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A neutron diode for subcritical multistage multipliers with special reference in tritium breeding

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In this paper the interaction between a magnetic field and the neutron spin magnetic moment is explored for use in the design of a neutron diode or valve that allows a neutron flux to pass in one direction, while preventing a neutron flux in the opposite direction. A neutron diode that ensures the unidirectional movement of neutrons could be used in the design of a subcritical multistage neutron multiplier, a device that has thus far not been realised. With a subcritical multistage neutron multiplier, an initial source of neutrons could be multiplied substantially in a very small area. Such a device could have potential applications in tritium breeding in a fusion reactor, in medicine, in space exploration, etc. Utilizing a simplified geometrical model, a first preliminary study is performed to assess the feasibility of this concept.

Keywords. *Subcritical multistage neutron multiplier, spin-magnetic field interaction.*

I. INTRODUCTION

Despite being discussed in patents since the mid 20th century [1, 2], and more recently, see for example [3–7], subcritical multistage neutron multipliers have been relegated to academic text books and practical applications have been limited to two stages. However, the potential benefits of this technology (the ability to attain large neutron multiplication factors from limited neutron sources) are particularly attractive for the fusion reactor project, and most specifically in the breeding of tritium.

The reasons for the limitation of a practical subcritical multistage neutron multiplier are easy to grasp when considering the bases behind of technology, which is depicted schematically in Fig. 1. Essentially, the device consists of two multiplying sections (generally separated spatially) with asymmetric coupling, in such a way that neutrons produced in the first section easily penetrate the second while those produced in the second section have little influence over the first [7], thus preventing an uncontrolled chain reaction. So, the key factor in a subcritical multistage neutron multiplier is to ensure the unidirectional movement of neutrons, in which neutrons are progressively multiplied in each stage but never return to the previous stage. If this is not prevented, a state of supercriticality would result in which the number of neutrons grows exponentially and in an uncontrolled way, i.e. a runaway explosion.

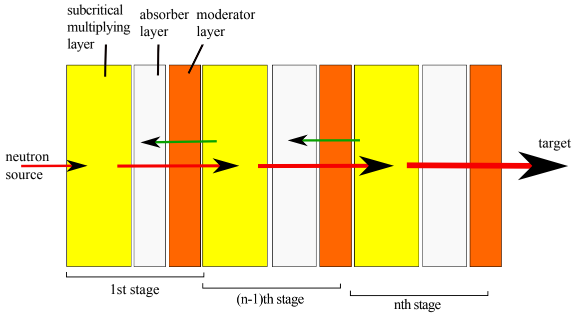

FIG. 1: The principle behind a classical subcritical multistage neutron multiplier.

Until now, the only way to ensure the unidirectional movement of neutrons has been by deploying a sandwich-like combination of selective multiplying-absorber-moderator layers and energetic spectral shifting [8] (see Fig. 1). In this multiplier, the unidirectionality of the motion of neutrons is attained as follows: fast neutrons produced in the first fissile subcritical layer pass through an absorber or neutron poison layer (which only absorbs thermal neutrons); these fast neutrons then pass through a second layer where they are moderated before reaching the second fissile subcritical multiplying layer as thermal neutrons. Reverse operation is not possible, because the asymmetry of the system: fast neutrons produced in the second fissile layer travelling to the left (in the reverse direction) are moderated before they reach the absorber layer, and then captured before they can reach the first fissile layer. Thus, the unidirectionality of the motion of neutrons is attained through a complex combination of layers, and the safety of the system relies entirely on spectral shifting. If this were to fail – for example, due to thermal instability of the layers, resonances, etc. – there would be nothing to prevent the neutrons produced in a later stage returning to the previous stage, and then a prompt criticality condition could

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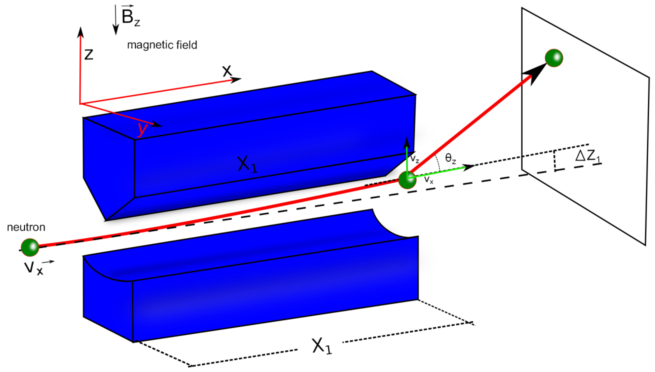

FIG. 2: Neutron deflection due to the coupling of an external magnetic field gradient and the neutron spin.

ensue. For this reason, this technology is, in practice, limited to two stages.

