Master's Thesis

Master in Chemical Engineering

Optimization of the Long-term Planning of Supply Chains with Decaying Performance

REPORT

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Summary

This master's thesis addresses the optimization of supply and distribution chains considering the effect that equipment aging may cause over the performance of facilities involved in the process. The decaying performance of the facilities is modeled as an exponential equation and can be either physical or economic, thus giving rise to a novel mixed integer non-linear programming (MINLP) formulation.

The optimization model has been developed based on a typical chemical supply chain. Thus, the best long-term investment plan has to be determined given production nodes, their production capacity and expected evolution; aggregated consumption nodes (urban or industrial districts) and their lumped demand (and expected evolution); actual and potential distribution nodes; distances between the nodes of the network; and a time horizon.

The model includes the balances in each node, a general decaying performance function, and a cost function, as well as constraints to be satisfied. Hence, the investment plan (decision variables) consists not only on the start-up and shutdown of alternative distribution facilities, but also on the sizing of the lines satisfying the flows.

The model has been implemented using GAMS optimization software. Results considering a variety of scenarios have been discussed. In addition, different approaches to the starting point for the model have been compared, showing the importance of initializing the optimization algorithm.

The capabilities of the proposed approach have been tested through its application to two case studies: a natural gas network with physical decaying performance and an electricity distribution network with economic decaying performance. Each case study is solved with a different procedure to obtain results.

Results demonstrate that overlooking the effect of equipment aging can lead to infeasible (for physical decaying performance) or unrealistic (for economic decaying performance) solutions in practice and show how the proposed model allows overcoming such limitations thus becoming a practical tool to support the decision-making process in the distribution sector.

Resumen

Este proyecto final de máster trata sobre la optimización de cadenas de suministro y distribución considerando el efecto que el envejecimiento de los equipos puede causar sobre el rendimiento de las instalaciones. La pérdida de eficiencia de las instalaciones puede ser física o económica y se modela a través de una ecuación exponencial, dando lugar a una formulación no lineal con variables mixtas enteras MINLP.

El modelo de optimización se ha desarrollado en base a una cadena de suministro de productos químicos. Se debe determinar el mejor plan de inversión a largo plazo dados: los nodos de producción, su capacidad máxima y evolución prevista; los nodos de consumo (zonas urbanas o industriales) y su demanda agregada (y evolución esperada); los nodos de distribución actuales y potenciales; las distancias entre los nodos de la red; y el horizonte de tiempo.

El modelo incluye los balances en cada nodo, una función genérica de pérdida de eficiencia, la función de costes y los requisitos a satisfacer. El plan de inversión (con sus correspondientes variables de decisión) consiste no sólo en la puesta en marcha y cierre de las instalaciones de distribución, sino también en el diseño de las líneas de conexión.

El modelo se ha implementado en el programa de optimización GAMS. Se han discutido los resultados considerando una gran variedad de escenarios. Además, se han comparado diferentes alternativas para la inicialización del modelo, resaltando la importancia del punto inicial en el algoritmo de optimización.

Se ha validado el modelo a través de su aplicación sobre dos casos de estudio: una red de gas natural con pérdida de eficiencia física y una red de distribución de electricidad con pérdida de eficiencia económica. Cada caso de estudio se debe resolver con un procedimiento adaptado a sus características.

Los resultados demuestran que pasar por alto el efecto del envejecimiento de los equipos puede llevar a diseños inviables (cuando existe pérdida de eficiencia física) o estimaciones poco realistas (cuando existe pérdida de eficiencia económica) en la práctica. El modelo propuesto permite superar tales limitaciones, convirtiéndose así en una herramienta práctica para apoyar el proceso de toma de decisiones en el sector de la distribución.

Resum

Aquest projecte final de màster tracta sobre l'optimització de cadenes de subministrament i distribució considerant l'efecte que l'envelliment dels equips pot causar sobre el rendiment de les instal·lacions. La pèrdua d'eficiència de les instal·lacions pot ser física o econòmica i es modela a través d'una equació exponencial, donant lloc a una formulació no lineal amb variables mixtes enteres MINLP.

El model d'optimització s'ha desenvolupat a partir d'una cadena de subministrament de productes químics. S'ha de determinar el millor pla d'inversió a llarg termini donats: els nodes de producció, la seva capacitat màxima i evolució prevista; els nodes de consum (zones urbanes o industrials) i la seva demanda agregada (i evolució esperada); els nodes de distribució actuals i potencials; les distàncies entre els nodes de la xarxa; i l'horitzó de temps.

