

Near-Far Resistant CML Propagation Delay Estimation and Multi-user Detection for Asynchronous DS-CDMA Systems

Francesc Rey, Gregori Vázquez, Jaume Riba

Department of Signal Theory and Communications, Polytechnic University of Catalonia

UPC Campus Nord-Mòdul D5, c/ Jordi Girona 1-3, 08034 Barcelona (Spain)

Tel. +34-93-401.64.51 Fax. +34-93-401.64.47

E-mail: {frey, gregori, jriba}@gps.tsc.upc.es

ABSTRACT

Multi-user receivers in asynchronous Direct Sequence Code Division Multiple Access (DS-CDMA) systems require the knowledge of several parameters such as timing delay between users. The goal of this work is to present a *near-far* resistant joint multi-user synchronization and detection algorithm for DS-CDMA systems.

The solution is based on the Conditional Maximum Likelihood (CML) estimation method (classically used in the context of sensor array processing) that leads to a fast convergence algorithm to estimate the time delays among users. At the same time the estimator implements the decorrelating detector, identifying the transmitted symbols for the different users.

I. INTRODUCTION

Relative Propagation Delay among users is one of the most important and challenging parameters that must be estimated for a correct detection of the transmitted information in DS-CDMA. The estimation of this parameter must be very accurate because detectors are very sensitive to it, degrading rapidly its performance in the presence of propagation delay errors. To estimate this timing parameter for each user, it is possible to find in the literature several works e.g., [1],[2],[3] using different estimation techniques.

Maximum Likelihood estimation techniques are classically used to solve the problem of timing estimation. One of the reasons that justify this use is their good performance against noise. Unconditional ML (UML) and Conditional ML (CML) methods can be used to design the propagation delay estimator. Both methods have been studied and applied in array signal processing works [4], but only UML has been classically employed in the field of digital communications [2]. Nevertheless UML methods require considering low SNR scenarios to avoid mathematical difficulties, and so the impact of the self-noise is not taken into account. Although CML solutions have been classically used to solve the

Direction Of Arrival (DOA) problem in sensor arrays area, we will see that our problem can be easily studied under that point of view. The CML estimation method applied to frequency and timing estimation has already been treated in [5],[6]. In the timing estimation case it purposes to work in the Fourier transform domain to abord the problem. After that transformation, the delay parameters τ_k , which appear as an exponential term, play the same role as the directions of arrival in sensor array processing.

In multiuser detection, further parameters to be considered are the received powers from different users. When the received powers become similar and the processing gain is high enough, the receiver complexity can be reduced simply by using a filter-bank, each one matched to a specific user. Nevertheless when there are large received power differences among users, and the orthogonality in the users' sequences is not achieved, the power levels are drastically influenced on the timing delay estimation and on the performance of the receiver. This problem is known as the *near-far* problem.

The goal of this paper is to present an algorithm categorized as Non-Data-Aided (NDA) method, which estimates the timing delay for each user. The proposed solution is based on the CML estimation technique, which does not present self-noise (allowing good performance at high SNR), and is robust in *near-far* scenarios. The algorithm not only estimates the propagation delay among users, but also implements the decorrelating detector to deliver the transmitted information in a multi-user system. Furthermore the use of the Hessian matrix in the parameter estimation makes the convergence of the algorithm very fast, requiring only a few iterations.

Next section studies the signal model used to derive our propagation delay estimator. Section 3 applies the CML estimation method to this signal model and analyzes the solution. It justifies why the CML algorithm is self-noise free and analyzes why it is *near-far* resistant. It also demonstrates that the present contribution, at the same time that estimates the delay parameters, it implements the decorrelating detector. Finally section 4 presents some numerical results and evaluates its performance comparing the results with the Cramér-Rao Bound (CRB).

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II. DISCRETE-TIME SIGNAL MODEL

The received signal in a multi-user DS-CDMA system contains the sum of the K users' signals, each one with a different delay τ_k and power level P^k . The expression for the received waveform is:

$$r(t) = \sum_k \sqrt{P^k} s^k(t - \tau_k) + n(t) \quad (1)$$

where $s^k(t)$ is the k user transmitted signal and $n(t)$ represents the received additive white gaussian noise term. For each user, the transmitted signal can be modeled as:

$$s^k(t) = \sum_{n=0}^{N_c-1} d_n^k b^k(t - nT) \quad (2)$$

where $b^k(t)$ is the signature of the k -user, T the bit duration and d_n^k are the transmitted information bits. The user signature is defined as the convolution of the code $\{c_c^k(i) = \pm 1\}$ and the shaping pulse:

$$b^k(t) = \sum_{i=1}^{L_c} c_c^k(i) p(t - iT_c) \quad (3)$$

where T_c is the chip duration and $p(t)$ the pulse. It is important to remark the band-limited characteristic of the waveform $p(t)$, a required condition to approach the proposed estimation technique.

