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Integrated Game-Theory Modelling for Multi Enterprise-Wide Coordination and Collaboration under Uncertain Competitive Environment

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Abstract

In this work, an integrated Game Theory (GT) approach is developed for the coordination of multi-enterprise Supply Chains (SCs) in a competitive uncertain environment. The conflicting goals of the different participants are solved through coordination contracts using a non-cooperative non-zero-sum Stackelberg game under the leadership of the manufacturer. The Stackelberg payoff matrix is built under the nominal conditions, and then evaluated under different probable uncertain scenarios using a Monte-Carlo simulation. The competition between the Stackelberg game players and the third parties is solved through a Nash Equilibrium game. A novel way to analyze the game outcome is proposed based on a win-win Stackelberg set of “Pareto-frontiers”. The benefits of the resulting MINLP tactical models are illustrated by a case study with different vendors around a client SC. The results show that coordinated decisions lead to higher expected payoffs compared to the standalone case, while also leading to uncertainty reduction.

Keywords: Decentralized multi-participant SC, Coordination, Game Theory, Uncertainty, Competition, Pareto-frontiers.

1. INTRODUCTION

The dynamic competitive nature of the Supply Chain (SC) underscores the interest of the Process System Engineering (PSE) and Operations Research (OR) communities in the SC optimization considering all participants (decentralized decision-making). Such approach should take into consideration individual and global objectives in order to achieve the Enterprise-Wide Optimization (EWO) (Hjaila et al., 2015, 2016a). A SC is a set of entities distributed along different sites to produce intermediate/final products for other SCs and/or final markets (Figure 1). SC tactical managers aim to synchronize and coordinate the resources (physical/economic) and information flows among the SC entities over a specified planning horizon, so as to ensure profitability for the entire company/companies. When the tactical decisions of a SC are synchronized under a common objective function of a single enterprise, a centralized SC takes place (Hjaila et al., 2016b). However, a decentralized SC network takes place when the SC entities belong to different enterprises, the tactical decisions have to be synchronized under the different goals set by the different enterprises involved (Hjaila *et al.*, 2016a). A decentralized SC is represented in Figure 1. The dashed arrows represent the economic sales for one SC enterprise and cost for other SC enterprise, and thus a conflict of interest arises and the whole system becomes difficult to coordinate, especially in a competitive uncertain environment. Since the EWO decisions are included among the Supply Chain Management (SCM) tactical decisions, the coordination between the participating enterprises becomes a necessity. This challenge can be addressed using coordination/collaboration contracts.

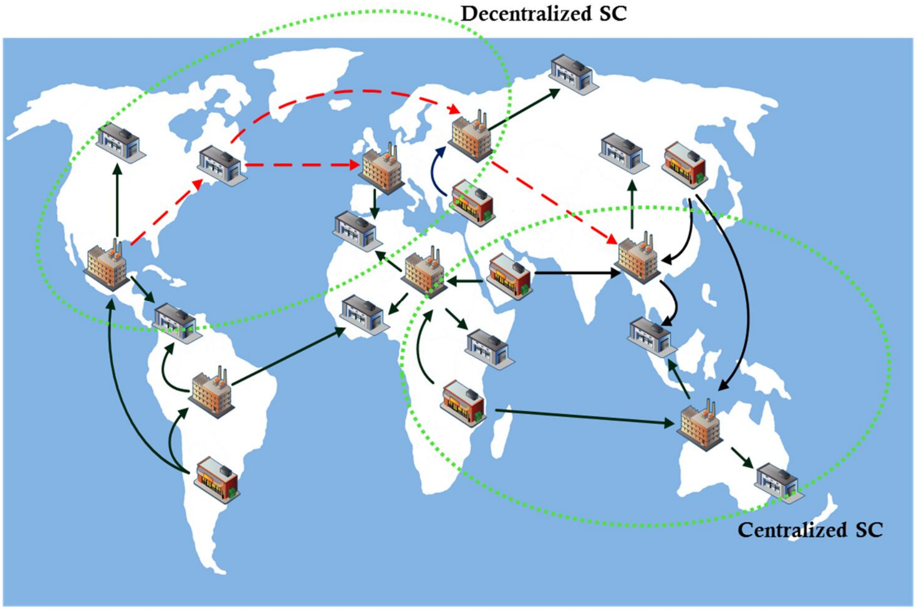


Figure 1- Centralized and decentralized SC

Most current SC tactical models focus on a monopoly market situation where the decisions are guided by one decision-maker, namely a “central decision-maker”, under an overall “centralized” target. However, in a decentralized (multi-enterprise) SC, different independent decision-makers participate with their individual objectives and policies, and each one pursues to optimize its individual performance (non-cooperative). Usually, the decentralized SC process is carried out without considering the risk that may be faced due to (i) the overlapping conflicting decisions, and (ii) the way other enterprises may react, especially when all are interacting with competitive 3rd parties under an uncertain market situation. In such a scenario, two main issues arise: (i) Competition: different enterprises compete for limited supply or limited demand in order to improve their individual benefits, (ii) Conflict of interests: the selling company seeks maximum sale value, while the buyer company seeks a minimum cost value. Such conflicting interactions are represented by the red arrows in Figures 1 and 2. To address these issues and reach an agreement in which each company takes satisfactory decisions in an environment of competing and conflicting goals, Game Theory (GT) provides a suitable platform. Hjaila (2016c) coins for the first time the “multi enterprise-wide coordination (M-EWC)” to cope with the presence of different actors with their operations and finance management problems in large-scale chemical SCs.

During the past decade, GT has witnessed an increased interest from the PSE, OR and management science communities, as its necessity to incorporate various decision makers into the planning problems escalates. This can be seen from the proliferation of game-theoretic publications in SCM (i.e. Leng and Parlar, 2010; Zhao et al., 2010; Banaszewski et al., 2013; Zhao et al., 2013; Cao et al., 2013; Li et al., 2013; Chu and You, 2014; Yue and You, 2014, Chu et al., 2015; Hjaila et al., 2015, 2016b).

Next, we would like to highlight some important definitions that are used throughout this manuscript. Within the GT perspective, the enterprises with conflicting or competing objectives are considered as game players. The objective function in GT is called the "payoff function" in case of maximizing the profits, or "loss function" in case of minimizing the cost. The possible actions/reactions of the game players are referred as "strategies". A game can be either a zero-sum-game or non-zero-sum game. For zero-sum-games, the amount gained by one player is the same as the amount lost by the other player, in this case, it is not possible determining when a player should cooperate to obtain a cumulative benefit. For non-zero-sum games, the amount gained by one player is not the same as the amount lost by the other players, so the gains of one player cannot be deduced from the gains of the other players. The game can be considered as "dynamic" when the game is repeated sequentially. Depending on the interaction among players, games can be classified as cooperative or non-cooperative. For cooperative games, the players are supposed to agree on forming a coalition towards optimizing one shared objective under a given set of conditions. For non-cooperative games, the game players seek, independently, to optimize their individual benefits. Nash Equilibrium (NE) (Nash, 1951) and Stackelberg (Stackelberg, 2011) games are approaches to solve non-cooperative games. NE is used when the roles of the game players are symmetric (i.e., no one is leading the game), and they simultaneously make their decisions. The NE solution is reached when none of the players can improve her/his benefits by changing just her/his own strategy, unilaterally. On the other hand, the Stackelberg game can be played when there is a conflict of interests among different players and their roles are not symmetric. That is, one of the players moves before the others; this player is leading the game by playing the first move to achieve its best results taking into consideration that the other players are seeking the same objective.

Many works have been carried out to optimize decentralized SCs through GT based on cooperative and non-cooperative systems. Based on cooperative systems, Banaszewski et al. (2013) propose a cooperative multi-agent auction-protocol for a Brazilian oil SC to identify the oil products distribution plan (types, amounts, and allocation). However, the multi-agent-based systems are built on cooperative enterprises, in which they agree to form a coalition towards a shared objective, regardless of the individual objectives.

Zhao et al., (2010) develop a cooperative game model for the optimization of a decentralized manufacturer-retailer SC based on option-contracts under the condition that the manufacturer maximum production matches with the retailer reserved quantities. Later on, Zhao et al. (2013) develop a bi-directional option contract (call option or put option) as a cooperative game strategy for optimizing a manufacturer- retailer SC network. For the call option, the manufacturer must assign a specific price to a specific amount of products; while for the put option, the retailer must pay a penalty or an allowance for returning or cancelling an order. However, the games which are built on cooperative negotiations focus on the global assessment of the payoffs regardless of the individual behavior of the enterprises and the way how they react to different scenarios in a competitive uncertain market situation, which may lead to an inaccurate assessment and ultimately to suboptimal decision-making.

On the other hand, few works deal with the tactical decision-making of decentralized SCs based on non-cooperative games. Leng and Parlar (2010) use the Nash equilibrium game to find the optimal production levels of different competitive suppliers which provide a single manufacturer in an assembly SC. They study different scenarios to identify (i) the optimal production levels of each supplier and (ii) the retail price of the manufacturer. Li et al. (2013) solve the conflicting objectives between a single seller and a single buyer through a non-cooperative game based on the shortage penalty where the seller has to pay an allowance in case of any shortage in the supply. However, their works are based on a simple SC structure where there is one vendor selling to one client. The later acts as the game leader. Moreover, the existence of third parties is not considered in their game models, and the competition among different vendors or different clients is not considered either. Considering different clients, Cao et al. (2013) develop a non-cooperative Stackelberg game model based on revenue sharing for one manufacturer and different retailers SC under the leading role of the manufacturer and considering the uncertainty of the manufacturer production cost. However, in their work, the manufacturer is leading the game based on its SC uncertain conditions regardless of the uncertain reaction of the retailers, which also can lead to SC disruptions. Furthermore, the retailers in their model are obliged to buy from one manufacturer giving them a narrow space of options to negotiate or reject.

Considering different suppliers and different retailers, Yue and You (2014) solve the interaction between different suppliers/retailers and one manufacturer at the strategic level using a model based on GT. The competition between the suppliers/retailers is solved using a NE game, while the interaction between the manufacturer and the suppliers/retailers is modeled as a non-cooperative Stackelberg game under the leading role of the manufacturer. The resulting model is a bi-level optimization model. The follower model is replaced by its Karush–Kuhn–Tucker (KKT) conditions in the leader model. However, the competition among different clients is not considered giving the vendors a narrow set of options, which may lead to partners withdrawing from the game. Furthermore, the follower SC model has to be simplified to use the KKT conditions approach. In addition, the leader constraints the quantity and price on the follower based on deterministic information regardless of its SC uncertain behavior and the uncertain reaction of the follower. This scenario may lead to disruptions that can affect the overall decentralized SC decision-making.

Recently, at the SCM tactical level, Hjaila *et al.* (2016a) propose a non-zero-sum Scenario-Based Dynamic Negotiation (SBDN) for the coordination between different vendors interacting with a manufacturing SC. The authors analyze different scenarios based on cooperative and non-cooperative negotiations and compare them with the standalone scenario. It is worth mentioning that the results of Hjaila *et al.* (2016a) will be used for comparison purposes in this manuscript.

As previously described, most of the literature on GT for decentralized SCs coordination based on either cooperative or non-cooperative games focus on simple SC topologies, where the existence of different competing vendors/clients requires further study.

Moreover, most of the literature tends to linearize the mathematical formulations in order to simplify and mitigate the computational efforts which may lead to lose some practicality and result in sub-optimal decisions. Current non-cooperative GT models for decentralized SCs coordination allow to provide individual decisions based on static cases, without considering the whole SC perspective. Again, it is important to understand how the other participating enterprises react when the monopoly is given to one player (leader “vendor or client”) considering the uncertain interaction among them and their 3rd parties. Therefore, effective games that are able to deal with the firms’ conflicting objectives and their corresponding interaction including their competitive 3rd parties are necessary in order to enhance the enterprise-wide decision-making and to avoid any potential disruption that may lead to lose important partners from the whole system.

Consequently, this paper is aiming to develop an integrated GT method for the optimization of decentralized SCs, by suggesting the best terms for the coordination contracts between enterprises with conflicting/overlapping objectives. The proposed method considers different vendors and different clients, which gives a wide set of options for the game players to negotiate. The conflicting objectives between the vendor (supplier production-distribution SC) and the client (manufacturing-distribution SC) are captured through a non-cooperative non-zero-sum Stackelberg game which is built on the expected win-win principles. It is important to highlight that the proposed approach takes into consideration the uncertain behavior of the enterprises unfolding from the competitive nature of their 3rd parties. The competition between different vendors and clients is expected to lead to NE situation in which the enterprises of main interest (main vendors and clients “Stackelberg game players”) are also competing players. The game outcome is represented as a Stackelberg set of “Pareto frontiers”, where each point corresponds to a possible coordination contract. The Stackelberg set of Pareto frontiers gives a wider set of options for the game players to later negotiate. Such options represent the tradeoff between their different preferences and risk behavior. In this paper, we will examine the effect of the uncertainty of the 3rd parties, on the game players SCs and the decentralized SCs coordination, from different point of views: from the follower side, leader side, and both sides. We will also examine the relationship between the SCs coordination and the uncertainty reduction and how this affects the players’ willingness to collaborate. It is worth mentioning that our integrated GT approach does not seek equilibrium, as in reality, equilibrium does not exist.

The objectives of this work can be summarized as follows:

- To integrate the Stackelberg and NE games in a single comprehensive GT approach.
- To bring the competing 3rd parties into the game, and consider the uncertainty of their market prices.
- To represent the game players through their full SCs together with their competing 3rd parties.
- To develop a Stackelberg set of “Pareto frontiers”.

- To analyze the relationship between the possible coordination contracts and the "uncertainty reduction".

2. Problem Statement and methodology

When addressing decentralized SCs, the SC definition can be extended to a set of enterprises with their facilities/SCs interacting within a global SC network (Figure 2). The SC tactical managers have to identify the resources and the cash flows through the SC nodes that result in acceptable financial returns over a discrete planning horizon. The red arrows in Figure 2 represent the cash flows between the enterprises, where the conflict of interest arises, as each line represents a sale for one enterprise and a cost for other enterprise/s.

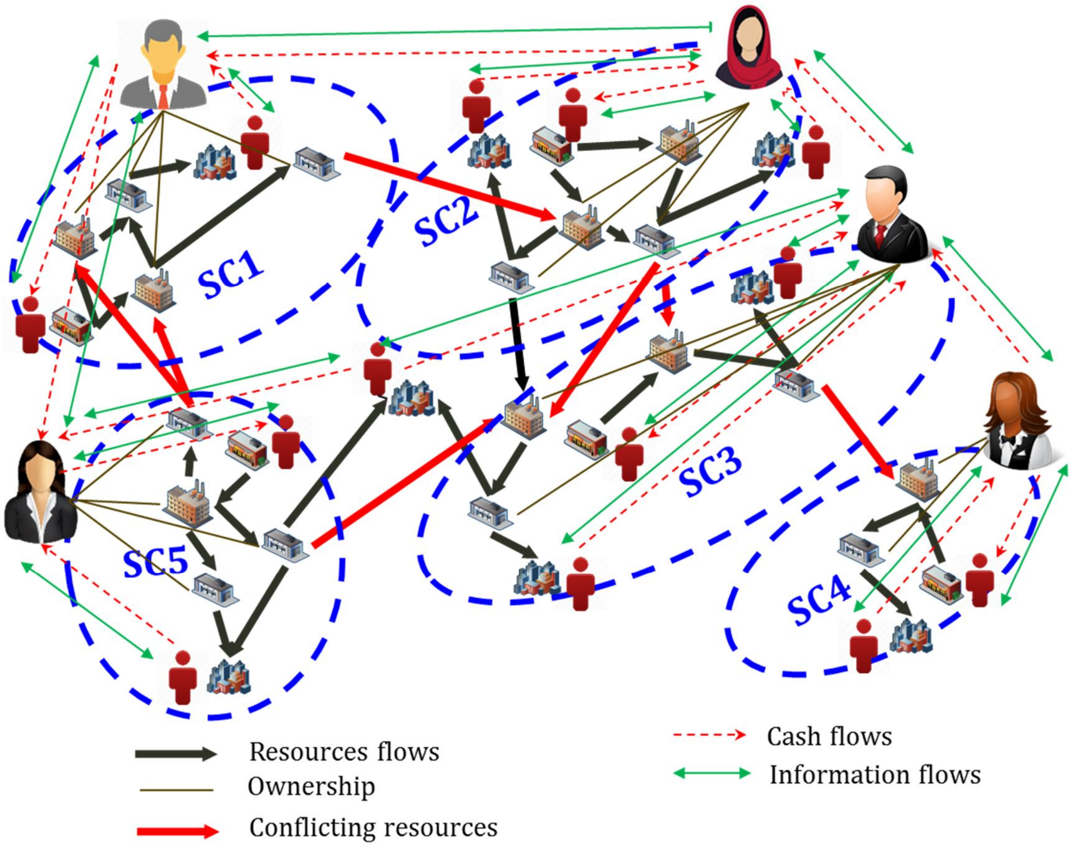


Figure 2- Decentralized SC network

Figure 3 illustrates the problem statement of this paper. Two main enterprises with their full SCs are considered for this study: the vendor and the client with their 3rd parties. The main actors are the main client SC and the main vendor SC. The main vendor is supposed to sell products to the main client (inner component) and to external clients (3rd party). The main client is supposed to purchase this inner component from the main vendor and from external vendors (3rd party). The competition arises between the vendors: (i) the main vendor and the external vendor "3rd party", and (ii) between the clients: main client and the external clients

"3rd party" (Figure 3). Conflicting objectives exist between the main vendor and the main client on the inner component flows and values.

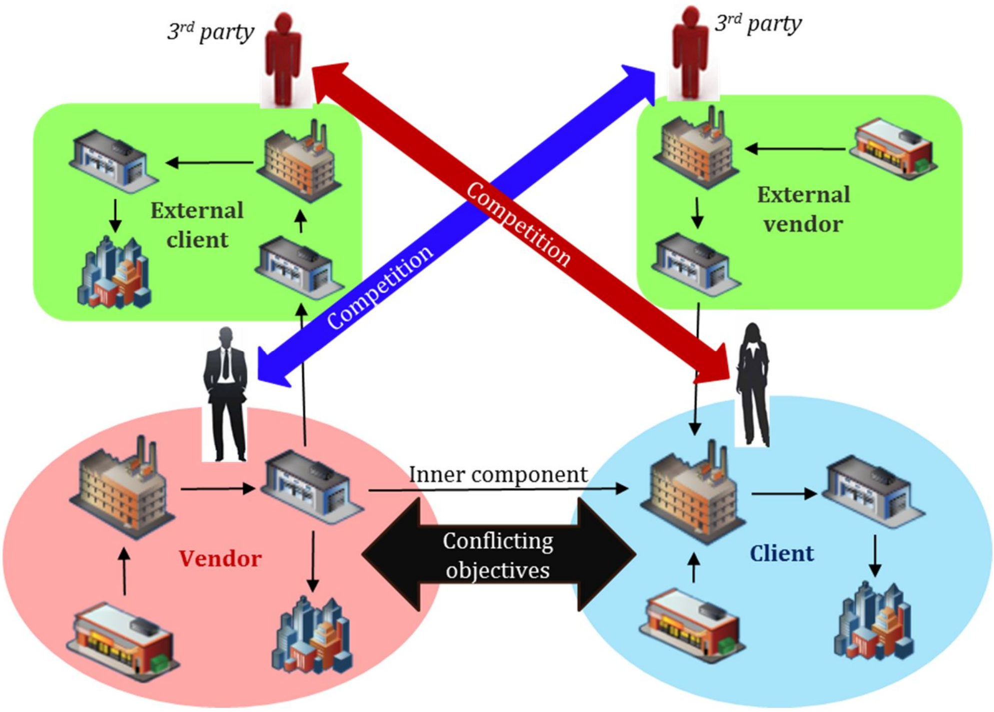


Figure 3- Decentralized SC participants

In order to represent the individual objectives of each enterprise, we model the conflicting objectives and the competition between the different actors as an integrated Stackelberg-NE approach based on non-cooperative games. The Stackelberg game is to capture the conflicting objectives, while the NE game is to capture the competition among the players of interest.

