Analytical solutions for quasi-geostrophic inertial modes and onset of thermal convection in rapidly rotating spheroids

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The flows in the fluid cores of rapidly rotating planetary bodies can be conveniently described as being invariant along the direction parallel to the rotation axis. This description, also referred to as columnar, is based on the quasi-geostrophic approximation and it holds for timescales longer than the rotation period as long as other forces acting on the fluid are of secondary importance with respect to rotation. A significant effort of the community is presently spent in the development of quasi-geostrophic numerical models of planetary cores, the final goal being to run numerical simulations in realistic parameters regimes. The development of such models has proven fundamentally challenging, especially when magnetic forces are present. Therefore, analytical solutions to simple dynamical problems will be of paramount importance for benchmarking purposes.

We present an analytical and explicit solution to the problem of the columnar inertial modes in rapidly rotating sphere and spheroids in absence of viscosity. We find that the oblateness of the spheroid significantly alters the frequency of the low order inertial modes for high azimuthal wavenumbers. However the geometry of the flow is the same as for the spherical case. Excellent agreement with known 3-D solutions has been found. Typically, given the geometry of the columnar flows, the axial vorticity equation is assumed to be a valid description of the dynamics of quasi-geostrophic flows. Based on a recently developed projection technique, we found the axial vorticity equation to be appropriate only in the case of highly oblate spheroids.

This analytical solution can be used to calculate the critical Rayleigh number and the shape of the flow at the onset of thermal convection. We do so by following an asymptotic procedure already applied to the spherical case and for 3-D flows.