Finally the received signal expressed as a function of the user's signatures, can be written as:

$$r(t) = \sum_k \sum_n \sqrt{P^k} d_n^k b^k(t - nT - \tau_k) + n(t) \quad (4)$$

As proposed in [6] the CML timing estimation can be treated as a DOA problem working in the frequency transformed domain in the case of band-limited waveforms. Applying the Fourier transform the timing delay appears as a complex exponential whose phase is proportional to the delay. The frequency domain representation of (4) is:

$$\mathcal{F}\{r(t)\} = \sum_{k,n} \sqrt{P^k} d_n^k B^k(f) e^{-j2\pi nT} e^{-j2\pi f\tau_k} + N(f) \quad (5)$$

where $B^k(f)$ is the Fourier transform of the signature.

The algorithm is derived in a discrete-time signal model obtained by sampling the received waveform at N_{sc} samples per chip. Choosing the sampling frequency as $f_s = 1/T_s = N_{sc} G_p 1/T$, and collecting M samples of $r(n)$, the vector \mathbf{r} can be defined as:

$$\mathbf{r} = [r(0) \dots r(M-1)T_s]^T \quad (6)$$

and the frequency transformed domain of equation (5) can be obtained for the discrete-time signal model using the FFT transformation as:

$$\mathbf{y} = FFT\{\mathbf{r}\} = \sum_{k,n} \sqrt{P^k} d_n^k \boldsymbol{\beta}^k \circ e^{jcG_p n f} \circ e^{jc\bar{\tau}_k f} + N(f) \quad (7)$$

where \circ is the Schur product, the processing gain is denoted as G_p , the normalized propagation delay for user k is:

$$\bar{\tau}_k = \frac{\tau_k}{N_{sc}} \quad (8)$$

B' is the number of the FFT points, and c is a constant defined as:

$$c = -\frac{2\pi}{B'} N_{sc} \quad (9)$$

Finally $\boldsymbol{\beta}^k$ is the FFT of the k user signature b^k :

$$\boldsymbol{\beta}^k = [b^k(0) \dots b^k(M-1)T_s]^T \quad (10)$$

$$\boldsymbol{\beta}^k = FFT\{b^k\}$$

And denoting \mathbf{f} as the vector:

$$\mathbf{f} = [0 \ 1 \dots B'-1]^T \quad (11)$$

the notation: $e^{jc\bar{\tau}_k f}$

refers to the vector:

$$[e^{jc\bar{\tau}_k 0} \ e^{jc\bar{\tau}_k 1} \ \dots \ e^{jc\bar{\tau}_k (B'-1)}]^T \quad (12)$$

At this point let us consider equation (7) in a matrix formulation to obtain the following signal model:

$$\mathbf{y} = \mathbf{A}(\bar{\boldsymbol{\tau}})\mathbf{x} + \mathbf{n} \quad (13)$$

where \mathbf{y} , is the observation vector; $\mathbf{A}(\bar{\boldsymbol{\tau}})$ is the model transfer matrix that contains the parameters to estimate; \mathbf{x} contains the set of unknown parameters, that includes the transmitted symbols and the received amplitude for the k users (and, if considered, the signal phase); and finally \mathbf{n} models the additive gaussian noise term. Notice that the proposed model can consider the impact of a slow time-varying frequency flat fading channel (introducing its influence into vector \mathbf{x}). The only condition is that the channel coefficients remain constant along a symbol period. That is, when the coherence time is higher than the symbol period:

$$\Delta t_c \approx 1/fd \gg T \quad (14)$$

Identifying (13) with (7), the vector \mathbf{y} is the FFT of the received vector \mathbf{r} in (6):

$$\mathbf{y} = FFT\{\mathbf{r}\} \quad (15)$$

The unknown parameters (i.e. the transmitted symbols and the received amplitudes) for user k define the vector \mathbf{x}^k :

$$\mathbf{x}^k = [\sqrt{P^k} d_1^k \ \sqrt{P^k} d_2^k \ \dots \ \sqrt{P^k} d_{N_s}^k]^T \quad (16)$$

and collecting \mathbf{x}^k for all users, vector \mathbf{x} is obtained:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}^{1T} & \mathbf{x}^{2T} & \dots & \mathbf{x}^{KT} \end{bmatrix}^T \quad (17)$$

All the terms containing the parameters to estimate τ_k and the known user information (i.e. the user signature) are introduced in matrix $A(\bar{\tau})$.

