The Multi-objective Assembly Line Worker Integration and Balancing Problem of Type-2

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Abstract

The consideration of worker heterogeneity in assembly lines has received a fair amount of attention in the literature in the past decade. Most of this exploration uses as motivation the example of assembly lines in sheltered work centers for the disabled. Only recently has the community started looking at the situation faced in assembly lines in the general industrial park, when in the presence of worker heterogeneity. This step raises a number of questions around the best way to incorporate heterogeneous workers in the line, maximizing their integration while maintaining productivity levels. In this paper we propose the use of Miltenburg's regularity criterion and cycle time as metrics for integration of workers and productivity, respectively. We then define, model and develop heuristics for a line balancing problem with

these two goals. Results obtained through an extensive set of computational experiments indicate that a good planning can obtain trade-off solutions that perform well in both objectives.
Keywords: Assembly Line Balancing, disabled workers, worker integration, regularity criterion.

1. Introduction

Traditional *assembly line balancing* (ALB) research focuses on the *simple assembly line balancing problem* (SALBP) initially defined by Baybars (1986) through several well-known simplifying hypotheses. This classical single-model problem consists of finding the best feasible assignment of tasks to stations so that precedence constraints are fulfilled. Two basic versions of this problem are called type-1, in which the cycle time, c, is given, and the aim is to minimize the number of needed workstations; and type-2, used when there is a given number of workstations, m, and the goal is to minimize the cycle time (Scholl, 1999).

Research focus has later changed in order to consider features present in more realistic industrial settings (Scholl and Becker, 2006; Becker and Scholl, 2006; Boysen et al., 2007, 2008; Battaïa and Dolgui, 2013) such as different line layouts, specific assignment constraints and multi-model lines, to cite a few. An important subset of this research has been interested in generalizing one of the main assumptions of the SALBP that states that workers have equal task processing times. In this context, some authors have considered the workforce with different *levels* of performance (Mansoor, 1968; Bartholdi and Eisenstein, 1996; Gel et al., 2002; Corominas et al., 2008; Koltai et al., 2014; Sungur and Yavuz, 2015, e.g.). We are particularly interested in another variant, in which heterogeneity is more pronounced, configuring the so called *assembly line worker assignment and balancing problem* (ALWABP) (Miralles et al., 2007). In this problem, inspired by assembly lines in *sheltered work centers for the disabled* (SWDs), workers are highly heterogeneous. Indeed, not only each worker might have a specific processing time for each task, but also each worker has a set of tasks that they can not execute, called incompatible tasks.

Research on the ALWABP has concentrated on the following four issues: (i) the development of exact algorithms; (ii) the development of fast heuristic solution methods; (iii) the extension of the basic problem to incorporate different line structures and (iv) the consideration of multiple objectives. This literature is reviewed below:

(i) A first branch-and-bound method has been proposed by Miralles et al. (2008). More recently, Borba and Ritt (2014) proposed a task-oriented branch-and-bound while Vilà and Pereira (2014) proposed a branch-bound-and-remember. These two latter methods are the state-of-the-art with respect to exact algorithms for the ALWABP.

(ii) A number of heuristics have been proposed for the ALWABP, including versions of tabu search (Moreira and Costa, 2009), beam search (Blum and Miralles, 2011; Borba and Ritt, 2014), genetic algorithms (Moreira et al., 2012; Mutlu et al., 2013), priority-based constructive heuristics (Moreira et al., 2012), ant-colony optimization (Bouajaja and Dridi, 2015) and variable neighborhood search (Polat et al., 2016). The best results so far have been obtained by the 2-phase variable neighborhood search of Polat et al. (2016) and the iterated probabilistic beam search of Borba and Ritt (2014).

(iii) Extensions of the ALWABP to incorporate specific line conditions have also received a fair amount of attention in the literature. These include the consideration of job rotation (Costa and Miralles, 2009; Moreira and Costa, 2013), parallel layouts for either stations or lines (Araújo et al., 2012, 2015), cooperation between workers (Araújo et al., 2012) and mixed-model assembly lines (Cortez and Costa, 2015).

(iv) Finally, different objectives other than the minimization of the number of stations (ALWABP-1) or the cycle time (ALWABP-2) have been considered, including minimization of operation costs (Ramezanian and Ezzatpanah, 2015), homogeneous distribution of load along the line (Zacharia and Nearchou, 2016) and ergonomic risks (Akyol and Baykasoğlu, 2016).

All these studies have kept the original motivation of sheltered work centers and therefore consider a full set of heterogeneous workers. In spite of the extreme social importance of SWDs and its significant contribution in providing job opportunities for persons with disabilities, it still configures a somehow segregating space, not completely achieving the ultimate goal of providing a fully societal integration. This question has recently started to be addressed: Moreira et al. (2015b) extended some ALWABP models and algorithms to conventional companies, defining the *assembly line worker integration and balancing problem* (ALWIBP). In this study, the authors consider the balancing of lines in which only a percentage of workers have heterogeneous capabilities, mimicking the situation of assembly lines in conventional industrial settings that open job opportunities for persons with disabilities. Moreira et al. (2015a) have also considered the robustness of the obtained solutions when faced with task time variability.

These two articles have opened a new avenue for research, in which particularities associated with the operation of mixed (with conventional and heterogeneous workers) lines must be addressed. One new practical aspect of the problem is the need to evenly distribute workers with special characteristics along the lines. This becomes an important feature in conventional industries mixed (as defined above) lines, in which one usually aims to evenly distribute workers with disabilities. This generates configurations that do not segregate workers with disabilities in clusters and allow them to fully cooperate with more experienced workers. Interestingly, the even distribution of a subset of workers along the line may also be an important ingredient in some SWDs, in which a few number of more experienced workers - or monitors - might also be present and one would like to have them evenly spaced throughout the line.

Given the context presented above, we are interested in extending existing approaches, both to the ALWABP as well as to the ALWIBP, in order to consider a regular distribution of a special set of workers in the line. The main scientific contributions of this paper are, therefore:

- The proposal of a metric for evaluating a proper distribution of workers along the assembly line. This metric is based on Miltenburg (1989) production variation rate criteria.

- The linearization of the above mentioned criteria within a mixed-integer program that can be used to obtain the $\tilde{p}$ (user-defined parameter) best possible configurations.

- The proposal of two fast heuristics in order to assign workers and tasks to stations while attempting to minimize cycle time and respecting worker-assignment additional constraints derived from the proposed regularity criterion.

These proposals are evaluated through a large set of computational experiments which, in a nutshell, indicate not only their computational efficiency but also the viability of using such procedures in practical contexts without sacrificing productivity levels. As observed in other assembly line balancing problems (Kara et al., 2011; Manavizadeh et al., 2012; Chutima and Naruemitwong, 2014; Saif et al., 2014; Triki et al., 2014, e.g.), the ALWIBP when considering a specific distribution of workers has a multi-objective nature. Therefore, the evaluation of solutions has to be done using adequate tools.

In this paper, we use a number of such tools to evaluate the quality of the proposed methods with respect to the natural trade-off between the conflicting objectives. More specifically, we use the following metrics: *error ratio* (Deb, 2001; Coello et al., 2002; Coello and Lamont, 2004), *hyper area covered* (Zitzler and Thiele, 1998, 1999), *hyper area ratio* (Coello et al., 2002; Coello and Lamont, 2004), *coverage relation* (Zitzler and Thiele, 1999), *spacing* (Deb, 2001; Coello et al., 2002; Coello and Lamont, 2004) and *generational distance* (Deb, 2001; Deb et al., 2002; Coello et al., 2002; Coello and Lamont, 2004), are used to evaluate the quality of the proposed methods with respect to the natural trade-off between the two conflicting objectives regarding worker distribution and productivity.