In this paper, a totally different approach to attaining the unidirectional movement of neutrons is proposed. Here, the neutrons produced in one stage will never be able to return to the previous stage because a real spatial deflection in their path is produced between stages. This allows not only a safe design of a 2-stage neutronic multiplier but also of an n-stage multiplier where substantial multiplication factors can be achieved. The potential application of this technology for tritium breeding in a fusion reactor (which is already providing the initial neutron source) is very interesting.

In the next section, the core idea for the spatial deflection of neutrons between stages is presented, after which the potential multiplication factor attainable using this approach will be discussed.

II. STATEMENT OF THE CORE IDEA: THE USE OF THE SPIN OF NEUTRONS IN A MULTISTAGE NEUTRON MULTIPLIER DIODE

Neutrons possess spin and a magnetic dipole moment and therefore in the presence of a external magnetic field there would be a coupling between them. As a result of this coupling, if the magnetic field features a gradient then a force acting on the magnetic dipole will result (see Fig. 2). The core idea in this paper is to use this force to control the neutron trajectory between stages. The idea of using this magnetic force on neutrons is not new. It has been used in several applications, as, for example, in the design of neutron polarizers, neutron focusers, neutron spectrometers and monochromators [9]. However, the potential use of this force has not been proposed, as far as the authors know, for a multistage neutron multiplier. Our idea is that the magnetic deflection of neutrons can provide the key to the design of a neutron diode or valve which ensures spatial unidirectionality of neutrons.

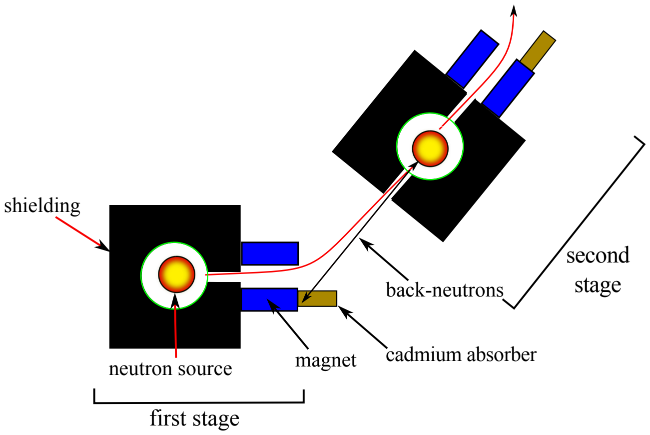

FIG. 3: The use of the spin of neutron for two-stages subcritical multiplier.

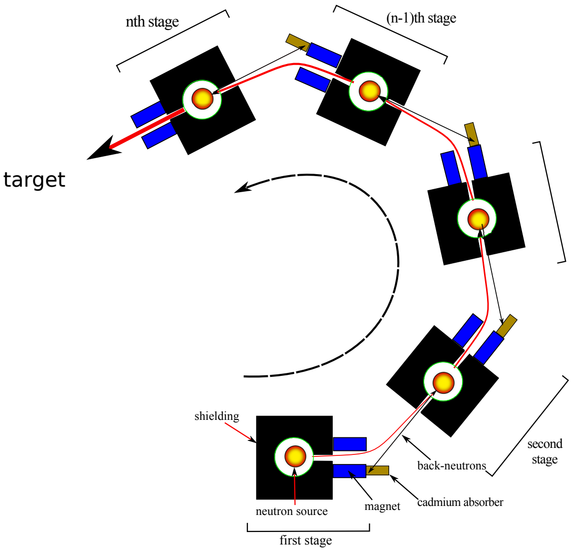

FIG. 4: The use of the spin of neutron for n-stage subcritical multiplier.

The reader can appreciate the possibilities of this idea by looking at Fig. 3 where a 2-stage device is depicted, and at Fig. 4 where an n-stage device is shown. In contrast to the classical approach (see Fig. 1), in Figs. 3 and 4 there is a real spatial angular separation between each stage, and a neutron from one stage could never reach the previous stage.

In the next section we will develop the theory behind this concept and derive an expression to allow the feasibility of the idea to be assessed.

III. NEUTRON DEFLECTION DUE TO SPIN-MAGNETIC FIELD COUPLING

We are interested in finding an expression for the displacement of a neutron under a magnetic field gradient, as depicted in Fig. 2. This displacement enables us to find the angular separation between stages, and is thus a parameter which is a direct measure of the feasibility of the proposed concept.