2.1 Stackelberg Game:

Under win-win (nominal/expected) principles, the conflicting goals of the main vendor and the main client have been modeled through non-cooperative non-symmetrical roles, non-zero-sum single-leader single-follower Stackelberg game, under the leading role of the main client. The Stackelberg game players are the main client "as the leader" and the main vendor "as the follower". The Stackelberg game item is the inner component and the coordination contract must include the transfer price of the game item and the inner component flows (physical/economic) between their SCs over a discrete planning horizon. The game reaction function is identified to be the physical flows of the inner component from the follower SC to each manufacturing plant of the leader SC. Based on the available information that each player possesses about the other player, each one acts to optimize its SC individual payoff by taking into account that the other player is pursuing the same objective. The leader player makes the first move of the Stackelberg game anticipating the reaction of the follower by offering the transfer price of the game item. Consequently, the follower player reacts by optimizing its production plan to provide the offered amount of the game item (Figure 4). This is repeated until the Stackelberg Payoff matrix is built. Each cell of the Stackelberg Payoff matrix corresponds to a possible coordination contract (i.e. transfer price and quantity demanded flows). It is worth mentioning that the Stackelberg Payoff matrix depends on the knowledge that each player has previously acquired about the other, and therefore different solutions might be found.

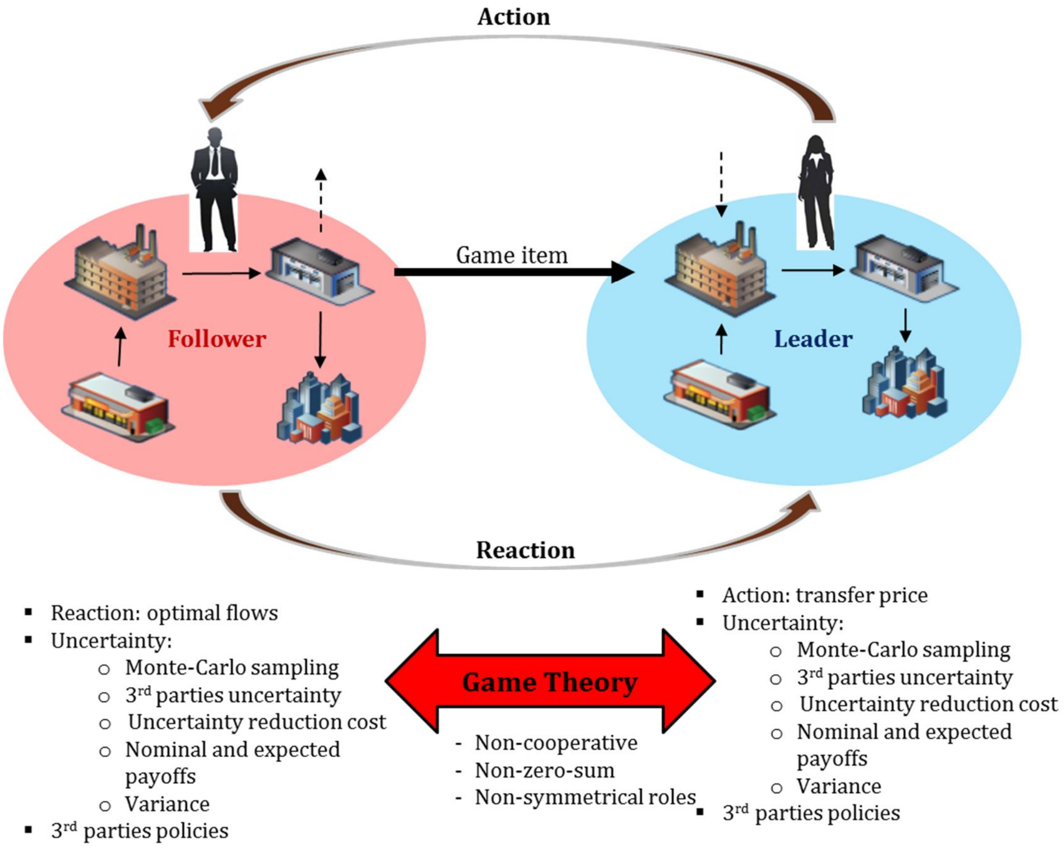


Figure 4- Integrated-GT methodology

Next, the Stackelberg's Payoff matrix is evaluated using a Monte Carlo Simulation which considers:

i) The uncertain behavior of the follower SC resulting from the uncertain prices of the resources to/from its 3rd party.

ii) The uncertain behavior of the leader SC resulting from the uncertain prices of the resources from its 3rd party.

iii) The uncertain behavior of both the follower and leader SCs 3rd parties.

The expected payoffs of the game players are obtained on the basis of generated scenarios for their external conditions (3rd parties). The Stackelberg game output is represented as a Stackelberg set of “Pareto frontiers” that guarantees win-win outcomes (nominal/expected). Both game players must carefully evaluate the game outcome, based on their expected payoffs and respective variances.

2.2 Nash-Equilibrium (NE):

The NE game is used to find the best strategy for the competing enterprises, in which none of the NE game players can improve her/his payoff by changing only her/his strategy while the other player's strategies remain unchanged. The competing players are (Figure 3):

i) The vendors: the main vendor (Stackelberg game follower player) and the external vendor compete to sell resources to the main cient (Stackelberg game leader player).

ii) The clients: the main client (Stackelberg game leader player) and the external client compete to purchase resources from the main vendor (Stackelberg game follower player).

The idea of the NE game is that each player is playing her/his best move taking into consideration that the other player is playing also his/her best in a simultaneous way. To do so, each NE competing game player must consider the best strategy of the other competing player. The NE solution "equilibrium" is achieved when none of them can improve his/her payoff by making any more move.

2.3 Integrated Stackelberg-NE Game:

To integrate the Stackelberg and the NE games, each Stackelberg game player must consider the optimal strategy of the competing 3rd party (NE-game player) when making the Stackelberg move (offering transfer price/amounts). In other words, the main client must consider the optimal price of the competing external client (3rd party) when offering the price, and the main vendor also as a Stackelberg game player must consider the optimal quantity that the external vendor (3rd party) offers to the main client. This means that the main client and the main vendor play different roles: Stackelberg and NE game players.

Finally, the decisions achieved are the raw material (RM) acquisition and 3rd party prices, the inner component production, storage, and distribution levels.

3. Mathematical model

A generic tactical model is developed which integrates the Stackelberg-game with the NE game in one mathematical formulation. In the next sections, we will elaborate the GT theoretical models separately (sections 3.1 & 3.2). Then, both models will be integrated into a single novel GT theoretical model (section 3.3). Afterward, the single model will be translated into a SC tactical multi-enterprise model which is able to capture the competition and the conflicting objectives among various participants.

3.1 Stackelberg-game theoretical model

Mathematically, a single-leader single-follower Stackelberg game forms a bi-level model (Colson *et al.*, 2007), where the leader SC model is considered at the upper-level problem, and the follower SC model is considered at the lower-level problem. The idea of the bi-level formulation is that the leader makes her/his action taking into consideration the optimal decisions of the follower, as both the upper-level and the lower-level problems are solved simultaneously. Eqns. (1) & (2) summarize the bi-level model formulation. The terms *Z* and *z* are the upper-level and lower-level objective functions, respectively. *X* and *Y* represent the upper-level and lower-level decision variables; *G* and *H* represent the upper-level inequality and equality constraints, while *g* and *h* represent the lower-level inequality and equality constraints. It can be noticed that the constraints of the upper-level problem depend on both the upper and lower levels decision variables (*x* and *y*). The Stackelberg-game leader player is represented by *L*, and the Stackelberg-game follower player is represented by *F*.

$$\max_{x\in X,y} Z_L(x,y)\begin{cases} H_L(x,y)=0 \\ \\ G_L(x,y)\le 0 \end{cases} \tag{1}$$

$$\text{Where } y\in\max_{y\in Y,x} z_F(x,y)\begin{cases} h_F(x,y)=0 \\ \\ g_F(x,y)\le 0 \end{cases} \tag{2}$$

In case the follower SC model is convex and regular, the bi-level model can be formulated by replacing the lower-level model by its Karush-Kuhn-Tucker (KKT) conditions (Bard, 1998; Colson *et al.*, 2007), thus transforming it into constraints in the leader SC optimization model (upper-level). This manipulation results in a monolithic model that can be solved at once.

3.2 Nash-Equilibrium game theoretical model

Assuming a NE game with *i* number of players, where $i\in\{1,2,...,I\}$, k_i is the strategy of player *i*, and k_{-i} is the strategy of the rest of the competing players (all players except *i*). The NE equilibrium is achieved when all competitive players make their strategies simultaneously by taking into consideration the strategies of the rest of the players. The objective function $f_i(k)$ is to maximize the payoff of player *i* (Eq. (3)).

$$\text{O.F.}\quad \max_{k_i} f_i(k_i,k_{-i}) \qquad \forall i\in I \tag{3}$$

The NE equilibrium strategy k^* is achieved when none of the players can improve her/his payoffs by changing only her/his own strategy (Eq. (4)).

$$f_i(k_i^*,k_{-i}^*)\ge f_i(k_i,k_{-i}^*) \qquad \forall i\in I \tag{4}$$

3.3 Integrated Stackelberg Game:

Here, we integrate the Stackelberg and NE theoretical game models into one algorithm (Eqns. (5)-(10)), considering that the Stackelberg-game players are also NE players (see Figure 3). When (*i*) represents the NE-game competing vendors' players and *j* represents the NE-game competing clients' players. Then, the Stackelberg-game Leader player as NE-game player $L \equiv j$ ” competes with (-j) players and the Stackelberg-game follower player *F* “as NE-game player $F \equiv i$ competes with (-*i*) players. So, to maximize the payoff of the Stackelberg-game leader ($L \equiv j$), the strategy k_{-j} of the rest of the clients must be considered in the objective function of the Stackelberg game leader (Eq. (5)). To maximize the payoff of the Stackelberg-game follower *F*, as NE-game player, the strategy of the rest of the vendors NE-game players q_{-i} must be considered in the objective function of the follower (Eq. (6)).

$$\max_{x\in X, k\in X, y} Z_{L\in J}(x,y,k_L,k_{-j}) \begin{cases} H_L(x,y,k_L,k_{-j}) = 0 \\ G_L(x,y,k_L,k_{-j}) \le 0 \end{cases} \tag{5}$$

$$\textit{Where } y \in \max_{y\in Y, q\in Y, x} z_{F\in I}(x,y,q_F,q_{-i}) \begin{cases} h_F(x,y,q_F,q_{-i}) = 0 \\ g_F(x,y,q_F,q_{-i}) \le 0 \end{cases} \tag{6}$$

From the Stackelberg leader side, her/his NE equilibrium strategy k_L^* is achieved when she/he cannot improve her/his payoff by changing only her/his own strategy k_j (Eq. (7)). From the Stackelberg-follower side as NE-game player, her/his NE equilibrium strategy (q_F^*) is achieved when she/he cannot improve her/his payoff by changing her/his own strategy (q_F) (Eq. (8)).

$$Z_{L\in J}(x^*,y^*,k_L^*,k_{-j}^*) \ge Z_{L\in J}(x,y,k_L,k_{-j}^*) \qquad \forall j \in J \tag{7}$$

$$z_{F\in I}(x^*,y^*,q_F^*,q_{-i}^*) \ge z_{F\in I}(x,y,q_F,q_{-i}^*) \qquad \forall i \in I \tag{8}$$

The NE equilibrium strategy k_{-j}^* for the external clients (-j) competing with the Stackelberg-game leader is achieved when none of them can improve her/his benefits by changing only her/his own strategy (Eq. (9)). The NE equilibrium strategy q_{-i}^* for the external vendors (-*i*) competing with the Stackelberg-game follower (*F*) is achieved when none of the external vendors NE game players can improve her/his benefits by changing only her/his own strategy (Eq. (10)).

$$f_{-j}(k_{-j}^*,k_L^*) \ge f_{-j}(k_{-j},k_L^*) \qquad \forall j \in J \tag{9}$$

$$f_{-i}(q_{-i}^{*}, q_F^{*}) \geq f_{-i}(q_{-i}, q_F^{*}) \qquad \forall i \in I \tag{10}$$

Then the solution of the integrated GT algorithm can be considered as the Stackelberg-NE equilibrium. In the next section, we incorporate the above integrated-GT algorithm within a practical multi-enterprise SC tactical game model considering the uncertainty of the competing third parties.

3.4 The tactical integrated-GT model

To represent the integrated-GT approach (Stackelberg-NE) within a decentralized SC framework, a set of enterprises supply chains (*sc*1, *sc*2... *SC*) is considered with their new subsets linking each *SC* to its corresponding enterprise game player SC: the Stackelberg-leader "NE- game player" (*L*), the Stackelberg-follower "NE-game player" (*F*), the external vendor "NE-game player" (*V*), and the external client "NE-game player" (*C*). The model formulation also includes the set of resources *r*, suppliers *s*, production plants *pl*, warehouse/distribution centers *w*, and markets *m* (Figure 5). The Stackelberg-game item to be negotiated is the inner component, which is represented by the resource subset (*r'*).The Stackelberg players' strategies are represented in the model formulation as follows (see Figure 5):

i) The Stackelberg-game leader strategy is the action which corresponds to the unit transfer price $pL_{r',sc}$ of the game item (the strategy *x* in Eqns. (5)-(8))

ii) The Stackelberg-game follower strategy is the reaction $QF_{r',sc,t}$ which corresponds to the resource amounts offered to the leader from the follower SC each time period *t*. $QF_{r',sc,t}$, represents the follower strategy (*y*) in Eqns. (5)-(8).

The NE-game competitive players are represented in the mathematical formulation as the Stackelberg leader (L) and the external client (*C*) on one side, and the Stackelberg follower (*F*) and the external vendor (*V*) on the other side (see Figure 5). $L \,\&\, C \in J$, and $F \,\&\, V \in I$, in Eqns. (5)-(10). The NE-game players' strategies are represented as follows:

i) The prices that each client offers to the main vendor, $pL_{r',sc}$ and $pC_{r',sc,t}$, represent the NE-game strategies k_L and k_{-j} in Eqns. (5)-(10).

ii) The quantities $QF_{r',sc,t}$ and $VL_{r',sc,t}$ that each vendor offers to the Stackelberg leader represent the NE –game strategies q_F and q_{-i} in Eqns. (5)-(10).

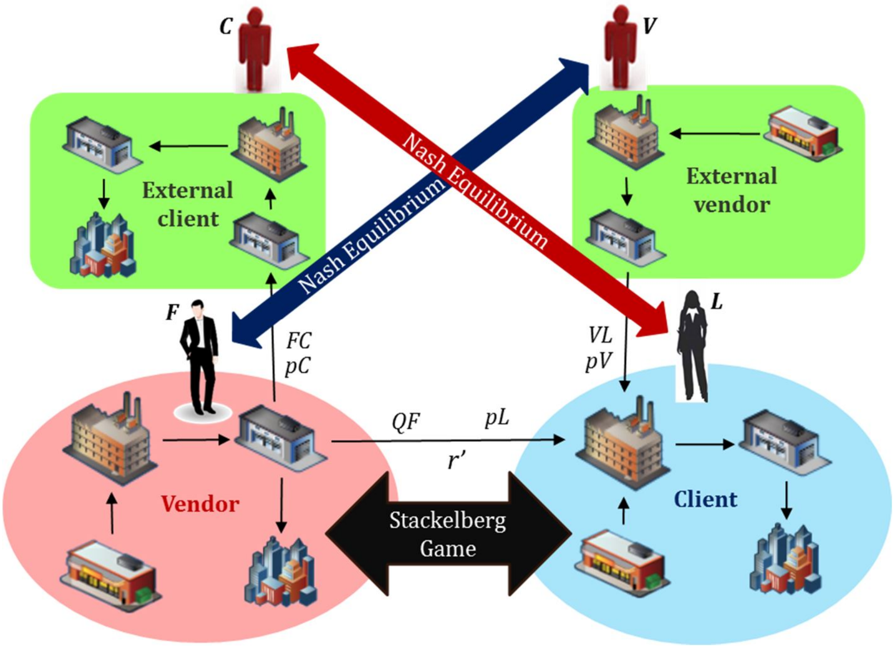


Figure 5- Integrated-Game Theory main items

As the game is non cooperative, the objective function is to maximize the individual payoffs ($Payoff_{sc}$) of the game players SCs (Eq. (11)),

$$Payoff_{sc} = SALE_{sc} - COST_{sc} \qquad \forall sc \in SC \tag{11}$$

The final customer demand ($xdem_{r,sc,m,t}$) of a resource may be satisfied from any participating supply chains (Eq. (12)). $MK_{r,w,sc,m,t}$ represents the resource flows from the warehouses *w* to the final customers *m*.

$$\sum_{w \in W} MK_{r,w,sc,m,t} \le xdem_{r,sc,m,t} \qquad \forall sc \in SC; r \in R; m \in M; t \in T \tag{12}$$

Eq. (13) illustrates the mass balance of the game item resource r' at the warehouses of the game players SCs. $ST_{r',w,sc,t}$ corresponds to the storage levels of r' at warehouse w each time period t; while, $FPD_{r',pl,sc,t}$ corresponds to the follower SC production levels of r' in the production plants pl each planning time period t. $QFL_{r',w,sc,w',t}$ represents the quantity flows of r' from the warehouses w of the follower SC to the warehouses w' of the leader SC. $FC_{r',w,sc,w',t}$ represents the quantity flows of r' from the warehouses w of the follower SC to the the warehouse w' of the external client C each planning time period t.