For each user k and for each symbol n , we can define the vector $\mathbf{a}_\beta^k(c\bar{\tau}_k)$ as:

$$\mathbf{a}_\beta^k(c\bar{\tau}_k) = \boldsymbol{\beta}^k \circ e^{jc\bar{\tau}_k f} \quad (18)$$

and collecting for each user all symbols, matrix $A^k(\bar{\tau}_k)$ is obtained:

$$A^k(\bar{\tau}_k) = [\mathbf{a}_\beta^k(c\bar{\tau}_k) \quad \mathbf{a}_\beta^k(c\bar{\tau}_k + L) \quad \dots \quad \mathbf{a}_\beta^k(c\bar{\tau}_k + L(N_s - 1))] \quad (19)$$

Finally the whole matrix that considers all users by means of the union of all the $A^k(\bar{\tau}_k)$ submatrices is:

$$A(\bar{\tau}) = [A^1(\bar{\tau}_1) \quad A^2(\bar{\tau}_2) \quad \dots \quad A^K(\bar{\tau}_K)] \quad (20)$$

Notice that in the temporal domain, the columns of $A^k(\bar{\tau}_k)$ are scrolled versions of the k -user signature delayed $\bar{\tau}_k$.

Finally the last term to identify is the AWGN term which corresponds to the n vector:

$$\begin{aligned} \mathbf{n} &= FFT\{[n(0) \quad \dots \quad n(M-1)T_s]^T\} \\ E[\mathbf{n}\mathbf{n}^H] &= \sigma_w^2 \mathbf{I} \end{aligned} \quad (21)$$

III. CML BASED PROPAGATION DELAY ESTIMATOR

The cost function in CML estimation parameters for the signal model in (13) is derived from the joint ML cost function that corresponds with:

$$L(\bar{\tau}, \mathbf{x}) = \frac{1}{(\pi\sigma_w^2)^{B'}} e^{-\frac{1}{\sigma_w^2} \text{tr}\{(y - A(\bar{\tau})\mathbf{x})^H (y - A(\bar{\tau})\mathbf{x})\}} \quad (22)$$

The ML function depends on the parameter estimation vector $\bar{\tau}$ and also on the vector \mathbf{x} . Notice that the vector \mathbf{x} contains the set of unknown parameters and thus it is necessary to take some considerations on this vector. The UML method takes into account some statistical information and maximizes the expectation on (22):

$$L(\bar{\tau}) = E_{\mathbf{x}}\{L(\bar{\tau}, \mathbf{x})\} \quad (23)$$

Nevertheless we do not desire to require information on the contents of vector \mathbf{x} . To solve this problem a joint estimation of $\bar{\tau}$ and \mathbf{x} should be done at the expense of increasing the algorithm complexity. In

order to solve this difficulty the CML estimation method is proposed.

The philosophy of the CML consists in estimating the vector \mathbf{x} assuming that $\bar{\tau}$ is known, and then maximize the resulting conditioned ML function. The ML estimation of \mathbf{x} is [7]:

$$\hat{\mathbf{x}}_{ML} = A(\bar{\tau})^\# \mathbf{y} \quad (24)$$

and replacing (24) in (22) the conditioned ML function only depends on $\bar{\tau}$:

$$L(\bar{\tau}) = \frac{1}{(\pi\sigma_w^2)^{B'}} e^{-\frac{B'}{\sigma_w^2} \text{tr}\{P_A^\perp \hat{\mathbf{R}}\}} \quad (25)$$

Maximizing the previous expression is equivalent to minimize the exponent, and so the CML cost function can be obtained as [4]:

$$L_{CML}(\bar{\tau}) = \text{tr}(P_A^\perp \hat{\mathbf{R}}) = \mathbf{y}^H P_A^\perp \mathbf{y} \quad (26)$$

where $P_A^\perp = \mathbf{I} - AA^\#$ is the orthogonal projection matrix onto the orthogonal signal subspace, and $A^\# = (A^H A)^{-1} A^H$ the pseudoinverse of $A(\bar{\tau})$ ¹.

