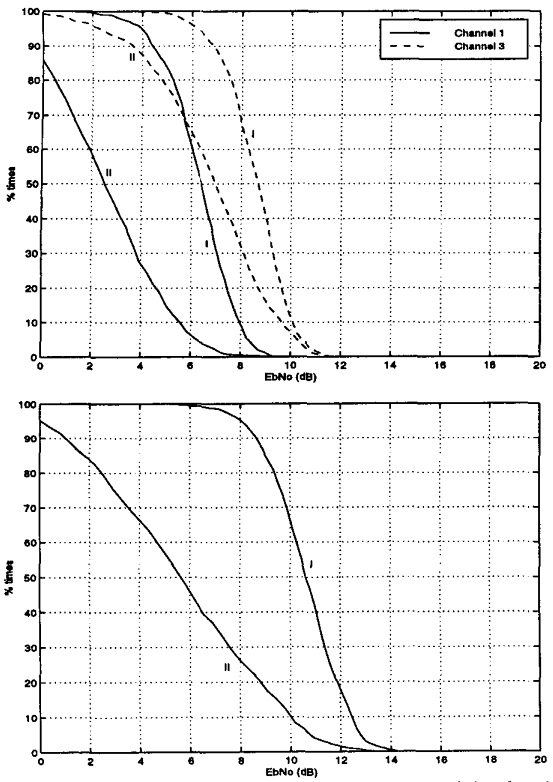

Figure 2. Algorithm performance in temporal diversity (upper plot) and spatial diversity (lower plot) for an *EbNo*=12dB. Comparison of the algorithm with (I) and without (II) the use of linear block codes.

IV. CONCLUSIONS

In this paper linear block codes in the complex field have been introduced for blind equalization of mobile channels in spatial and temporal diversity receivers.

The proposed criterion, combining a blind equalization technique ([1] and [10]) with the redundancy introduced by a systematic linear code, can be applied over TDMA structures with frequency selective mobile channels, DS-CDMA systems and OFDM modulation with frequency flat fading channels.

Spatial and temporal diversity receivers over TDMA structures, in frequency selective mobile channels, have been considered. The results show the performance improvement of this new equalization method over the previous scheme presented in [1] and [10].

The results presented in the current paper can be achieved introducing codes defined over the complex fields ([5],[6]) that, following the structure presented in section 3, could be used to correct the errors at the output of the equalizer.

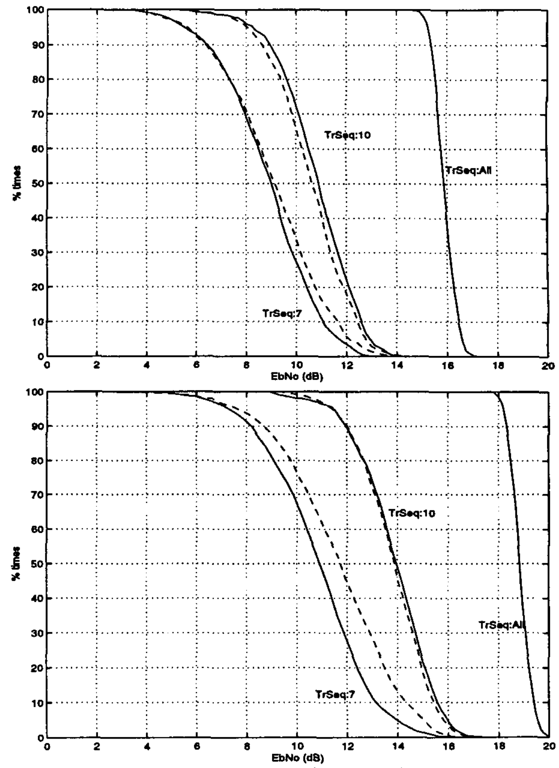

Figure 3. Comparison of the proposed method with a time reference m.m.s.e. equalizer (upper plot for an EbNo=12dBand lower plot for an EbNo=15dB).

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