El model inclou els balanços a cada node, una funció genèrica de pèrdua d'eficiència, la funció de costos i els requisits a satisfer. El pla d'inversió (amb les seves corresponents variables de decisió) consisteix no només en la posada en marxa i el tancament de les instal·lacions de distribució, sinó també en el disseny de les línies de connexió.

El model s'ha implementat amb el programa d'optimització GAMS. S'han discutit els resultats considerant una gran varietat d'escenaris. A més, s'han comparat diferents alternatives per a la inicialització del model, ressaltant la importància del punt inicial en l'algorisme d'optimització.

S'ha validat el model a través de la seva aplicació sobre dos casos d'estudi: una xarxa de gas natural amb pèrdua d'eficiència física i una xarxa de distribució d'electricitat amb pèrdua d'eficiència econòmica. Cada cas d'estudi ha de ser resolt amb un procediment adaptat a les seves característiques.

Els resultats mostren que passar per alt l'efecte de l'envelliment dels equips pot portar a dissenys inviables (quan existeix pèrdua d'eficiència física) o estimacions poc realistes (quan existeix pèrdua d'eficiència econòmica) a la pràctica. El model proposat permet superar aquestes limitacions, convertint-se així en una eina pràctica per recolzar el procés de presa de decisions en el sector de la distribució.

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1. Introduction

In this chapter, the motivation for the master's thesis is exposed and the goals to achieve are stated. In addition, it also includes the basic concepts of the field required to meet the objectives.

1.1. Motivation

The Committee on Challenges for the Chemical Sciences in the 21st Century, made up of 17 experts in different fields of chemistry and chemical engineering, gathered in 2003 (see References) the goals and challenges that the chemical sector will face in this century. They present computer technology advances as an opportunity to develop novel mathematical models and methods for simulation and optimization of the chemical supply chain.

Their understanding of the concept of chemical supply chain is broader than the usual one. It includes an enormous span of scales of space and time, from atoms and molecules to industrial-scale processes, as shown in Fig. 1.

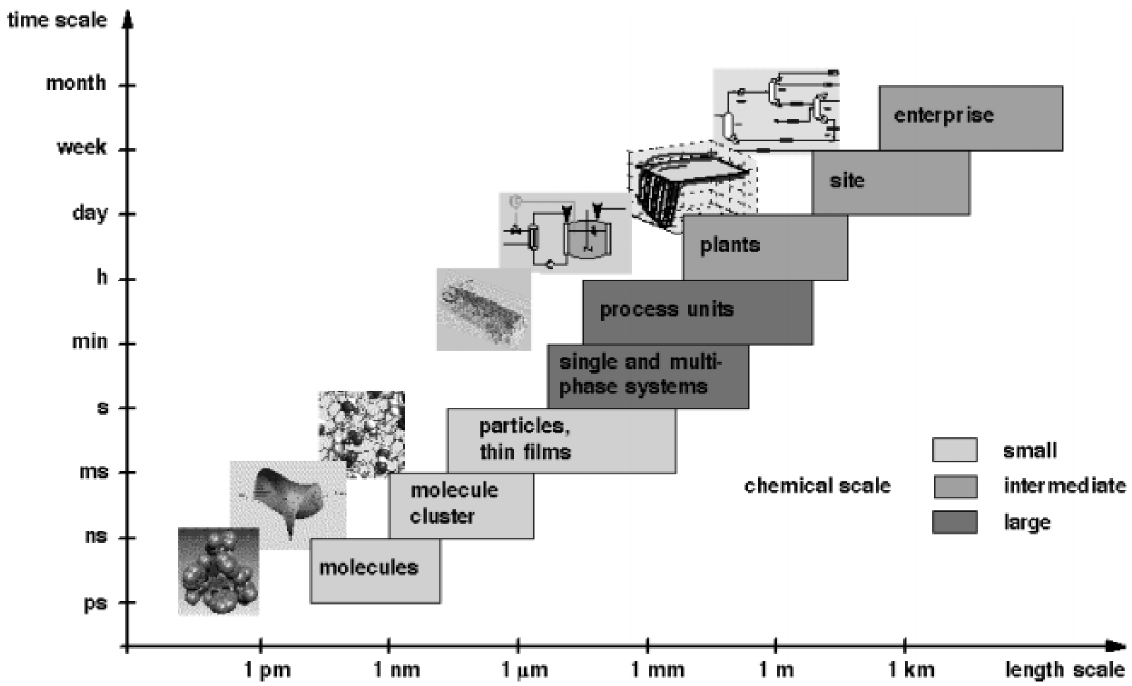

Fig. 1. The chemical supply chain by Professor Wolfgang Marquardt (Committee on Challenges for the Chemical Sciences in the 21st Century, 2013).

