Analysis of optimal control problem formulations in skeletal movement predictions

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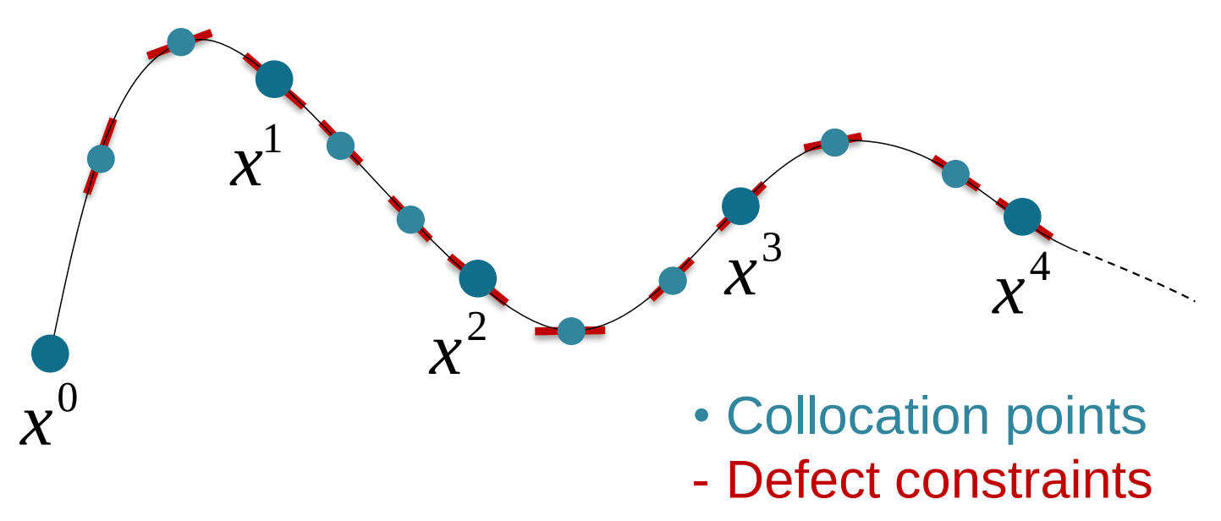
Introduction

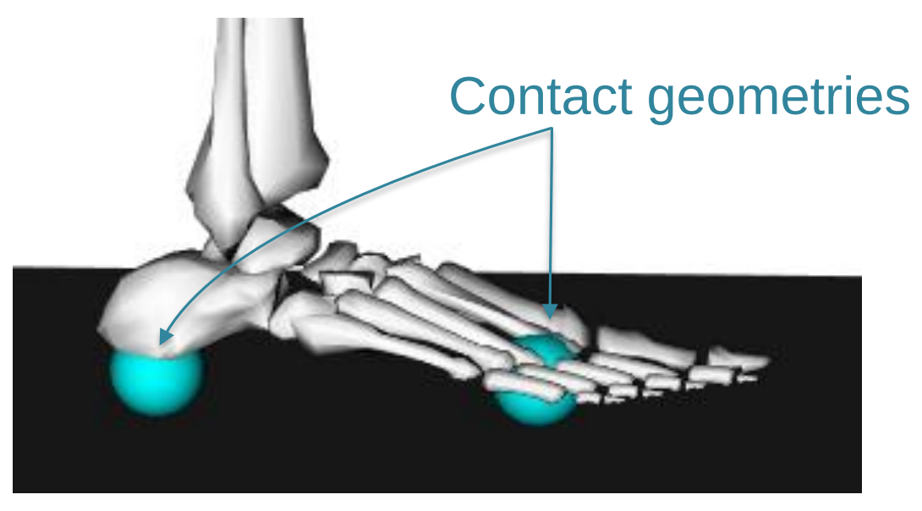
Prediction of movement in biomechanics is typically obtained by optimizing performance criteria to solve the kinematic redundancy, e.g. performing a task with minimal effort. **Direct collocation** based on an implicit formulation of the dynamic equations allows efficiently solving the kinematic [1] and muscle [2] redundancy problems.