$QL_{r',w',w,sc,t}$ corresponds to the quantity flows of r' at the warehouses w of the leader SC from the follower SC warehouses w'; while, $VL_{r',w',w,sc,t}$ represents the quantity of r' purchased from the external vendors SC. $LPRD_{r',r,pl,sc,t}$ is the production levels of resource r (intermediate product, final product, etc.) from r' in the leader SC production plants pl each time period t, based on the production recipe represented by $fac_{r',r,sc}$, and assuming linear correlations. $FC_{r',w',w,sc}$ is the quantity flows of r' to the warehouses w of the client SC from the warehouses w' of the follower SC. $CP_{w,pl,sc,t}$ corresponds to the quantity flows from warehouses w to the client *sc* production plants pl. $VL_{r',w,w',sc}$ represents the quantity flows of r' from the warehouses w of the external vendor SC to the warehouses w' of the leader SC, and $VP_{w,pl,sc,t}$ corresponds to the quantity flows from warehouses w of the external vendor SC to production plants pl each time period t.

$$
\begin{aligned}
& ST_{r',w,sc,t} + ST^{stock}_{r',w,sc,t} - ST_{r',w,sc,t-1} = \sum_{sc\in F}\sum_{pl\in PL} FPD_{r',pl,sc,t} - \sum_{sc\in F}\sum_{\substack{w'\in W\\ w\neq w'}} QFL_{r',w,sc,w',t} - \sum_{sc\in F}\sum_{\substack{w'\in W\\ w\neq w'}} FC_{r',w,sc,w',t} \\
& + \sum_{sc\in L}\sum_{\substack{w'\in W\\ w\neq w'}} QL_{r',w',w,sc,t} + \sum_{sc\in L}\sum_{\substack{w'\in W\\ w\neq w'}} VL_{r',w',w,sc,t} - \sum_{sc\in L}\sum_{\substack{r\in R\\ r\neq r'}}\sum_{pl\in PL} LPRD_{r',r,pl,sc,t}\,.\,fac_{r',r,sc} \\
& + \sum_{sc\in C}\sum_{\substack{w'\in W\\ w\neq w'}} FC_{r',w',w,sc} - \sum_{sc\in C}\sum_{pl\in PL} CP_{w,pl,sc,t} - \sum_{sc\in V}\sum_{\substack{w'\in W\\ w'\neq w}} VL_{r',w,w',sc} + \sum_{sc\in V}\sum_{pl\in PL} VP_{w,pl,sc,t} \\
& \forall r\in R; sc\in SC; w\in W; t\in T
\end{aligned}
\tag{13}
$$

Here, it can be seen the generality of the tactical game model, as it can cope with all kind of game players (Stackelberg and NE) SCs in the same model formulation, including 3rd parties. For example, if the SC of interest corresponds to a specific game player, then simply, the other terms of the other game players can be eliminated. The model formulation is flexible enough to consider all possible links around the leader/follower SCs. It can also be used for standalone cases, by eliminating the interaction flows. The model formulation can also be adapted to centralized SCs by eliminating the inner component costs between the participating SCs.

Eqns. (14) & (15) represent the production and storage capacities, respectively.

$$PRD^{\min}_{r,pl,sc} \leq prd_{r,pl,sc,t} \leq PRD^{\max}_{r,pl,sc} \qquad \forall r\in R; pl\in PL; sc\in SC; t\in T \tag{14}$$

$$ST^{\min}_{r,w,sc} \leq ST_{r,w,sc,t} \leq ST^{\max}_{r,w,sc} \qquad \forall r\in R; w\in W; sc\in SC; t\in T \tag{15}$$

Eq. (16) represents the NE-game strategies of the vendors ($QF_{r',sc,t}$ & $VL_{r',w',w,t}$). The quantity offered to the main client by the Stackelberg-game follower $QF_{r',sc,t}$ must be more than or equal to the total quantity needed for the leader SC manufacturing processes minus the quantity flows $VL_{r',w',w,t}$ that are offered from the competitive external vendor. So, if the optimal quantity that the NE-game player V offers to the main client is known, then this optimal value can be substituted in Eq. (16), thus resulting in the NE-game equilibrium between the main vendors.

$$QF_{r',sc,t} \ge \sum_{\substack{r\in R\\ r\neq r'}} \sum_{pl\in PL} LPRD_{r',r,pl,t}.fac_{r',r,pl} - \sum_{\substack{w'\in W\\ w'\neq w}} \sum_{w\in W} VL_{r',w',w,t} \qquad \forall sc \in F; r' \in R; t \in T \tag{16}$$

To avoid infeasible solutions in the leader SC model when considering the follower Stakelberbg-strategy, the follower Stackelberg resources flows $QF_{r',sc,t}$ must be less than the maximum storage capacity and higher than the minimum storage capacity in warehouse *w'* of the leader SC of the game item *r'* (Eq. (17))

$$ST^{min}_{r',w',t} \le QF_{r',sc,t} \le ST^{max}_{r',w',t} \qquad \forall w' \in W; sc \in F; r' \in R; t \in T \tag{17}$$

The total SC sales $SALE_{sc}$ (Eq. (18)) are the sales to the final markets plus the sales to the external clients plus the sales to the leader SC. Here, rp_r is the retail price of the final product (*r*). The term ($QF_{r',sc,t}.pL_{r',sc}$) denotes the sales to the leader player SC when the SC belongs to the follower player. The term ($VL_{r',sc,t}.pV_{r',sc,t}$) represents the sales to the leader when the SC belongs to the external vendor (*V*).

$$\begin{aligned} SALE_{sc} = \sum_{r\in R}\sum_{m\in M}\sum_{t\in T} MK_{r,sc,m,t}, rp_{r,sc,m} &+ \sum_{sc\in F}\sum_{r'\in R}\sum_{t\in T} QF_{r',sc,t}.pL_{r',sc} \\ &+ \sum_{sc\in V}\sum_{r'\in R}\sum_{t\in T} VL_{r',sc,t}.pV_{r',sc,t} \\ &\forall sc \in SC \end{aligned} \tag{18}$$

The SC Cost (Eq. (19)) is the summation of the RM purchase (CRM_{sc}), production ($CPRD_{sc}$), storage (CST_{sc}), transport (CTR_{sc}), Stackelberg-game item ($QF_{r',sc,t}.pL_{r',sc}$) contract cost, purchase cost of resource *r'* from external vendors ($VL_{r',sc,t}.pV_{r',sc,t}$), and purchase cost at the client SC ($FC_{r',sc,t}.pC_{r',sc,t}$), respectively. The term ($QF_{r',sc,t}.pL_{r',sc}$) and the term ($VL_{r',sc,t}.pV_{r',sc,t}$) are the inner component cost from the follower SC and from the external vendor in case the SC of interest belongs to the leader L. The term ($FC_{r',sc,t}.pC_{r',sc,t}$) is the purchase cost of r' from the follower SC in case the SC of interest corresponds to the external client C.

Here, it can be understood the conflicting objectives between the game players, as the same term ($pL_{r'} \cdot QF_{r',sc',t}$) is considered as a sale when the SC belongs to the follower (Eq. (18)), while it is considered as a cost when the SC belongs to the leader (Eq. (19)).

$$
\begin{aligned}
COST_{sc} &= CRM_{sc} + CPRD_{sc} + CST_{sc} + CTR_{sc} \\
&+ \sum_{sc \in L} \sum_{r' \in R} \sum_{t \in T} QF_{r',sc,t} . pL_{r',sc} + \sum_{sc \in L} \sum_{r' \in R} \sum_{t \in T} VL_{r',sc,t} . pV_{r',sc,t} \\
&+ \sum_{sc \in C} \sum_{r' \in R} \sum_{t \in T} FC_{r',sc,t} . pC_{r',sc,t} \\
&\qquad \forall sc \in SC
\end{aligned}
\tag{19}
$$

The RM purchase cost CRM_{sc} (Eq. (20)) from the external suppliers s is the RM purchased quantity ($RM_{r,s,sc,t}$) multiplied by the RM unit price ($vrm_{r,s,sc,t}$), which is computed following the piecewise pricing model proposed in Hjaila et al., (2016b), where different unit prices are offered by the external suppliers s depending on the quantity demanded, based on the elasticity demand theory.

$$
CRM_{sc} = \sum_{r \in R} \sum_{s \in S} \sum_{t \in T} RM_{r,s,sc,t} . vrm_{r,s,sc,t} \qquad \forall sc \in SC \tag{20}
$$

The SC production cost $CPRD_{sc}$ is calculated on the basis of the unit production cost $uprd_{r,pl}$ or resource *r* in each production plant *pl* (Eq. (21))

$$
CPRD_{sc} = \sum_{t \in T} \sum_{pl \in PL} \sum_{r \in R} PRD_{r,pl,sc,t} \cdot uprd_{r,pl} \qquad \forall sc \in SC \tag{21}
$$

The SC storage cost CST_{sc} is computed on the basis of the unit storage cost $ust_{r,w}$ of resource *r* in warehouse *w* each time period *t* (Eq. (22)).

$$
CST_{sc} = \sum_{t \in T} \sum_{w \in W} \sum_{r \in R} ST_{r,w,sc,t} \cdot ust_{r,w} \qquad \forall sc \in SC \tag{22}
$$

The SC transport cost CTR_{sc} is calculated as a function of the travel distance $dist_{r,sc}$ of the resource *r* and the unit transport cost $utr_{r,sc}$ (Eq. (23)).

$$CTR_{sc} = \sum_{r \in R} \sum_{t \in T} Q_{r,sc,t} \cdot dist_{r,sc} \cdot utr_{r,sc} \qquad \forall sc \in SC \qquad (23)$$

Uncertainty Evaluation

The expected payoff $ExPayoff_{sc}$ (Eq. (24)) of the SC is evaluated using *N* generated probable uncertain scenarios around a specific mean (μ) and standard deviation (σ). A Monte Carlo sampling method is used for this regard.

$$ExPayoff_{sc} = \frac{\sum_{n \in N} Payoff_{sc,n}}{N} \qquad \forall sc \in SC \qquad (24)$$

A simplified way to calculate the probability of acceptance $prob_{sc}$ is proposed in Eq. (25) based on the follower payoffs successful scenarios. SN_{sc} and N_{sc} correspond to the successful and total number of the follower SC payoffs scenarios (Monte-Carlo sampling) as in Hjaila *et al.* (2016a).

$$prob_{sc} = \frac{SN_{sc}}{N_{sc}} \qquad \forall sc \in F \qquad (25)$$

As a result of the integrated GT mathematical formulation, an MINLP non-convex model is obtained. The rigorous way to address the system under study requires to solve a bi-level optimization problem. However, in this case, both the leader and the follower models are MINLP non-convex due to the complexity added by the Nash Equilibrium game integration. As a consequence, current methods based on the traditional Karush–Kuhn–Tucker (KKT) conditions to address this type of problems (Yue & You, 2014), even just considering the nominal scenario (without uncertainty in the market conditions) cannot be applied to the resulting model, or would require the simplification of, at least, the follower's model (Bard, 1998; Colson et al., 2007). As a consequence, a comparison of the eventual results to be obtained would not be fair and consistent, and with loss of practicality.

The integrated-GT tactical model is generic and flexible enough to be applied when different clients and different vendors participate in a decentralized SC. It is able to capture the conflicting and competing objectives (Eqns. 16-20) in one single comprehensive approach. The mathematical model is able to cope with the different roles that the same game player may act. Each SC can act as a vendor for other buyers' SC/s, and as a client for other vendors' SC/s. The flexibility of the generic model and its ability to contain all possible SCs (centralized/decentralized, standalone/non-cooperative) including the 3rd parties SCs add to the PSE and OR researches a new comprehensive approach able to solve complex decentralized structures.

4. Case study: results and discussion

To illustrate the practicality of the proposed integrated-GT approach, the resulting MINLP models are implemented and solved for a case study adapted from Hjaila et al., (2016a) in order to compare the obtained results.

4.1 Case study

The decentralized SC under study (Figure 6) consists of two main multi-product SCs with their own markets/suppliers and 3rd parties: polystyrene manufacturing SC (as the main client) and energy generation SC (as the main vendor). The energy generation SC consists of 6 renewable energy generators (*g*1, *g*2... *g*6) fed by one RM supplier (*s*1) of 4 alternative resources (wood pellets *b*1, coal *b*2, petcoke *b*3, and marc waste *b*4). The main vendor sells energy to the local Grid as external client (3rd party), two energy markets, and to the polystyrene manufacturing SC. The main client SC consists of 3 polystyrene manufacturing plants (*pl*1, *pl*2 and *pl*3) producing two different products (*A* and *B*) using 4 alternative resources (*rm*1, *rm*2, *rm*3 and *rm*4); *rm*1 and *rm*2 to produce product A, *rm*3 and *rm*4 to produce product B, supplied by 4 alternative competing suppliers (*sup*1, *sup*2, *sup*3, and *sup*4) plus energy from the local Grid as external vendor (3rd party). The final products (*A* and *B*) are stored in 2 warehouses (*w*1 and *w*2) to be distributed later to final polystyrene markets (*m*1, *m*2, and *m*3). The polystyrene manufacturing-distribution SC has its own Waste Water Treatment Plant (WWTP). The energy needed for treating the WWTP is considered in the energy demand, with a treatment factor of 0.43 kWh/m^3.

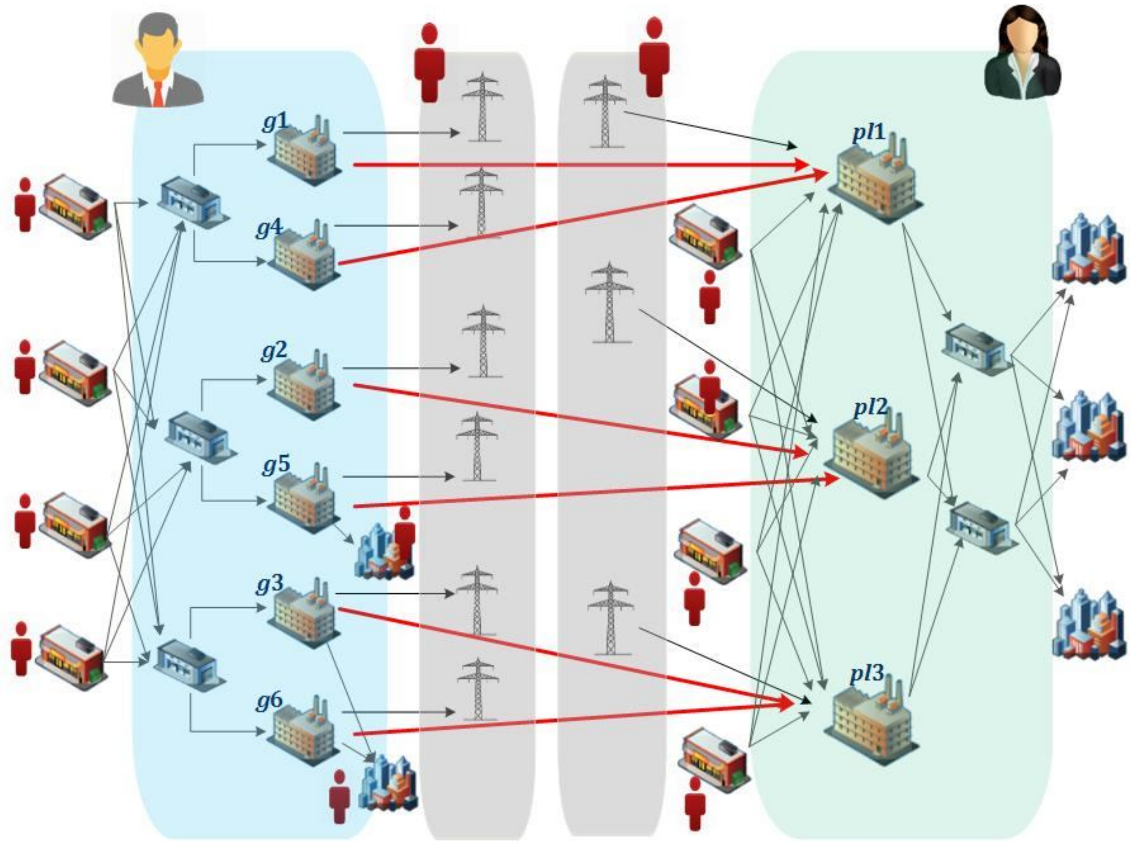


Figure 6- Decentralized SC Network

To be more practical, the RM suppliers participate in the decision-making by their pricing policies following the piecewise pricing model of Hjaila *et al*. (2016b). Tables (1-6) illustrate the main parameters of the decentralized SC as in Hjaila et al. (2016b).

Table 1- Distance between suppliers and polystyrene production sites (km)

Polystyrene SC supplier	Distance to polystyrene production plants (km)		
	*pl*1	*pl*2	*pl*3
*sup*1	100	150	145
*sup*2	200	120	130
*sup*3	110	70	80
*sup*4	170	220	215

Table 2- Polystyrene manufacturing plants production capacities

Production site	Product A (ton)	Product B (ton)
*pl*1	195	225
*pl*2	240	270
*pl*3	90	120

Table 3- Polystyrene production unit costs

		Unit production cost (€/kg)
Product *A*	*rm*1	0.64
	*rm*2	0.62
Product *B*	*rm*3	0.58
	*rm*4	0.53

Table 4- Polystyrene SC RM initial prices

RMs	Price (€/kg)
*rm*1	1.00
*rm*2	0.90
*rm*3	0.90
*rm*4	0.85

Table 5- Biomass initial prices

Biomass	Price (€/kg)
b1	0.060
b2	0.040
b3	0.065
b4	0.055

Table 6- Energy generation efficiency and cost

	Efficiency (kWh/kg)		Generation cost (€/kWh)	
	*g*1-*g*3	*g*4-*g*6	*g*1-*g*3	*g*4-*g*6
*b*1	0.73	1.50	0.26	0.13
*b*2	2.00	2.60	0.20	0.14
*b*3	0.85	1.80	0.21	0.15
*b*4	0.80	2.00	0.23	0.14

The main Stackelberg-game players are: the polystyrene manufacturing-distribution SC enterprise as the leader and the energy generation-distribution SC as the follower. The leader action is the internal energy transfer price. The leader offers between 0.14€/kWh and 0.22€/kWh. The Stackelberg-game follower player is supposed to sell energy to the Stackelberg-game leader player SC and also to the local Grid as external client (3rd party). The polystyrene manufacturing SC is supposed to purchase energy from the energy generation SC and from the local Grid as external vendor (3rd party). The reaction function of the Stackelberg game is the internal energy amounts that the follower send to each production site of the leader SC along a planning horizon of 6 time periods.

The NE-game competing players are: the polystyrene production-distribution SC enterprise “Stackelberg-game leader” vs. the local Grid as external client on one side, and between the energy generation SC enterprise “Stackelberg-game follower” and the local Grid as external vendor on the other side. The Spanish local Grid is considered for this work with current (selling/purchasing) prices as in Table 7.

Table 7- Current energy prices around the decentralized SC

	Energy price (€/kWh)
Energy price to fixed markets	0.20
Energy price to local Grid (demand <2GWh)	0.21
Energy price to local Grid (2GWh<demand< 4GWh)	0.20
Energy price to local Grid (4GWh<demand < 6GWh)	0.19
Local Grid energy price to energy markets	0.22
Local Grid energy price to Polystyrene SC (demand<2GWh)	0.22
Local Grid energy price to Polystyrene SC (2GWh<demand<4GWh)	0.21
Local Grid energy price to Polystyrene SC (4GWh<demand<8GWh)	0.20

To obtain the expected payoffs of the game players, 500 scenarios are generated for the energy prices of the 3rd parties, using Monte-Carlo Sampling method, assuming normal distribution with equal probabilities: σ (standard deviation)=0.03; the mean (μ) for the energy prices of the 3rd parties around the follower SC is equal to the current energy prices as in Table 7. For the leader SC, the mean (μ) is the external vendor energy prices according to the quantity demanded as in Table 8.

Table 8- Monte Carlo mean (μ) of the Local Grid external vendor

	Mean μ (€/kWh)
Local Grid energy price to Polystyrene SC (demand<2GWh)	0.20
Local Grid energy price to Polystyrene SC (2GWh<demand<4GWh)	0.19
Local Grid energy price to Polystyrene SC (4GWh<demand<8GWh)	0.18

Assumptions:

- The optimal strategies of the 3rd parties are known within a range (optimal zones). These zones are considered in the mathematical model formulations (Eq. (18)).
- The transport and the storage costs of the RM from the suppliers are charged by the RM buyers (energy generation enterprise/polystyrene manufacturing enterprise).
- The energy sold/purchased has no storage.

The resulting non-cooperative non-zero-sum integrated GT model has been solved for the abovementioned case study, and the Stackelberg-payoff matrix is built under the nominal conditions (energy prices around the decentralized SC as in Table 7), considering the NE-game competing players strategies. Then the nominal Stackelberg-payoff matrix is evaluated under different uncertain disruptions:

i) The follower's uncertain conditions resulted from the uncertain nature of its 3rd party. 500 probable scenarios are generated for the energy prices around the follower SC (σ=0.03, μ= energy prices as in Table 7).

ii) The leader's uncertain conditions resulted from the uncertain nature of its 3rd party. 500 probable scenarios are generated for the energy prices of its 3rd party ($\sigma=0.03$, μ= energy prices as in Table 8).

iii) The uncertain conditions of the leader and the follower resulting from both of the aforementioned cases above.

