

Reliability Importance Measures for a Health-Aware Control of Drinking Water Networks

Jean C. Salazar*¹, Fatiha Nejjari¹, Ramon Sarrate¹, Philippe Weber² and Didier Theilliol²

Abstract—This work focuses on a health-aware Model Predictive Control (MPC) scheme, which aims at enhancing the availability of the system. The objective is to extend the uptime of the system by delaying, as much as possible the system reliability decay. The weights of the MPC cost function are set according to some reliability importance measures. This work describes the main reliability importance measures and studies which of them are best suited for a health-aware MPC strategy applied to a Drinking Water Network. The overall system reliability as well as the reliability importance measures are computed online through a Dynamic Bayesian Network.

Index Terms—Availability, Reliability, Model Predictive Control, Dynamic Bayesian Network, Reliability Importance Measures

I. INTRODUCTION

The research on performance degradation in a control system design has gained a lot of interest in the last decade [1]–[4]. The objective is to extend the operation time of the system as far as possible. This can be achieved by considering the level of actuators reliability and their importance for the reliability of the system in the control algorithm. Then, it is possible to redistribute the control effort among the available actuators to relieve the load on devices in the worst conditions avoiding their break down. Consequently, an appropriate rule to redistribute the control effort should be implemented [5], [6].

In this work a Dynamic Bayesian Network (DBN) is used to model the overall system reliability. This approach has been recently considered in some works [7]–[12]. It uses a DBN which includes a temporal dimension and takes into account observations (evidences) about the state of the components to compute the system reliability.

Reliability Importance Measures (RIMs) offer an evaluation of the relative importance of individuals components or groups of components constituting a system, with respect to its safety, reliability, availability and performance. RIMs can be defined on the basis of the system structure or component reliability [13]. In this work, the RIMs are used to improve

the system reliability through the control algorithm and are computed using the DBN [6].

This paper presents, through an illustration dedicated to Drinking Water Networks (DWNs), the benefits of integrating the overall system and components reliability by means of their reliability importance measures in the control algorithm. The objective is to distribute the control efforts within the actuators to extend their useful life and improve the overall DWN reliability. DWNs are multivariable dynamic constrained systems that are composed by the interconnection of several subsystems (i.e.: tanks, actuators, intersection nodes, water sources and consume sectors), whose main objective is to satisfy the consumer demand.

To perform an optimal management of the DWN and supply the consumer demand a multi-criteria problem is formulated through a Model Predictive Control (MPC) approach [14]. MPC has proved to be an efficient technique that can predict the appropriate control actions to achieve optimal performance according to a defined criteria in the cost function [16].

The objective is to provide control performance while preserving the DWN reliability which depends on many factors such as the quality and quantity of the water and its availability at the sources, the failure rates of the pumps, valves, among others.

The paper is organized as follows: Section II presents the DWN system. Section III presents the reliability modelling using a DBN. In Section IV a review of the reliability importance measures and its integration in the control algorithm is presented. In Section V, some results are given and discussed. Finally, some conclusions are provided in Section VI.

II. DRINKING WATER NETWORK DESCRIPTION

A Drinking Water Network is a system composed by sources (water supplies), sinks (water demand sectors), pipelines that link sources to sinks through pumps and valves. The network consists of 5 sources and 1 sink (Fig. 1). It is assumed that the demand forecast (d_m) at the sink is known and that any single source can satisfy this required water demand (Fig. 2).

The aim of using MPC techniques for controlling water distribution networks is to compute, ahead of time, the input commands to achieve the optimal performance of the network according to a given set of control goals. MPC has been widely applied in the management of the urban water cycle [15].

*This work was supported by Spanish Government (MINISTERIO DE ECONOMÍA Y COMPETITIVIDAD) and FEDER under project DPI2014-58104-R (HARCRICS).

¹Jean C. Salazar, Fatiha Nejjari and Ramon Sarrate are with the Research Center for Supervision, Safety and Automatic Control (CS2AC), Universitat Politècnica de Catalunya (UPC), 10 Rambla Sant Nebridi, 08222, Terrassa - Spain. {jean.salazar, fatiha.nejjari, ramon.sarrate}@upc.edu

²Philippe Weber and Didier Theilliol are with the Centre de Recherche en Automatique de Nancy (CRAN), Université de Lorraine, FST BP 70239, 54506 Vandoeuvre-lès-Nancy - France. {philippe.weber, didier.theilliol}@univ-lorraine.fr