It is, however, unclear how using a **foot-ground contact model** influences the numerical condition and convergence for different formulations of the optimal control problem.

Direct collocation

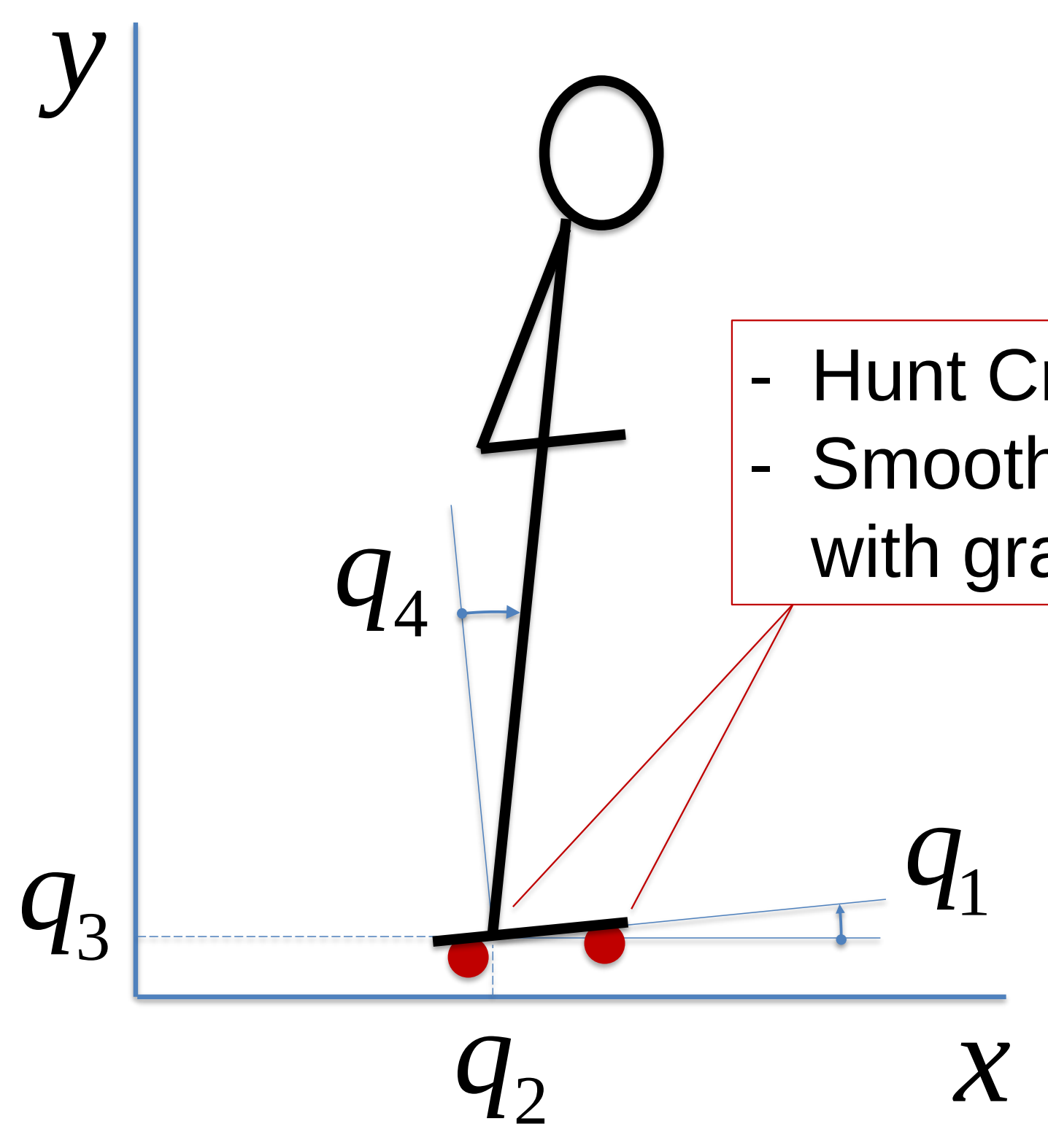


Contact model



Methods

A simple **planar torque-driven model** with two segments and **four degrees of freedom**, **similar** to the ones used to study **balance control**, was implemented to test different optimal control problem formulations.