The remainder of this paper presents a formal definition of the problem (Section 2), a discussion of the use of Miltenburg's criteria (Section 3), the description of the proposed heuristics (Section 4) and a thorough computational study (Section 5). Conclusions are presented in Section 6. For convenience, a list of symbols and acronyms is provided in the appendix.

2. Formal definitions and a mathematical model for the ALWIBP-2

The goal of the ALWIBP-2 is, given a fixed number of workstations, m, to find an assignment of tasks and workers minimizing the cycle time, c, such that precedence relationships and incompatibilities are respected. As follows, we present the notations and the ALWIBP-2 mathematical model introduced by Moreira et al. (2015b).

Data:

$S = \{1, ..., m\}$	set of workstations;
$W = \{1, ..., d\}$	set of heterogeneous workers;
$N = \{1, ..., n\}$	partially ordered set of tasks;
$F_i = \{j \in N \vert i \lessdot j\}$	set of immediate successors of task i, where $i \lessdot j$ indicates that task i is an immediate predecessor of task j;
t_i (t_{wi})	execution time of task i when assigned to a conventional (heterogeneous) worker;
$W_i \subseteq W$	subset of heterogeneous workers that are able to execute task i;
$N_w \subseteq N$	set of tasks that worker $w \in W$ can execute;
l_w	a positive constant, $w \in W$.

Variables:

$x_{si} \in \{0, 1\}$	equal to 1 only if task $i \in N$ is assigned to workstation $s \in S$;
$y_{sw} \in \{0, 1\}$	equal to 1 only if worker $w \in W$ is assigned to workstation $s \in S$;
c	cycle time.

Model M1:

$$\text{minimize } c \tag{1}$$

subject to

$$\sum_{s \in S} x_{si} = 1 \qquad i \in N \tag{2}$$

$$\sum_{i \in N} x_{si} \geq 1 \qquad s \in S \tag{3}$$

$$\sum_{s \in S} y_{sw} = 1 \qquad w \in W \tag{4}$$

$$\sum_{w \in W} y_{sw} \leq 1 \qquad s \in S \tag{5}$$

$$\sum_{\substack{s \in S; \\ s \geq k}} x_{si} \leq \sum_{\substack{s \in S; \\ s \geq k}} x_{sj} \qquad i \in N, j \in F_i, k \in S \backslash \{1\} \tag{6}$$

$$\sum_{i \in N} t_i x_{si} \leq c \qquad s \in S \tag{7}$$

$$\sum_{i \in N_w} t_{wi} x_{si} \leq c + l_w (1 - y_{sw}) \qquad s \in S, w \in W \tag{8}$$

$$y_{sw} \leq 1 - x_{si} \qquad s \in S, i \in N, w \notin W_i \tag{9}$$

$$x_{si} \in \{0, 1\}, \qquad s \in S, i \in N \tag{10}$$

$$y_{sw} \in \{0, 1\} \qquad s \in S, w \in W \tag{11}$$

$$c \geq 0. \tag{12}$$

The objective function (1) minimizes the cycle time of the assembly line. Constraints (2)–(3) guarantee that each task is assigned to a single workstation and that all workstations have at least one task, respectively. Constraints (4) state that each heterogeneous worker is assigned to a workstation, while Constraints (5) establish that a workstation can receive at most one worker. Precedence relations among tasks are enforced by Constraints (6). Constraints (7) and (8) ensure that the cycle time is respected in workstations without or with heterogeneous workers, respectively. Note that parameter l_w defined earlier must be sufficiently large to deactivate these latter constraints if $y_{sw} = 0$. We assume that t_i is the reference value for the execution time of task i and $t_i \leq t_{wi}$, for each $w \in W$. Then, we use $l_w = \sum_{i \in N_w} t_{wi} - t_i$ which indicates the maximum additional time that a heterogeneous worker w spends at a station in comparison to a "conventional" worker. Constraints (9) imply that tasks are not assigned to heterogeneous workers who are not able to execute them.

3. Incorporating Miltenburg's criteria

In this paper, we consider the ALWIBP-2 which minimizes the cycle time while trying to obtain a more appropriate assignment of workers. This second goal aims at distributing as evenly as possible the heterogeneous workers along the line, so that: (i) "conventional" workers can assist them in the execution of their tasks (in SWDs, for example) or (ii) clusters of heterogeneous workers are avoided. In both cases, the main idea is to obtain a line balancing in which workers from a special set (monitors, in the case of lines in SWDs or heterogeneous workers, in the case of mixed lines) are evenly distributed along the line .

In order to evaluate this criterion, we adapt one of the regularity criteria proposed by Miltenburg (1989). The author introduces the *product rate variation problem* (known in the literature as PRV), which appears in *just-in-time* production systems (JIT), in particular in mixed-model assembly lines. The goal of a regular distribution of products is relevant in these environments since they aim at producing only needed quantities and therefore require important to maintain the usage rate of the line.

In mathematical terms, let p be the quantity of different products and $\tilde{u}$ be the total number of units to sequence. Also, consider u_i as the number of units of each product and β_{ij} as the quantity of product i sequenced until position j. Then, we can measure the regularity criterion of a schedule, r, by:

$$r = \sum_{i=1}^{p} \sum_{j=1}^{\tilde{u}} \left(\beta_{ij} - \frac{u_i}{\tilde{u}} j\right)^2 \tag{13}$$

Kubiak and Sethi (1991) proposed an assignment formulation for the PRV which can be extended for more general objective functions. Later, Bautista et al. (1996a) considered relations between the PRV and the apportionment problem in order to state some useful properties to solve the former one. More studies concerning other JIT scheduling problems as well as different sequencing metrics can be found in (Monden, 1983; Miltenburg and Sinnamon, 1989; Kubiak, 1993; Bautista et al., 1996b; Duplaga and Bragg, 1998; Lebacque et al., 2007; Boysen et al., 2009).

In our context and considering the case of a mixed line, let v_s (z_s) be the number of "conventional" (heterogeneous) workers assigned in stations up to (and including) station s. Then, r is computed as follows:

$$r = \sum_{s \in S} \left[\left(z_s - \frac{|W|}{|S|} s\right)^2 + \left(v_s - \frac{|S| - |W|}{|S|} s\right)^2 \right] \tag{14}$$

Note that we do not need to consider both terms in (14), since the regularity of the distribution of workers with disabilities implies a proper placement of the others. For this purpose, see that $v_s = s - z_s$, $s \in S$. Therefore equation (14) can be rewritten as:

$$\begin{aligned} \mathrm{r} = \sum_{s\in S}\left[\left(z_s - \frac{|W|}{|S|}s\right)^2 + \left(s - z_s - \frac{|S|-|W|}{|S|}s\right)^2\right] = \\ \sum_{s\in S}\left[\left(z_s - \frac{|W|}{|S|}s\right)^2 + \left(-z_s + \frac{|W|}{|S|}s\right)^2\right] = \\ \sum_{s\in S}\left[2\left(z_s - \frac{|W|}{|S|}s\right)^2\right] = 2\sum_{s\in S}\left[\left(z_s - \frac{|W|}{|S|}s\right)^2\right]. \end{aligned} \tag{15}$$

In order to present the worker regularity problem formulation, we define:

Variables:

$\pi_s \in \{0,1\}$	equal to 1 if a heterogeneous worker is assigned to workstation $s \in S$;
$z_s \in \mathbb{Z}$	number of heterogeneous workers assigned in stations up to (and including) station s.

and write the model as:

Model M2:

$$\text{minimize} \sum_{s\in S}\left(z_s - \frac{|W|}{|S|}s\right)^2 \tag{16}$$

subject to

$$\sum_{s\in S}\pi_s = |W| \tag{17}$$

$$z_s = \sum_{\substack{s'\in S;\\ s'\le s}}\pi_{s'} \qquad \forall s \in S \tag{18}$$

$$\pi_s \in \{0,1\} \qquad \forall s \in S \tag{19}$$

$$z_s \in \mathbb{Z} \qquad \forall s \in S. \tag{20}$$

The objective function (16) prioritizes the uniform distribution of workers. Constraints (17) guarantee the assignment of all heterogeneous workers in the assembly line, while constraints (18) compute their cumulative amount at each position of the line. Constraints (19) and (20) are integrality constraints.

