MAXIMUM LIKELIHOOD TIME-OF-ARRIVAL ESTIMATION USING ANTENNA ARRAYS: APPLICATION TO GLOBAL NAVIGATION SATELLITE SYSTEMS*

Gonzalo Seco, Juan A. Fernández-Rubio
Dept. of Signal Theory and Communications, Polytechnic University of Catalonia.
Campus Nord UPC, Edifici D5, C/ Jordi Girona, 1-3. 08034 Barcelona, SPAIN.
Tel: +34 93 4016454; fax: +34 93 4016447. e-mail:{gonzalo,juan}@tsc.upc.es

ABSTRACT

The problem of estimating the time-of-arrival (TOA) of a known signal in the presence of interferences and multipath propagation is addressed. This problem is essential in high precision receivers of the Global Navigation Satellite Systems. This paper presents the maximum likelihood TOA estimator when an antenna array is used in the receiver. The desired signal impinges the array with a known direction-of-arrival (DOA) vector, which allows to model all the undesired signal as unknown and arbitrary spatially correlated noise. This simplified model makes only the desired parameters remain in the formulation explicitly, then avoiding complex maximization schemes needed by other models. The fact that estimator is formulated in the frequency domain permits the introduction of the temporal correlation of the noise. Simulation results illustrate the satisfactory performance of the estimator.

1 INTRODUCTION

Time-of-arrival (TOA) estimation is a key task in diverse areas such radar, sonar and communications. Some systems that arouse great interest at present and wherein the propagation-delay estimation is fundamental are the Global Navigation Satellite Systems (GNSS). The multipath propagation and the interferences are the main error sources in the TOA measurement, and they are the responsible for the present GNSS not satisfying the availability, integrity and accuracy requirements of some applications. Many single-antennas techniques to measure the TOA have been investigated , being the most widely used the Delay Lock Loop (DLL) and the Multipath Estimating DLL [Nee94]. However the use of antenna arrays probably yields the most effective techniques to combat the degradation caused by the interferences and the multipath [SF97].Some works have been devoted to the study of array beamforming algorithms in order to cancel only the interferences [MB96], but the problem of mitigating both the interferences and the multipath is more challenging because the coherence in the scenario makes the conventional array techniques fail [SF97]. Other works tackle the problem of estimating the times and directions of arrival of all the received reflections [WL97] and yield to complex optimization algorithms. Moreover these techniques do not consider the particular characteristics of the GNSS, where the only desired TOA is that of the direct signal, whose direction-of-arrival is available in most applications. This paper presents the maximum likelihood TOA of the direct signal estimator when an antenna array is used in the receiver, and it is tailored to the characteristics of the GNSS.

2 ML TIME-OF-ARRIVAL ESTIMATION

2.1 Problem Formulation

Consider an antenna array composed of N sensors arranged in an arbitrary geometry and having arbitrary responses. Assume that M_I interferences and M delayed reflections of the direct signal impinge on the array. For a narrow-band array the received complex N-vector is given by

$$\mathbf{x}(t) = \sum_{i=0}^{M} \alpha_i \mathbf{a}_i s(t-\tau_i) + \sum_{j=1}^{M_I} \mathbf{a}_{I,j} i_j(t) + \mathbf{w}(t) \qquad (1)$$

where sub-index 0 stands for the line-of-sight signal (or signal of interest, SOI) and

$\mathbf{w}(t)$	$Nx1$ thermal noise vector.
α_i	complex amplitude.
$\mathbf{a}$	$Nx1$ steering or DOA vectors.
$s(t)$	arbitrary but known narrow-band signal.
$i_j(t)$	interferences.

All the parameters and signals in the right-hand side of (1) are unknown, except for the steering vector and the shape of the SOI, that is, $\mathbf{a}_o$ and $s(t)$. However, only one parameter is of interest: the time-of-arrival of the direct signal τ_o. Therefore, we are interested in a technique that only estimates this parameter and not all of them in order to reduce the computational load.