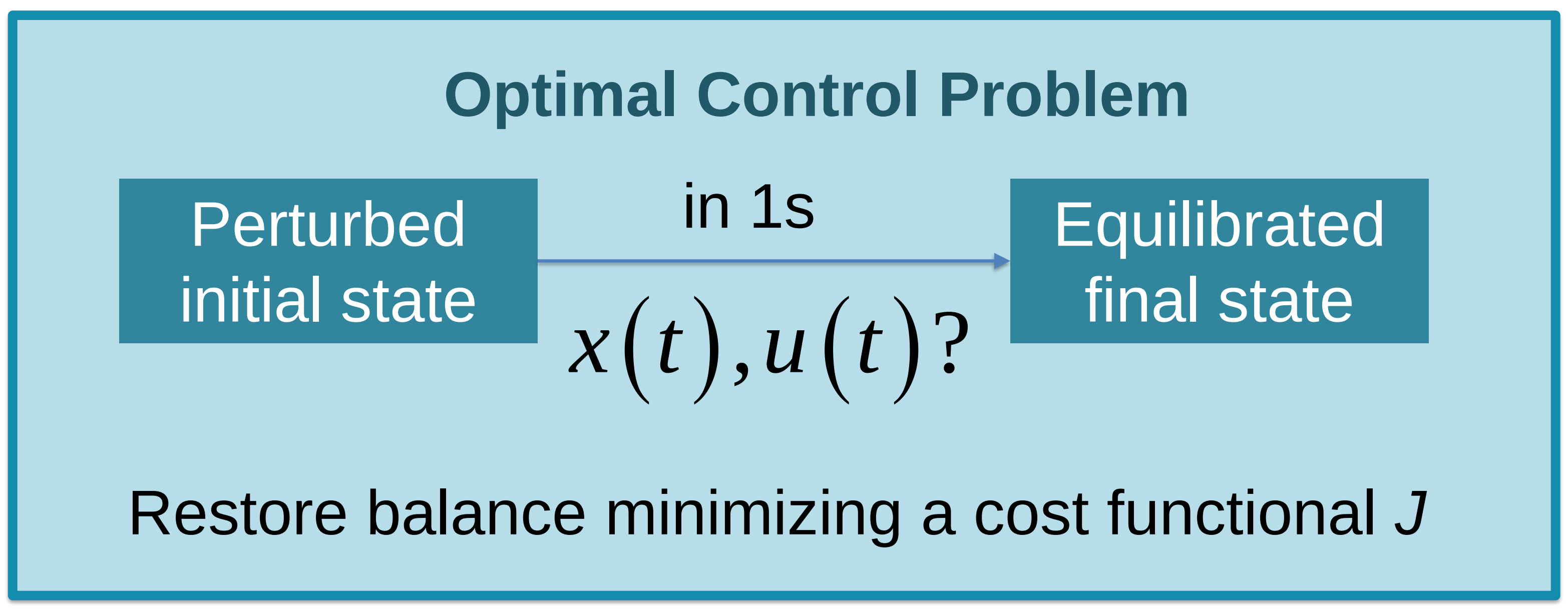


- Hunt Crossley contact model [3]
- Smooth approximation for use with gradient method

Dynamic optimization problem

Independently of the formulation, **the optimal control problem aimed at predicting controls and kinematics** to restore equilibrium from an initial perturbed state within 1s.