The case study is modeled using the General Algebraic Modeling System GAMS 24.2.3 on a Windows 7 computer with Intel® Core™ i7-2600 CPU 3.40GHz processor with 16.0 GB of RAM. The resulting MINLP tactical models have been solved for 6 time periods; 1000 working hours each, using Global mixed-integer quadratic optimizer "GloMIQO (Misener & Floudas, 2013)". The R–project program 3.2.1 is used for statistical computing. Table 9 summarizes the model statistics of each game player model. The CPU times when considering uncertainty is multiplied by the number of the generated scenarios.

Table 9- Model statistics

Game player SC	Model	Single equations	Single variables	Discrete variables	CPU each action (sec)
Leader	MINLP	964	1653	126	7.95
Follower	MINLP	1202	1289	180	3.85

4.2 Results: nominal conditions

The abovementioned case study is solved at the nominal conditions (at the energy prices around the decentralized SC, as in Table 7). The nominal payoffs of the Stackelberg-game players are obtained for each leader action (energy transfer price) and follower response (internal energy flows) (Table 10). The highlighted payoffs values in Table 10 are obtained based on the proposed leader transfer price and the follower optimal amounts. When the leader offers transfer prices from 0.14€/kWh to 0.16 €/kWh, the follower responds with 0 GWh energy amounts, returning to its SC standalone case (payoff= 2.44 M€). But, when the leader increases the transfer price to 0.17 €/kWh, the best for the follower is to provide 6 GWh distributed among the leader manufacturing plants along 6 time periods. When the leader offers 0.18-0.19 €/kWh, the best for the follower is to provide 23.10 GWh. When the leader offers up to 0.22 €/kWh, the follower is ready to sell all the energy amounts needed for the leader SC production.

Table 10- Stackelberg Payoff matrix (nominal conditions)

Leader action (€/kWh) →	Leader (L) payoff - Follower (F) Payoff (M€)																	
	0.14		0.15		0.16		0.17		0.18		0.19		0.2		0.21		0.22	
Follower response (GWh)↓	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L
0	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47
3.00	2.36	7.66	2.41	7.66	2.43	7.61	2.46	7.59	2.50	7.57	2.51	7.53	2.54	7.51	2.59	7.46	2.62	7.43
6.00	2.29	7.87	2.35	7.85	2.41	7.75	2.47	7.69	2.53	7.63	2.59	7.60	2.65	7.51	2.71	7.47	2.77	7.41
9.00	2.20	8.09	2.29	7.98	2.37	7.90	2.46	7.81	2.56	7.72	2.64	7.63	2.74	7.54	2.83	7.47	2.91	7.38

12.00	2.10	8.27	2.22	8.17	2.34	8.04	2.46	7.94	2.58	7.79	2.70	7.67	2.82	7.57	2.94	7.46	3.06	7.33
15.00	2.01	8.48	2.16	8.36	2.31	8.21	2.46	8.04	2.61	7.89	2.76	7.74	2.91	7.58	3.06	7.43	3.21	7.28
18.00	1.89	8.71	2.07	8.53	2.25	8.36	2.43	8.18	2.61	8.00	2.79	7.82	2.97	7.64	3.15	7.45	3.33	7.28
21.00	1.77	8.93	1.98	8.72	2.19	8.52	2.40	8.31	2.61	8.09	2.82	7.87	3.03	7.67	3.24	7.46	3.45	7.25
23.10	1.69	9.04	1.92	8.83	2.15	8.60	2.38	8.36	2.62	8.14	2.85	7.91	3.08	7.67	3.31	7.44	3.54	7.21
24.71	1.61	9.19	1.85	8.96	2.10	8.70	2.35	8.43	2.60	8.21	2.84	7.95	3.09	7.70	3.34	7.46	3.58	7.21

The Stackelberg-payoff matrix is represented in Figure 7. The leader and the follower nominal standalone payoffs are obtained to be used as benchmarks for bounding the winning zone. It is noticed that at energy prices from to 0.14-0.17 €/kWh, the leader is winning while the follower is losing (conflicting objectives) until reaching to the prices 0.17-0.20 €/kWh where the win-win zone lies. The collaboration among their SCs is viable in the win-win zone as their willingness to collaborate increases.

If the price offered from the leader is below 0.17 €/kWh, the current conditions will lead the corresponding follower to decline any coordination contract (the probability to find alternative utility clients who will pay more than 0.17 €/kWh is high enough), so both players will return to their respective standalone cases.

Certainly, the leader still may try to establish a coordination contract at this price with other utility vendors, which in the study are considered as “third parties” and represented by the “utility network”. If the internal and/or external conditions of any of these third parties differ from the ones faced by the “original follower”, this possibility may be studied following the same proposed procedure by just replacing the conditions of the “old” follower with the ones of the “new” potential follower.

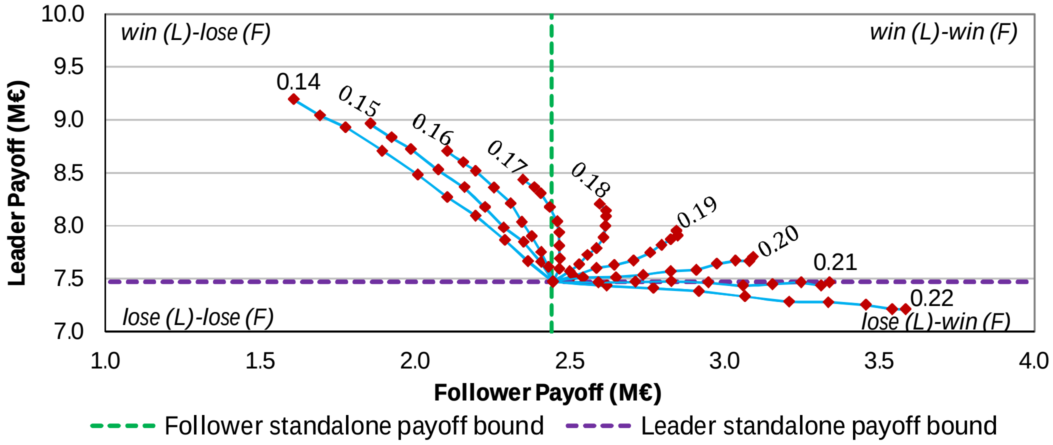


Figure 7- Leader payoffs vs. follower payoffs

Since the transfer prices (0.17-0.20 €/kWh) with their follower's corresponding amounts lead to a win-win coordination, more transfer price possibilities have been examined within the same range (0.175-0.205 €/kWh) in order to reach a decent coordination contract (Figure 8). Different Stackelberg set of Pareto solutions can be established (the dark lines/points) as in Figure 8, where each point on the graph corresponds to an optimal contract of coordination, but they do not guarantee the equilibrium.

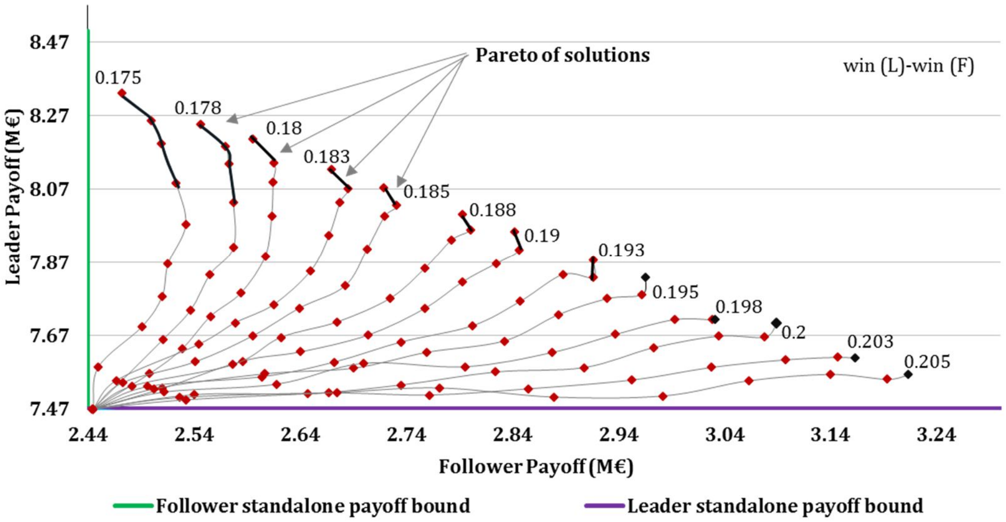


Figure 8- Stackelberg set of Pareto solutions (nominal conditions)

As shown in Figure 9, a Pareto trade-off between the players' benefits can be established if the Leader offers are between 0.175 and 0.205 €/kWh. Accordingly, the Stackelberg equilibrium is represented as a set of Pareto frontiers, the so called Stackelberg set of "Pareto frontiers" (Figure 9). The "Pareto frontiers" is meant to give the game players a wide range of options to be negotiated, simultaneously, based on more data available, risk behavior, and preferences.

Analyzing the extreme points on the Pareto frontiers, the highest leader payoff is 8.33 M€ (at price 0.175 €/kWh) is 11.5 % higher than its SC standalone payoff. This value results in the lowest follower Payoff (2.47 M€); 1.1 % higher than its nominal standalone payoff. On the other side, the highest follower payoff (Figure 9) is 3.21 M€ (at price 0.205 €/kWh and energy amount 24.71 GWh); 31.4 % higher than its standalone payoff. This point corresponds to the lowest leader payoff (7.56 M€); 1.2 % higher than its standalone payoff. In any of the solution points between those extreme solutions on the Stackelberg set of "Pareto frontiers", the follower shall be able to offer all the required energy to the leader (24.71 GWh) with a significant profit potential with respect to the standalone case, the leader would also obtain benefits from this deal. Other equilibrium points can be found in this game, but they do not take the maximum profit of this win-win potential.

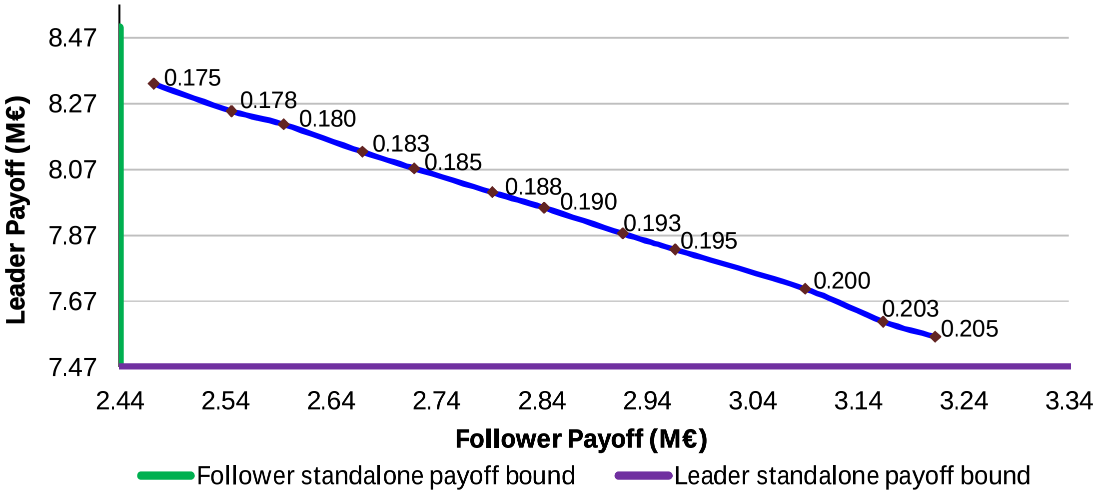


Figure 9- Stackelberg set of Pareto frontiers (nominal conditions)

It is worth mentioning that each point on Figures 7 & 8 leads to a NE solution but does not guarantee a Stackelberg equilibrium, However, just the points on the Pareto frontiers (Figure 9) lead to Stackelberg-NE equilibrium.

The Stakelberg payoff matrix is built between the main vendor (Stackelbeg follower) and the main client (Stackelberg leader). Each Stackelberg game player is competing with external third parties. When optimizing each Stackleberg game player model, the optimal Nash strategies of the competitive third parties are considered, until reaching to the equilibrium, in which none of them can modify its payoff by changing its own strategy. For example, The NE competing vendors are competing for the total demand of 24.71 GWh. However, the equilibrium is that the follower sells the entire amount to the leader SC, while the local Grid (as NE vendor) sells zero amount as its optimal price range is high. So that if the local Grid (as NE-game player vendor) tries to change its strategy through its restricted price policy (Figure 10a), still the leader will buy all the energy amounts form the follower, as the follower doesn't restrict any quantity limits to specific prices. The local Grid (as NE-game client) has its optimal price strategy restricted to specific Energy amounts (Figure 10b). So, the equilibrium is that the follower sells the 24.71GWh without price restrictions to the leader.

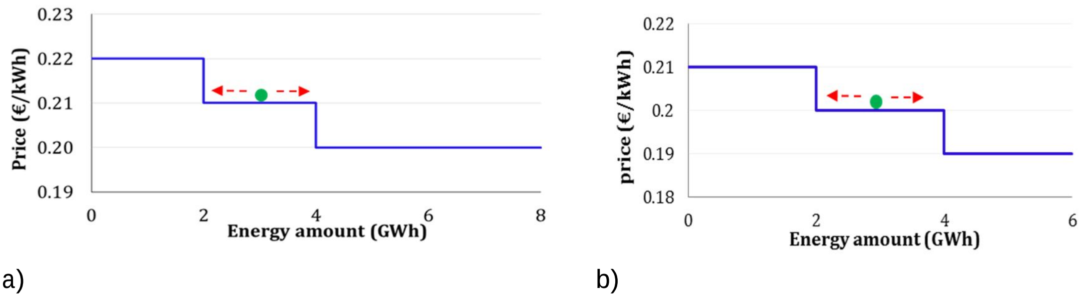


Figure 10- Local Grid as NE competitive players

a) Competing vendor b) Competing client

4.2.1 Tactical decisions: Leader SC

The tactical decisions, associated to the trade-off between the benefits of the different players, are affected by the Stackelberg-NE equilibrium. For example, Figure 11 shows the internal energy flows from the follower SC to the leader SC polystyrene manufacturing plants at a possible coordination contract: 24.71GWh at 0.185 €/kWh, namely "the 5th point on the Pareto frontiers from the left of Figure 9" (Figure 11a), comparing with the standalone case (Figure 11b). It can be seen that in the standalone case, the leaders' decision should be to shutdown polystyrene plant *pl*3 (Figures 11b & 12b) because of the local Grid energy market prices, which are higher at low demand levels (see Figure 10a). However, a proper coordination contract would enable to maintain the polystyrene plant *pl*3 working (Figure 11a & 12a) so both leader and follower may get higher benefits. Furthermore, the coordination contract leads to function the polystyrene manufacturing plant *pl*1 all time periods at its manufacturing capacity to produce product B (Table 2), in which the energy will be provided from the energy generation plant *g*4 (see Figure 6). Functioning the polystyrene manufacturing plant *pl*1 leads to higher benefits, as it is the closest to the rm4 supplier (Table 1) which dominates the RM purchase levels for producing polystyrene product B (Figure 13a).

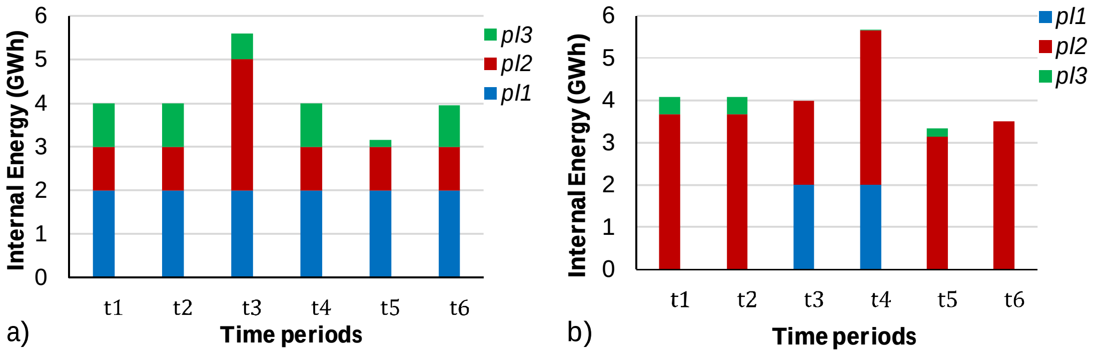
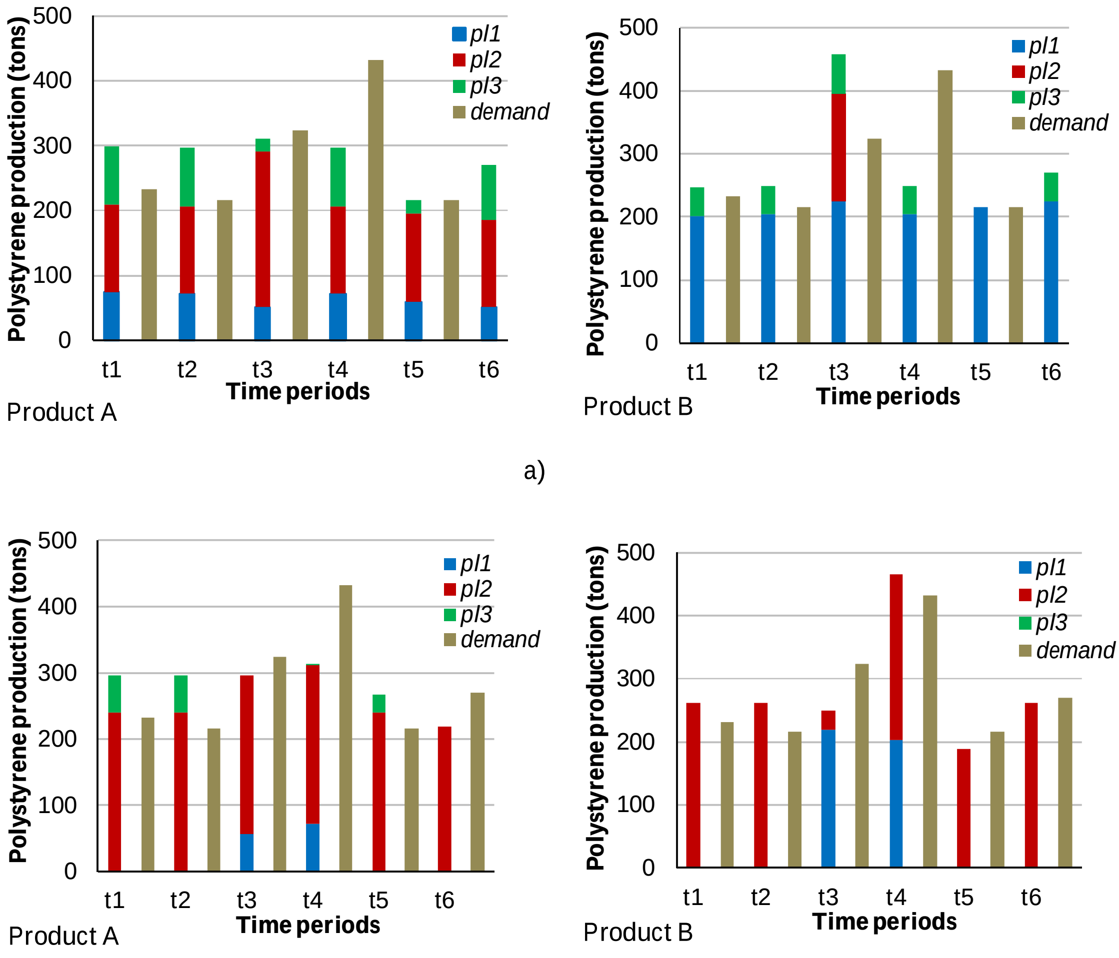


Figure 11- Internal energy flows to polystyrene manufacturing sites
a) Coordination contract: 24.71 GWh at 0.185€/kWh b) Standalone

It is worth noticing from Figure 12 that in order to produce product A, the polystyrene production plant *pl*2 dominates the production levels: 54.1 % of product A under the coordination contract (0.185€/kWh for 24.71 GWh) (Figure 12a), and 83.9 % under the standalone case (Figure 12b). Unlike the standalone case, the collaboration between the follower and the leader would maintain all the polystyrene manufacturing plants working at all time periods to produce product A (Figure 12a) following the internal energy provided from the follower. The dominance of the polystyrene production plant *pl*2 can be explained by the short distance between *pl*2 and the RM supplier *sup*2 (Table 1), as the RM *rm*2 dominates the RM purchase levels for producing product A (see Figure 13a).