In order to derive the maximum likelihood estimator,

*This work has been partially supported by a grant of the *Generalitat de Catalunya* and by National Research Plan of Spain CICYT under grant TIC96-0500-C10-01.

the signal in (1) is modeled as the addition of two terms

$$\mathbf{x}(t) = \alpha_o\, \mathbf{a}_o\, s(t-\tau_o) + \mathbf{n}(t) \tag{2}$$

The first term is the line-of-sight signal (LOSS). The second one, $\mathbf{n}(t)$, is an equivalent noise that includes the contribution of all the undesired signals, that is, the reflections, the interferences and the thermal noise. This simplified model is appropriate for our goals because only the desired parameter remains explicitly, meanwhile the equivalent noise vector contains all the nuisance parameters. This simplification is at the expense of a certain model mismatching in a real scenario, which may result in a bias in the estimates.

Suppose that K samples of the received signal are taken with sampling interval T_s. Then the sampled data can be expressed as[1]

$$\mathbf{X} = \alpha_o \mathbf{a}_o \mathbf{s}^T(\tau_o) + \mathbf{N} \tag{3}$$

where $\mathbf{X} = [\mathbf{x}(t_o) \cdots \mathbf{x}(t_o + (K-1)T_s)]$, $\mathbf{N} = [\mathbf{n}(t_o) \cdots \mathbf{n}(t_o + (K-1)T_s)]$ and $\mathbf{s}(\tau_o) = [s(t_o - \tau_o) \cdots s(t_o + (K-1)T_s - \tau_o)]^T$. The vector $\mathbf{n}(t)$ is assumed to be a zero-mean wide-sense stationary complex Gaussian vector with unknown covariance matrices and independent of the SOI,

$$E\left\{\mathbf{n}[m]\,\mathbf{n}^H[k]\right\} = \mathbf{Q}[m-k] \tag{4}$$

The covariance matrices, which attempts to model both thermal noise and all other interferences and multipath components, are split into an unknown spatial covariance matrix $\mathbf{Q}$ and into a known correlation time series: $\mathbf{Q}[n] = \mathbf{Q}\cdot p[n]$. In order to facilitate the introduction of the temporal correlation into the probability density function (PDF) of the data, the signals are transformed into the frequency domain by means of the FFT, resulting in

$$\mathbf{X}_F = \alpha_o \mathbf{a}_o \mathbf{s}_F^T(\tau_o) + \mathbf{N}_F \tag{5}$$

where $\mathbf{X}_F = [\mathbf{x}_F(\omega_o) \cdots \mathbf{x}_F(\omega_{K-1})]$, $\mathbf{N}_F = [\mathbf{n}(\omega_o) \cdots \mathbf{n}(\omega_{K-1})]$ and $\mathbf{s}_F(\tau_o) = \frac{1}{\sqrt{K}} FFT\{\mathbf{s}(\tau_o)\}^T$. Under the above assumptions, the noise vectors $\mathbf{n}(\omega_i)$ are asymptotically (as the time spanned by the signal samples is much greater than the correlation time of the noise) uncorrelated, zero mean, and Gaussian with covariance matrix $\mathbf{S}(\omega_i) = \mathbf{Q}\cdot P(\omega_i)$.

The problem addressed in this paper may be stated as follows: Given the collection of data $\mathbf{X}_F$ as defined in (5), the vector $\mathbf{a}_o$, the signal $s(t)$ and the power spectral density (PSD) $P(\omega_i)$, estimate the time delay τ_o.