The resulting model is clearly nonlinear. Considering:

Data:

$h_{(s)} = max\{0, \|W\| - \|S\| + s\}$	minimum possible number of heterogeneous workers that can be assigned up to station s;
$\kappa_{(s)} = min\{s, \|W\|\}$	maximum possible number of heterogeneous workers that can be assigned up to station s;
$K_s = \{h_{(s)}, ..., \kappa_{(s)}\}$	set of possible values for the number heterogeneous workers assigned up to station s.

Variables:

$\gamma_{s\kappa} \in \{0, 1\}$	equal to 1 only if there are κ heterogeneous workers allocated until workstation s.

A linear version of model *M2* can be written as:

Model M3:

$$\text{minimize} \sum_{s \in S} \sum_{\kappa \in K_s} \left[\kappa^2 - 2\kappa \left(\frac{|W|}{|S|} s \right) + \left(\frac{|W|}{|S|} s \right)^2 \right] \gamma_{s\kappa} \tag{21}$$

subject to

(17), (19) and

$$\sum_{\kappa \in K_s} \gamma_{s\kappa} = 1 \qquad s \in S \tag{22}$$

$$\sum_{\substack{s' \in S; \\ s' \leq s}} \pi_{s'} = \sum_{\kappa \in K_s} \kappa \gamma_{s\kappa} \qquad s \in S \tag{23}$$

$$\gamma_{s\kappa} \in \{0, 1\} \qquad s \in S, \kappa \in K_s. \tag{24}$$

The objective function (21) is a linearized version of the sum of the squared deviations of ideal and real number of heterogeneous workers and "conventional" ones. Constraints (22)–(23) compute the number of heterogeneous workers partially distributed along the assembly line.

Observe that this model is more appropriate if we deal with assembly lines with more "conventional" workers than heterogeneous ones, since the cardinality of set K_s, $s \in S$, will be lower and hence, the model will be more compact. For assembly lines with a larger number of heterogeneous workers (and a few monitor workers, such in SWDs), we indicate a formulation using the second term of equation (14) as objective function in order to reduce the number of variables in the process of linearization, which is analogous to the previous one. We also point out that the methods and models are valid when there is the general objective of distributing a given subset of workers along the line.

We can obtain solutions by minimizing the cycle time and adding a set of constraints to ensure that heterogeneous workers are assigned to the indicated positions. This yields model *M4*, which is presented below.

Data:

$s^p = ({s_1}^p \ {s_2}^p \ \dots \ {s_m}^p)$	m-tuple representing the positions of heterogeneous workers in the line such that ${s_k}^p = 1$ if there is a heterogeneous worker $w \in W$ in station $k \in S$.

Model M4:

$$\text{minimize } c \tag{25}$$

subject to

(2)–(12) and

$$\sum_{w \in W} y_{sw} - {s_s}^p = 0 \qquad \forall s \in S. \tag{26}$$

Constraints (26) force heterogeneous workers to be assigned to a pre-determined set of workstations, defined by the m-tuple s^p.

Finally, the *worker regularity generator model* (WRGM) extends model *M3* in order to avoid the same configuration being found more than once.

Data:

$\overline{p}$	number m-tuples s^p to be generated;
$P_{\overline{p}}$	set of of worker configurations, where $p \in \{1, ..., \overline{p}\}$.

Model WRGM:

$$\text{minimize} \sum_{s \in S} \sum_{\kappa \in K_s} \left[\kappa^2 - 2\kappa \left(\frac{|W|}{|S|} s \right) + \left(\frac{|W|}{|S|} s \right)^2 \right] \gamma_{s\kappa} \tag{27}$$

subject to

$$(17), (19), (22)\text{–}(24) \text{ and}$$

$$\sum_{k \in S} s_k^{p'} \pi_k \leq |W| - 1 \qquad p' \in \{1, ..., p-1\}, s^{p'} \in P_{p-1}. \tag{28}$$

Constraints (28) prohibit that worker distribution π is identical to the ones already in set $P_{\overline{p}}$. In order to evaluate the cycle time for the worker distributions, we call model *M4* for each $s^p \in P_{\overline{p}}$, $p = 1, ..., \overline{p}$. Note that in the end of each execution, we will have the information of which worker is assigned to each workstation. Algorithm 1 summarizes the procedure.

In the Algorithm, during the loop in lines 3–5, we apply the auxiliary function *solveM4*(s^p) that has as input a tuple s^p and solves the *M4* model. In the end, the algorithm returns the best cycle time value for each tested distribution s^p.

4. Heuristic methods

In this section, we present three *constructive heuristics* for the ALWIBP-2 with productivity and integration objectives. As in the previous section,

Algorithm 1 Evaluation of the cycle time for each worker regularity distribution

1: **given** $P_{\overline{p}}$ (obtained with model WRGM);
2: Consider $c^{\overline{p}} = \{\hat{c}^1\ \hat{c}^2\ ...\ \hat{c}^{\overline{p}}\}$ a $\overline{p}$-tuple that measures the cycle time for each configuration $p \in \{1, ..., \overline{p}\}$;
3: **for all** $p \in \{1, \ldots, \overline{p}\}$ **do**
4: $c^p \leftarrow solveM4(s^p)$;
5: **end for**
6: **return** $c^{\overline{p}}$.

all three procedures start by generating $P_{\overline{p}}$, the set of tuples containing the heterogeneous workers positions to be tested. The second step concerns the generation of task and worker assignments respecting these configurations. The second step differs in the three different heuristics, as explained in the following.

4.1. Phase 1: Generation of the type worker configurations

The size of set $P_{\overline{p}}$ grows exponentially with the number of workstations. For instance, if we take an assembly line with 30 workstations and 9 heterogeneous workers, we have $|P_{\overline{p}}| = C_9^{30} = 14.307.150$ possible configurations. Using model WRGM presented earlier, we consider the $\tilde{p} = 30$ best solutions in terms of the adaptation of Miltenburg's criterion proposed earlier.

4.2. Phase 2: Task and worker assignments

Given set $P_{\overline{p}}$ obtained in Phase 1, we perform the task and worker assignments according to three different procedures: a *M4-based heuristic* (MH, Section 4.2.1), a *worker regularity constructive heuristic* (WRCH, Section 4.2.2) and a *worker regularity constructive heuristic with randomness* (WRCHR, 4.2.3).

4.2.1. MH: M4 model based heuristic

The *MH* performs the task and worker assignments using Algorithm 1 for $\overline{p}$ equal to $\tilde{p}$. For each configuration $s^p \in P_{\tilde{p}}$, $p \in \{1, ..., \tilde{p}\}$, the Algorithm is allowed to run for $\mathcal{T}_p$ seconds. If the model is not solved to optimality in $\mathcal{T}_p$ seconds, we take the best incumbent solution found by time $\mathcal{T}_p$. In these runs, CPLEX was tuned for feasibility.

4.2.2. WRCH: worker regularity constructive heuristic

The task assignment phase of the WRCH is based on the ALWABP-2 *constructive heuristic* (CH) proposed by Moreira et al. (2012). The CH initially estimates a range of cycle times. For each cycle time value (in increasing order), the algorithm tries to assign tasks and workers to stations subject to precedence constraints. Assignment is made sequentially, in a workstation-oriented fashion and makes use of worker and task priority rules (Moreira et al., 2012). The procedure stops when the solution found is feasible for the current cycle time. We consider all 16 task priority rules and 3 worker allocation criteria proposed in (Moreira et al., 2012).