The dynamic optimization was solved using **direct collocation** (in GPOPS-II [4]), on a grid with 20 mesh intervals and 4 Legendre-Gauss-Radau collocation points at each interval.



Optimal Control Problem

Perturbed initial state → in 1s $x(t),u(t)$? → Equilibrated final state

Restore balance minimizing a cost functional *J*

Acknowledgments

The support of the MIRAD project (IWT-SBO no 120057) is gratefully acknowledged.

Problem formulations

Formulation A: implicit dynamics + minimal accelerations

States: $x=[q,\dot{q}]$ Joint coordinates and velocities

Controls: $u=[u_T,u_a]$ Torque controls and joint accelerations

State derivatives: $\dot{x}=[\dot{q},u_a]$

Path constraints: $\mathrm{M}u_a+C(q,\dot{q})+G(q)-\tau_{GRF}-u_T T_{\max}=0$ Equations of motion

Cost function: $J=\int_{t_0}^{t_f}\left(u_T^2+u_a^2\right)dt$

- NLP derivatives computed with finite differences.
- Time scale factor of 3.

Formulations B to E, like A with slight changes:

Formulation B: implicit dynamics + minimal jerks

Formulation C: A with automatic differentiation

Formulation D: A with time scale factor of 10.

Formulation E: explicit dynamics

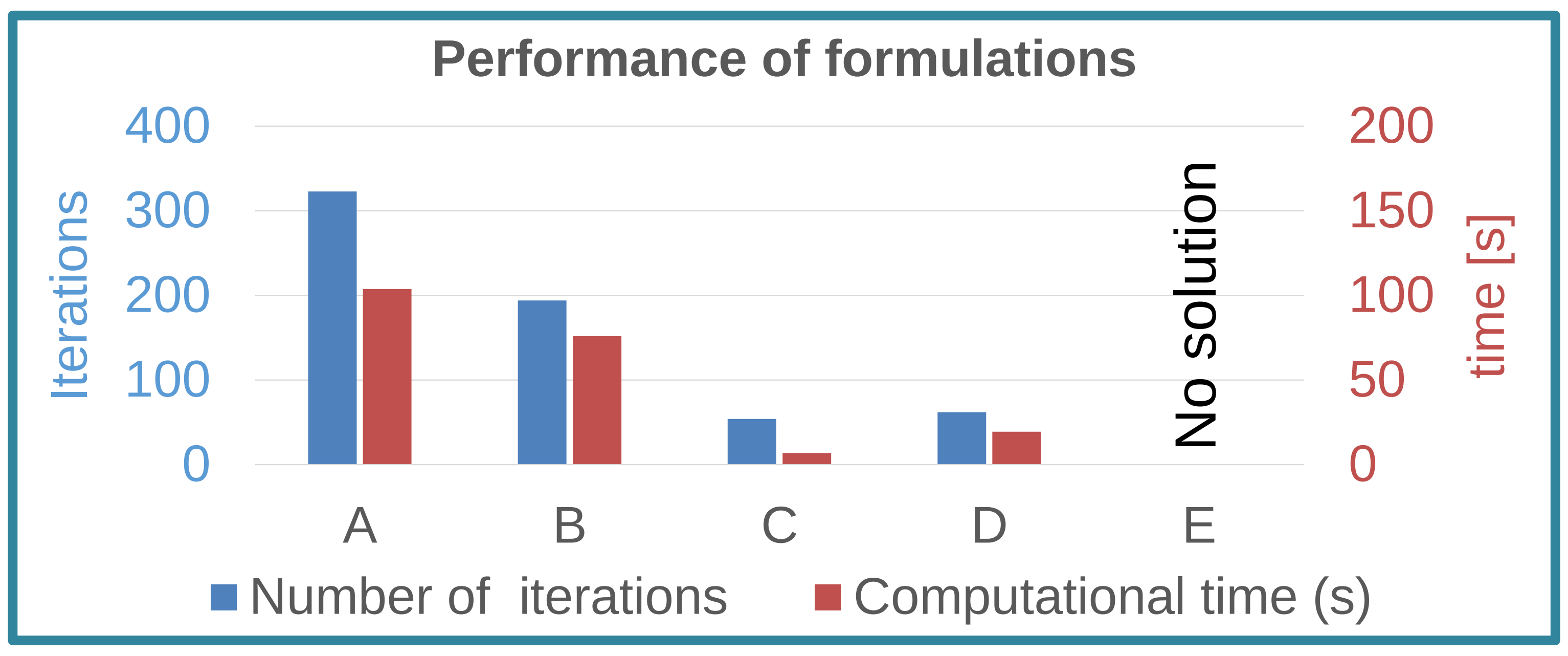
$$\ddot{q}=\mathrm{M}^{-1}\left[\tau_{GRF}+u_T T_{\max}-C(q,\dot{q})-G(q)\right]$$