2.2 The ML Estimator

Using the asymptotic PDF of the frequency domain data, the log-likelihood function takes the form

$$L(\tau_o, \alpha_o, \mathbf{Q}) = -KN \ln\pi - K\ln|\mathbf{Q}| - N\sum_{i=0}^{K-1} \ln P(\omega_i) - tr\left\{\mathbf{Q}^{-1}\left(\mathbf{X}_F - \alpha_o\mathbf{a}_o\mathbf{s}_F^T(\tau_o)\right)\mathbf{P}^{-1}\left(\mathbf{X}_F - \alpha_o\mathbf{a}_o\mathbf{s}_F^T(\tau_o)\right)^H\right\} \tag{6}$$

where $\mathbf{P}$ is a KxK diagonal matrix whose entries are the $P(\omega_i)$'s. The problem dimensionality can be reduced analytically. To this end, it can be shown that (6) is maximized, for fixed τ_o and α_o, by

$$\widehat{\mathbf{Q}}_{ML} = \frac{1}{K}\left(\mathbf{X}_F - \alpha_o\mathbf{a}_o\mathbf{s}_F^T(\tau_o)\right)\mathbf{P}^{-1}\left(\mathbf{X}_F - \alpha_o\mathbf{a}_o\mathbf{s}_F^T(\tau_o)\right)^H \tag{7}$$

Now we define the sample correlation as

$$\begin{aligned} \widehat{\mathbf{R}}_{x_F} &= \tfrac{1}{K}\mathbf{X}_F\mathbf{P}^{-1}\mathbf{X}_F^H \quad \widehat{\mathbf{r}}_{xs_F}(\tau_o) = \tfrac{1}{K}\mathbf{X}_F\mathbf{P}^{-1}\mathbf{s}_F^*(\tau_o) \\ \widehat{P}_{s_F} &= \tfrac{1}{K}\mathbf{s}_F^T(\tau_o)\mathbf{P}^{-1}\mathbf{s}_F^*(\tau_o) \end{aligned} \tag{8}$$

$$\widehat{\mathbf{R}}_{n_F}(\tau_o) = \widehat{\mathbf{R}}_{x_F} - \frac{1}{\widehat{P}_{s_F}}\widehat{\mathbf{r}}_{xs_F}(\tau_o)\widehat{\mathbf{r}}_{xs_F}^H(\tau_o) \tag{9}$$

and substituting (7) into (6) yields the concentrated log-likelihood function $L(\tau_o, \alpha_o) = -K\ln\left|\widehat{\mathbf{Q}}_{ML}\right|$, which developed according to the determinant identity $\left|\mathbf{I} + \mathbf{B}^H\mathbf{C}\right| = \left|\mathbf{I} + \mathbf{C}^H\mathbf{B}\right|$ provides the ML amplitude estimate

$$\widehat{\alpha}_{o,ML} = \frac{1}{\widehat{P}_{s_F}} \frac{\mathbf{a}_o^H\widehat{\mathbf{R}}_{n_F}^{-1}(\tau_o)\widehat{\mathbf{r}}_{xs_F}(\tau_o)}{\mathbf{a}_o^H\widehat{\mathbf{R}}_{n_F}^{-1}(\tau_o)\mathbf{a}_o} \tag{10}$$

It can be proved that substituting this value back into the concentrated log-likelihood function it results in

$$L(\tau_o) = K\ln\left(1 + \left(\mathbf{a}_o^H\widehat{\mathbf{R}}_{x_F}^{-1}\mathbf{a}_o\right)^{-1} f(\tau_o)\right) \tag{11}$$

$$f(\tau_o) = \frac{\left|\mathbf{a}_o^H\widehat{\mathbf{R}}_{x_F}^{-1}\mathbf{X}_F\mathbf{P}^{-1}\mathbf{s}_F^*(\tau_o)\right|^2}{\mathbf{s}_F^T(\tau_o)\left(K\mathbf{P}^{-1} - \mathbf{P}^{-1}\mathbf{X}_F^H\widehat{\mathbf{R}}_{x_F}^{-1}\mathbf{X}_F\mathbf{P}^{-1}\right)\mathbf{s}_F^*(\tau_o)} \tag{12}$$

so finally the ML time-of-arrival estimator is given by

$$\boxed{\widehat{\tau}_{o,ML} = \arg\max_{\tau_o} f(\tau_o)} \tag{13}$$

Neglecting the error produced by the finite number of samples in the FFT, the signal vector can be expressed as[2]

$$\mathbf{s}_F(\tau_o) = \mathbf{S}_F\mathbf{v}^*\left(e^{j\frac{2\pi}{KT_s}\tau_o}\right) \tag{14}$$