We perform task and worker assignments according to configurations $s^p \in P_{\tilde{p}}$, $p \in \{1, ..., \tilde{p}\}$. The worker-assignment procedure in CH is modified in order to only assign heterogeneous workers to the positions indicated in s^p.

4.2.3. WRCHR: worker regularity constructive heuristic with randomness

We extend the WRCH by including randomness in task assignment. Let $\mathcal{N} \subseteq N$ be the set of candidate tasks such that $i \in \mathcal{N}$ if and only if all predecessors of i have already been assigned and the insertion of i on the current station respects the cycle time constraints. Consider $\mathcal{R} \subseteq \mathcal{N}$ as the set of tasks with the best value of the priority rule being used. The *worker regularity constructive heuristic with randomness* (WRCHR) adapts the task selection of CH randomly choosing with the same probability a task $j \in \mathcal{R}$ at each step. The task and worker priority rules remain the same as presented in the WRCH.

4.3. A numerical example

In this section, we illustrate how the heuristics run through an instance called Example 1, generated for this purpose. Consider an assembly line with 5 workstations, 3 "conventional" workers and 2 heterogeneous ones (w_1 and w_2). The task precedence graph is shown in Figure 1, while the task execution times by worker are presented in Table 1.

In Table 2, we can see the worker placements generated by the WRGM. For example, in the first sequence, there are heterogeneous workers at stations 2 and 4. As follows, we take this sequence in order to show the solutions obtained by the three heuristics.

The task assignment prioritizes candidates with the largest number of followers, while worker allocation takes into account the one who obtained the minimum value of restricted lower bound. Table 3 shows the number of

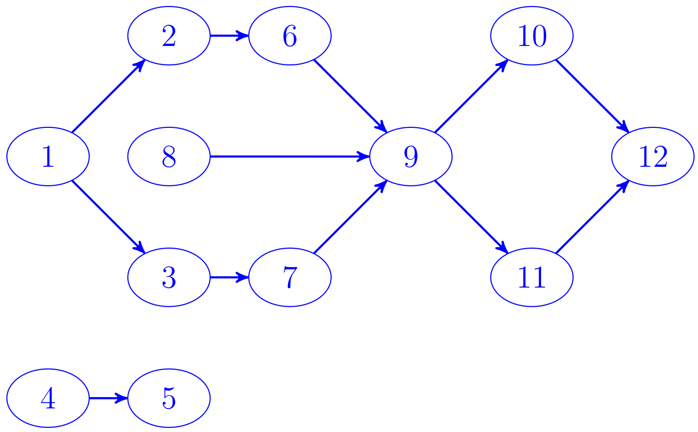

Figure 1: Task precedence graph of Example 1.

Table 1: Task times of Example 1.

i	Task time		
	t_i	t_{w_1}	t_{w_2}
1	10	10	20
2	12	12	12
3	15	23	19
4	15	20	31
5	10	25	10
6	7	18	35
7	8	20	15
8	15	∞	15
9	12	26	∞
10	20	∞	40
11	30	31	32
12	40	50	∞

followers for each task of Figure 1. Tables 4, 5 and 6 present informations about the solutions obtained by MH, WRCH and WRCHR, respectively. We indicate the worker, the workload and the tasks placed at each station. Since "conventional" workers do not distinguish themselves in terms of task times, we represent them by the symbol "-".

Table 4 contains the lowest cycle time given an optimal solution concerning Miltenburg's regularity criterion. The cycle time obtained by WRCH

Table 2: Results obtained by the WRGM for Example 1. Column r presents the value of the worker distribution criteria.

s^p	Workstations					r
	1	2	3	4	5	
1	0	1	0	1	0	0.4
2	1	0	0	1	0	0.6
3	0	1	0	0	1	0.6
4	1	0	0	0	1	0.8
5	0	0	1	1	0	1.0
6	0	1	1	0	0	1.0
7	1	0	1	0	1	1.2
8	0	0	1	0	1	1.2
9	1	1	0	0	0	2.6
10	0	0	0	1	1	2.6

Table 3: Number of followers of each task of Example 1.

Task	Number of followers
1	8
2	5
3	5
4	1
5	0
6	4
7	4
8	4
9	3
10	1
11	1
12	0

(Table 5) is worse than the aforementioned, with the same worker sequence. Note that the task priority rule scheme assigns tasks with the largest number of followers while the current station capacity is not exceeded. See that the choice of w_1 in this case is due to lower idle time that this worker will produce in this station, since the restricted lower bound of both workers is undefined (there are unassigned tasks that both are not able to perform).

Table 4: Results obtained by the MH for Example 1.

Workstations	Workers	Workload	Tasks
1	-	44	1 2 4 6
2	w_2	44	3 5 8
3	-	40	7 9 10
4	w_1	31	11
5	-	40	12
$c = 44$	$r = 0.4$		

Table 5: Results obtained by the WRCH for Example 1.

Workstations	Workers	Workload	Tasks
1	-	44	1 2 3 6
2	w_1	40	4 7
3	-	47	8 9 10
4	w_2	42	5 11
5	-	40	12
$c = 47$	$r = 0.4$		

Table 6 shows two repetitions of WRCHR. Note that the randomness of this algorithm can produce better solutions, as we see in the task assignment of station 2 of the first solution (Repetition 1). In this case, the worker w_2 is the only candidate for the assignment, since worker w_1 is not able to execute task 8.

5. Experimental study

We have carried out an experimental study over a new benchmark proposed for the ALWIBP-2. In Section 5.1, we introduce this set of instances. Section 5.2 presents the numerical tests concerning the ALWIBP-2 model (Section 5.2.1) and heuristics (Section 5.2.2). Section 5.2.3 compare the effectiveness of the algorithms using some multi-objective optimization measures.

5.1. Benchmark scheme

The proposed benchmark for the ALWIBP-2 considers conventional assembly lines with a parcel of heterogeneous workers. We select the 100 in-

Table 6: Results obtained by the WRCHR for Example 1.

Rep. 1				
	Workstations	Workers	Workload	Tasks
	1	-	44	1 2 4 6
	2	w_2	44	3 5 8
	3	-	40	7 9 10
	4	w_1	31	11
	5	-	40	12
$c = 44$	$r = 0.4$			

Rep. 2				
	Workstations	Workers	Workload	Tasks
	1	-	40	4 5 8
	2	w_1	45	1 2 3
	3	-	47	6 7 9 10
	4	w_2	32	11
	5	-	40	12
$c = 47$	$r = 0.4$			

stances from (Otto et al., 2013) for each family of instances with 50 (middle-sized) and 100 (large-sized) tasks. These examples were the same used in (Moreira et al., 2015b), and have the following characteristics: (i) mixed precedence graphs (with chain and bottleneck structures); (ii) "high" and "low" order strengths; and (iii) task times generated according to "peak at the bottom" and "bimodal" distributions.

Precedence graphs are kept as in the original instances, as well as task execution times t_i. In order to generate task execution times for heterogeneous workers, we use uniform distributions that depend on the kind of instance being generated (low or high task variability and low or high task $\times$ worker incompatibilities). The parameters used are as follows:

- Variability of task execution times (Var): "low" ($U[t_i, 2t_i]$) and "high" ($U[t_i, 5t_i]$);
- Task/worker incompatibilities (Inc): "low" (10% of total of tasks) and "high" (20% of total of tasks);

Taking both factors, we have a total of 4 instances generated for each original instance from (Otto et al., 2013). The number of stations m considered for each of them is the same as its corresponding SALBP instance. The quantity of "conventional" and heterogeneous workers is defined by $m - \lceil \mu \times m \rceil$ and $\lceil \mu \times m \rceil$, where μ is a parameter indicating the estimated percentage of heterogeneous workers in the assembly line. We use three values for μ, given by 10%, 20% and 30%, generating a total of 1200 instances for each group of 50 and 100 tasks.