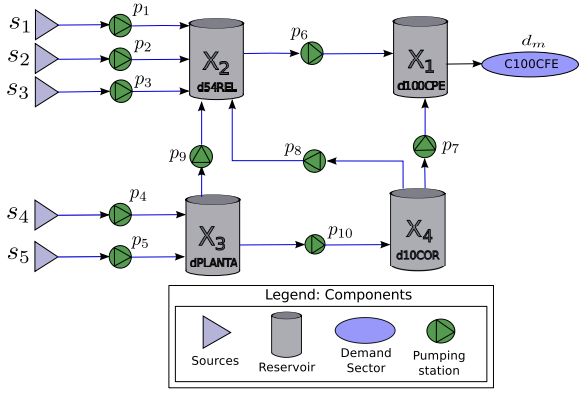


Figure 1. Drinking Water Network System Example

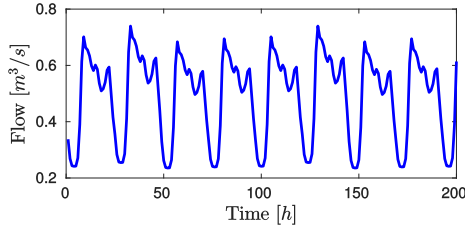


Figure 2. Drinking water demand.

Consider that the DWN is modelled by the following linear discrete-time dynamic model described in the state-space form by applying mass balance to each tank:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_d d_m(k) \\ y(k) &= Cx(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^p$ is the control input, $y(k) \in \mathbb{R}^q$ is the measured output, $A \in \mathbb{R}^{n \times n}$ is the state matrix, $B \in \mathbb{R}^{n \times p}$ is the input matrix and $C \in \mathbb{R}^{q \times n}$ is the output matrix, and B_d and d_m are the matrix and vector of water demand, respectively.

The MPC algorithm [14] uses the model of the system (1) to predict the future output of the system and compute the optimal control actions aimed to optimize a given cost function over a prediction horizon H_p . This cost function is minimized subject to a set of physical and operational constraints over a control horizon $H_c \leq H_p$. Once the minimization is performed, a vector of control actions is obtained and just the first component is applied to the system. The procedure is repeated for the next time instant following a receding-horizon strategy and taking into account feedback system measurements and future set-points.

In this paper, the multiobjective optimization problem is formulated as follows:

$$\begin{aligned} \min_{\substack{(\Delta \hat{u}(k|k), \dots, \\ \Delta \hat{u}(k+H_c-1|k))}} J(k) &= \sum_{j=0}^{H_c-1} \sum_{i=1}^p \Delta \hat{u}_i(k+j|k)^2 \\ &+ \sum_{j=0}^{H_c-1} \sum_{i=1}^p \rho_i(k) \hat{u}_i(k+j|k)^2 \end{aligned} \quad (2)$$

$$\begin{aligned} \text{subject to: } \underline{u} &\leq \hat{u}(k+j|k) \leq \bar{u} \quad j = 0, \dots, H_c - 1 \\ \underline{x} &\leq \hat{x}(k+l|k) \leq \bar{x} \quad l = 1, \dots, H_p \end{aligned}$$

where $\rho_i(k)$ is a weight, \underline{u} and \bar{u} denote the minimum and maximum flow capacities, and \underline{x} and \bar{x} denote the minimum and maximum volume capacities. The notation $k+j|k$ allows a future time instant $k+j$ to be referred at current time instant k , and $\Delta \hat{u}_i(k) \triangleq \hat{u}_i(k) - \hat{u}_i(k-1)$.

The first term in the cost function aims at guaranteeing a smooth pump operation whereas the second term penalizes pump operation according to their weights $\rho_i(k)$.

In [15] an application of MPC for a DWN taking into account economical factors, level service and degradation criteria is developed, where ρ_i represents the economic weight of pumping depending on the electric tariffs along the day. In this work, ρ_i will be used to introduce the reliability dependence in the computation of the control law, as will be explained in Section IV-F.

The simulation parameters are presented in Table I.

Table I
SIMULATION PARAMETERS

Parameter	Value				
H_p / H_c	24 / 8				
T_s [h]	1				
\bar{u} [m^3/s]	0.75	0.75	0.75	1.20	0.85
\underline{u} [m^3/s]	0	0	0	0	0
\bar{x} [m^3]	65200	3100	14450	11745	
\underline{x} [m^3]	25000	2200	5200	3500	
x_0 [m^3]	45100	2650	9825	7622	

III. RELIABILITY MODELLING

A. Components Reliability

In this paper the reliability is modelled as an exponential function,

$$R_i(t) = e^{-\lambda_i t} \quad (3)$$

where λ_i is the failure rate of the i th component modelled using the Cox's expression [17],

$$\lambda_i = \lambda_i^0 \times g_i(\ell, \vartheta) \quad (4)$$

where λ_i^0 is the baseline failure rate (nominal failure rate) for the i th component and $g_i(\ell, \vartheta)$ represents the effect of stress on the component known as covariate, where ℓ represents an image of the load applied and ϑ is a component parameter.