where $\mathbf{S}_F$ is a diagonal matrix whose entries are proportional to $\mathbf{s}_F(0)$ and $\mathbf{v}(z) = \left(1, z, \ldots, z^{K-1}\right)^T$. Therefore, the MLE is obtained by maximizing the quotient of two polynomials evaluated at the unit circle

$$\widehat{\tau}_{o,ML} = \arg\max_{\tau_o} \left.\frac{p(z)}{q(z)}\right|_{z=e^{j\frac{2\pi}{KT_s}\tau_o}} \tag{15}$$

$$p(z) = \mathbf{v}^T(1/z)\,\mathbf{S}_F\mathbf{P}^{-1}\widehat{\mathbf{R}}_{x_F}^{-1}\mathbf{X}_F\mathbf{a}_o\mathbf{a}_o^H\widehat{\mathbf{R}}_{x_F}^{-1}\mathbf{X}_F\mathbf{P}^{-1}\mathbf{S}_F^H\mathbf{v}(z) \tag{16}$$

[1] The superscripts ^{H}, *, ^{T} denote conjugate transpose, conjugate and transpose, respectively.

[2] The FFT bins are reordered so that the frequencies are in increasing order.

$$q(z) = \mathbf{v}^T(1/z)\mathbf{S}_F\left(K\mathbf{P}^{-1} - \mathbf{P}^{-1}\mathbf{X}_F^H\widehat{\mathbf{R}}_{x_F}^{-1}\mathbf{X}_F\mathbf{P}^{-1}\right)\mathbf{S}_F^H\mathbf{v}(z) \quad (17)$$

If the SINR is not too low, the number of frequency bins with appreciable signal content is moderate and there is an a priori estimate of the TOA then the ML estimate of τ_o can be obtained from the phase of the root z_o of the polynomial $h(z)$ which is the nearest to the value of z computed with the a priori TOA estimate, that is

$$\widehat{\tau}_{o,ML} = \frac{KT_s}{2\pi}\angle z_o \quad (18)$$

being $h(z) = \frac{dp(z)}{dz}q(z) - \frac{dq(z)}{dz}p(z)$.

3 SIMULATION RESULTS

Although the formulation of the MLE has been absolutely general, we are going to apply it to the problem of estimating the TOA of the line-of-sight signal received from a GPS satellite. We deal with the L1-C/A signal, which is a DS-CDMA signal with a $1.023 Mchips/s$ code and a SNR of about $-20dB$. In order that the antenna array is sensitive to the signal reflections, and not only to the noise and the interferences, $s(t)$ is the despreaded GPS signal (the incoming signal after passing through a filter matched to the code) with a SNR approximately equal to $23dB$. This filter colors the noise and the interferences. In all the simulations the array is a uniform linear array with half-wavelength spacing, the DOA of the line-of-sight signal is 30^o and the thermal noise is spatially white. The number of samples is $K = 21$ and rough previous synchronization with an error up to a tenth of a chip is assumed. The RMS (Root Mean Square) errors are computed from 200 Monte Carlo simulations.

3.1 Time-of-arrival estimation variance

Consider first three scenarios where, in addition to the line-of-sight GPS signal, an interference is received from different angles and different powers. Figure 1 shows the RMSE and the CRB in the TOA estimate as a function of the number of antennas. It is clear from this figure the performance improvement achieved thanks to the use of an antenna array, that is, the RMSE is greatly reduced with respect to the single-antenna case. The RMSE approaches the CRB, so the MLE is nearly optimum and the CRB can be used to predict its performance. It can be observed that for a high enough number of antennas the CRB do not depend on the interference power or the interference DOA, but it depends only on the noise power and the number of antennas. Figure 2 represents the RMSE and the CRB vs the interference power for a fixed number of antennas. The RMSE tends to a constant value when the SIR decreases, so the receiver can hold up arbitrarily strong interferences thanks to the use of an antenna array.