Instead, to produce polystyrene product B, the coordination contract (0.185€/kWh for 24.71 GWh) would lead to the dominance of production plant *pl*1 (75.4% of the total product B). The *pl*1 is the closest to the *rm*4 supplier which dominates the RM purchase levels for producing product B (see Table 2 and Figure 13a). However, at the standalone case, the production plant *pl*2 dominates producing product B (75 % of the total production of product B) due to the local Grid energy (as external vendor) higher prices at low demand levels. So, the leader's decision is to purchase higher energy amounts for the production plant *pl*2 to gain lower prices and functioning it up to its production capacity to produce products A and B.



a)

b)

Figure 12- Polystyrene production levels

a) Coordination contract: 24.71 GWh at 0.185€/kWh b) Standalone

Figure 13 shows the RM purchase levels at the possible coordination contract, 0.185€/kWh:24.71 GWh (Figure 13a) comparing with the standalone case (Figure 13b). It is noticed that *rm*2 dominates the RM purchase levels; 49 % (1690.91 tons) of the total RM purchase levels to produce polystyrene product *A*. The dominance of *rm*2 is due to its lower price and higher capacity compared with *rm*1 (Table 4). Under the coordination contract (0.185€/kWh for 24.71 GWh), the *rm*2 purchase levels are the highest (785.83 tons, namely, 22.8 %) at time period *t*3 (Figure 13a), as the production levels of product A are the highest (Figure 12a). However, under the standalone case, the *rm*2 purchase levels are the highest (796.36 tons, i.e. 23.10 %) at time period *t*4 (Figure 13b) so to satisfy the high polystyrene production levels (Figure 12b).

For producing product B, *rm*4 dominates the RM purchase levels (1298.39 tons, or else 37.6 %) comparing with the standalone case, to satisfy the production levels of *pl*1 (Figure 12a) which is the closest to *rm*4 supplier (see Table 2). Furthermore, the decision is to purchase *rm*4 up to its supplying capacity (240 tons), so to get the highest price discount. At the standalone case, *rm*3 dominates the RM purchase levels (1320.11 tons, namely 38.3 %) for producing polystyrene product B (Figure 13b), as its supplier (*sup*3) is the closest to the polystyrene production plant *pl*2 (Table 2) which dominates the production levels of product B (Figure 12b). Moreover, at the standalone case, the polystyrene production plant *pl*2 is working up to its production capacity, stressing the necessity to buy higher amount of RM with higher supplying capacity to get the highest possible discount (see Hjaila et al., 2016b). The excess of the production will be stored for later distribution (Figure 14).

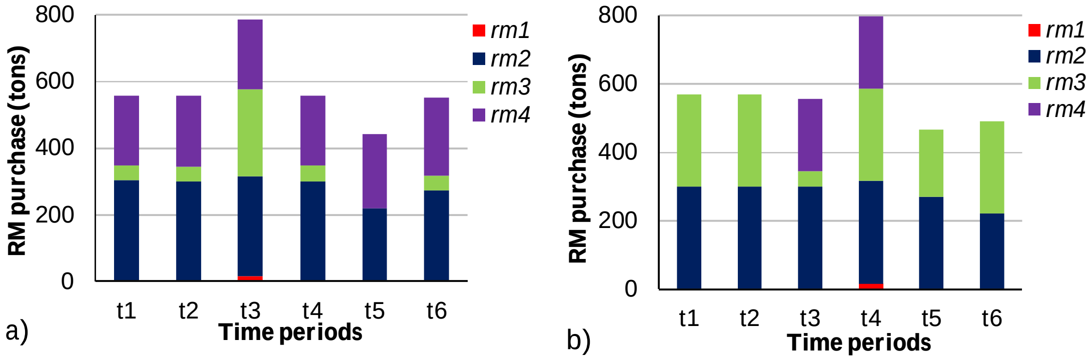


Figure 13- Polystyrene RM purchase levels

a) Coordination contract: 24.71 GWh at 0.185€/kWh b) Standalone

Consequently, the storage decisions (Figure 14) are affected by the manufacturing levels (see Figure 12) resulting from the coordination contract. The coordination contract (24.71 GWh at 0.185€/kWh) would result in 8.8 % decrease in the total storage of product A (350.05 tons) (Figure 14a), as the production activities are distributed among the 3 polystyrene manufacturing plants to produce product A (Figure 14a). The inventory levels of product B increases by 246.30 tons; 64.8 % higher than the standalone case, and this is due to the high production levels of product B at time period *t*3 (Figure 12a) following the high internal energy provided by the follower (Figure 11a). The excess of the polystyrene production will be stored for later distribution (Figure 14a).

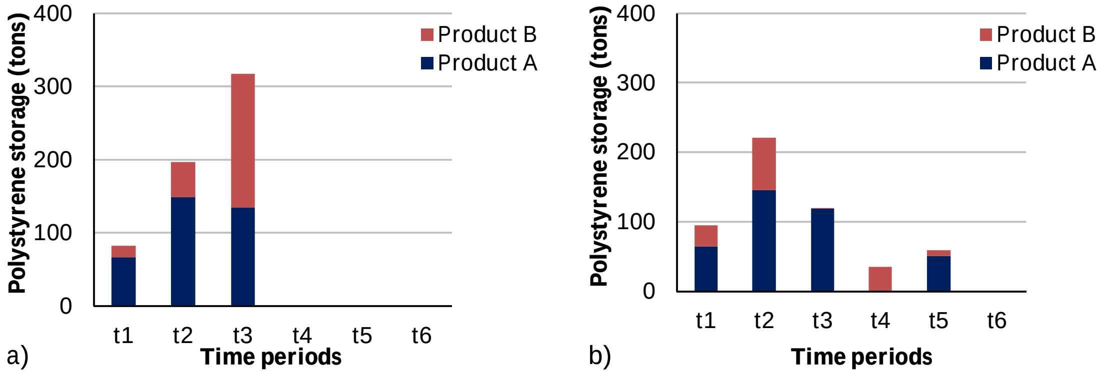


Figure 14- Polystyrene storage levels

a) Coordination contract: 0.185€/kWh:24.71 GWh b) Standalone

Table 11 summarizes the economic decisions of the leader. The total economic sales are the same (18.59 M€), as the decision is to fulfill the final polystyrene markets demands. The coordination based on 24.71 GWh internal energy at price 0.185€/kWh would improve the total cost of the leader SC with 5.7 % (with 0.60 M€ savings), in comparison with the standalone case. This leads to 7.5 % gains in 6 time periods, in comparison with the standalone case at the nominal conditions.

Table 11- Economic summary of the leader (nominal conditions)

	Coordination contract 24.71 GWh at 0.185€/kWh	Standalone
Cost (M€)	10.52	11.12
Sales (M€)	18.59	18.59
Profit (M€)	8.07	7.47

4.2.2 Tactical decisions: Follower SC

It is noticed from Figure 15 that the coordination would lead the follower to produce more, as the energy sales increases by 16.71 GWh; 18.60 % more than the standalone case. It is also worth noticing that the energy sales to the local Grid as external client has been reduced by 11.4 % (8GWh) due to the coordination contract (0.185 €/kWh to 24.71 GWh), comparing with the standalone case. In the standalone case, the follower has to sell 11.40 % (8GWh) more energy to the local Grid in order to compensate the lack of the contract. However, the follower couldn't be able to sell higher amounts of energy to the local Grid as the cost becomes higher and the market prices do not compensate.

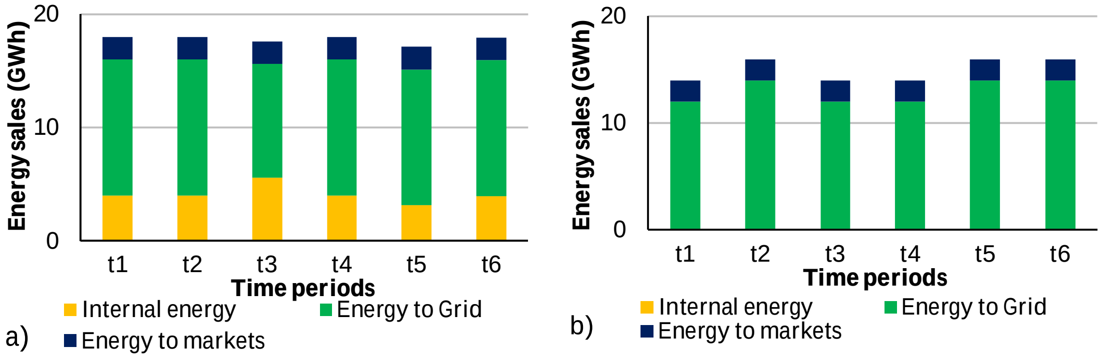


Figure 15- Follower SC energy sales

a) Coordination contract: 24.71 GWh at 0.185€/kWh b) Standalone

Figure 16 illustrates the energy generation levels along the planning horizon resulting from the coordination contract (0.185€/kWh to 24.71 GWh) (Figure 16a) and the standalone case (Figure 16b). The coordination contract would lead to functioning the energy generation plants (*g*4, *g*5, and *g*6) up to their generation capacities (6 GWh) all time periods in order to sell 24.71 GWh to the leader SC. However, in the standalone case, the follower decides not to function the energy generation plants up to their generation capacities as the cost of producing 1GWh is high (0.17 €/kWh) and the local Grid higher prices are restricted to lower energy amounts (Figure 10b), which does not compensate the follower.

Here is an example to understand this point. Assume that the follower at the standalone case wants to sell the highest possible amount (up to energy generation capacities). The best option is to operate the energy generation plants (*g*4, *g*5, and *g*6). So, the maximum energy generation = 3x6x6 = 108 GWh to be distributed between the local Grid as external client and the fixed energy markets. The energy sales to the energy markets = 12 GWh (fixed demand 2 GWh per time period). So, the sales to the local Grid will be 96 GWh (16 GWh per time period). According to Figure 10b, to sell higher energy amounts (>4 GWh), the price is 0.19 €/kWh. So, the energy economic sales = 96 x 0.19 + 12 x 0.20 = 20.60 M€. The cost of producing 108 GWh = 108 x 0.17 = 18.36 M€. This means that the follower SC payoff is equal to 2.24 M€, so the follower loses. This explains why at the standalone case, the follower couldn´t generate energy up to the generation capacity.

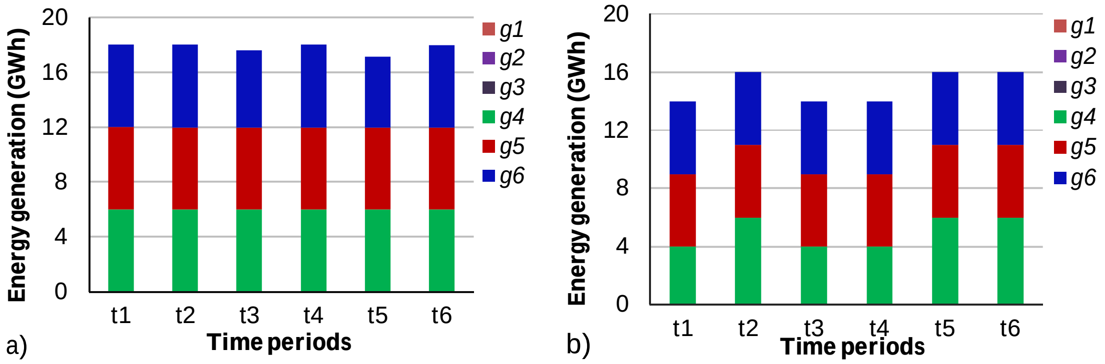


Figure 16- Energy generation levels

a) Coordination contract: 24.71 GWh at 0.185€/kWh b) Standalone

The follower SC RM purchase levels (Figure 17) follow the energy generation levels. The RMs *b*2 and *b*4 dominate the RM purchase levels due to their lower prices (see Table 4). However, the coordination would lead to 20.8 % (7.94 kilotons) higher RM purchase amounts in order to follow the higher levels of energy generation (Figure 16a).

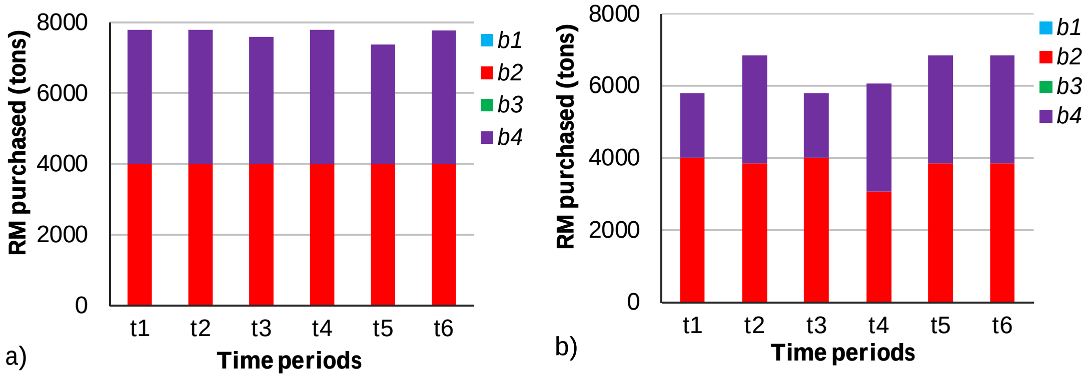


Figure 17- Follower RM purchase levels

a) Coordination contract: 24.71 GWh at 0.185€/kWh b) Standalone

Table 12 summarizes the follower economic decisions resulting from the coordination contract (24.71 GWh at 0.185€/kWh) in comparison with the standalone case. The coordination contract achieves 17.8 % improvement in the total economic sales leading to 18.8 % increase in the follower SC cost and 11.48 % total gains.

Table 12- Follower economic summary (nominal conditions)

	Coordination contract 24.71 GWh at 0.185€/kWh	Standalone
Cost (M€)	18.27	15.38
Sales (M€)	20.99	17.82
Profit (M€)	2.72	2.44

4.3 Results: under follower uncertain conditions

In this section, the integrated GT outcome is evaluated under the uncertain conditions of the follower resulting from the uncertain nature of its 3rd party. To do so, the Stackelberg payoff matrix in Table 10 is evaluated using a Monte Carlo sampling method. The follower SC model is solved for 500 scenarios generated from the energy prices around its SC considering: mean (μ) = energy prices as in Table 7; standard deviation (σ = 0.03). The expected payoffs of the follower are obtained. Table 13 illustrates the Stackelberg payoff matrix considering the leader payoffs and the follower expected payoffs. It is noticed that the expected follower payoff at the standalone case (2.74 M€) increases; 12.20 % more than its standalone nominal payoff (2.44 M€). To understand the behavior of the game players, the payoff matrix has been visualized for further analysis (Figure 18).

Table 13- Stackelberg-NE leader payoff-follower expected payoff matrix

Leader action (€/kWh) →	**Follower Expected Payoff vs. Leader Payoff (M€)**																	
	0.14		0.15		0.16		0.17		0.18		0.19		0.20		0.21		0.22	
Follower response (GWh) ↓	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L
0	2.74	7.47	2.74	7.47	2.74	7.47	2.74	7.47	2.74	7.47	2.74	7.47	2.74	7.47	2.74	7.47	2.74	7.47
3.00	2.64	7.66	2.66	7.66	2.65	7.61	2.72	7.59	2.75	7.57	2.79	7.53	2.81	7.51	2.84	7.46	2.87	7.43
6.00	2.52	7.87	2.58	7.85	2.64	7.75	2.69	7.69	2.76	7.63	2.81	7.60	2.88	7.51	2.94	7.47	3.00	7.41
9.00	2.40	8.09	2.41	7.98	2.55	7.90	2.67	7.81	2.76	7.72	2.86	7.63	2.93	7.54	3.03	7.47	3.09	7.38
12.00	2.29	8.27	2.35	8.17	2.53	8.04	2.65	7.94	2.78	7.79	2.89	7.67	3.01	7.57	3.13	7.46	3.25	7.33
15.00	2.18	8.48	2.32	8.36	2.48	8.21	2.63	8.04	2.78	7.89	2.93	7.74	3.08	7.58	3.23	7.43	3.38	7.28
18.00	2.07	8.71	2.25	8.53	2.42	8.36	2.61	8.18	2.79	8.00	2.97	7.82	3.15	7.64	3.33	7.45	3.51	7.28
21.00	1.96	8.93	2.16	8.72	2.37	8.52	2.58	8.31	2.79	8.09	2.99	7.87	3.19	7.67	3.43	7.46	3.64	7.25
23.10	1.86	9.04	2.08	8.83	2.31	8.60	2.56	8.36	2.78	8.14	3.02	7.91	3.25	7.67	3.48	7.44	3.71	7.21
24.71	1.74	9.19	1.99	8.96	2.20	8.70	2.43	8.43	2.76	8.21	3.00	7.95	3.26	7.70	3.49	7.46	3.72	7.21

The Stackelberg leader payoff-follower expected payoff matrix is represented in Figure 18. The leader standalone payoff and the follower expected standalone payoff are obtained to mark the winning zone. It is noticed that the follower expected standalone payoff has been shifted (0.29 M€) to the right side (Figure 18) comparing with Figure 7. This means that the leader has to offer higher prices in order to compensate the follower in case of any possible disruptions. The follower is expected to gain more under the uncertain conditions, and thus his/her expectations from the leader becomes higher, excluding the leader price offer 0.17 €/kWh from the game (it becomes inside the losing zone of the follower). The new win-expected win zone starts from the leader price 0.18 €/kWh (Figure 18).