Different definitions of the load function $g(\ell, \vartheta)$ exists in the literature, basically it depends on the nature of the component and its degradation process, for example: the number of cycles for a on/off actuator (e.g. a valve, a switch, etc.); the amount of effort which is related with the work load of the actuator (e.g., a pump). In addition, environmental factors could be taken into account such as humidity, temperature, etc. In this paper, it is assumed that the failure rate depends on the use the component, hence, $g_i(\cdot)$ is defined as:

$$g_i(u_i(k)) = \frac{u_i(k) - \underline{u}_i}{\bar{u}_i - \underline{u}_i} \quad (5)$$

where $u_i(k)$ is the control effort at time instant k , \underline{u}_i and \bar{u}_i are the minimum and maximum control efforts allowed for the i th actuator. Then (4) can be rewritten as:

$$\lambda_i(k) = \lambda_i^0 \times g_i(u_i(k)) \quad (6)$$

The major actuator load corresponds to $u_i(k) = \bar{u}_i$, which leads to the worst failure rate $\lambda_i(k) = \lambda_i^0$.

The failure rates of pumps in the DWN system are presented in Table II.

Table II
SIMULATION PARAMETERS

$\lambda^0 [h^{-1} \times 10^{-4}]$									
p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
9.85	10.70	10.50	1.40	0.85	0.80	11.70	0.60	0.74	0.78

It is possible then, to obtain the system reliability from the reliability of its components, as detailed in the next Section.

B. System Reliability

A system is defined as a set of components whose states are described by the binary random variable X_i , as:

$$P(X_i = 1) = p_i, \quad P(X_i = 0) = 1 - p_i \quad \forall i \in [1, n]. \quad (7)$$

Therefore, it is considered that they can have two states: operational (up) and failed (down). The state of i th component is described by the binary variable X_i : $X_i = 1$ if the component is UP, $X_i = 0$ if the component is down.

It is assumed that all components are mutually independent, this means that the joint distribution of the system state vector $\mathbf{X} = (X_1, \dots, X_n)$ is determined by the reliability of the components p_1, \dots, p_n .

The dependence of the state of the system with respect to the state of its components is determined by means of the structure function $\Phi(\mathbf{X})$. Then, if $\Phi(\mathbf{X}) = 1$ means the system is up, and if $\Phi(\mathbf{X}) = 0$ means the system is down.

Generally, $\Phi(\cdot)$ is determined by the structure of the system, it could be serial, parallel or a more complex structure (i.e. a combination of serial and parallel structure). This could be a problem in the case of systems with high amount of components or in complex structures.

In this work the DWN reliability is computed from its components reliability using a Bayesian Network. Thus, the complexity of computing the system reliability using the structure function is avoided. The BN nodes represent random variables and the edges represent the influences between them.

The strength of the dependences are quantified by conditional probabilities. Based on these relations and the structure of dependencies of the network, the BN is used to estimate the a posteriori probability of unknown variables given the other (or knowledge i.e. evidences) by a probabilistic reasoning based on the Bayes theorem [18].

The procedure described in [11] was followed to obtain the Dynamic Bayesian Network for the drinking water system (Fig.3). This DBN is used here not only to compute the system reliability but also to compute some reliability

importance measures of each actuator, that will be introduced in next section.

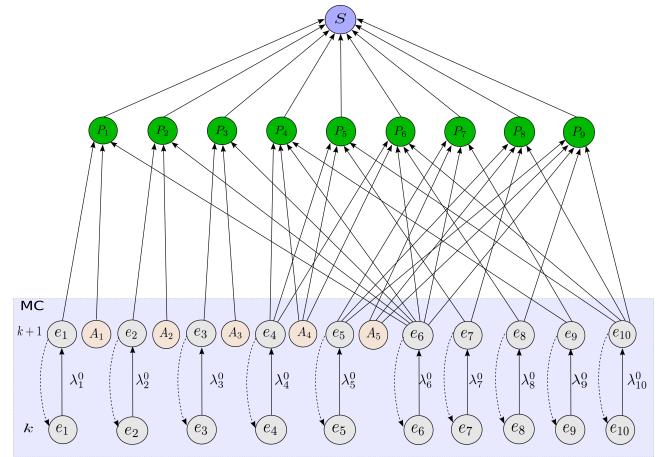


Figure 3. Dynamic Bayesian Network for the Drinking Water Network.

It consists of building the BN based on the minimal path sets of the system. A minimal path set is a minimum set of components whose functioning (i.e. being up) ensures the functioning of the system: if all elements of a minimal path set are up, the path is available and then the system is up.