Results and conclusions

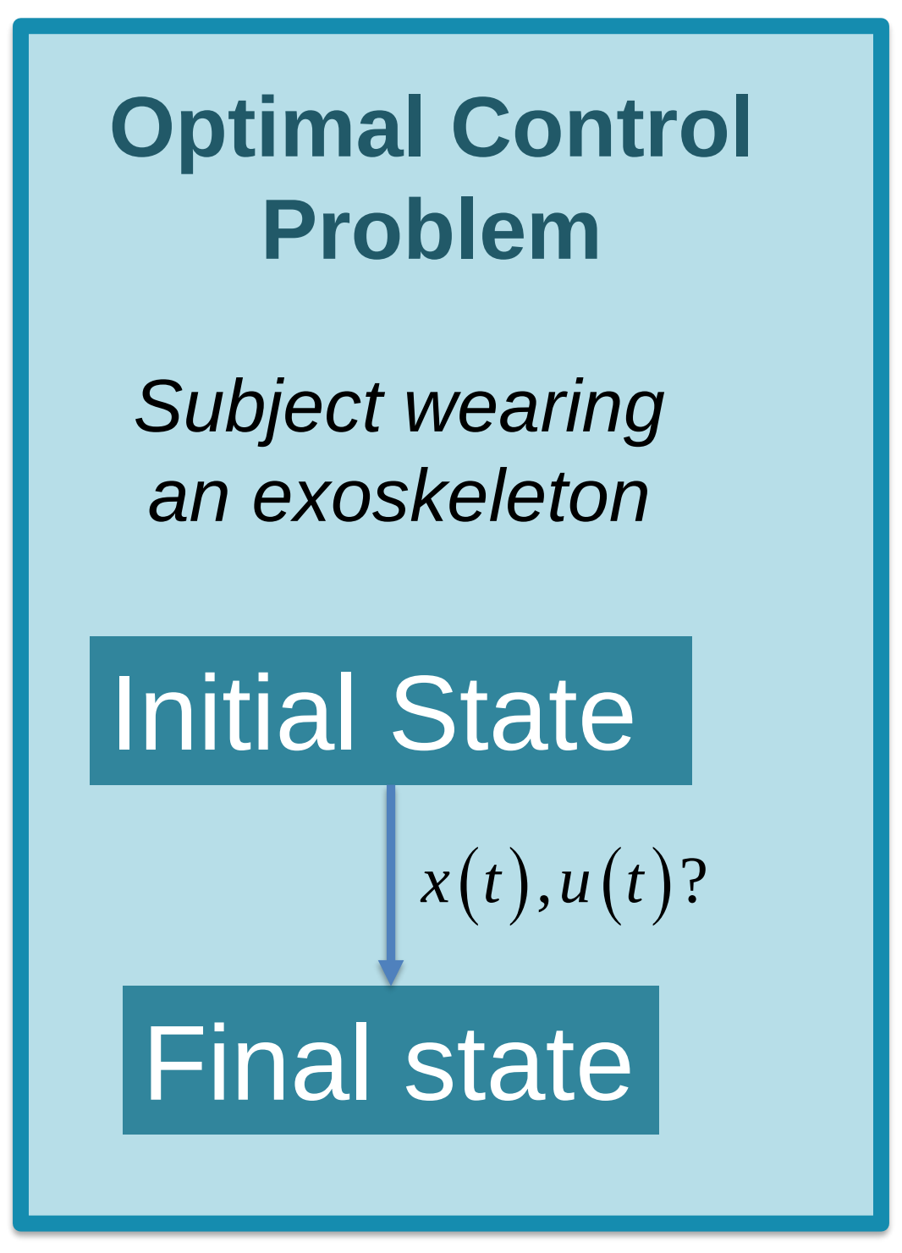
Implicit dynamic formulations in combination with a penalty on coordinate accelerations or jerks **resulted in better convergence** than explicit dynamic formulations minimizing torque controls only.

