Quasi-linear approximation of the HMRI

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The magnetorotational instability (MRI) is one of the most important processes in astrophysics, and is the leading candidate for the mechanism by which outward angular momentum transport occurs in magnetised accretion disks. Notably, the presence of an axial magnetic field allows for instability when angular momentum profiles increase with radius, for which purely hydrodynamic flows are stable.

Unfortunately, it is not feasible to run direct numerical simulations (DNS) at the relevant parameters for an astrophysical disk, and so additional approaches are required when investigating the instability.

One such approach is the use of laboratory experiments, whereby an axial magnetic field is applied to liquid metal in the Taylor-Couette geometry. For instability, a magnetic Reynolds number $\mathrm{Rm} \geq O(10)$ is required, which, coupled with the fact that liquid metals typically have magnetic Prandtl number $\mathrm{Pm} \approx O(10^{-6})$, necessitates a Reynolds number $\mathrm{Re} \geq O(10^{7})$. This is problematic; under such strong rotation the Taylor-Proudman theorem states that the flow will be dominated by the conditions at the end-plates. As such, this setup has not yet succeeded in producing the standard MRI (SMRI). However, by imposing an additional azimuthal magnetic field, the necessary requirement for instability changes from $\mathrm{Rm} \geq O(10)$ to $\mathrm{Re} \geq O(10^{3})$. The resulting helical MRI (HMRI) is continuously connected to the SMRI, has been experimentally reproduced, and is numerically well understood.

Another approach that has recently received much attention is that of direct statistical simulation (DSS), where the low-order statistics are obtained directly from a hierarchy of cumulant equations. It offers a number of advantages over DNS when probing the long time behaviour of astrophysical phenomena. However, it is not currently universally applicable, and there have been a number of examples of inconsistencies with fully nonlinear DNS; current research is focussed on improving the performance of DSS in such cases.

Motivated by this, we perform DNS on the HMRI under the generalised quasi-linear approximation (GQL), which is essentially equivalent to the cumulant expansions utilised by DSS. The GQL approximation improves upon the standard quasi-linear (QL) approximation by incorporating fully self-consistent interactions of large-scale modes, whilst still being formally linear in the small scales.

Here, we address whether GQL can produce the low-order statistics of axisymmetric HMRI more accurately than the corresponding QL approximation, via diagnostics such as the energy spectra in addition to the first and second cumulants. We find that GQL performs notably better than QL in producing the statistics of the HMRI, even with relatively few large-scale modes. We conclude that DSS based on GQL (GCE2) should be significantly more accurate than that based on QL (CE2).