According to the Committee, challenges are to gain a better understanding of the structure and information flows underlying the chemical supply chain and to develop quantitative models that can be used to better coordinate and optimize the chemical enterprise. To tackle these problems successfully, new concepts will be required for developing systematic modeling techniques that can describe parts of the chemical supply chain at different levels of abstraction.

To sum up, the main goal is to increase profitability and efficiency in the chemical industry through supply chain management, and technology advances will play a central role to achieve it. This Master's Thesis will focus on developing a model that represents more precisely this efficiency and controls the effects of the loss of efficiency in supply chain planning.

Process efficiency can be compromised by events of very different origin. Fouling, the unwanted accumulation of materials on a surface, can cause a wide range of malfunctions in a process, from flow obstruction to reduced heat transfer. Some sensitive equipment suffer from efficiency reductions with age, due to loss of precision or motion.

All those behaviors fall under the concept of decaying performance (DP), which generally considers the gradual loss of efficiency of a process. The most common types of loss of performance are:

- *Physical decaying performance* (Fig. 2.a): A physical property of the system decreases as a function of operating time (e.g. the capacity of a compressor station).
- *Economic decaying performance* (Fig. 2.b): A cost increases owing to the age of an equipment (e.g. maintenance cost).

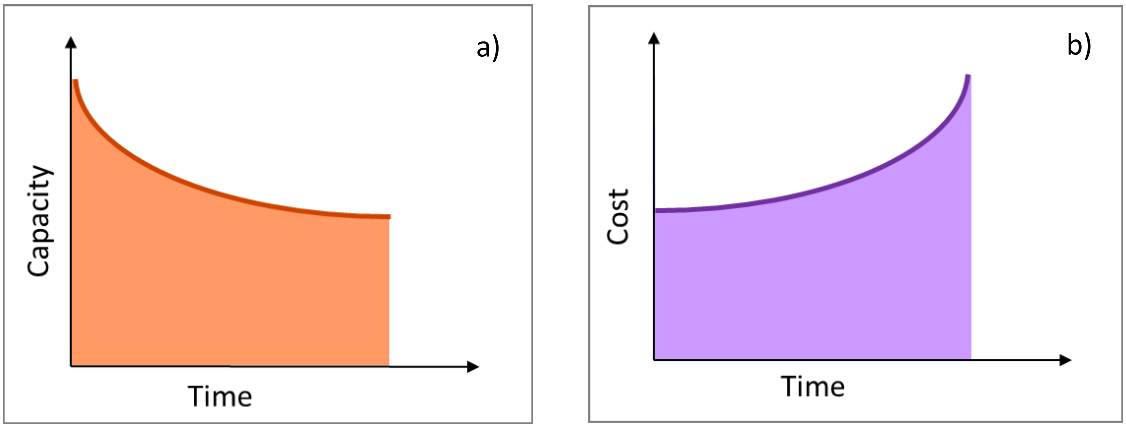

Fig. 2. Decaying performance. a) Physical and b) Economic.

Models addressing the optimization of networks from the viewpoint of distribution companies should take into account the effect that equipment aging may cause not only on the performance of the system but also on the investment planning regarding capacity expansions and maintenance. Overlooking the decaying performance of the equipment can lead to infeasible solutions involving networks which are incapable of satisfying the demand in practice and which significantly underestimate the real cost of a network with the capacity of satisfying such needs.

This work addresses this gap and the decision-making problems faced by companies managing distribution facilities within global or local supply chains subject to decaying performance.

1.2. Process Systems Engineering

Process Systems Engineering is the branch of engineering that deals with design, operation control and optimization of chemical, physical and biological processes through computer-aided methods and tools.

It includes a vast range of applications. Some of them are:

- Forecasting
- Process simulation and optimization
- Decision analysis
- Project management
- Supply chain management
- Uncertainty management

This work will focus on supply chain management and optimization.