3.2 Time-of-arrival estimation bias

In the presence of coherent multipath the received signal can not be modeled as in (2) and the TOA estimator presented herein suffers from a certain bias. However, this bias is much smaller than that of the conventional-single antenna techniques. A quite usual method in the GPS literature to asses the robustness of a certain technique against the multipath propagation is to evaluate the errors produced by a single reflection. This is the reason why in figure 3 we compare the bias produced with a single antenna to that produced with a 7-antenna array. Even if the DOA of the reflection is close to the DOA of the direct signal, there is an important reduction of the bias, specially for relative delays greater than 0.2 chips. If the reflection and the direct signal arrive from well separated directions, the bias is reduced by several orders of magnitude. Finally, figure 4 shows that the use of an increasing number of antennas is effective for reducing the bias.

4 CONCLUSIONS

A technique to estimate the time-of-arrival of a known signal has been presented. It is based on the use of an antenna array and on the knowledge of the direction-of-arrival of the direct signal. A simplified model of the received signal is used, which allows to reduce the computational load and avoids iterative maximization schemes. The simulations show that the errors produced by the interferences and the multipath can be reduced in several orders of magnitude using the investigated ML time-of-arrival estimator together with antenna arrays.

References

[MB96] D.J. Moelker, Y. Bar-Ness. "Adaptive antenna arrays for interference cancellation in GPS and GLONASS receivers". *Proc. IEEE Position, Location and Navigation Symp.* pp. 191-198, 1996.

[Nee94] R.D.J. van Nee, J. Siereveld, P.C. Fenton, B.R. Townsend. "The Multipath Estimating Delay Lock Loop: Approaching theoretical accuracy limits".*Proc. IEEE Position, Location and Navigation Symp.,* pp. 246-251, 1994.

[SF97] G. Seco, J.A. Férnandez Rubio. "Multipath and interference errors reduction in GNSS by joint pseudorange measurement and array beamforming", *First European Symp. on Global Navigation Satellite Systems (GNSS'97),* pp: 606-614, Germany, April 1997.

[WL97] M. Wax, A. Leshem. "Joint estimation of time delays and directions of arrival of multiple reflections of a known signal". *IEEE Trans. on Signal Processing,* **45**(10): 2477-2484, October 1997.

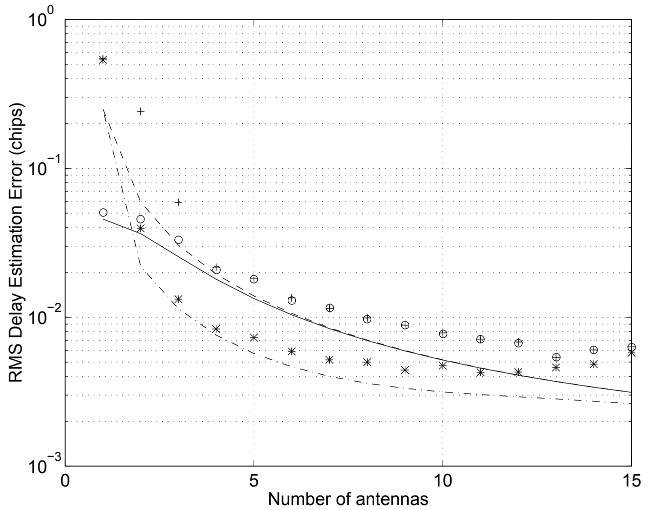

Figure 1: RMSE and CRB in the TOA when the direct signal and one interference are received. $K = 21$ samples. DOA of the direct signal 30^o. In solid line and 'o' the CRB and RMSE, respectively, when the interference arrives from 35^o and $SIR = 10dB$ after the despreading. The dashed line and '+' correspond to the case of interference DOA$= 35^o$ and $SIR = -5dB$.The dash-dotted line and the '$*$' are for interference DOA$= 45^o$ and $SIR = -5dB$.