5.2. Computational study

In the following sections, we measure the performance of both models and algorithms proposed for the ALWIBP-2 and its variant with worker regularity goal. The methods were coded in C++, in the Linux operational system. Computational tests were conducted in a Intel(R) Xeon(R) X5675 3.07GHz 96 GB RAM machine, using IBM CPLEX 12.6 with 1 thread and 6 GB as the size limit of the search tree.

5.2.1. Experiment 1: M1 model

We set up the solution time limit for model *M1* as 3,600s. Tables 7 and 8 use the following criteria as average performance measures:

- Δ_c(%): percentage of proved optimal solutions;
- Υ_1^*(%): percentage increase in the ALWIBP-2 cycle time when compared to the reference solution for the SALBP-2;
- Gap(%): optimality gap obtained by CPLEX in the allowed time limit;
- t(s): run time.

In Table 7, the results show that the model proved optimal solutions in approximately 69% of instances. Note that the reduced gaps indicate the possibility that many of the heuristic solutions are actually optimal, even though gaps of optimality have not been closed. Incorporating heterogeneous workers and regularity distribution criteria had an effect of degrading the cycle time, but in our tests this was kept in reasonable values (an average 7.5% increase in cycle time). This seems like a reasonable price to pay, specially considering that the generated instances include more heterogeneous workers

Table 7: ALWIBP-2 model – results of middle instances (50 tasks).

μ	Var	Inc	$\Delta_c(\%)$	$\Upsilon_1^*(\%)$	Gap(%)	t(s)
10	2	10	70	2.2	0.3	1,168
		20	68	2.6	0.3	1,277
	5	10	72	5.3	0.2	1,102
		20	75	5.7	0.2	1,087
20	2	10	61	4.0	0.5	1,487
		20	67	4.6	0.4	1,264
	5	10	73	10.0	0.3	1,067
		20	73	10.7	0.3	1,068
30	2	10	66	6.3	0.7	1,335
		20	61	6.6	0.7	1,482
	5	10	74	15.2	0.6	1,116
		20	70	16.6	0.8	1,175
	Avg.		69	7.5	0.4	1,219

(a minimum of 10%, in the easiest instances) than it is usually required by most national legislations.

Table 8 shows the results obtained for instances with 100 tasks. As expected, the number of optimal solutions proved and the average gap of optimality have worsened due to the higher complexity of these examples. However, the greater flexibility to assign tasks to workstations in this context reduced the increased percentage of cycle time compared with SALBP-2. Finally, the computational time to solve both groups of instances proved to be reasonable and applicable in operational planning in ALWIBP-2 environments.

5.2.2. Experiment 2: ALWIBP-2 approaches

This section addresses comparisons among the three heuristics applied to the ALWIBP-2 with worker regularity criterion. Preliminary tests have been conducted to establish the parameters of the algorithms. According to our experiments, we take $\tilde{p}$ and $\mathcal{T}_p$, $p \in \{1, ..., \tilde{p}\}$, equal to 30 solutions and 1,800s, respectively. Furthermore, the WRCHR was run 20 times in order to

Table 8: ALWIBP-2 model – results of large instances (100 tasks).

μ	Var	Inc	Δ_c(%)	Υ_1^*(%)	Gap(%)	t(s)
10	2	10	32	1.6	1.2	2,511
		20	34	1.8	1.3	2,492
	5	10	26	3.8	2.1	2,693
		20	28	4.0	2.1	2,623
20	2	10	21	3.0	2.3	2,945
		20	26	3.2	2.4	2,771
	5	10	22	6.7	4.2	2,884
		20	23	7.2	4.5	2,796
30	2	10	3	5.0	4.4	3,388
		20	6	5.4	4.6	3,358
	5	10	2	11.4	9.0	3,491
		20	2	12.3	9.6	3,466
	Avg.		19	5.5	4.0	2,951

deal with its stochastic characteristic. Still considering this algorithm, the maximum quantity of candidate tasks (i.e. the cardinality of set $\mathcal{R}$) selected in the assignment phase was fine-tuned to 5. The columns of Table 9 present the evaluation criteria:

- $\delta_c(\%)$: average percentage of proved optimal solutions;
- $\Upsilon_2^*(\%)$: average gap of the best cycle time found in all type worker configurations compared with SALBP-2 reference solution;
- $\Upsilon_2^{*\rho}(\%)$: average gap of the best cycle time found in ρ best type worker configurations compared with SALBP-2 reference solution;
- $\overline{\Upsilon}_2^{\rho}(\%)$: average gap of the cycle times found in ρ best type worker configurations compared with SALBP-2 reference solution;
- $\overline{\Upsilon}_2(\%)$: average gap of the cycle times found in all type worker configurations compared with SALBP-2 reference solution;

In our experiments, we adopt ρ equal to 20%. In other words, we consider averaged results for the top 20% solutions respect to the worker distribution criteria (without repetition). In the metrics, we consider worker placements for which the three heuristic procedures found feasible solutions. According to Column Υ_2^* of Table 9, we pay a price of approximately 8% of increase cycle time when we also prioritize the regular distribution of workers along the line, comparing with a productive rate of an ordinary system. Note that these figures do not change even if we take the ρ best distribution configurations. Analyzing the overall average (Column $\overline{\Upsilon}_2$), we see that the MH performed well in all scenarios tested. The results also show that the algorithms WRCH and WRCHR obtained solutions close to the ones obtained by the MH. It is important to highlight that the randomness introduced in WRCHR proved to be effective, improving the obtained results. As depicted in Column "δ_c", the ALWIBP-2 model was solved to optimality for most of type worker placements.

Table 10, which presents the results for larger instances, corroborates the good performance of the three algorithms implemented. In Column "δ_c", due to the complexity to solve problems of this magnitude, the number of optimal solutions found by the ALWIBP-2 model reduced. As we observed in the previous section, cycle time gaps with respect to SALBP reference

Table 9: Multi-objective ALWIBP-2 heuristics – results of middle instances (50 tasks).

τ	Var	Inc	MH					WRCH				WRCHR			
			$\delta_c(\%)$	$\Upsilon_2^*(\%)$	$\Upsilon_2^{*\rho}(\%)$	$\overline{\Upsilon}_2^{\rho}(\%)$	$\overline{\Upsilon}_2(\%)$	$\Upsilon_2^*(\%)$	$\Upsilon_2^{*\rho}(\%)$	$\overline{\Upsilon}_2^{\rho}(\%)$	$\overline{\Upsilon}_2(\%)$	$\Upsilon_2^*(\%)$	$\Upsilon_2^{*\rho}(\%)$	$\overline{\Upsilon}_2^{\rho}(\%)$	$\overline{\Upsilon}_2(\%)$
10	2	10	79.7	2.1	2.6	3.5	3.7	4.0	5.0	6.2	9.7	3.2	3.8	4.8	6.7
		20	80.4	2.6	3.1	4.1	4.3	4.5	5.2	6.7	10.3	3.6	4.1	5.3	7.2
	5	10	84.5	5.4	6.1	7.0	7.3	8.1	9.2	10.7	14.2	6.4	7.2	8.4	10.8
		20	84.4	5.8	6.4	7.5	7.9	8.2	9.4	10.8	14.6	6.8	7.5	8.8	11.2
20	2	10	83.6	4.1	4.5	5.7	6.0	6.8	7.7	9.3	13.2	5.6	6.2	7.6	10.1
		20	84.5	4.7	5.1	6.5	6.9	7.4	8.2	10.1	14.2	6.2	6.7	8.3	10.9
	5	10	91.2	10.2	10.9	12.8	13.1	14.1	15.4	17.8	22.3	12.1	13.0	15.2	18.5
		20	92.2	10.9	11.7	13.5	14.0	14.8	16.0	18.5	23.1	12.7	13.8	15.8	19.2
30	2	10	83.5	6.5	7.0	8.2	8.4	9.9	11.0	12.5	16.6	8.6	9.3	10.7	13.5
		20	85.6	7.0	7.6	9.0	9.5	10.3	11.4	13.4	18.0	9.1	9.9	11.5	14.6
	5	10	96.4	15.7	16.7	18.4	18.9	20.6	22.5	25.1	30.5	18.5	19.9	22.0	26.2
		20	98.1	17.3	18.2	20.1	20.6	22.0	23.9	26.7	32.2	19.8	21.1	23.4	27.6
	Average		87.0	7.7	8.3	9.7	10.0	10.9	12.1	14.0	18.2	9.4	10.2	11.8	14.7

Table 10: Multi-objective ALWIBP-2 heuristics – results of large instances (100 tasks).