Once, the network is obtained, the components reliabilities (computed using a Markov Chain model) are propagated to the network and the system reliability could be estimated using the Bayes theorem [18] given the parallel functioning of the minimal path sets and components reliabilities values.

IV. RELIABILITY IMPORTANCE MEASURES

Importance Measures (IMs) were first introduced by [19]. IMs are classified in two groups: Reliability Importance Measures (RIMs) and Structural Importance Measures (SIMs). The RIMs evaluate the relative importance of a component taking into account its contribution to the overall system reliability and the SIMs provide the relative importance of a component taking into account its position into the system structure.

These metrics can be defined either according to their functional aspect, taking into account the minimal path sets, or according to their dysfunctional aspect, considering the minimal cut sets. As both are equivalent, in this work only the functional aspect is used.

The aim from the system reliability analysis point of view, is to use the RIMs to identify the weakness or strengths in the system and to quantify the impact of component failures over system functioning.

A. Birnbaum's Importance Measure

The Birnbaum importance measure [19] also known as Marginal Importance Factor (MIF) is related to the probability of a component to be critical for the system functioning. It is defined as:

Definition 1. The B -reliability importance of component i for the functioning of the system, denoted as I_{MIF_i} , for a coherent system with independent components is defined as:

$$\begin{aligned} I_{MIF_i} &= \mathbb{P}(\Phi(\mathbf{X}) = 1 | X_i = 1) - \mathbb{P}(\Phi(\mathbf{X}) = 1 | X_i = 0) \\ &= \frac{\partial R(\mathbf{p})}{\partial p_i} \\ &= R(1_i; \mathbf{p}) - R(0_i; \mathbf{p}). \end{aligned} \quad (8)$$

The notation $R(1_i, \mathbf{p})$ denotes the reliability of the system in which the i th components is replaced by an absolutely reliable one, while $R(0_i, \mathbf{p})$ denotes the reliability of the system in which the i th component is failed.

The Birnbaum's measure is the probability that the failure or functioning of component i coincide with system failure or functioning. This approach is well known from classical sensitivity analysis. Moreover, it can be interpreted as the maximum lost in system reliability when i th component changes from the condition of perfect functioning to a failed condition.

Note that Birnbaum's importance measure (I_{MIF_i}) of the i th component depends only on the structure of the system and the reliabilities of the other components which means that it is independent of the actual reliability of the i th component.

B. Criticality Reliability Importance Measure

The Criticality Reliability Importance also know as Critical Importance Factor (CIF) was introduced by [20] and it is defined as:

Definition 2. The criticality reliability importance of component i for system functioning, denoted by I_{CIF_i} , is defined as the probability that i th component functions and is critical for the system functioning given that the system is functioning.

$$\begin{aligned} I_{CIF_i} &= p_i \frac{\mathbb{P}(\Phi(\mathbf{X}) = 1 | X_i = 1) - \mathbb{P}(\Phi(\mathbf{X}) = 1 | X_i = 0)}{\mathbb{P}(\Phi(\mathbf{X}) = 1)} \\ &= \frac{p_i}{R(\mathbf{p})} I_{MIF_i} \end{aligned} \quad (9)$$

Moreover, this can be interpreted as the probability that the i th component has caused a system failure when it is known that the system is failed.

C. Fussell-Vesely Reliability Importance Measure

The Fussell-Vesely importance measure also known as the Diagnostic Importance Factor (DIF) was proposed initially in the context of fault tree [21], [22]. It takes into account the contribution of a component to system functioning, and it is derived from the minimal path sets.

Definition 3. The Fussell-Vesely reliability importance measure of the i th component, denoted by I_{FV_i} , is defined as the probability that a minimal path containing component i exist and makes the system to be operative.

$$\begin{aligned} I_{FV_i} &= \mathbb{P}\{\exists P \in \overline{\mathcal{P}}_i \text{ s.t. } X_j = 1 \forall j \in P | \Phi(\mathbf{X}) = 1\} \\ &= \frac{p_i \mathbb{P}\{(1_i, \mathbf{X}) : \exists P \in \overline{\mathcal{P}}_i \text{ s.t. } X_j = 1 \forall j \in P\}}{R(\mathbf{p})} \\ &= \mathbb{P}(X_i = 1 | \phi(\mathbf{X}) = 1) \end{aligned} \quad (10)$$

where $P \in \overline{\mathcal{P}}_i$ denotes the minimal path containing component i .

The Fussell-Vesely importance measure can be explained as the probability that component i fails given that the system is failed.