If the magnetic field is applied along the z-axis, as shown in Fig. 2, then the force acting on the neutron is along the z-axis. A neutron has a spin 1/2. The force due to the nuclear magnetic moment of the neutron μ_s interacting with the magnetic field is given by the classical expression:

$$F_z = \mu_{S_z} \frac{d\mathbf{B}_z}{dz} \tag{1}$$

where μ_{S_z} is the z-component of the magnetic moment of the neutron and $\mathbf{B}_z$ is the magnetic field applied along the z-axis.

The acceleration of the neutron is then:

$$a_z = \frac{F_z}{m_n} = \frac{\mu_{S_z}}{m_n} \frac{d\mathbf{B}_z}{dz} \tag{2}$$

where m_n is the mass of the neutron. Now, if X_1 is the distance travelled inside the field (see Fig. 2) and v_x is the velocity in the x-direction, then the time spent by the neutron in the field is:

$$t_1 = \frac{X_1}{v_x} \tag{3}$$

The displacement of the neutron in the z-direction Z_1 is given by:

$$Z_1 = \frac{1}{2} a_z t_1^2 = \frac{1}{2} \frac{\mu_{S_z}}{m_n} \frac{d\mathbf{B}_z}{dz} \frac{X_1^2}{v_x^2} \tag{4}$$

and the velocity v_z in the z-direction at the moment the neutron leaves the field is given by:

$$v_z = a_z t_1 = \frac{\mu_{S_z}}{m_n} \frac{d\mathbf{B}_z}{dz} \frac{X_1}{v_x} \tag{5}$$

A measure of the power of angular separation between stages is the angle θ_z in Fig. 2, which is given by:

$$\theta_z = \tan^{-1}\left[\frac{v_z}{v_x}\right] = \tan^{-1}\left[\frac{\mu_{S_z}}{m_n} \frac{d\mathbf{B}_z}{dz} \frac{X_1}{v_x^2}\right] \tag{6}$$

where $0 \leq \theta_z \leq \frac{\pi}{2}$. So, the most dangerous situation is where $v_z = 0$ and $\theta_z = 0$, meaning that there is no angular separation between neutrons leaving the nth stage and those coming back from the $(n+1)$th stage. The ideal case is where $\theta_z = \frac{\pi}{2}$.

Taking into account that

$$E_n = \frac{1}{2} m_n v_x^2 \tag{7}$$

where E_n is the initial energy of the neutron, Eq. (6) can be rewritten as:

$$\theta_z(E_n) = \tan^{-1}\left[\frac{\mu_{S_z}}{2} \frac{d\mathbf{B}_z}{dz} \frac{X_1}{E_n}\right] \tag{8}$$

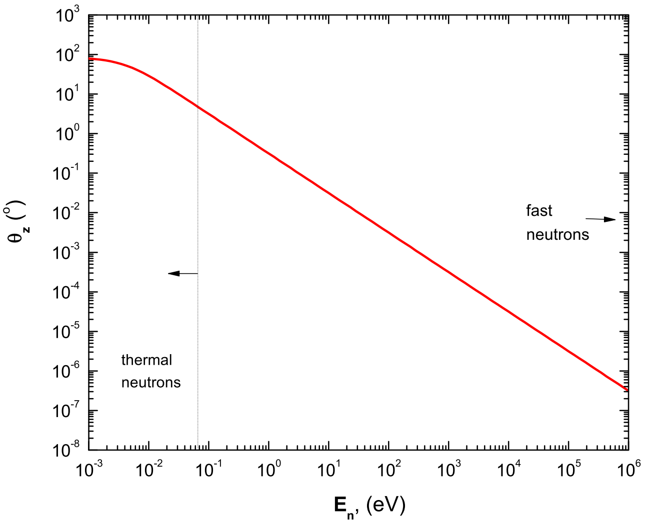

FIG. 5: Angular separation θ_z predicted by Eq. (8) as a function of neutron energy.

To obtain some idea of the angular separation parameter predicted by Eq. (8), we assume some typical values of the parameters. The best available measurement for the value of the magnetic moment of the neutron is -1.91304272 Bohr magnetons [10]. Let us assume a magnetic gradient of $\frac{d\mathbf{B}_z}{dz} = 100\ \mathrm{Tm}^{-1}$, a value that is easy to obtain taking advantage of the use of strong magnetic fields in fusion reactors with magnetic confinement, and a length for each stage of 50 cm. The resulting curve is shown in Fig. 5. It can be seen that, for fission neutrons with energies in the order of MeVs, the resulting angular separation would be $\theta_z \approx 10^{-7}\,^\circ$, which is a very small angle and would then require very large physical separation between stages in order to obtain appreciable spatial separation between forward and backward travelling neutrons; the spatial separation between stages is proportional to $\theta_z \times L$ with θ_z in radians and L being the distance between stages, and the dimensions of the device for multiple stages could then be unpractical.