τ	Var	Inc	MH					WRCH				WRCHR			
			δ_c(%)	Υ_2^*(%)	$\Upsilon_2^{*\rho}$(%)	$\overline{\Upsilon}_2^{\rho}$(%)	$\overline{\Upsilon}_2$(%)	Υ_2^*(%)	$\Upsilon_2^{*\rho}$(%)	$\overline{\Upsilon}_2^{\rho}$(%)	$\overline{\Upsilon}_2$(%)	Υ_2^*(%)	$\Upsilon_2^{*\rho}$(%)	$\overline{\Upsilon}_2^{\rho}$(%)	$\overline{\Upsilon}_2$(%)
10	2	10	45.0	1.6	1.9	2.4	2.4	3.9	4.4	5.1	8.8	3.5	3.9	4.5	6.5
		20	46.0	1.8	2.1	2.7	2.7	4.1	4.6	5.5	9.2	3.7	4.0	4.7	6.8
	5	10	48.5	3.9	4.2	4.9	5.0	7.2	7.8	8.9	12.7	6.3	6.7	7.6	10.2
		20	49.1	4.0	4.5	5.2	5.2	7.5	8.2	11.0	12.9	6.5	7.0	9.7	10.4
20	2	10	37.4	2.9	3.2	3.8	3.8	6.2	6.6	7.4	11.0	5.6	6.0	6.6	8.9
		20	40.1	3.2	3.5	4.1	4.2	6.5	7.0	7.8	11.6	5.8	6.3	7.0	9.4
	5	10	46.1	6.5	6.9	7.8	7.9	11.5	12.3	13.4	17.6	10.3	11.0	11.9	15.1
		20	47.6	7.2	7.7	8.4	8.5	12.2	13.0	14.2	18.3	10.9	11.5	12.5	15.7
30	2	10	27.5	4.6	5.0	5.7	5.7	8.9	9.5	10.3	14.1	8.3	8.7	9.5	12.2
		20	28.9	5.1	5.4	6.3	6.3	9.5	10.1	11.1	14.9	8.9	9.4	10.2	12.8
	5	10	35.8	10.9	11.4	12.5	12.6	17.7	18.7	20.4	25.2	16.8	17.5	18.7	22.8
		20	37.2	11.7	12.2	13.3	13.4	18.7	19.6	21.3	26.1	17.5	18.3	19.6	23.6
	Average		40.8	5.3	5.7	6.4	6.5	9.5	10.2	11.4	15.2	8.7	9.2	10.2	12.9

solutions improve when we deal with large-sized instances. Note that average represented by Column Υ_2^* of MH is even lower than the one found by the ALWIBP-2 model (Column Υ_1^* of Table 8), due to the model inability to find better feasible solutions in the allowed time limit.

We also investigate the impact of the priority rules proposed by Moreira et al. (2012) behind WRCH and WRCHR. Considering the first algorithm, we observed that the criteria based on task execution times obtained the best results. On the other hand, the stochastic features present in the task assignment phase of the WRCHR resulted in more robust results regardless of the used rules.

The idea of pre-determined stations to receive "conventional" workers and disabled ones can generate infeasible solutions, since there may not be task assignments that respect cycle time constraints or even task/worker feasibilities. Taking the middle-sized instances, we observed that the three heuristics have not found solutions in less than 0.8% of the configurations tested. For problems with 100 tasks, the MH obtained feasible solutions for all worker placements, while WRCH and WRCHR have not succeeded in 0.8%, on average. The average computational time (in seconds) spent by the MH, WRCH and WRCHR for solving the ALWIBP-2, for each configuration, was 354, 0.02, 0.20 seconds (middle-sized instances), and 1,170, 0.09, 1.64 seconds (large-sized instances), respectively.

5.2.3. Experiment 3: ALWIBP-2 approaches through multi-objective measures

Cycle time and Miltenburg's regularity criterion can be conflicting objective functions. We illustrate the interaction between them by comparing the three methods through a Pareto dominance concept. Let $f_1, ..., f_{obj}$ be functions to be minimized. A solution x dominates y ($x \preceq y$) if $f_i(x) \leq f_i(y), i = 1, ..., obj$ and $f_i(x) < f_i(y)$, for at least one objective i. We say that x is Pareto optimal if there is no y such that y dominates x. Define reference set $\mathcal{L}$ as a Pareto list composed by the dominant points obtained by the methods. In this study, we consider four binary metrics and two unary ones in order to measure the performance of the three algorithms. In Table 11, we present a summary of these metrics.

Tables 12 and 13 compare the heuristics concerning ER, HA and HAR criteria. Analyzing the ER metric, we see that MH obtained in average 89% and the totality of the dominant solutions in middle-sized and large-sized instances, respectively. However, when we take the overall average of

Table 11: Multi-objective metrics.

Error ratio (ER)

Definition: percentage of solutions in a given Pareto list that are in the reference set $\mathcal{P}$.
Formula: $\frac{\sum_{i=1}^{|P|} e_i}{|P|}$, where $e_i = 1$ if the solution i of a Pareto set P belongs to $\mathcal{L}$.
Type: binary.
Reference: Deb (2001); Coello et al. (2002); Coello and Lamont (2004).

Hyper area covered (HA)

Definition: defines the area of objective value spaced covered by a given Pareto list P.
Formula: $\bigcup_{i=1}^{|P|} A_i$, where A_i is the area covered by the i^{th} solution.
Type: unary.
Reference: Zitzler and Thiele (1998, 1999).

Hyper area ratio (HAR)

Definition: portion of the area occupied by a given Pareto set compared with the area occupied by the reference set.
Formula: $\frac{HA_i}{HA_{\mathcal{L}}}$, where HA_i and $HA_{\mathcal{L}}$ are the HA metrics obtained by an algorithm i and the reference set, respectively.
Type: binary.
Reference: Coello et al. (2002); Coello and Lamont (2004).

Coverage relation (Cv)

Definition: considering non-dominated sets A and B, we computed the fraction of the solutions from B which are covered by at least one solution in A.
Formula: $\frac{\{b \in B; \exists a \in A, a \preceq b\}}{|B|}$.
Type: binary.
Reference: Zitzler and Thiele (1999).

Spacing (Sp)

Definition: calculates the uniformity of the spread of the points of the solution set $|P|$.
Formula: $\sqrt{\frac{\sum_{i=1}^{|P|}(\overline{d} - d_i)^2}{n-1}}$, where $d_i = \min_j\{|f_1(i) - f_1(j)| + |f_2(i) - f_2(j)|\}$ and $\overline{d} = \frac{\sum_i d_i}{|P|}$.
Type: unary.
Reference: Deb (2001); Coello et al. (2002); Coello and Lamont (2004).