The use of **automatic differentiation** and a well chosen **time scale factor** have a **high impact on the computational efficiency.** In addition, time scaling slightly improves the accuracy of the solution.

Future work aims at predicting human-exoskeleton interaction. Accounting for the contact pressures between the subject and exoskeleton is important, since contact pressures are directly related to comfort.



Application

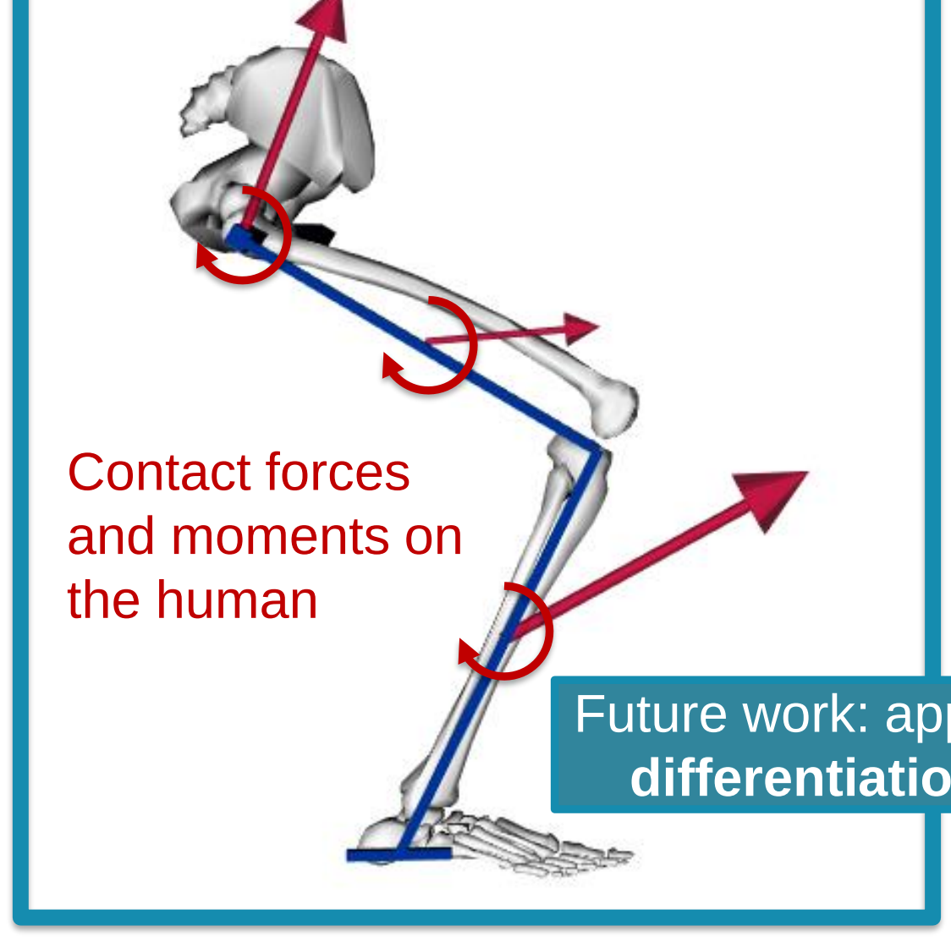


Optimal Control Problem

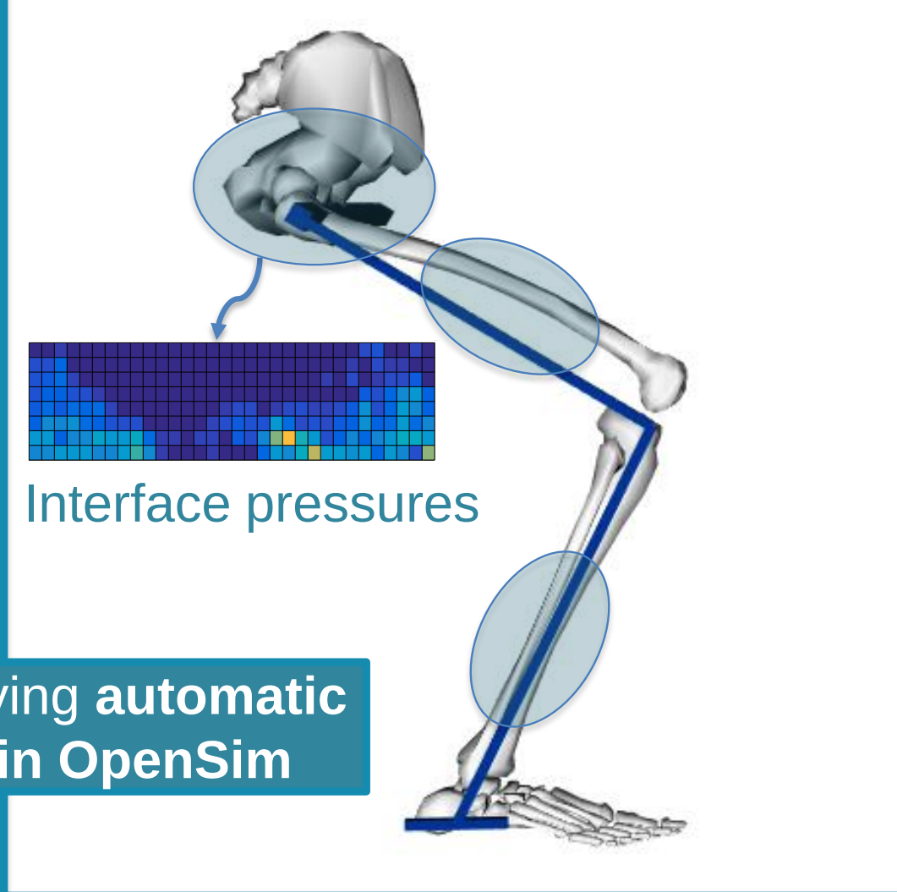
Subject wearing an exoskeleton

Initial State → $x(t),u(t)$? → Final state

Prediction of contact wrenches



Prediction of contact pressures



Future work: applying **automatic differentiation in OpenSim**

References

[1] A.J. Van den Bogert, D. Blana and D. Heinrich. *Procedia IUTAM* 2:297-316, 2011.
[2] F. De Groote, A.L. Kinney, A.V. Rao and B.J. Fregly. *Ann. Biomed. Eng.* Published online (DOI: 10.1007/s10439-016-1591-9).
[3] K. H. Hunt and F. R. E. Crossley. *ASME J Appl. Mech.* 42:440-445, 1975.
[4] M. A. Patterson, and A. V. Rao. *ACM Trans. Math. Soft*. 41(1): Article 1 (37 pages), 2014.