However, for thermal neutrons, with energies below 10^{-3} eV, good angular separations of $\theta_z \approx 70\,^\circ$ are obtained. This implies that the use of a moderator is necessary to slow down the energetic neutrons released in fission. For example, for the case where fission neutrons with initial energies of 1 MeV or thereabouts are moderated to thermal energies of ~0.03 eV, then, according to Fig. 5, an angular separation of ~5 ° or 0.1 radians will be obtained. For a distance between stages of $L = 10$ cm, we will obtain a spatial separation of neutrons between stages of ~1 cm, which is good enough.

IV. SUBCRITICAL MULTIPLICATION

The subcritical multiplication factor for one stage, M_1, from an initial neutron source S_0 is given by [8]:

$$M_1 = \frac{1}{1-\kappa} \tag{9}$$

where κ is the multiplication factor (< 1) of the system. To err on the conservative side, let us allow for losses and uncertainties in the subcritical device, and thus multiply Eq. (9) by a factor ϵ where $0 \leq \epsilon \leq 1$. Then, Eq. (9) becomes:

$$M_1 = \frac{\epsilon}{1-\kappa} \tag{10}$$

Next, considering n stages, the subcritical multiplication factor becomes:

$$M_n = \left[\frac{\epsilon}{1-\kappa}\right]^n \tag{11}$$

In applying the proposed neutron diode concept we must take into account the following special considerations:

1. The neutrons can have either spin up ↑ or spin down ↓ which can be assumed to be equally likely. However, according to Fig. 2, we will be only able to control one component of spin in the desired manner (up or down but not both), and the other component (or fraction of neutrons) will be lost. This means that, even in the best case scenario, we only will be able to 'capture' 50% of the neutronic flux passing through the magnetic field, and thus an additional loss factor of $\frac{1}{2}$ must be considered for each stage.
2. According to Fig. 3, of the neutrons that are created by fission in each stage, 50% would be going forwards towards the next stage and 50% going in the opposite direction, where they will finally be absorbed by a neutronic poison. Thus, we must include another additional loss factor of $\frac{1}{2}$.

Accounting for these loss factors, Eq. (11) becomes:

$$M_n = \left[\frac{\epsilon}{1-\kappa}\right]^n \cdot \left[\frac{1}{2}\right]^n \cdot \left[\frac{1}{2}\right]^n \tag{12}$$

or

$$M_n = \left[\frac{1}{1-\kappa}\right]^n \cdot \left[\frac{\epsilon}{4}\right]^n \tag{13}$$

Thus, taking Eq. (11) into account, the multiplication factor for n stages can be expressed as:

$$M_n = \left[\frac{M_1}{4}\right]^n \tag{14}$$

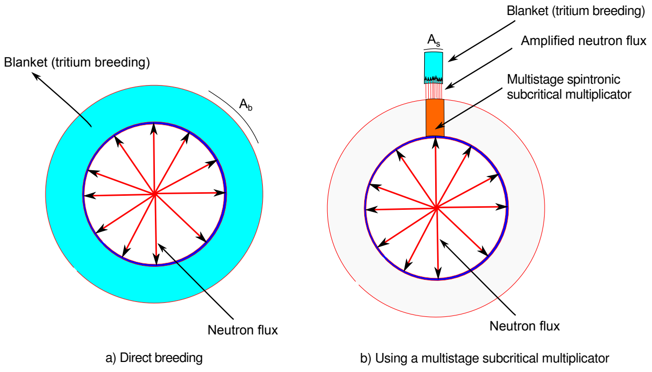

FIG. 6: Tritium breeding: **(a)** Classical approach with direct irradiation; **(b)** Using a multistage subcritical amplifier.

V. BREEDING OF TRITIUM

In this section, we will consider the practical application of the proposed idea for the breeding of tritium in a fusion reactor. Fig. 6 shows a generalized scheme for the breeding of tritium by neutronic irradiation of a blanket. On the left-hand side, we have the classical approach where neutrons coming from the fusion chamber are being directly absorbed by the blanket, which is located along the periphery of the chamber. On the right-hand side of the figure, we have an application of the proposed idea, where the neutrons from the fusion chamber are first amplified by a multistage subcritical multiplier and finally deposited in the blanket target.