Generational Distance (GD)

Definition: given a Pareto list P, calculates how far it is from the reference set $\mathcal{L}$.
Formula: $\frac{\sum_{i=1}^{|P|} d_i}{n}$, where d_i is the Euclidean distance between solution i and the closest one which belongs to the reference set $\mathcal{L}$.
Type: binary.
Reference: Deb (2001); Deb et al. (2002); Coello et al. (2002); Coello and Lamont (2004).

HA and HAR measures, we note that solutions from WRCH and WRCHR were relatively closer to the area covered by the reference set, especially in instances with 100 tasks.

Concerning Cv metric, all pairwise comparisons between the three algorithms are shown in Tables 14 and 15. The first line of both tables indicates the values of Cv applied to algorithms A-B, where $A, B \in \{$MH, WRCH and WRCHR$\}$. The MH heuristic covers most of solutions comparing with the other methods. In some cases, WRCHR obtains a solution that covers another one from MH (see Column 5 of Table 14). This is due to computational time limit imposed to CPLEX when it solves the model *M4*, resulting in a solution with less quality than the one coming from WRCHR.

In attempt to investigate the distribution of dominant points over the Pareto set of each algorithm, we evaluate Sp and GD metrics in the solutions obtained by MH, WRCH and WRCHR (Tables 16 and 17). These results corroborate the excellence of MH in front of WRCH and WRCHR, obtaining well spread solutions and closer to the dominant reference set. We also highlight the satisfactory influence of randomness during the task assignment of workers, that led the WRCHR be better than the WRCH.

Table 12: ALWIBP-2 heuristics – results of middle instances for ER, HA and HAR metrics (50 tasks).

τ	Var	Inc	ER			HA			HAR		
			MH	WRCH	WRCHR	MH	WRCH	WRCHR	MH	WRCH	WRCHR
10	2	10	0.90	0.07	0.39	1126.66	1403.69	1298.51	1.00	1.38	1.27
		20	0.88	0.12	0.45	1154.43	1339.37	1307.80	1.00	1.27	1.20
	5	10	0.89	0.09	0.37	1208.55	1449.58	1397.29	1.00	1.35	1.27
		20	0.89	0.09	0.43	1162.30	1488.98	1337.40	1.00	1.39	1.21
20	2	10	0.84	0.05	0.43	1238.53	1888.40	1580.10	1.00	1.81	1.38
		20	0.84	0.04	0.42	1474.19	1722.06	1659.59	1.00	1.37	1.22
	5	10	0.85	0.04	0.44	1523.28	1872.83	1730.83	1.00	1.46	1.28
		20	0.84	0.03	0.38	1521.37	1933.26	1879.32	1.00	1.55	1.45
30	2	10	0.93	0.06	0.34	1325.34	1599.62	1512.23	1.00	1.38	1.30
		20	0.92	0.07	0.35	1324.94	1448.27	1456.51	1.00	1.20	1.14
	5	10	0.93	0.09	0.37	1464.17	1828.65	1786.22	0.99	1.45	1.38
		20	0.92	0.08	0.38	1458.83	1730.68	1679.50	1.00	1.30	1.24
	Average		0.89	0.07	0.40	1331.88	1642.12	1552.11	1.00	1.41	1.28

Table 13: ALWIBP-2 heuristics – results of middle instances for ER, HA and HAR metrics (100 tasks).

τ	Var	Inc	ER			HA			HAR		
			MH	WRCH	WRCHR	MH	WRCH	WRCHR	MH	WRCH	WRCHR
10	2	10	1.00	0.03	0.12	1965.04	2029.27	2033.74	1.00	1.06	1.06
		20	1.00	0.01	0.12	1961.43	2036.51	2032.32	1.00	1.06	1.06
	5	10	1.00	0.00	0.14	1981.34	2064.73	2057.87	1.00	1.06	1.06
		20	1.00	0.00	0.09	2020.87	2092.70	2106.77	1.00	1.06	1.07
20	2	10	1.00	0.00	0.04	1877.13	1922.90	1952.62	1.00	1.03	1.05
		20	1.00	0.00	0.04	1861.77	1957.96	1961.26	1.00	1.07	1.06
	5	10	1.00	0.00	0.03	1928.77	2067.06	2057.83	1.00	1.09	1.08
		20	1.00	0.00	0.04	1943.54	2066.76	2080.58	1.00	1.08	1.09
30	2	10	1.00	0.00	0.05	1898.31	1988.45	1980.65	1.00	1.06	1.06
		20	1.00	0.01	0.05	1889.59	2021.75	1998.21	1.00	1.08	1.06
	5	10	1.00	0.00	0.05	1999.33	2167.77	2144.79	1.00	1.09	1.08
		20	1.00	0.00	0.04	2045.09	2195.53	2195.19	1.00	1.07	1.08
	Average		1.00	0.00	0.07	1947.68	2050.95	2050.15	1.00	1.07	1.07

Table 14: ALWIBP-2 heuristics – results of middle instances for Cv metric (50 tasks).

τ	Var	Inc	WRCH-MH	WRCHR-MH	MH-WRCH	WRCHR-WRCH	MH-WRCHR	WRCH-WRCHR
10	2	10	0.00	0.10	0.89	0.75	0.61	0.06
		20	0.00	0.12	0.85	0.74	0.55	0.05
	5	10	0.00	0.11	0.89	0.78	0.63	0.03
		20	0.00	0.11	0.87	0.80	0.57	0.04
20	2	10	0.00	0.16	0.92	0.78	0.57	0.02
		20	0.00	0.16	0.91	0.78	0.58	0.03
	5	10	0.00	0.15	0.91	0.79	0.56	0.04
		20	0.00	0.16	0.91	0.78	0.62	0.03
30	2	10	0.00	0.07	0.94	0.85	0.66	0.05
		20	0.00	0.08	0.88	0.80	0.65	0.03
	5	10	0.00	0.07	0.90	0.78	0.63	0.05
		20	0.00	0.08	0.89	0.84	0.62	0.05
	Average		0.00	0.11	0.90	0.79	0.60	0.04

Table 15: ALWIBP-2 heuristics – results of large instances for Cv metric (100 tasks).

τ	Var	Inc	WRCH-MH	WRCHR-MH	MH-WRCH	WRCHR-WRCH	MH-WRCHR	WRCH-WRCHR
10	2	10	0.00	0.00	0.97	0.72	0.88	0.14
		20	0.00	0.00	0.99	0.73	0.88	0.13
	5	10	0.00	0.00	1.00	0.85	0.86	0.05
		20	0.00	0.00	1.00	0.87	0.91	0.06
20	2	10	0.00	0.00	1.00	0.70	0.96	0.15
		20	0.00	0.00	1.00	0.72	0.96	0.16
	5	10	0.00	0.00	1.00	0.84	0.97	0.07
		20	0.00	0.00	1.00	0.86	0.96	0.05
30	2	10	0.00	0.00	1.00	0.74	0.95	0.19
		20	0.00	0.00	0.98	0.75	0.95	0.14
	5	10	0.00	0.00	1.00	0.73	0.95	0.18
		20	0.00	0.00	1.00	0.81	0.96	0.13
	Average		0.00	0.00	1.00	0.78	0.93	0.12

Table 16: ALWIBP-2 heuristics – results of middle and large instances for Sp metric.