Minimizing expression (26) can be computationally inefficient, so a solution based on its gradient is suggested. In sensor array context, with the same signal model as defined in (13), an expression for the gradient and approximate Hessian are derived in [8]:

$$g_i(\theta) = -2\text{Re}\left[\text{tr}\left[A^\# y y^H P_A^\perp \frac{\partial}{\partial \theta_i} A\right]\right] \quad (27)$$

$$H_{ij}(\theta) \equiv 2\text{Re}\left[\text{tr}\left[A^{\#H} \left(\frac{\partial}{\partial \theta_i} A\right)^H P_A^\perp \frac{\partial}{\partial \theta_j} A A^\# y y^H\right]\right] \quad (28)$$

If the estimation delay error is small (in tracking schemes) with (27) and (28) it is possible to obtain an iterative algorithm for CML estimation as:

$$\bar{\tau}_k = \bar{\tau}_{k-1} - \mathbf{H}(\bar{\tau}_{k-1})^{-1} \mathbf{g}(\bar{\tau}_{k-1}) \quad (29)$$

The use of the Hessian matrix focuses the gradient term towards the minimum and allows a very fast algorithm convergence to the correct value (i.e. only a few iterations are required to obtain an accurate estimation of the timing delay).

A more accurate study of the gradient expression shows that it is computed by the product of two terms. The first term is:

$$A(\bar{\tau})^\# \mathbf{y} \quad (30)$$

¹ To simplify the notation we drop in matrix A its dependence with the parameter estimation vector $\bar{\tau}$.

which corresponds to the estimation of vector x (that contains the transmitted symbols). Observe that this expression is the decorrelating detector solution [9], so the algorithm not only estimates the propagation delay but also implements this sub-optimum detector. The second term is:

$$y^H P_A^\perp \frac{\partial}{\partial \theta_i} A \quad (31)$$

The presence of this term justifies that the proposed algorithm is self-noise free. In [6] the problem is analyzed in timing estimation and it is shown how in the evaluation of the timing delay the possible interference of the adjacent symbols are blocked.

At this point let us justify that the proposed solution is a robust *near-far* estimator. Analyzing (16) and (19) we can see that the k -user received powers P^k are not introduced in the model transfer matrix $A(\bar{\tau})$, but considered in the transmitted information vector x . Following (24), if the different users' signals are received with large power differences (due to the *near-far* problem), it is guaranteed that the method will estimate correctly the different powers and transmitted symbols, which justifies that CML is resistant to *near-far* scenarios.

Finally we derive the Cramér-Rao Bound (CRB) to analyze the performance of the estimator. In [10] the Fischer information matrix (FIM) and the CRB corresponding to the conditional model are derived.

$$J_c(\bar{\tau}) = \frac{2}{\sigma_n^2} \text{Re} \left\{ \left(D^H P_A^\perp D \right) \circ \Gamma^T \right\} \quad (32)$$

$$CRB_c(\bar{\tau}) = J_c^{-1}(\bar{\tau})$$

where the matrix D , defining the columns of matrix $A(\tau)$ as $a(\tau_1), \dots, a(\tau_N)$, is:

$$D = \begin{bmatrix} \frac{\partial}{\partial \tau_1} a(\tau_1) & \dots & \frac{\partial}{\partial \tau_N} a(\tau_N) \end{bmatrix} \quad (33)$$

and Γ is:

$$\Gamma = E \{ x x^H \} \quad (34)$$

In our case, and focussing on user k , the FIM is:

$$J_c(\tau_k) = 2SNR \text{Re} \left\{ \text{tr} \left(\left(\frac{\partial}{\partial \tau_k} A \right)^H P_A^\perp \left(\frac{\partial}{\partial \tau_k} A \right) \right) \right\} \quad (35)$$

IV. SIMULATION RESULTS

In this section we present several simulations and evaluate the root-mean square error (RMSE) of the proposed estimator, comparing the results with the Cramér-Rao Bound (CRB) to analyze the performance of the system.

The number of users in simulations were 2 and 3 users. The spreading codes were Gold sequences with 15 chips per bit. The pulse shaping was a square-root raised cosine pulse with roll-off factor equal to 0.5 and the considered modulation was BPSK. As the performance of the estimator can depend on the particular simulated delays τ , several propagation delays have been considered in the interval:

$$\tau_k \in \left[\frac{-1}{2N_{sc}}, \frac{1}{2N_{sc}} \right] \quad k = 1 \dots K$$

At the receiver the signal was oversampled at 4 samples per chip $N_{sc}=4$. Finally the received signal has been collected in 8 symbols blocks ($M=8G_p N_{sc}$).

Figure 1 shows the convergence of the algorithm with 2 users, $E_b/N_0=15\text{dB}$ and *near-far* ratio $NF=0\text{dB}$. The number of iterations for each block was 2, and a total of 12 symbols were simulated. In order to evaluate the capacity of the algorithm to track delay changes, a sudden change in the timing delays was done after 6 transmitted symbols (a pessimistic situation which does not illustrate a real case).