D. Reliability Achievement Worth, RAW

The Reliability Achievement Worth (RAW) describes the increase of the system reliability if component i is replaced by a perfect reliable one. It is defined as:

Definition 4. The RAW, denoted by I_{RAW_i} qualifies the maximum possible percentage of system reliability increase generated by component i . It is expressed as:

$$\begin{aligned} I_{RAW_i} &= \frac{\mathbb{P}(\Phi(1_i, \mathbf{X}) = 1)}{\mathbb{P}(\Phi(\mathbf{X}) = 1)} \\ &= \frac{\mathbb{P}(\Phi(\mathbf{X}) = 1 | X_i = 1)}{\mathbb{P}(\Phi(\mathbf{X}) = 1)} \\ &= 1 + \frac{q_i}{R(\mathbf{p})} I_{MIF_i} \end{aligned} \quad (11)$$

E. Reliability Reduction Worth, RRW

The Reliability Reduction Worth (RRW) measure [23] reflects the reduction of system reliability if component i is failed. It is defined as:

Definition 5. The RRW, denoted as I_{RRW_i} expresses the potential damage caused to the system by a failure in the component i .

$$\begin{aligned} I_{RRW_i} &= \frac{\mathbb{P}(\Phi(\mathbf{X}) = 1)}{\mathbb{P}(\Phi(0_i, \mathbf{X}) = 1)} \\ &= \frac{\mathbb{P}(\Phi(\mathbf{X}) = 1)}{\mathbb{P}(\Phi(\mathbf{X}) = 1 | X_i = 0)} \\ &= \frac{1}{1 - \frac{p_i}{R(\mathbf{p})} I_{MIF_i}} \end{aligned} \quad (12)$$

F. Reliability Importance Measures as MPC weights

The reliability-aware MPC approach consists in setting $\rho_i(k)$ weights in the cost function (2) to redistribute the control effort among the actuators [11] based on the RIMs. These RIMs will be computed through the DBN reliability model described in Section III-B.

In this paper, different assignments of ρ_i are proposed. On the one hand, an approach focused in the reliability of the components (local approach) in which the weights are set as:

$$\rho_i(k) = 1 - R_i(k) \quad (13)$$

On the other hand, a global approach that focuses on the system reliability where a representation of the pump criticality through the RIMs is used.

Table III
PUMPS RELIABILITY IMPORTANCE MEASURES AT T=2000

Pump	λ [$\times 10^{-4}$]	R_i [%]	I_{MIF} [%]	I_{DIF} [%]	I_{CIF} [%]	I_{RAW} [%]	I_{RRW} [%]
p_1	9.85	13.94	4.54	14.61	0.77	0.10	0.10
p_2	10.70	11.76	4.43	12.32	0.63	0.10	0.10
p_3	10.50	12.24	4.45	12.83	0.66	0.10	0.10
p_4	1.40	75.57	8.78	77.54	8.05	0.10	0.11
p_5	0.85	84.37	13.72	86.56	14.04	0.10	0.11
p_6	0.80	85.21	87.48	98.58	90.39	0.11	1.04
p_7	11.70	9.63	12.89	10.99	1.50	0.11	0.10
p_8	0.60	88.69	5.81	89.40	6.25	0.10	0.11
p_9	0.74	86.24	12.66	88.06	13.24	0.10	0.11
p_{10}	0.78	85.55	8.11	86.77	8.42	0.10	0.11

V. RESULTS AND DISCUSSION

For the computation of the RIMs in the DWN system it is considered that sources, tanks and pipelines are perfectly reliable and only actuators are affected by a loss of reliability according to (3).

First of all, a static RIM analysis is performed in order to get better knowledge on them. Component reliability is assumed to follow (3) with $\lambda_i = \lambda_i^0$ and $t = T_M$ (2000 hours) and no RIM information is integrated into the control algorithm (i.e., $\rho_i = 1$). The corresponding results are presented in Tables III and IV.

In Table IV pumps are sorted according to their reliability importance measures. Remark that pump 6 is the most critical according to all RIMs. It is also interesting to highlight that some RIMs give a similar pump criticality order: CIF and RRW are equivalent, and MIF provides a close result.

Table IV
A priori CLASSIFICATION OF THE PUMPS

λ_i	R_i	I_{MIF}	I_{DIF}	I_{CIF}	I_{RAW}	I_{RRW}
p_7	p_8	p_6	p_6	p_6	p_6	p_6
p_2	p_9	p_5	p_8	p_5	p_7	p_5
p_3	p_{10}	p_7	p_9	p_9	p_1	p_9
p_1	p_6	p_9	p_{10}	p_{10}	p_2	p_{10}
p_4	p_5	p_4	p_5	p_4	p_3	p_4
p_5	p_4	p_{10}	p_4	p_8	p_4	p_8
p_6	p_1	p_8	p_1	p_7	p_5	p_7
p_{10}	p_3	p_1	p_3	p_1	p_9	p_1
p_9	p_2	p_3	p_2	p_3	p_{10}	p_3
p_8	p_7	p_2	p_7	p_2	p_8	p_2

This *a priori* knowledge will be later used to decide how to distribute the control effort in the system following the MPC scheme. Next, a dynamic RIM analysis is performed.