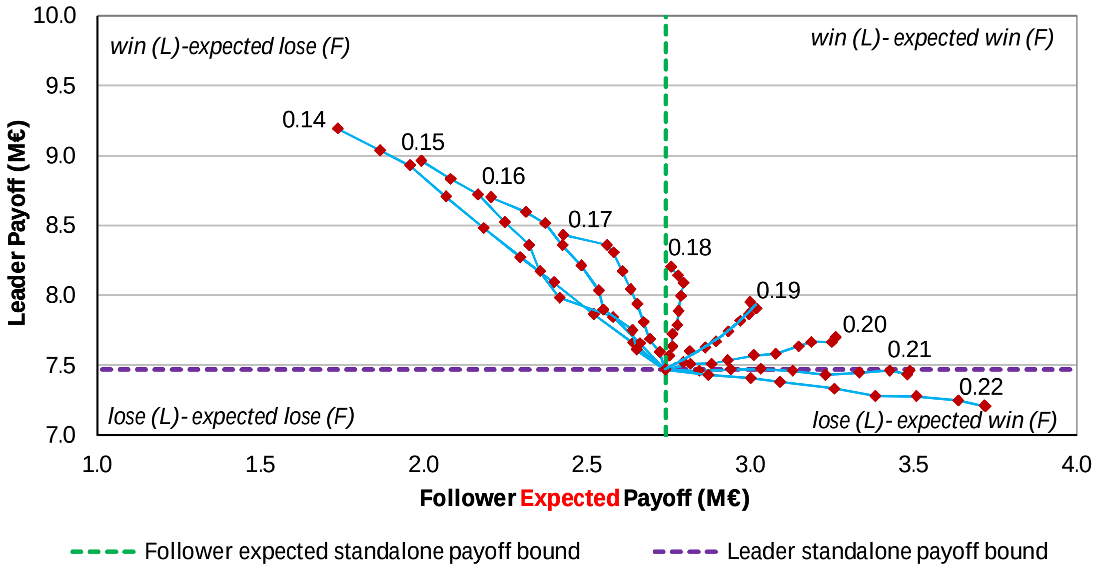


Figure 18- Leader payoff vs. follower expected payoff

Then, a different set of leader prices offers are examined between 0.18 €/kWh and 0.21 €/kWh, and the leader win and the follower expected win payoffs are obtained (Figure 19). The Stackelberg set of "Pareto frontiers" is established for each scenario (Figure 19). Each point on the "Pareto frontiers" corresponds to a possible coordination contract. Compared with the payoffs under the nominal conditions of Figure 8, It is noticed that the Pareto frontiers resulting from the leader price offers between 0.175 €/kWh and 0.178 €/kWh are also excluded from the game.

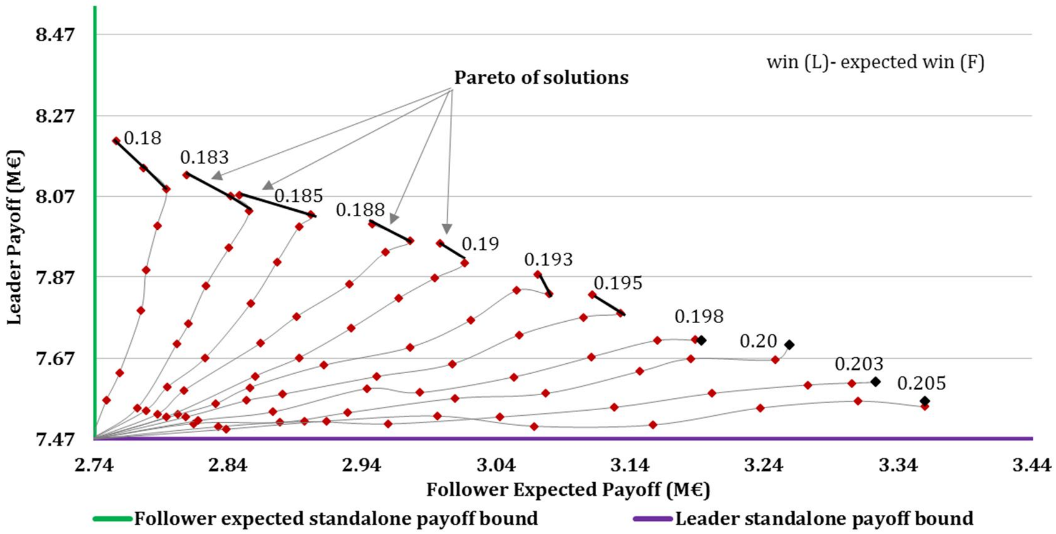


Figure 19- Leader win-follower expected win solutions

The Stackelberg set of "Pareto frontier" under the follower uncertain conditions can be established if the leader offer is between 0.180 €/kWh and 0.205 €/kWh (Figure 20). In any of these cases, the follower shall be able to offer between 21.00 GWh and 24.71 GWh to the leader with a significant profit potential respect to the standalone case, the leader would also gain from this deal.

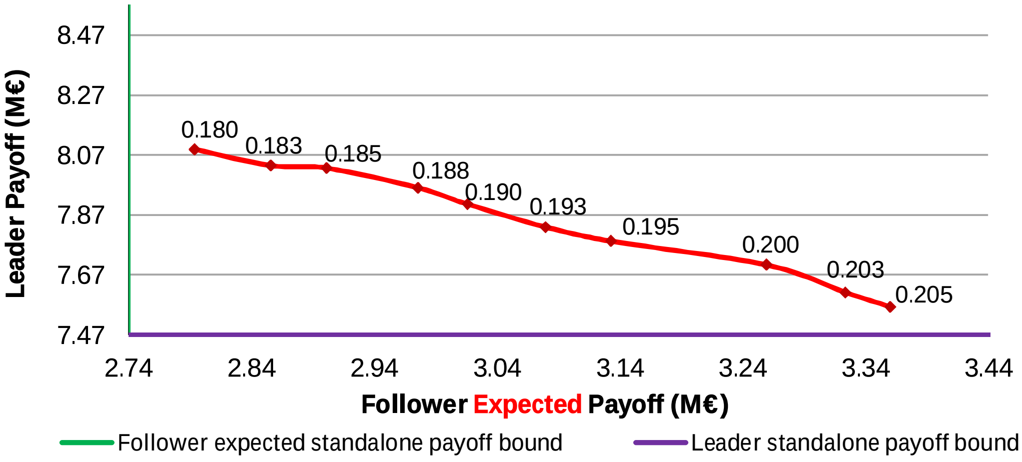


Figure 20- Stackelberg set of "Pareto frontiers" (follower uncertain conditions)

Figure 21 shows the follower nominal and expected payoffs. Here, it can be seen clearly the positive shift of the follower expected payoff up to its nominal payoff. An increase of 12.2 % (0.29 M€) of the follower expected standalone payoff leads to 6.6 % increase in the follower's profit expectations. This results in excluding the prices offers (<0.18 €/kWh) as they lead to follower's payoffs below its expected standalone payoff (Figure 21).

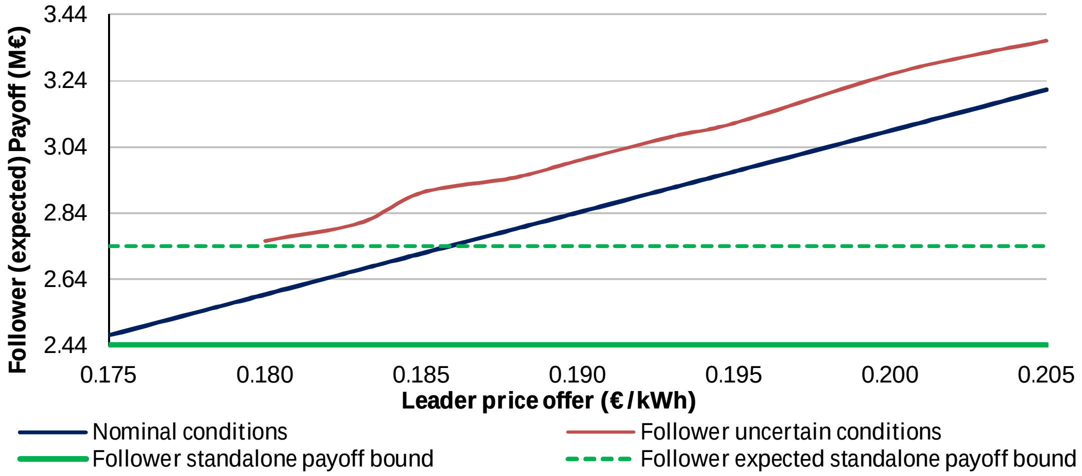


Figure 21- Follower nominal vs. expected payoffs

Follower uncertainty reduction

The probability of acceptance (Eq. 28) of the follower is calculated for each coordination contract possibility. The variance (σ^2) of the follower's expected payoffs is calculated and plotted on Figure 22. It is noticed that the probability of acceptance increases when the leader price offer increases, resulting in higher expectations of the follower payoffs. It is worth noticing that the coordination reduces the variance of the expected payoff of the follower of about 27.34 % (Figure 22) compared with the variance at the expected standalone case. The variance using the different coordination contracts is not the same, as the internal energy amounts (contract amounts) are not the same; lower energy amounts lead to higher uncertainty and thus to higher variance (Figure 22). This means that the coordination guarantee an “uncertainty effect reduction” of the expected payoffs of the follower: expected payoff = contract payoff “fixed” + market payoff “uncertain”. The coordination assures stable benefits “contract payoff” regardless of the uncertain conditions along the planning horizon (Figures 22 & 23). Such market stability stresses the willingness of the game players to collaborate considering that they can improve the quality of their SCs, such as reducing operational cost, so to assure higher benefits than the contract benefits.

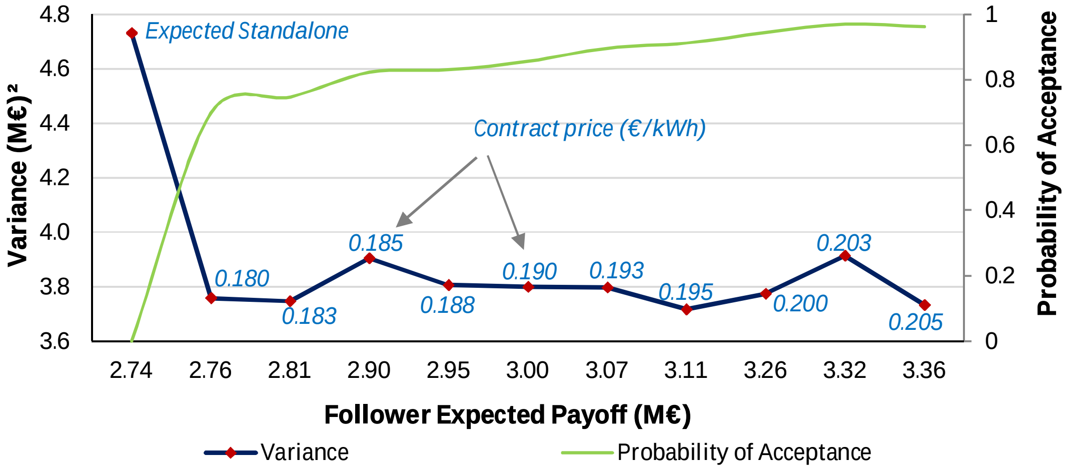


Figure 22- Probability of acceptance and variance (Follower uncertain conditions)

Figure 23 shows the breakdown of the expected payoff of the follower under the different coordination contracts. It is to be noticed that the standalone case results in zero uncertainty effect reduction which is risky for the follower. However, the uncertainty effect reduction increases (lower variance) when the coordination contract prices increases.

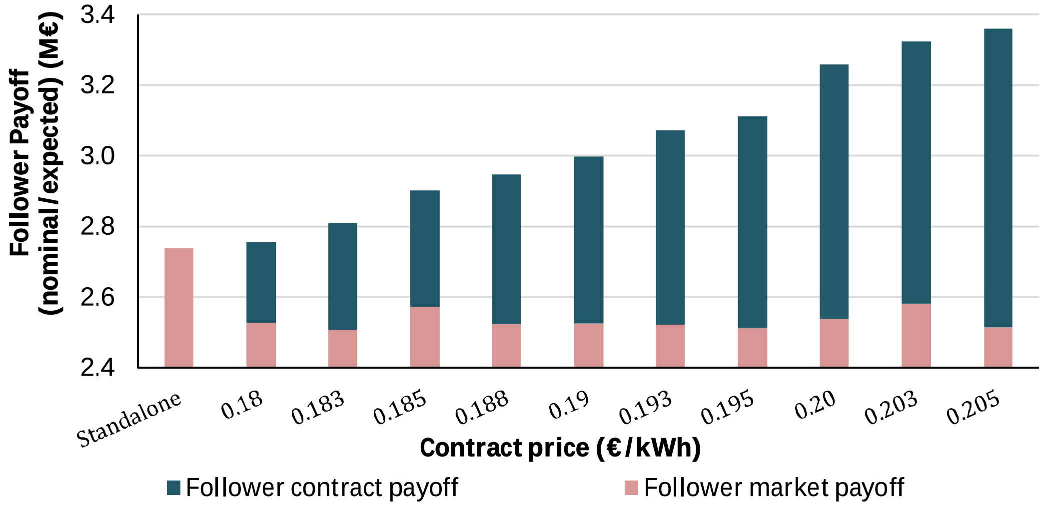


Figure 23- Follower contract and market payoffs (follower uncertain conditions)

Stackelberg vs. SBDN approach:

The results obtained using the integrated-GT approach are compared with the ones found by the Scenario Based Dynamic Negotiations (SBDN) approach proposed by Hjaila et al. (2016a) on the same case study.

Using the SBDN, the best coordination contract offered by the leader under the uncertainty of the follower corresponds to the transfer price 0.18 €/kWh for 24.71 GWh. This solution is the same best solution resulting in this paper (see Table 13 & Figure 20). Table 14 summarizes the coordination contract (transfer price 0.18 €/kWh for 24.71 GWh) results from the SBDN and the integrated-GT approach. It is noticed that integrated-GT approach leads to better follower expected payoff with a difference of 1.7 %. Using the integrated-GT approach, the interchanged amounts are decided by the follower according to her/his best conditions so, given a certain price, the most profitable solution for the follower player is obtained. On the other hand, using the SBDN approach, the leader decides the amounts and prices according to her/his best conditions considering the probability of acceptance of the follower, resulting in a higher expected profit for the leader partner; that is 0.02 M€ more payoffs for the leader in 6 time periods.

Table 14- Stackelberg approach vs. SBPN

		SBDN (Hjaila *et al.*, 2016a)		**Integrated-GT (this paper)**	
Contract price (€/kWh)	Internal energy (GWh)	Leader payoff (M€)	Follower expected payoff (M€)	Leader payoff (M€)	Follower expected payoff (M€)
0.18	24.71	8.23	2.71	8.21	2.76

4.4 Results: under leader uncertain conditions

In this section, the Stackelberg payoff matrix at the nominal conditions (Table 10) is evaluated considering the uncertain behavior of the leader player resulting from the uncertain nature of its 3rd party (Local Grid as NE vendor). The MINLP tactical model of the leader SC is solved for the generated 500 prices scenarios of the local Grid using Monte Carlo method: mean (μ) = local Grid energy prices as in Table 8; standard deviation ($\sigma = 0.03$). Table 15 illustrates the Stackelberg payoff matrix considering the leader expected payoff and the follower nominal payoff for each coordination contract offer. It can be noticed that the coordination would lead to 6.80 % (0.50 M€) higher leader expected payoffs at the standalone case (7.98 M€) compared with the nominal standalone payoff (7.47 M€).

Table 15- Stackelberg leader expected payoff-follower payoff matrix conditions)

Leader action (€/kWh) →	**Follower Payoff - leader Expected Payoff (M€)**																	
	0.14		0.15		0.16		0.17		0.18		0.19		0.2		0.21		0.22	
Follower response (GWh) ↓	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L
0	2.44	7.98	2.44	7.98	2.44	7.98	2.44	7.98	2.44	7.98	2.44	7.98	2.44	7.98	2.44	7.98	2.44	7.98
3.00	2.36	8.13	2.41	8.10	2.43	8.06	2.46	8.04	2.50	8.01	2.51	7.98	2.54	7.95	2.59	7.92	2.62	7.88
6.00	2.29	8.27	2.35	8.22	2.41	8.16	2.47	8.09	2.53	8.04	2.59	7.98	2.65	7.92	2.71	7.86	2.77	7.80
9.00	2.20	8.43	2.29	8.32	2.37	8.24	2.46	8.15	2.56	8.07	2.64	7.98	2.74	7.88	2.83	7.80	2.91	7.71
12.00	2.10	8.56	2.22	8.44	2.34	8.32	2.46	8.21	2.58	8.08	2.70	7.95	2.82	7.84	2.94	7.72	3.06	7.60
15.00	2.01	8.71	2.16	8.56	2.31	8.41	2.46	8.26	2.61	8.11	2.76	7.96	2.91	7.81	3.06	7.66	3.21	7.51
18.00	1.89	8.86	2.07	8.68	2.25	8.50	2.43	8.32	2.61	8.14	2.79	7.96	2.97	7.78	3.15	7.60	3.33	7.42
21.00	1.77	9.02	1.98	8.82	2.19	8.60	2.40	8.39	2.61	8.18	2.82	7.96	3.03	7.76	3.24	7.55	3.45	7.34
23.10	1.69	9.12	1.92	8.88	2.15	8.65	2.38	8.42	2.62	8.19	2.85	7.96	3.08	7.72	3.31	7.49	3.54	7.27

24.71	1.61	9.19	1.85	8.96	2.10	8.70	2.35	8.43	2.60	8.21	2.84	7.95	3.09	7.70	3.34	7.46	3.58	7.21

Given that the expected standalone payoff of the leader is higher than its nominal standalone payoff, and that unlike the case of the follower, the leader has to offer lower prices (Figures 24). The winning zone at the nominal, follower uncertain conditions, and leader uncertain conditions is summarized as below:

- 0.17 €/kWh < Winning zone "nominal conditions" < 0.21 €/kWh (Figure 7)
- 0.18 €/kWh < Winning zone "follower uncertain conditions" < 0.21 €/kWh (Figure 18)
- 0.17 €/kWh < Winning zone "leader uncertain conditions" < 0.19 €/kWh (Figure 24)

Here can be seen the conflicting objectives under the leader and the follower uncertain conditions, the leader offers higher prices and the follower seeks lower prices.

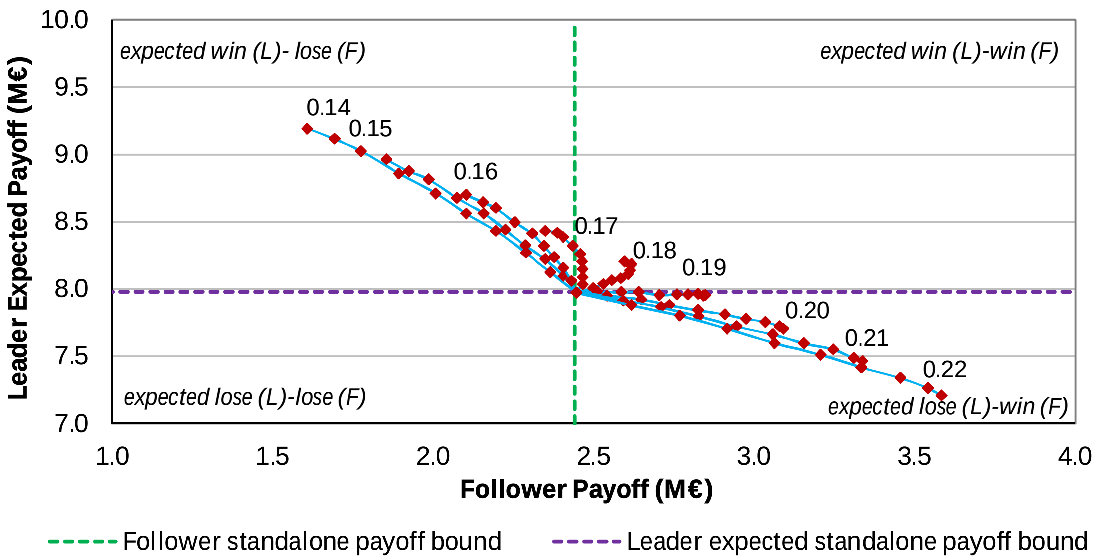


Figure 24- Leader expected payoffs vs. follower payoffs

Different sets of price offers are examined within the winning zone under the leader's uncertain conditions, to obtain the leader's expected win and follower's win payoffs (Figure 25). Different sets of "Pareto frontiers" are established for the leader price offers. Compared with the nominal payoffs in Figure 8, the uncertain conditions of the leader bring the price 0.173 €/kWh into the game while excluding the prices between 0.19 €/kWh and 0.205 €/kWh (Figure 25).