1.2.1. Supply chain management

The terms *Supply Chain* and *Supply Chain Management* are attributed to the British logistician and consultant Keith Oliver, who coined them in 1982 as the result of the merging of the concepts of transportation, distribution and materials management that was taking place (Blanchard, 2010).

A supply chain can be defined as a network of interdependent entities (i.e., retailers, distributors, transporters, storage facilities, and suppliers) constituting the processing and distribution channels of a product from the sourcing of its raw materials to delivery to the final consumer as illustrated in Fig. 3 (Laínez-Aguirre and Puigjaner, 2015).

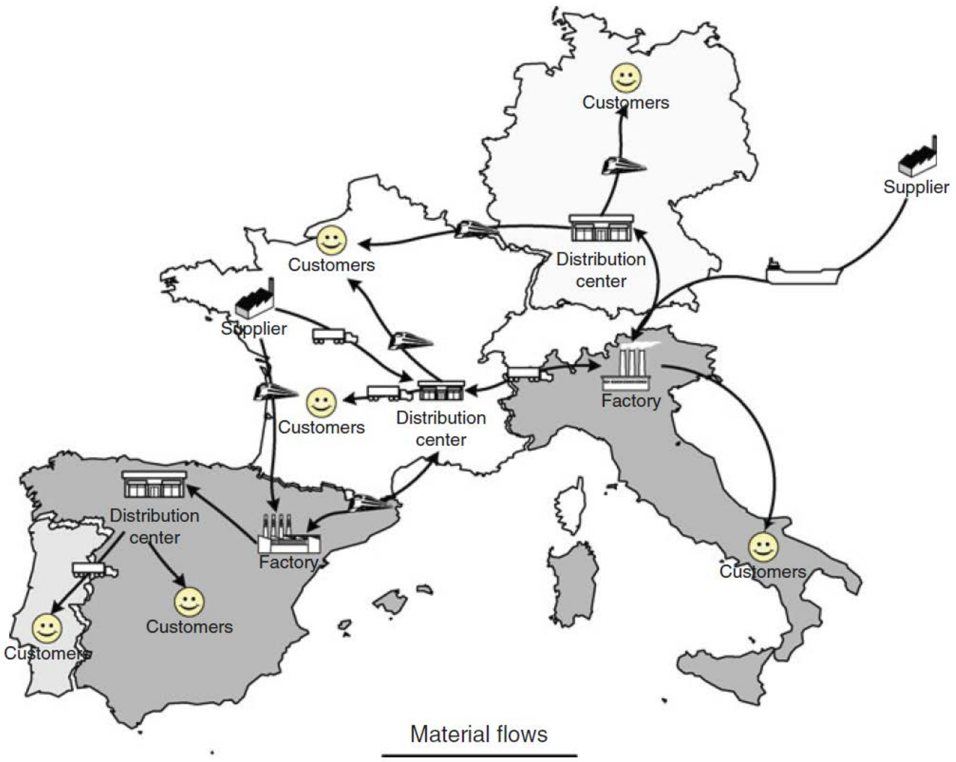

Fig. 3. Scheme of a supply chain network. (Laínez-Aguirre and Puigjaner, 2015)

The concept of supply chain management refers to the management of material, information and financial flows through a network that aims to produce and deliver products or services for the consumers (Tang, 2006).

1.3. Optimization

Mathematical optimization, or mathematical programming, is one of the best developed and most used tools in Process Systems Engineering. As defined by Edgar, Himmelblau and Lasdon in 2001, optimization is the use of specific methods to determine the most cost-effective and efficient solution to a problem or design for a process. It concerns the optimum allocation of limited resources among competing activities, under a set of constraints imposed by the nature of the problem being studied. These constraints could reflect financial, technological, marketing, organizational, or many other considerations. In broad terms, mathematical programming can be defined as a mathematical representation aimed at programming or planning the best possible allocation of scarce resources. (Bradley, Hax and Magnanti, 1977)

Benefits of optimization include reducing cost and increasing profit, sensible use of resources, reducing environmental pressure, etc. However, optimizing a known system does not mean total control on its behavior. Uncertainty in the data used, inaccuracy in estimations or unexpected facts can significantly affect real operation. Therefore, it is the engineer that is responsible of the final decision making process, discerning sensible actions from mathematical solutions.

Optimization problems can be divided in two main types:

- *Linear problems*

The term linear programming (LP) was coined by George Dantzig in 1947 to define problems that use exclusively linear functions (i.e. problems with objective function and constraints that are both linear) (Dantzig, 1998). Linear equations lead to a convex feasible region, which ensure the finding of the optimal solution, as the solution always occurs at the vertex of the feasible region (Edgar, Himmelblau and Lasdon, 2001). This results in problems that can be quickly and reliably solved with the available solvers.