The amount of tritium that is obtainable by the direct neutronic irradiation of the blankets is proportional to the total number of fusion neutrons reaching the blanket. This can be expressed as:

$$^3H \propto \Phi \times A_b \tag{15}$$

where ^3H is the rate of tritium breeding (atoms per unit time), Φ is the neutronic flux (neutrons per unit of area per unit time) and A_b is the total area of blanket.

Now let us imagine that we are using an n-stage subcritical multiplier system with an aperture area A_s, as depicted in Fig. 6, and with a multiplication factor M_n. Then, the amount of tritium obtained is given by:

$$^3H_s \propto \Phi \times A_s \times M_n \tag{16}$$

where ^3H_s is the rate of tritium breeding (atoms per unit time) obtained using the n-stage subcritical multiplier.

The ratio of tritium breeding rates obtained by using the subcritical multiplier and by direct irradiation is given by dividing Eq. (16) by Eq. (15), yielding the following expression:

$$\left[\frac{^3H_s}{^3H}\right] = \frac{A_s}{A_b} \times M_n \tag{17}$$

Inserting Eq. (14) into Eq. (17) one obtains:

$$\left[\frac{^3H_s}{^3H}\right] = \frac{A_s}{A_b} \times \left[\frac{M_1}{4}\right]^n \tag{18}$$

A. Discussion

According to Eq. (18), a significant reduction in the amount of blanket needed for tritium breeding in a fusion reactor is possible using a subcritical neutron multiplier with a large number of stages. For example, a typical safe value of the multiplication factor for a subcritical multiplier very often considered is $\kappa = 0.95$ [8]. Assuming $\epsilon = 0.5$, then according to Eq. (10) we will have a single-stage multiplication factor $M_1 \approx 10$, and then, Eq. (18) becomes:

$$\left[\frac{^3H_s}{^3H}\right] = \frac{A_s}{A_b} \times \left[\frac{5}{2}\right]^n \tag{19}$$

Therefore, if we are using a multistage subcritical multiplier system with, say, $n = 10$, the area of blanket needed will be substantially reduced: $A_s/A_b = 10^{-4}$.

A reduction in the size of blanket needed for tritium breeding in a fusion reactor is an attractive option not just from an economic point of view, but also because of the reduction of undesired by-products, especially the volatile polonium-210 resulting from the irradiation of the lead in liquid Pb-16Li blankets.

VI. APPENDIX

A. Minimum efficiency

Inserting Eq.(10) into Eq.(14) we have that the efficiency for a n-stage multiplier using the proposed neutron-diode is given by

$$M_n = \left[\frac{1}{1-\kappa}\right]^n \left[\frac{\epsilon}{4}\right]^n \tag{20}$$

On the other hand, from safety considerations the maximum subcriticality is around $\kappa = 0.95$, [8], and then Eq.(20) becomes,

$$M_n = [5\epsilon]^n \tag{21}$$

because $n > 1$, then, it is easy to see that for $M_n > 1$ as, of course, is desired, it is necessary that $\epsilon > 0.2$, i.e., the efficiency of the device must be higher than a 20% or in other words, up to a 80% of the neutronic flux can be lost. This losses will include all the parasite absorptions, but most importantly the losses due to the isotropy in the flux in the neutron source, and then it can be assumed that only a 20% of the total flux is focused forward and backward.

VII. CONCLUSIONS

In this paper the use of a subcritical multistage neutron multiplier was proposed to facilitate a substantial reduction in the size of blankets for tritium breeding in a fusion reactor. To do this, a novel device, called the neutron diode, was first proposed. The neutron diode allows neutron currents from one stage to move to the next stage but not vice versa. Such a device exploits the interaction of the magnetic moment of the neutron and an external magnetic field and can potentially take advantage of the magnetic fields used in fusion reactors working with magnetic confinement.

NOMENCLATURE

a = acceleration
A = area
$\mathbf{B}$ = magnetic field
E_n = energy of the neutron
F = force
^3H = rate of tritium breeding
L = distance
m_n = mass of the neutron
M_n = n-stage subcritical multiplication factor
M_1 = single-stage subcritical multiplication factor
n = number of stages
t = time
v = velocity
x = x-axis coordinate
X_1 = displacement in x-direction
z = z-axis coordinate
Z_1 = displacement in z-direction

Greek symbols
ϵ = loss factor
θ = angular separation
κ = neutron multiplication factor
μ_{S_z} = spin-magnetic momentum of the neutron
Φ = neutronic flux form fusion core

Subscripts symbols
b = blanket
s = subcritical
n = n stages
1 = one stage

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