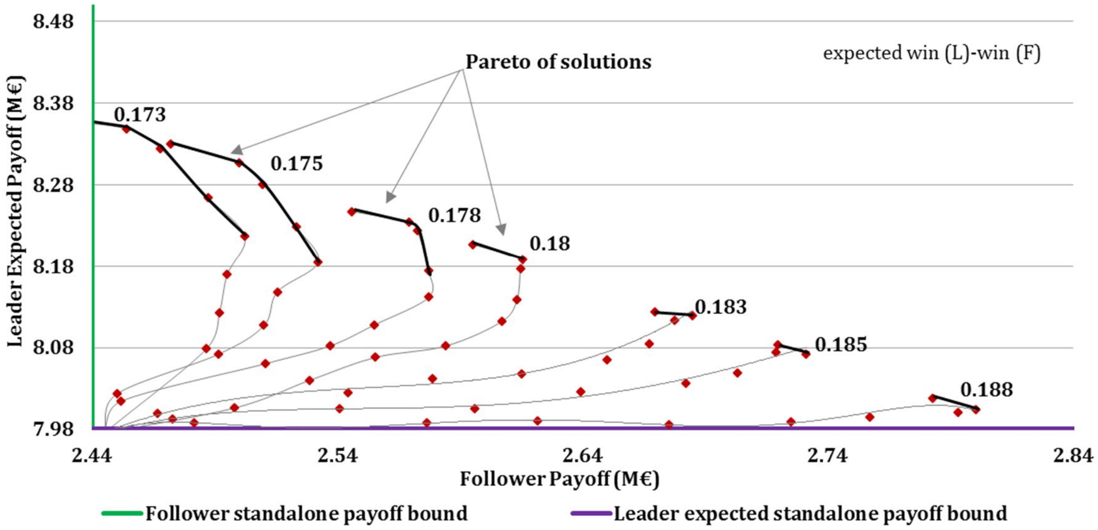


Figure 25- Leader expected win-follower win solutions

The tradeoff between the players' payoffs under the uncertain conditions of the leader player is represented as Stackelberg set of "Pareto frontier" (Figure 26). In any of the solution points, the follower shall be able to offer between 23.10 GWh and 24.71 GWh to the leader with a significant profit potential respect to her/his standalone case, the leader would also obtain expected benefits from this deal. The similar prices on the Pareto frontier correspond to different contract energy amounts (23.10-24.71 GWh).

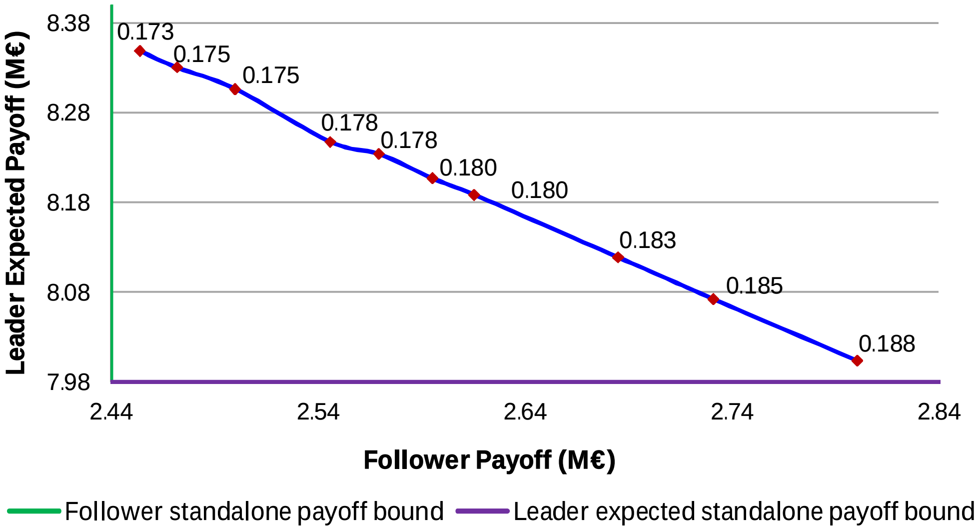


Figure 26- Stackelberg set of Pareto frontiers (leader uncertain conditions)

Leader uncertainty effect reduction

The variances (σ^2) of the leader's expected payoffs under the coordination contracts (resulted from the Stackelberg set of "Pareto frontier" of Figure 26) are obtained together with the leader's expected standalone payoff (Figure 27). It is noticed that the coordination would reduce the uncertainty effect that the leader may face as the variance of the expected payoffs decreases from 0.50 (M€)^2 to almost zero. This means that the coordination guarantees a confident payoff "contract payoff" whatever is the uncertain market situation around the leader SC.

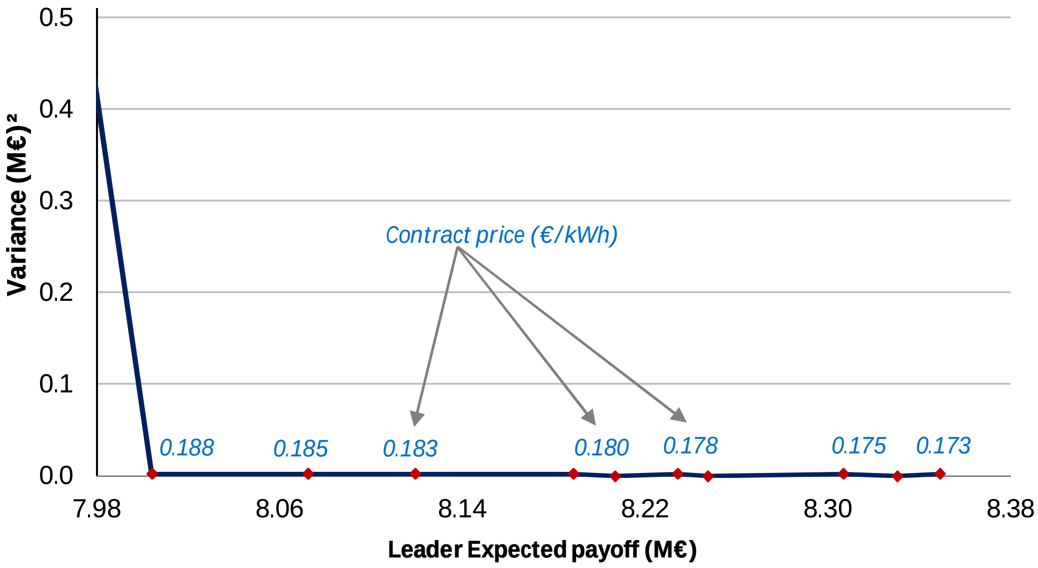


Figure 27- Leader expected payoffs variance

4.5 Results: under leader and follower uncertain conditions

The Stackelberg payoff matrix at the nominal conditions (Table 10) is evaluated considering the uncertain behavior of the leader and the follower players resulting from the uncertain nature of their 3rd parties. The Stackelberg matrix is built for the expected payoff leader-follower (Table 16). The follower's and the leader's expected payoffs are obtained from Table 13 and Table 15, respectively. The coordination under both players' uncertain conditions would lead to 6.80 % higher leader expected standalone payoffs (7.98 M€) compared with its nominal standalone payoff (7.47 M€). Also it leads to 12.20 % increase in the follower expected standalone payoff compared with its standalone nominal payoff (2.44 M€) in 6 time periods.

Table 16- Stackelberg leader expected payoff-follower expected payoff matrix

Leader action (€/kWh) →	Leader expected payoff vs. follower expected payoff (M€)																	
	0.14		0.15		0.16		0.17		0.18		0.19		0.2		0.21		0.22	
Follower response (GWh) ↓	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L
0	2.74	7.98	2.74	7.98	2.74	7.98	2.74	7.98	2.74	7.98	2.74	7.98	2.74	7.98	2.74	7.98	2.74	7.98
3.00	2.64	8.13	2.66	8.10	2.65	8.06	2.72	8.04	2.75	8.01	2.79	7.98	2.81	7.95	2.84	7.92	2.87	7.88

6.00	2.52	8.27	2.58	8.22	2.64	8.16	2.69	8.09	2.76	8.04	2.81	7.98	2.88	7.92	2.94	7.86	3.00	7.80
9.00	2.40	8.43	2.41	8.32	2.55	8.24	2.67	8.15	2.76	8.07	2.86	7.98	2.93	7.88	3.03	7.80	3.09	7.71
12.00	2.29	8.56	2.35	8.44	2.53	8.32	2.65	8.21	2.78	8.08	2.89	7.95	3.01	7.84	3.13	7.72	3.25	7.60
15.00	2.18	8.71	2.32	8.56	2.48	8.41	2.63	8.26	2.78	8.11	2.93	7.96	3.08	7.81	3.23	7.66	3.38	7.51
18.00	2.07	8.86	2.25	8.68	2.42	8.50	2.61	8.32	2.79	8.14	2.97	7.96	3.15	7.78	3.33	7.60	3.51	7.42
21.00	1.96	9.02	2.16	8.82	2.37	8.60	2.58	8.39	2.79	8.18	2.99	7.96	3.19	7.76	3.43	7.55	3.64	7.34
23.10	1.86	9.12	2.08	8.88	2.31	8.65	2.56	8.42	2.78	8.19	3.02	7.96	3.25	7.72	3.48	7.49	3.71	7.27
24.71	1.74	9.19	1.99	8.96	2.20	8.70	2.43	8.43	2.76	8.21	3.00	7.95	3.26	7.70	3.49	7.46	3.72	7.21

The expected standalone payoffs of the game players delimit the zone where the winning is expected (Figure 28). It can be seen that when both conditions of uncertainty are in force, this zone becomes reduced to contract prices from 0.18 €/kWh to 0.19 €/kWh.

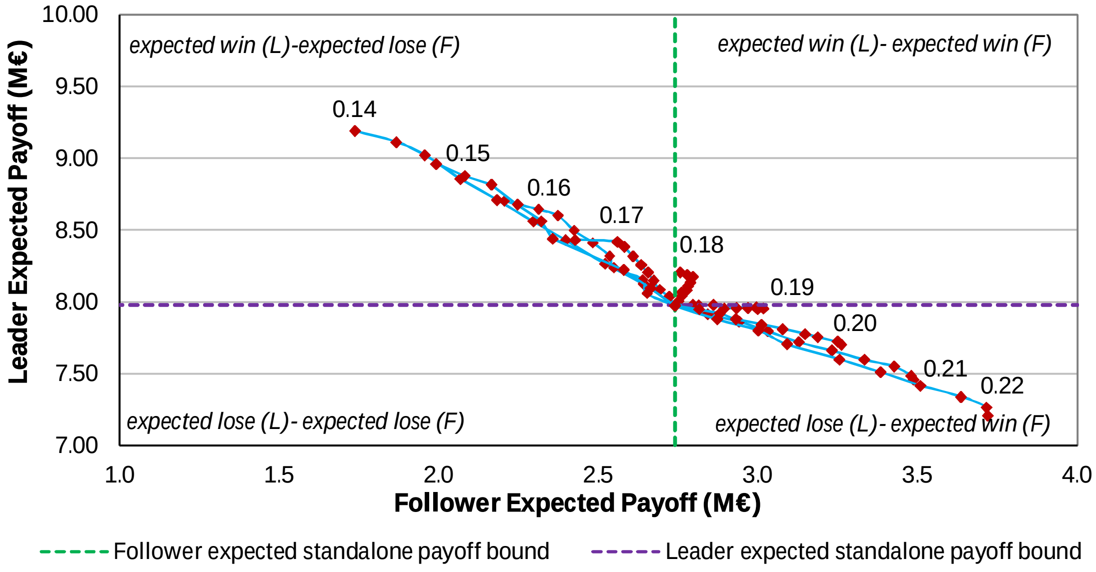


Figure 28- Leader expected payoffs vs. follower expected payoffs

The Stackelberg set of "Pareto frontier" is obtained (Figure 29) when the leader and the follower act under uncertain conditions. In any of these solution points, the follower shall offer the energy amounts between 21.00 GWh and 24.71 GWh to the leader with a significant profit expectations respect to the expected standalone case. The leader would also obtain expected benefits from this deal in case she/he offers prices from 0.18 €/kWh to 0.188 €/kWh. The rest of the energy amounts needed for the production SC of the leader is to be supplied from the local Grid. The similar prices on the Pareto frontiers correspond to different energy contract amounts.

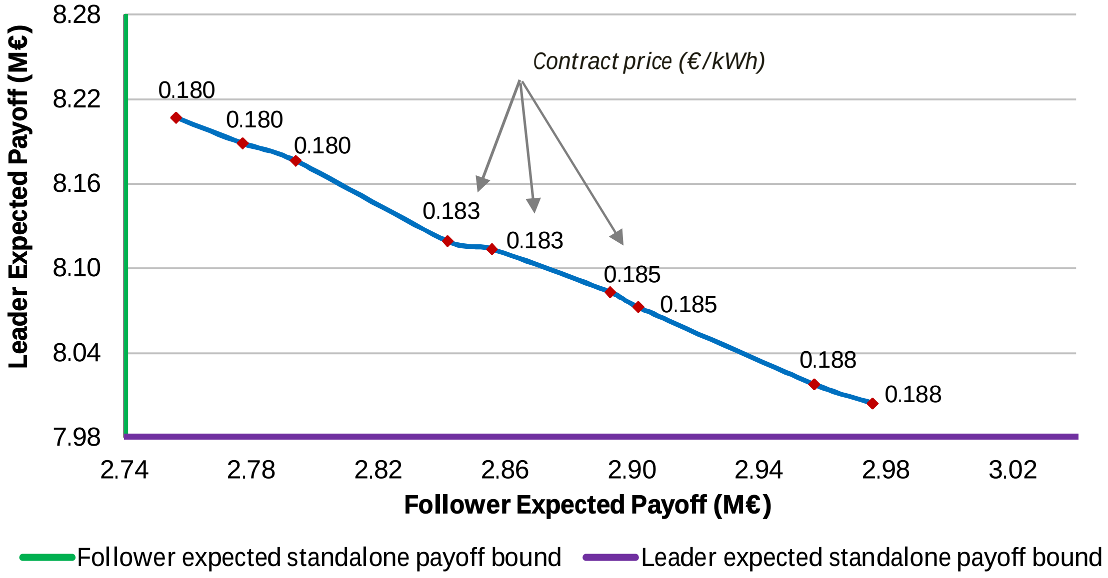


Figure 29- Stackelberg set of Pareto frontiers (leader & follower uncertain conditions)

Table 17 summarizes the possible coordination contracts based on the different uncertain conditions. It should be noticed, as it is mentioned in the above sections, that each game player seeks its SC individual nominal/expected benefits; the follower seeks higher prices and the leader seeks lower prices. When considering the uncertain conditions of both of them, a compromise can be reached for the tradeoff between their expected payoffs, excluding the prices < 0.18 €/kWh from the follower side and the prices >0.188 €/kWh from the leader side.

Table 17- Coordination contracts summary

	Coordination contract	
	Price (€/kWh)	Energy amount (GWh)
Nominal conditions	0.175-0.205	24.71
Follower uncertain condition	0.18-0.205	23.10-24.71
Leader uncertain conditions	0.173-0.188	23.10-24.71
Leader and follower uncertain conditions	0.18-0.188	21.00-24.71

From all the above-mentioned cases, the uncertain behavior of the 3rd parties affects the tradeoff between the conflicting objective partners and the equilibrium among the competing ones. Each coordination contract is able to mitigate the uncertainty of the 3rd parties resulting from the dynamic market situation. The coordination proves to be viable under the nominal and the uncertain conditions, thus stressing the SCs enterprises willingness to collaborate and negotiate the different proposed solutions.

4.6 Switching the game players roles

In the case of non-symmetrical games, which are the case of Stackelberg games, the role of each player depends on the power of its position. The reviewed literature usually considers that the client acts as the leader, so the case-study is initially solved in this way. But the proposed approach is applicable disregarding which partner is the leader and which one is the follower. In this section, the roles between the game players are switched, by considering the utility vendor as the leader and vice-versa, and the new results are analyzed in detail in order to demonstrate this capability. The vendor acts as the game leader and offers the coordination contract price; while, the client acts as the game follower and responds with the required energy amounts along the planning horizon (reaction function).

As in the opposite case, at the nominal conditions, the Stackelberg payoff matrix is built (Table 18 and Figure 30) for this situation. It is noticed that now the coordination contract price offer starts at 0.22 €/kWh.

Compared with Figure 7 at the same nominal conditions, Figure 30 shows that switching the roles of the game players leads to exclude the coordination contract price 0.17 €/kWh from the game. At price 0.17 €/kWh, before switching the roles (Table 10), the vendor payoff as follower was 2.35 M€; 7.8% more than when switching the roles (Table 18) for the same energy amount (24.71 GWh). This is because the follower is now the one who decides the internal energy flows (reaction function) according to its SC best conditions.

Table 18- Stackelberg payoff matrix (switched roles)

Leader action (€/kWh) →	Follower payoff vs. Leader payoff (M€)																	
	0.22		0.21		0.2		0.19		0.18		0.17		0.16		0.15		0.14	
Follower response (GWh) ↓	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L	F	L
0.00	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44	7.47	2.44
3.00	7.48	2.57	7.51	2.54	7.54	2.51	7.57	2.48	7.60	2.45	7.63	2.42	7.66	2.39	7.69	2.36	7.72	2.33
6.00	7.46	2.71	7.52	2.65	7.58	2.59	7.64	2.53	7.70	2.47	7.76	2.41	7.82	2.35	7.88	2.29	7.94	2.23
9.00	7.44	2.80	7.53	2.71	7.62	2.62	7.71	2.53	7.80	2.44	7.89	2.35	7.98	2.26	8.07	2.17	8.16	2.08
12.00	7.41	2.94	7.53	2.82	7.65	2.70	7.77	2.58	7.89	2.46	8.01	2.34	8.13	2.22	8.25	2.10	8.37	1.98
15.00	7.39	3.09	7.54	2.94	7.69	2.79	7.84	2.64	7.99	2.49	8.14	2.34	8.29	2.19	8.44	2.04	8.59	1.89
18.00	7.36	3.21	7.54	3.03	7.72	2.85	7.90	2.67	8.08	2.49	8.26	2.31	8.44	2.13	8.62	1.94	8.80	1.77
21.00	7.32	3.29	7.53	3.08	7.74	2.87	7.95	2.66	8.16	2.45	8.37	2.24	8.58	2.03	8.79	1.82	9.00	1.61
23.10	7.29	3.36	7.52	3.13	7.75	2.90	7.98	2.65	8.21	2.42	8.44	2.19	8.68	1.97	8.91	1.74	9.14	1.50
24.71	7.29	3.42	7.54	3.17	7.78	2.93	8.03	2.68	8.28	2.43	8.52	2.18	8.77	1.94	9.02	1.69	9.27	1.44

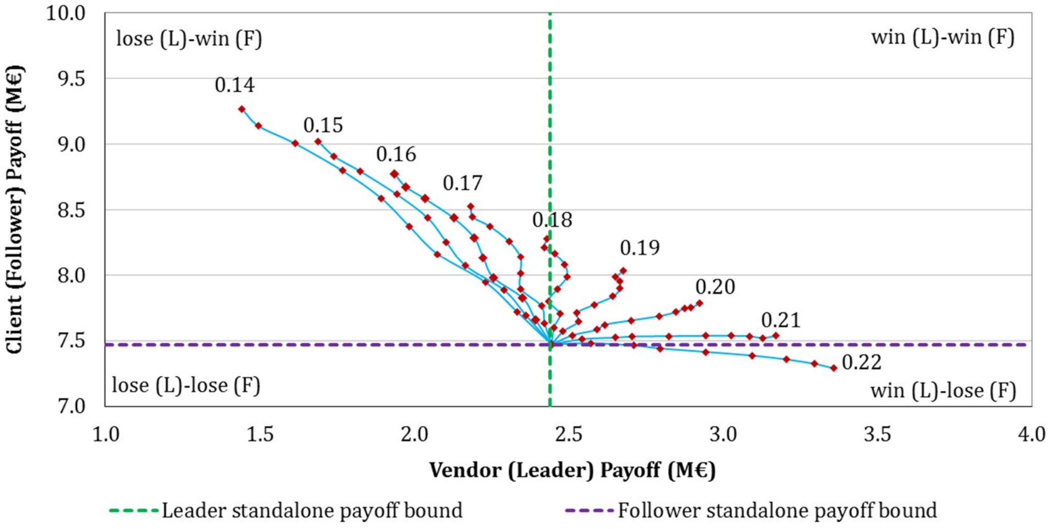


Figure 30- Leader vs. follower payoffs

The Stackelberg set of "Pareto frontier" is obtained from switching the roles (Figure 31). Compared with Figure 9 under the same nominal conditions, switching the roles leads to exclude lower prices 0.175-0.180 €/kWh from the game, and adding higher prices 0.208-0.210 €/kWh into the game for the same reason explained in the above paragraph. This is due to the leading role of the vendor, who seeks higher prices. This is because the client partner as follower decides the energy amounts, and she/he would exploit lower prices with higher energy amounts, stressing the vendor SC, and thus leading to less payoff for the vendor.