The dynamic analysis involves the computation of pump failure rate according to (5) and (6), as well as the pump RIMs through the DBN in Fig. 3. Then, following the methodology proposed in Section IV-F, several ρ_i assignments will be investigated.

In order to assess the performance of MPC scheme, three different indices will be used:

- 1) $R_s(T_M)$, the system reliability at the end of the mission. A higher value of this index will indicate a good performance in terms of preserving system reliability.

- 2) $JPR(T_M)$, the joint pump reliability index at the end of the mission. JPR measures the remaining overall pump reliability as follows:

$$JPR = \prod_{i=1}^p R_i(T_M) \quad (14)$$

- 3) U_{CUM} , the cumulative pump usage. U_{CUM} provides a measure of the pump energy consumption as follows:

$$U_{CUM} = T_s \sum_{k=0}^{T_{sim}/T_s} [u(k)^T u(k)] \quad (15)$$

Table V presents the performance indices corresponding to different simple ρ_i assignments. A nominal case corresponding to the situation where no reliability information is taken into account in the control loop (i.e., $\rho_i = 1$) is also considered.

Table V
SIMPLE ρ_i ASSIGNMENT PERFORMANCE

ρ_i	R_s [%]	U_{CUM} [$\times 10^6$]	JPR
1	97.50	1.537	0.093
$1 - R_i$	97.88	2.025	0.214
I_{MIF}	99.34	3.850	0.170
I_{DIF}	97.49	1.538	0.089
I_{CIF}	99.39	3.904	0.171
I_{RAW}	97.50	1.537	0.093
I_{RRW}	97.44	1.556	0.094

According to the system reliability index, the best results correspond to $\rho_i = I_{CIF_i}$ and $\rho_i = I_{MIF_i}$. These two RIMs provided a close pump criticality ordering in Table IV. However, $\rho_i = I_{RRW_i}$ does not provide a good performance, although CIF and RRW were expected to be equivalent according to Table IV.

Moreover, assigning $\rho_i = 1 - R_i$ produces the best remaining overall pump reliability, but does not provide the best overall system reliability.

Provided the results obtained in the static and dynamic RIM analyses some combined ρ_i assignments will be investigated. In particular, results corresponding to combinations of MIF, CIF and RRW are provided in Table VI. The best results correspond to $\rho_i = I_{CIF_i} \times I_{RRW_i}$ and $\rho_i = I_{MIF_i} \times I_{RRW_i}$,

improving the results obtained in assignment of ρ_i to a single RIM.

Table VI
COMBINED ρ_i ASSIGNMENT PERFORMANCE

ρ_i	R_s [%]	U_{CUM} [$\times 10^6$]	JPR
$I_{CIF_i} \times I_{RRW_i}$	99.44	3.971	0.172
$I_{MIF_i} \times I_{RRW_i}$	99.42	3.946	0.172
$I_{MIF_i} \times I_{CIF_i} \times I_{RRW_i}$	98.78	3.518	0.153
$I_{MIF_i} \times I_{CIF_i}$	98.73	3.463	0.152

Although the reliability is improved, the consumption of energy increases, this could be due to the fact that the controller tends to use more those pumps whose impact on the overall system reliability is low.

According to their definitions, when the combination of CIF and RRW is used, the objective is to preserve those components whose reliabilities are critical for the system functioning and those that can produce the largest system reliability reduction. In the case of $I_{CIF_i} \times I_{RRW_i}$, the objective is to preserve those components whose reliabilities changes would produce the higher variation and the higher reduction in the system reliability.

Fig. 4 illustrates the improvement of system reliability with respect to the nominal scenario.

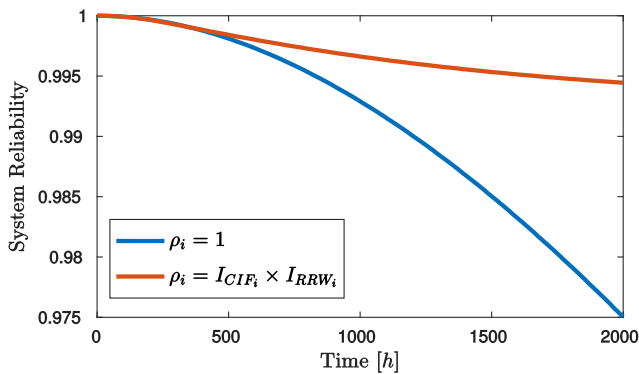


Figure 4. System reliability comparison.