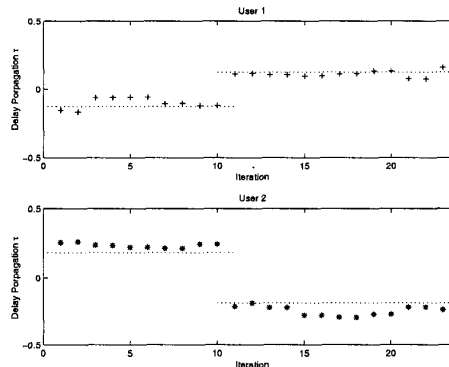


Figure 1. Estimated delay vector τ . $E_b/N_0=15\text{dB}$, $NF=0\text{dB}$, 2 users.

Dotted lines correspond to the users' delays while plot symbols '*' and '+' are the estimated delays. As it can be seen only a few iterations are required to obtain an accurate estimation of the delay parameters (in dotted line). Furthermore the algorithm can follow the sudden change in the received parameters.

Next simulations illustrate the performance of the algorithm in *near-far* environments. The bit-error rate (BER) for the first user is plotted for different NF ratios. Defining the *near-far* ratio as $\max(P^i/P^1)_{i=2,3}$; and computing the E_b/N_0 for the user with the lowest received power (user 1).

Figure 2 presents simulations considering 2 users. As it can be seen for *near-far* ratios equal to 10-20dB the algorithm is quite robust and the performance do not changes substantially. Only if the NF increases over 25-30dB the performance of the algorithm is degraded.

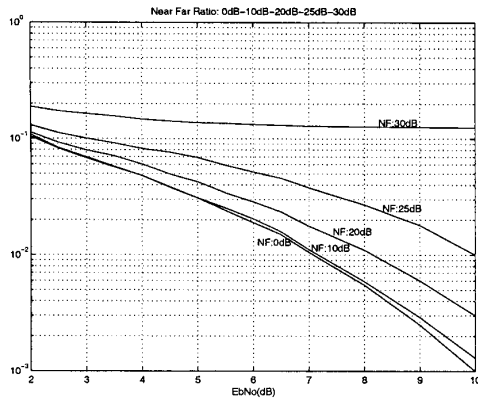


Figure 2. BER 2 users

Similar results are presented in figure 3. In that case the number of simulated users was 3 users. As it can be seen the algorithm is still robust in *near-far* ratios equal to 10-20dB, but the performance is degraded when the power ratio arrives over 20-25dB.

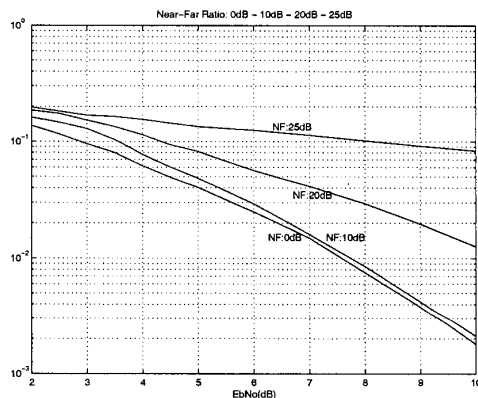


Figure 3. BER 3 users

Finally we analyze the performance of the algorithm comparing the RMSE with the CRB. Figure 4 illustrates, in a scenario with three users, the RMSE of the estimator for different Eb/No values.

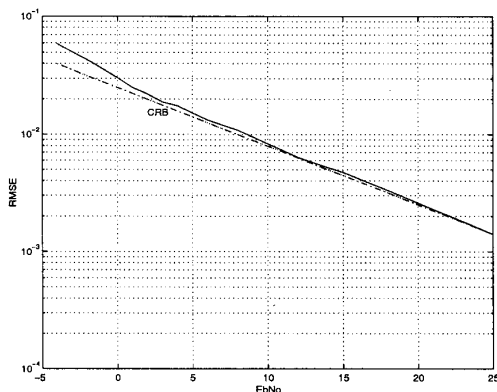


Figure 4. Root-Mean Square Error vs. Eb/No

As it can be seen the estimator attains the CRB derived in (35). Only for low Eb/No ratios the RMSE is higher than the CRB. That is because the ML estimation of x done in (24) for the CML estimator does not consider the noise term (which is important in low Eb/No values).

V. CONCLUSIONS

The proposed algorithm approaches a multi-user timing delay estimator and a symbol decorrelator detector for asynchronous DS-CDMA systems. The main contribution of this paper is the introduction of the CML techniques in the estimation of the users' delays. These estimation techniques have attractive properties because they do not present self-noise (allowing good performance in high SNR scenarios), and take into account the *near-far* problem.

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