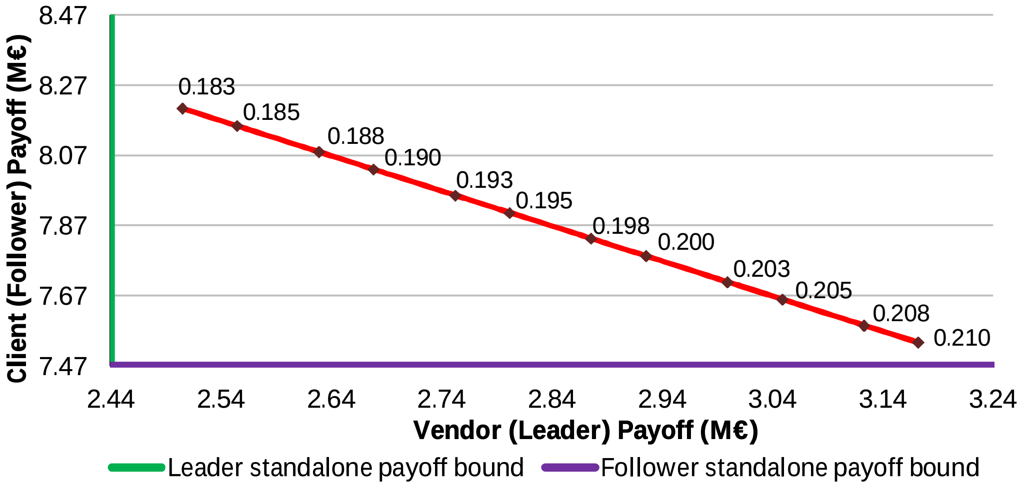


Figure 31- Stackelberg set of Pareto frontier: switching the game players roles

Figure 32 shows a comparison between the Stackelberg set of "Pareto frontiers" obtained from the original roles (client as leader: Figure 9) and from switching the roles (client as follower: Figure 31). Unexpectedly, the traditional belief that leading the game does not guarantee higher payoffs, although it affects the game outcome. According to the methodology discussed in this section, the follower player decides the internal energy amounts (reaction function) along the planning horizon according to its SC best conditions.

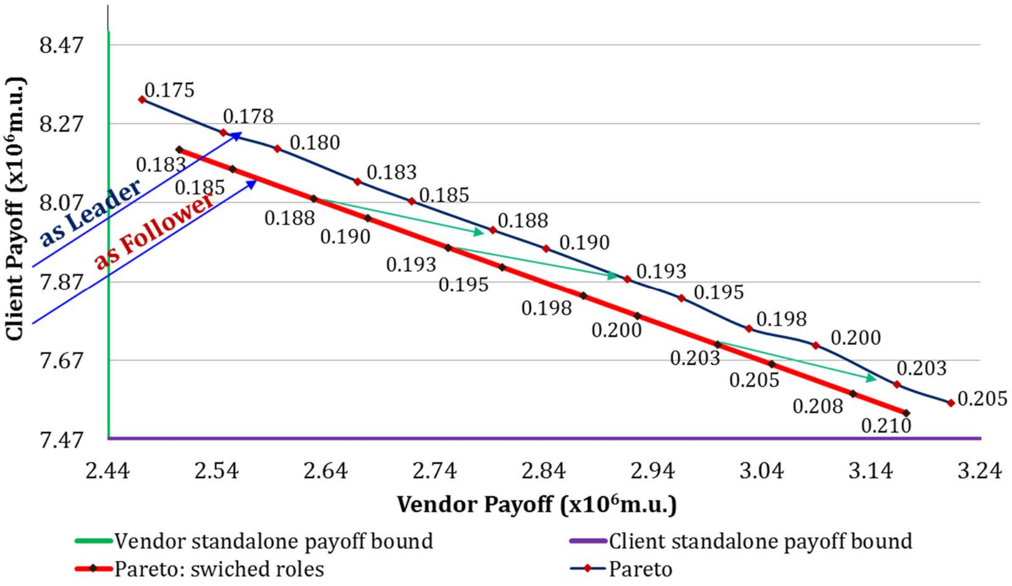


Figure 32- Stackelberg set of Pareto frontier: switching the game players roles

As can be noticed in Figure 33, the client role as follower leads to higher payoffs; ~ 1.32 % (100.74 x103m.u.) than when leading the game for the presented case study.

It is also noticed from Figure 34 that the vendor role as follower leads to ~ 6.5 % higher payoffs than when leading the game.

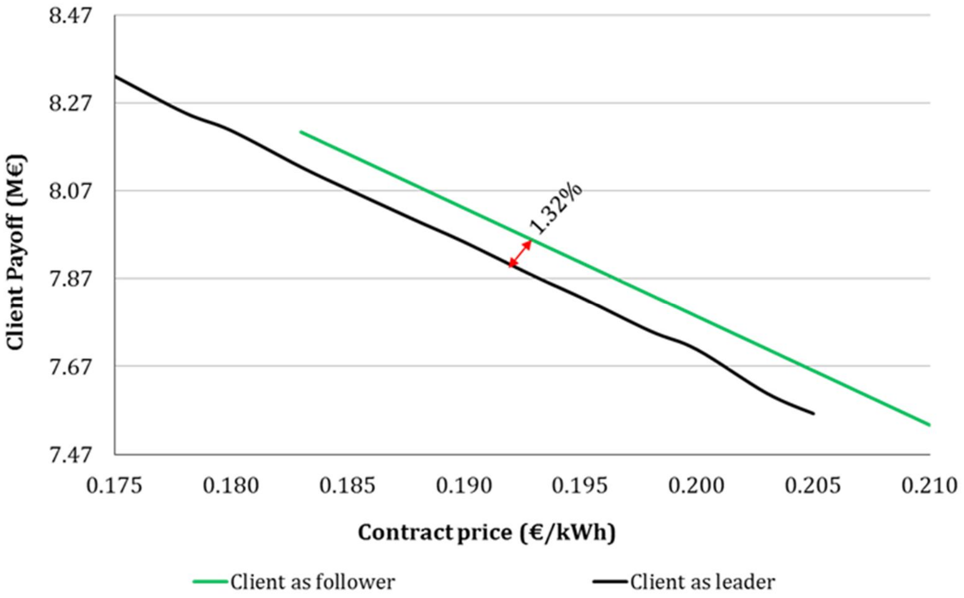


Figure 33- Client game role (Leader vs. follower)

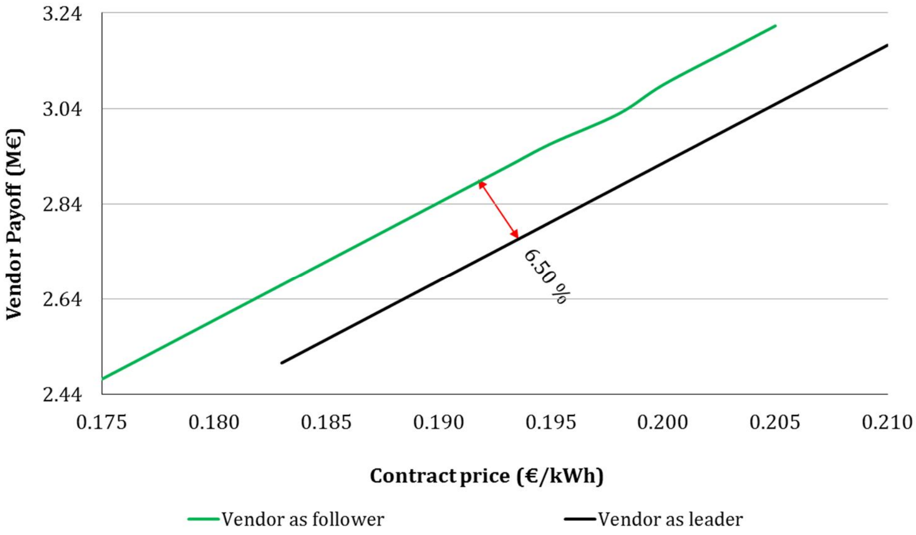


Figure 34- Vendor game role (Leader vs. follower)

Unlike the traditional myth, leading the game does not guarantee higher revenues. The game revenues depend not only on the game player's roles, but also on the reaction function, which causes the different outcomes.

5. Conclusions

This work presents an integrated-game (GT) method as a decision-support tool for Multi-Enterprise Wide Coordination (M-EWC) by determining the best coordination/collaboration contract between enterprises with conflicting objectives participating in a multi-enterprise global SC network. Based on non-symmetric roles, the interaction between enterprises with conflicting objectives is modeled as Stackelberg games non-cooperative non-zero-sum under the leading role of the manufacturer. The methodological framework of the Stackelberg game is based on building the payoff matrix under the nominal conditions. This payoff matrix then is evaluated using a Monte-Carlo sampling method by considering: (i) the uncertain conditions of the follower, ii) the uncertain conditions of the leader, and iii) the uncertain conditions of both game players (resulting from the uncertain nature of their third parties), so that the Stackelberg expected payoff matrix can be built. The competition between the clients on one side and between the vendors on the other side is solved using Nash Equilibrium (NE) game. The integrated GT outcome is represented using a novel Stackelberg set of "Pareto frontier", where each solution point is a Stackelberg-NE equilibrium coordination contract. The resulting coordination contracts mitigate the uncertainty effects of external conditions associated with each of the game players while keeping potential of higher profits expectations, compared with the standalone case. Furthermore, the resulting coordination contract reduces the risk that each game player may face, and thus stressing their willingness to collaborate for further negotiations.

The results of the integrated-GT approach are compared with the SBDN approach proposed by Hjaila et al. (2016a) considering the uncertain behavior of the follower game player using the same case study. The integrated-GT approach leads to better follower expected payoffs than the SBDN approach, while the SBDN leads to better profits for the leader partner. This difference is due to the different methodologies. Unlike the SBDN, the contract energy amounts of the coordination contracts resulted from the integrated-GT approach are the optimal amounts that the follower decides according to her/his best conditions. On the other side, using the SBDN, the leader decides the contract energy amounts according to her/his best conditions considering the probability of acceptance of the follower.

Certainly, if the follower rejects to collaborate at low prices, the leader still may try to establish a coordination contract with other utility vendors, which in this study are considered as "third parties" and globally represented by the "utility network". If the internal and/or external conditions of any of these third parties differ from the ones faced by the "original follower", this possibility may be studied following the same proposed procedure by just replacing the conditions of the "old" follower with the ones of the "new" potential follower.

The roles of the game players have been switched to study how this affects the game outcomes. The results on the presented case study show that leading the game does not guarantee higher payoffs: acting as the follower (so taking the last decision) results in higher payoffs. Actually, the game revenues depend not only on the game player's roles, but also on their reaction function.

The proposed approach adds to the PSE and OR communities a new practical decision-support tool towards improving the decentralized SC enterprise-wide decision-making. The proposed approach can be considered as a step-forward transitioning form from conceptual ideas, which the OR community has discussed for years, to actual implementation in realistic SCs applications.

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Financial support from the Spanish Ministry of Economy and Competitiveness and the European Regional Development Fund, both funding the Project SIGERA (DPI2012-37154-C02-01), and from the Generalitat de Catalunya (AGAUR FI program and grant 2014-SGR-1092-CEPEiMA), is fully appreciated.

Acronyms

CPU	Central Processing Unit
EWO	Enterprise-Wide Optimization
GAMS	The General Algebraic Modeling System
GB	Gigabyte
GHz	Gigahertz
GloMIQO	Global Mixed-Integer Quadratic Optimizer
GT	Game Theory
KKT	Karush–Kuhn–Tucker
LP	Linear Programming
M-EWC	Multi-Enterprise-Wide Coordination
MILP	Mixed Integer Linear Programming
MINLP	Mixed Integer Non-Linear Programming
MW	Megawatt
NE	Nash Equilibrium
NLP	Non-Linear Programming
OR	Operational Research
PSE	Process System Engineering
RM	Raw Material
SBDN	Scenario-Based Dynamic Negotiation
SCM	Supply Chain Management
SC	Supply Chain
SS	Standalone Scenario
WWTP	Wastewater treatment plant
μ	Mean
σ	Standard deviation

Nomenclature

Indexes

r	*resource (raw material, internal/final product, energy, manpower, ...)*
sc	*supply chain*
t	*time period*

Sets

C	*external client*
F	*follower*
i	*game player*
j	*game player*
L	*leader*
M	*external markets (final consumers)*
PL	*production plants*
R	*resources (raw materials, products, energy,...)*
r'	*game item*
S	*external RM suppliers*
SC	*supply chains*
T	*time periods*
V	*external vendor*
W	*warehouses/distribution center*
n	*scenarios*

Parameters

$dis_{r,sc}$	*travel distance of the resource* r *, supply chain* sc
$PRD^{min}_{r,pl,sc,t}$	*minimum production of resource* r *in production plant* pl *, time* t
$PRD^{max}_{r,pl,sc,t}$	*maximum production of resource* r *in production plant* pl *, time* t
$rp_{r,m}$	*retail price of resource* r *(final product)*
$ST^{max}_{r',w',t}$	*maximum storage capacity of resource* r' *at warehouse* w' *, leader SC, time* t
$ST^{min}_{r',w',t}$	*minimum storage capacity of resource* r' *at warehouse* w' *, leader SC, time* t
$uprd_{r,sc}$	*unit production cost of resource* r
$ust_{r,w,sc}$	*unit storage cost of resource* r *in warehouse* w
$utr_{r,sc}$	*unit transport cost of resource* r *, supply chain* sc
μ	*mean*
σ	*standard deviation*

Continuous variables

$COST_{sc}$	*cost, supply chain* sc
$CP_{w,pl,sc,t}$	*quantity flows from warehouses* w *to production plants* pl *, client SC, time* t
CPR_{sc}	*production cost*
CRM_{sc}	*external resources purchase cost*
CST_{sc}	*storage cost*

CTR_{sc}	*distribution cost*
CST_{sc}	*storage cost*
$ExPAYOFF_{sc}$	*expected payoff*
f_i	*objective function of player* i
$f_{r,r',sc}$	*production recipe of producing resource* r' *from resource* r *, follower Sc, time* t
$fac_{r',r,sc}$	*production recipe of resource* r *from resource* r' *,time* t
$FC_{r',sc,t}$	*resource* r' *flows from the follower SC to the external clients SC* C *, time* t
$FPD_{r',pl,sc,t}$	*production levels of* r' *in production plant* pl *, follower Sc, time* t
$FPRD_{r,r',pl,sc,t}$	*production levels of resource r' from resource r, production plant pl,follower SC, time* t
$g_F(x,y)$	*lower-level inequality constraints*
$G_L(x,y)$	*upper-level inequality constraints*
$h_F(x,y)$	*lower-level equality constraints*
$H_L(x,y)$	*upper-level equality constraints*
k_i	*strategy of player* i
k_{-i}	*strategy of the rest of the competitive players*
k_{-i}^{*}	*optimal strategy of the rest of the competitive players (* $-i$ *)*
k_i^{*}	*optimal strategy of player (* i
$Ldem_{r',pl,t}$	*demand of resource* r' *by production plant* pl *, leader SC, time* t
$LPRD_{r',r,pl,sc,t}$	*production levels of resource* r *(intermediate product, final product, etc.) from* r' *in production plants* pl *, leader SC, time* t
$MK_{r,w,sc,m,t}$	*resources* r *flows from the warehouses* w *to the final customers* m *, time* t
$PAYOFF_{sc}$	*aggregated payoff, supply chain* sc
$PAYOFF_{sc,n}$	*profit scenario*
$pC_{r',sc,t}$	*unit price of resource* r' *, client sc* C *, time* t
$pL_{r',sc}$	*unit transfer price of the game item* r'
$pV_{r',sc,t}$	*unit price of resource* r' *, vendor sc* V *, time* t
$PRD_{r,pl,sc,t}$	*production levels of resource* r *at production plant* pl *, time* t
$prob_{sc}$	*probability of acceptance*
$Q_{r,sc,t}$	*resources* r *flows, supply chain* sc *, time* t
q_F^{*}	*optimal strategy of the follower as NE-game player*
q_F	*strategy of the follower as NE-game player*
$QF_{r',sc,t}$	*amounts of resource* r' *from the follower SC each time period* t
$QFC_{r',w,sc,w',t}$	*quantity flows of* r' *from warehouse* w *of the follower SC to the the warehouse* w' *of the external client* C *, time* t

$QFC_{r',w',w,sc}$	*quantity flows of resource* r' *at the warehouses* w *of the client SC from the warehouses* w' *of the follower SC, time* t
$QL_{r',w',w,sc,t}$	*quantity flows of* r' *at warehouse* w *of the leader SC from the follower SC warehouses* w'*, time* t
$QVL_{r',w',w,sc,t}$	*quantity flows of* r' *from the external vendor warehouses* w' *to leader warehouses* w *, time* t
$RM_{r,s,sc,t}$	*resources* r *(i.e. RM) purchased from external suppliers* s *, time* t
$SALE_{sc}$	*economic sales, supply chain* sc
SN_{sc}	*number of payoffs successful scenarios*
$ST_{r',w,sc,t}$	*storage levels of* r' *at warehouse* w *, time* t
N_{sc}	*total number of generated scenarios (Monte-Carlo sampling)*
$VL_{r',sc,t}$	*amounts of resource* r' *purchased from the external vendor, time* t
$VP_{w,pl,sc,t}$	*quantity flows from warehouses* w *to to production plants* pl *, vendor SC, time* t
$xdem_{r,sc,m,t}$	*final customer demand of resource* r *, time* t
$Z_L(x,y)$	*upper-level objective function*
$z_F(x,y)$	*lower-level objective function*

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