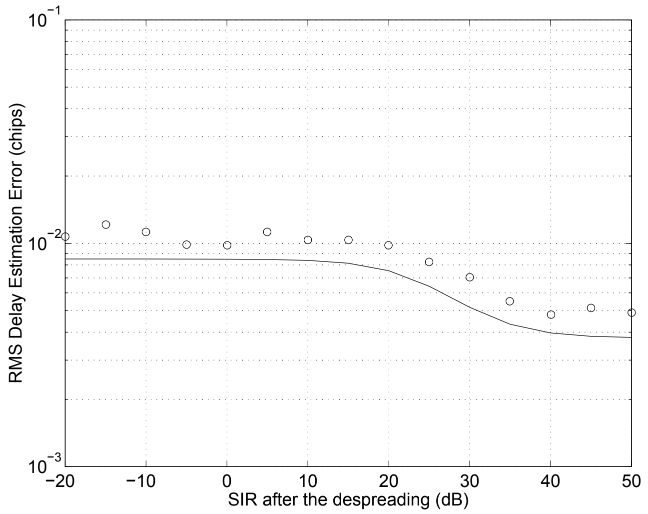

Figure 2: RMSE (solid line) and CRB ('o') in the time-delay estimates when the direct signal and one interference are received. $N = 7$ antennas. $K = 21$ samples. DOA of the direct signal 30^o. DOA of the interference 35^o.

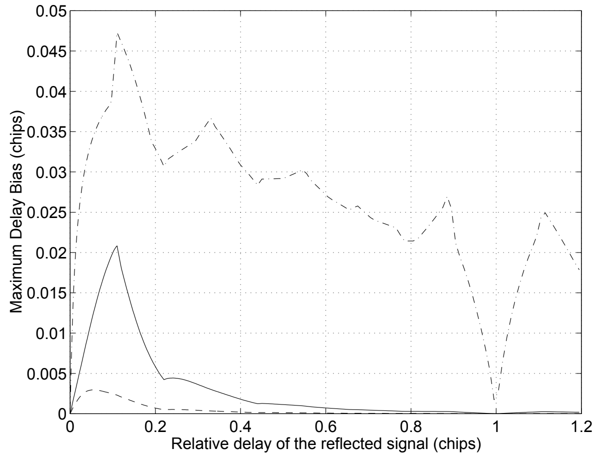

Figure 3: Mean error in the time-delay estimates produced by a single reflection as a function of its delay w.r.t the direct signal. $K = 21$ samples. The reflection is $1.6dB$ below the direct signal. The DOA of the direct signal is 30^o. In dash-dotted line, the error with 1 antenna. The solid and the dashed lines represent the error when $N = 7$ antennas are used, and the DOA of the reflection DOA is 35^o and 45^o, respectively.

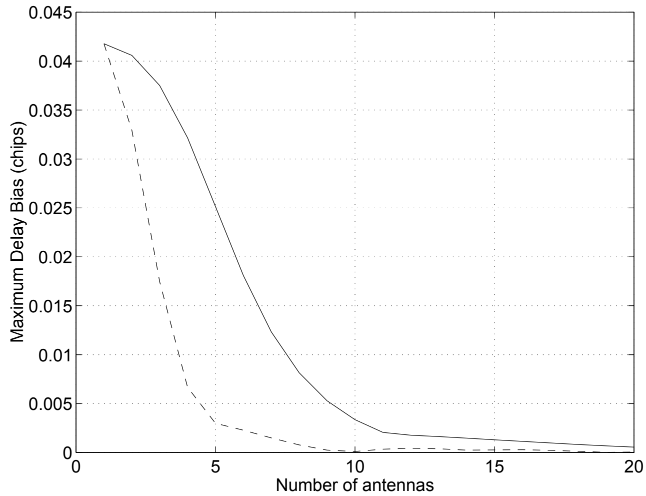

Figure 4: Mean error in the time-delay estimates produced by a single reflection. $K = 21$ samples. The DOA of the direct signal is 30^o. The reflection is delayed 0.15chips and attenuated $1.6dB$ w.r.t. the direct signal. The DOA of the reflection is 35^o for the solid line and 45^o for the dashed line.