τ	Var	Inc	50 tasks			100 tasks		
			MH	WRCH	WRCHR	MH	WRCH	WRCHR
10	2	10	2.74	3.54	3.41	1.55	1.93	1.75
		20	3.89	5.30	4.34	2.17	2.74	1.77
	5	10	4.23	5.19	5.70	1.94	3.12	2.26
		20	2.48	4.24	2.27	1.86	2.44	2.37
20	2	10	3.20	3.98	3.33	0.94	1.90	2.07
		20	3.25	4.60	3.39	1.39	2.07	1.35
	5	10	3.68	7.12	4.10	2.30	3.23	2.71
		20	4.95	6.64	5.52	1.50	2.60	2.12
30	2	10	2.88	4.13	3.94	1.70	1.87	1.54
		20	2.61	5.31	2.67	2.10	2.29	2.19
	5	10	3.37	7.33	5.10	2.20	3.55	2.44
		20	3.09	7.73	4.80	2.48	3.62	3.67
	Average		3.36	5.43	4.05	1.84	2.61	2.19

Table 17: ALWIBP-2 heuristics – results of middle and large instances for GD metric.

τ	Var	Inc	50 tasks			100 tasks		
			MH	WRCH	WRCHR	MH	WRCH	WRCHR
10	2	10	0.00	11.78	5.64	0.00	11.44	8.40
		20	0.00	12.49	5.25	0.00	11.60	7.84
	5	10	0.00	16.76	5.75	0.00	17.21	11.18
		20	0.00	14.84	5.69	0.00	17.90	11.27
20	2	10	0.00	15.34	6.89	0.00	18.83	14.29
		20	0.00	14.63	6.66	0.00	19.46	14.56
	5	10	0.00	20.82	8.28	0.00	29.56	20.80
		20	0.00	19.58	8.44	0.00	28.67	19.96
30	2	10	0.00	17.85	10.29	0.00	23.41	19.44
		20	0.00	17.09	9.97	0.00	23.68	19.57
	5	10	0.00	25.99	13.42	0.00	38.54	32.55
		20	0.00	25.60	12.81	0.00	38.91	28.77
	Average		0.00	17.73	8.26	0.00	23.27	17.39

6. Conclusions

In this paper we model and propose algorithms to solve an assembly line balancing problem where one aims to integrate workers with very different task execution times in the line. This problem is motivated by the situation faced in sheltered work centers for the disabled and in conventional assembly lines that employ a number of persons with disabilities.

We adapt Miltenburg's regularity criteria in order to obtain even distribution of a special set of workers (workers with disabilities or monitor workers) along the line, in order to avoid clustering of workers and promote a higher level of integration. A second objective is to maintain productivity levels.

Our algorithms present two stages. In the first stage, possible worker distributions are obtained and, in the second stage, they are used to obtain the worker and task assignments. Our results indicate that not only heterogeneous workers can be integrated in the line without major productivity losses, but also that the regularity distribution goal can be easily incorporated. This evidence becomes very important to encourage those companies that, within their CSR policies or pushed by emerging legal issues, are eager to integrate disabled people in their workforce.

A dominance analysis has shown that a simple heuristic obtained by truncating a branch-and-bound search presented the best overall performance, at the cost of (much) higher computational time. Nevertheless, the computational efficiency of the constructive heuristics justify their use in larger problems. They can also be easily adopted to cope with different line configurations.

Indeed, future research lines include the exploration of different assembly line aspects such as different layouts (U-lines, parallel stations, mixed-lines) and job rotation planning. In fact, regarding some recent studies concerning the impact of ergonomic policies in the production process (Bautista et al., 2016a,b; Akyol and Baykasoğlu, 2016), it would be interesting to also add ergonomic factors to the problem studied here and to its extension for planning the job rotation.

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Appendix A: Notations

Table A.18: Abbreviations used in this paper.

ALWABP	Assembly Line Worker Assignment and Balancing Problem
ALWIBP	Assembly Line Worker Integration and Balancing Problem
M1	ALWIBP-2 Model
M2	Model that optimizes the worker regularity distribution
M3	Linearized version of model *M2*
M4	Model that minimizes the cycle time given a predetermined position of heterogeneous workers
MH	*M4*-based heuristic
PRV	Product Rate Variation problem
SALBP	Simple Assembly Line Balancing Problem
SWDs	Sheltered Work Centers for Disabled
WRCH	Worker Regularity Constructive Heuristic
WRCHR	Worker Regularity Constructive Heuristic with Randomness
WRGM	Worker Regularity Generator Model

Table A.19: Parameters used in the ALWIBP-2 model.

m	number of workstations
S	set of ordered workstations
s, k	indexes for stations
d	number of heterogeneous workers
W	set of heterogeneous workers
w	index for heterogeneous worker
n	number of tasks
N	ordered set of tasks
i, j	indexes for tasks
$i \lessdot j$	task i is an immediate predecessor of task j
F_i	set of immediate successors of task i
$W_i \subseteq W$	subset of heterogeneous workers that are able to execute task $i \in N$
$Nw \subseteq N$	set of tasks that worker $w \in W$ is able to perform
t_i	execution time of task i when assigned to a conventional worker
t_{wi}	execution time of task i when assigned to a heterogeneous worker $w \in W$
l_w	a positive sufficient large constant.

Table A.20: Variables used in the ALWIBP-2 model.

$x_{si} \in \{0, 1\}$	equal to 1 only if task $i \in N$ is assigned to workstation $s \in S$
$y_{sw} \in \{0, 1\}$	equal to 1 only if worker $w \in W$ is assigned to workstation $s \in S$
$c \geq 0$	cycle time.

Table A.21: Parameters used in the regularity criterion approach for ALWIBP.

$h_{(s)}$	minimum possible number of heterogeneous workers assigned until station s
$\kappa_{(s)}$	maximum possible number of heterogeneous workers assigned until station s
$K_{(s)}$	set of possible values for the number of these workers assigned in station s or prior to it
$s^p = ({s_1}^p \ {s_2}^p \ \dots \ {s_m}^p)$	m-tuple representing the positions of heterogeneous workers in the line. ${s_k}^p = 1$ if there is a heterogeneous worker $w \in W$ in station $k \in S$.
$\overline{p}$	number of m-tuples s^p to be generated
$P_{\overline{p}}$	set of s^p configurations generated.

Table A.22: Variables used in the regularity criterion approach for ALWIBP.

$r \in \mathbb{R}$	Miltenburg's regularity criterion
$v_s \in \mathbb{Z}$	number of "conventional" workers assigned in stations up to (and including) station s
$z_s \in \mathbb{Z}$	number of heterogeneous workers assigned in stations up to (and including) station s
$\pi_s \in \{0, 1\}$	equal to 1 if a heterogeneous worker is assigned to workstation $s \in S$
$\gamma_{s\kappa} \in \{0, 1\}$	equal to 1 only if there are κ heterogeneous workers allocated until workstation s

Table A.23: Notations used in computational experiments.

Var	variability of task execution times (benchmark scheme)
Inc	task/worker incompatibilities (benchmark scheme)
μ	the estimated percentage of heterogeneous workers in the assembly line
Δ_c	percentage of proved optimal solutions
Υ_1^*	percentage increase in the ALWIBP-2 cycle time when compared to the reference solution for the SALBP-2
Gap	optimality gap obtained by CPLEX in the allowed time limit
t(s)	run time
δ_c	average percentage of proved optimal solutions by the MH
$\mathcal{T}$	computational time (in seconds) allowed to run each s^p configuration in the MH algorithm
Υ_2^*	average gap of the best cycle time found in all type worker configurations compared with SALBP-2 reference solution
$\Upsilon_2^{*\rho}$	average gap of the best cycle time found in ρ best type worker configurations compared with SALBP-2 reference solution
$\overline{\Upsilon}_2^{\rho}$	average gap of the cycle times found in ρ best type worker configurations compared with SALBP-2 reference solution
$\overline{\Upsilon}_2$	average gap of the cycle times found in all type worker configurations compared with SALBP-2 reference solution
ER	Error ratio
HA	Hyper area covered
HAR	Hyper area ratio
Cv	Coverage relation
Sp	Spacing
GD	Generational distance