Fig. 5 shows the tanks volume corresponding to $\rho_i = 1$ and $\rho_i = I_{CIF_i} \times I_{RRW_i}$. In both scenarios the MPC manages to keep the volume within the limiting bounds.

In Fig. 6 the pump commands corresponding to $\rho_i = 1$ and $\rho_i = I_{CIF_i} \times I_{RRW_i}$ are presented. Remark that different pump commands are produced in the two approaches.

VI. CONCLUSIONS

This paper has presented a model predictive control design based on reliability importance measures for a drinking water network. The use of reliability importance measures helped in identifying the relative importance of each actuator (pump) in the DWN with respect to the overall reliability of the system. The objective was to extend the uptime of the system by delaying, as much as possible the system reliability decay.

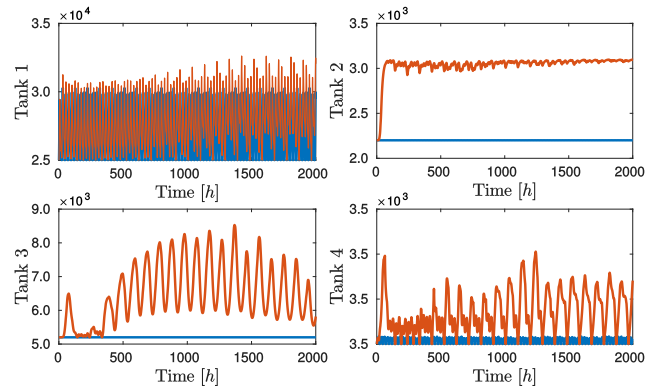


Figure 5. Tanks volume [m^3/h]: blue line corresponds to $\rho_i = 1$ and red line corresponds to $\rho_i = I_{CIF_i} \times I_{RRW_i}$.

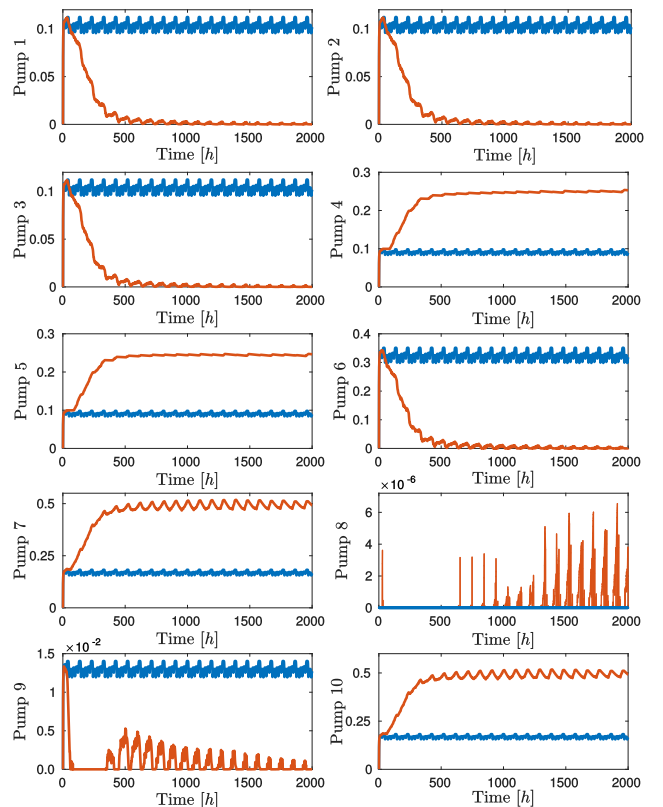


Figure 6. Pumping actions [m^3/h]: blue line corresponds to $\rho_i = 1$ and red line corresponds to $\rho_i = I_{CIF_i} \times I_{RRW_i}$.

A static and a dynamic analysis of the reliability importance measures have been performed using the DBN of the DWN. A combination of both analysis has helped in choosing the best parameter tuning of the MPC controller.

In this work, only the actuators use has been considered in the MPC cost function, tracking error issues will be considered and studied in future research.

