Bifurcation tracking techniques for periodic orbits of PDEs

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Continuation methods are commonly used to track curves of solutions depending on parameters. These methods have been used in Fluid Mechanics mainly to compute bifurcation diagrams of steady solutions. More recently they have also been used for periodic orbits.

Following our previous work we have developed a new efficient methodology for the continuation of the codimension-one bifurcations of periodic orbits, including pitchfork bifurcations present in reflection-symmetric systems. It is based on the combination of Newton-Krylov techniques applied to extended systems, and the integration of systems of variational equations up to second order. The extended systems are adapted from those usually found in the literature for fixed points of maps. It will be shown that to evaluate the action of the Jacobian it is only necessary to integrate systems of ODEs of dimension at most four times that of the original system. This minimizes the computational cost.

The thermal convection of a mixture of two fluids in a two-dimensional rectangular box is used as test problem. It is known that the onset of convection is oscillatory below a certain negative value of one of the parameters (the separation ratio), giving rise to a rich dynamics. A non-trivial diagram of periodic orbits is first deployed, by varying only the Rayleigh number, and some of the bifurcations found on the main branch of periodic orbits are followed by adding as second parameter the Prandtl number. Several codimension-two points have been found.