REFERENCES

- [1] Y. Eun, C. Gokcek, P. T. Kabamba, and S. M. Meerkov, "Selecting the level of actuator saturation for small performance degradation of linear

- designs,” in *Proc. of the 40th IEEE Conf. on Decision and Control*, 2001, pp. 1769–1774.
- [2] T. Perez, G. C. Goodwin, and M. M. Seron, “Performance degradation in feedback control due to constraints,” *IEEE Trans. on Automatic Control*, vol. 48, no. 8, pp. 1381–1385, 2003.
- [3] Y. M. Zhang and J. Jiang, “Fault tolerant control system design with explicit consideration of performance degradation,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 39, no. 3, pp. 838–848, 2003.
- [4] A. Khelassi, D. Theilliol, and P. Weber, “Reconfigurability analysis for reliable fault-tolerant control design,” *International Journal of Applied Mathematics and Computer Science*, vol. 21, no. 3, 2011.
- [5] A. Khelassi, D. Theilliol, P. Weber, and J.-C. Ponsart, “Fault-tolerant control design with respect to actuator health degradation: An LMI approach,” in *IEEE International Conference on Control Applications (CCA)*, Denver, USA, 2011, pp. 983–988.
- [6] F. Bicking, P. Weber, and D. Theilliol, “Reliability importance measures for fault tolerant control allocation,” in *Proceedings of the 2nd International Conference on Control and Fault-Tolerant Systems (SysTol 2013)*, Nice, France, 2013, pp. 104–109.
- [7] P. Weber and L. Jouffe, “Reliability modelling with dynamic bayesian networks,” in *5th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes (SAFEPROCESS’03)*, Washington, D.C., USA. IFAC, 2003, pp. 57–62.
- [8] A. Ben Salem, A. Muller, and P. Weber, “Dynamic bayesian networks in system reliability analysis,” in *6th IFAC SAFEPROCESS 2006*, Beijing, China, 2007, pp. 481–486.
- [9] C. Simon, P. Weber, and A. Evsukoff, “Bayesian networks inference algorithm to implement Dempster Shafer theory in reliability analysis,” *Reliability Engineering & System Safety*, vol. 93, no. 7, pp. 950–963, 2008.
- [10] P. Weber, C. Simon, D. Theilliol, and V. Puig, “Fault-tolerant control design for over-actuated system conditioned by reliability: A drinking water network application,” in *8th IFAC SAFEPROCESS*, 2012, pp. 558–563.
- [11] J. C. Salazar, P. Weber, F. Nejjari, D. Theilliol, and R. Sarrate, “MPC Framework for System Reliability Optimization,” in *Advanced and Intelligent Computations in Diagnosis and Control*, ser. Advances in Intelligent Systems and Computing, Z. Kowalczyk, Ed. Springer International Publishing, 2015, no. 386, pp. 161–177.
- [12] J. C. Salazar, P. Weber, R. Sarrate, D. Theilliol, and F. Nejjari, “MPC design based on a DBN reliability model: Application to drinking water networks,” in *Proceedings of the 9th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes*, vol. 48, no. 21, Paris, France, 2015, pp. 688–693.
- [13] W. Kuo and X. Zhu, *Importance measures in reliability, risk, and optimization: principles and applications*. Hoboken, N.J: John Wiley & Sons, Ltd, 2012.
- [14] J. M. Maciejowski, *Predictive control: with constraints*. Prentice Hall, 2002.
- [15] C. Ocampo-Martinez, V. Puig, G. Cembrano, and J. Quevedo, “Application of predictive control strategies to the management of complex networks in the urban water cycle,” *IEEE Control Systems Magazine*, vol. 33, no. 1, pp. 15–41, 2013.
- [16] J. C. Salazar, P. Weber, F. Nejjari, R. Sarrate, and D. Theilliol, “System reliability aware model predictive control framework,” *Reliability Engineering & System Safety*, to appear.
- [17] D. R. Cox, “Regression models and life-tables,” *Journal of the Royal Statistical Society. Series B (Methodological)*, vol. 34, no. 2, pp. 187–220, 1972.
- [18] F. Jensen, *An Introduction to Bayesian Networks*. Editions UCL Press, 1996.
- [19] Z. W. Birnbaum, “On the importance of different components in a multicomponent system,” in *Multivariate analysis*, P. R. Krishnaiah, Ed. Academic Press, 1969, vol. II, pp. 581–592.
- [20] H. E. Lambert, “Fault tree for decision making in system analysis,” Ph.D. dissertation, Lawrence Livermore Laboratory - University of California, 1975.
- [21] W. E. Vesely, “A time-dependent methodology for fault tree evaluation,” *Nuclear Engineering and Design*, vol. 13, no. 2, pp. 337–360, 1970.
- [22] J. Fussell, “How to Hand-Calculate System Reliability and Safety Characteristics,” *IEEE Transactions on Reliability*, vol. R-24, no. 3, pp. 169–174, 1975.
- [23] G. Levitin, L. Podofillini, and E. Zio, “Generalised importance measures for multi-state elements based on performance level restrictions,” *Reliability Engineering & System Safety*, vol. 82, no. 3, pp. 287–298, 2003.