One of the most well-known solvers for linear programs is the Simplex method, developed by Dantzig also in 1947. Its efficiency and robustness make it one of the most used algorithms in optimization (Bradley, Hax and Magnanti, 1977).

One of the most well-known algorithms for linear programs is the Simplex method, developed by Dantzig also in 1947. Its efficiency and robustness make it one of the most used algorithms in optimization (Bradley, Hax and Magnanti, 1977). A more efficient algorithm today is the one developed by Karmarkar in 1984, which is implemented in most of the commercial solvers.

A particular type of linear program is a mixed integer linear program (MILP), in which one or more of the decision variables are integers, commonly binary. Its computational difficulty is usually determined by the number of integer variables of the problem. Almost all LP solvers can solve MILPs, which increases its usefulness. (Edgar, Himmelblau and Lasdon, 2001)

- *Non-linear problems*

On the opposite, non-linear programming is the process of solving problems that include non-linear equations, either in the objective function or the constraints. The more complex and numerous non-linearities are, the more difficult becomes finding a solution. Usually, non-linear problems are time-consuming when being solved, even with powerful computers.

One of the most used method to solve non-linear programs is the Generalized Reduced Gradient algorithm, which was developed by Jean Abadie (1969).

As happens with linear programming, problems with integer decision variables are denominated mixed integer non-linear programs (MINLPs) and are computationally costly to be solved.

Mixed integer problems can be solved with the branch and bound algorithm (B&B), proposed by Land and Doig in 1960. It is represented as a branched tree, where branches that produce solutions far from the optimum are discarded.

1.4. State of the art

Nationwide chemical supply chain networks have been extensively studied (Elia et al., 2012; Elia and Floudas, 2014) in recent times. Much attention has been paid to sources and sinks (production and consumption), but less to distribution and associated needs and investments. Even though overlooking the effect that equipment aging may cause on the performance of the system can lead to infeasible solutions, the literature on works considering the decaying performance of processes is rather scarce. Some authors have considered this issue in the context of industrial scheduling (Jain and Grossmann, 1998; Smaïli et al., 1999; Pogiatzis et al., 2012), maintenance planning (Xenos et al., 2016) or process planning (Pan et al., 2016) but no study has been found in the context of supply and distribution networks.

In the case of energy systems, great effort has been devoted to the study of transmission networks (Villasana, Garver and Salon, 1985), (Alguacil, Motto and Conejo, 2003) and distributed energy systems (Ren and Gao, 2010). Previous research addresses the issues of generation and consumption (sources and sinks nodes). Equipment aging can significantly affect the performance of a distribution network. Moreover, further legal, administrative, physical or financial issues could also alter the investment performance along its life span. Thus, a decreasing performance should therefore be considered during the investment planning. Otherwise, the designed network may be actually unrealistic and may significantly underestimate the costs of a network that has to face equipment aging. Despite this, to the best of our knowledge, there are no previous studies which consider the depreciation of energy distribution networks.

1.5. Objectives

The aim of this thesis is to provide a general tool for the design and long-term planning of supply chains and distribution networks that considers their decaying performance as a function of operating time.

The objectives to achieve this goal include:

- To clearly state the problem to be solved.
- To formulate the mathematical model for the planning of a supply chain under decaying performance. This requires the definition of parameters, variables, constraints and objective function.
- To define case studies of different nature to validate the model under different circumstances. Case studies will address both physical decaying performance and economic decaying performance.
- To analyze the mathematical nature of the problem and the efficiency of the solution procedures and strategies.
- To design the best procedure to obtain solutions depending on the case study.
- To analyze and discuss the results obtained.

2. Problem statement

In this section, the details of the problem statement are defined.

A 3 echelon distribution network as in Fig. 4 is considered. It consists of a set of suppliers *p* whose maximum capacity cannot be exceeded, a set of substations or potential locations for them *s*, and a set of consumers *c* whose demand must be satisfied. Echelons in the network are connected through a series of piping (i.e., lines p-s and s-c).

Substations, lines p-s and lines s-c will be henceforth generally referred to as nodes. Each capacity expansion in a node is called a facility. This allows the separate treatment of every expansion, so that the age of each installation is recorded individually.

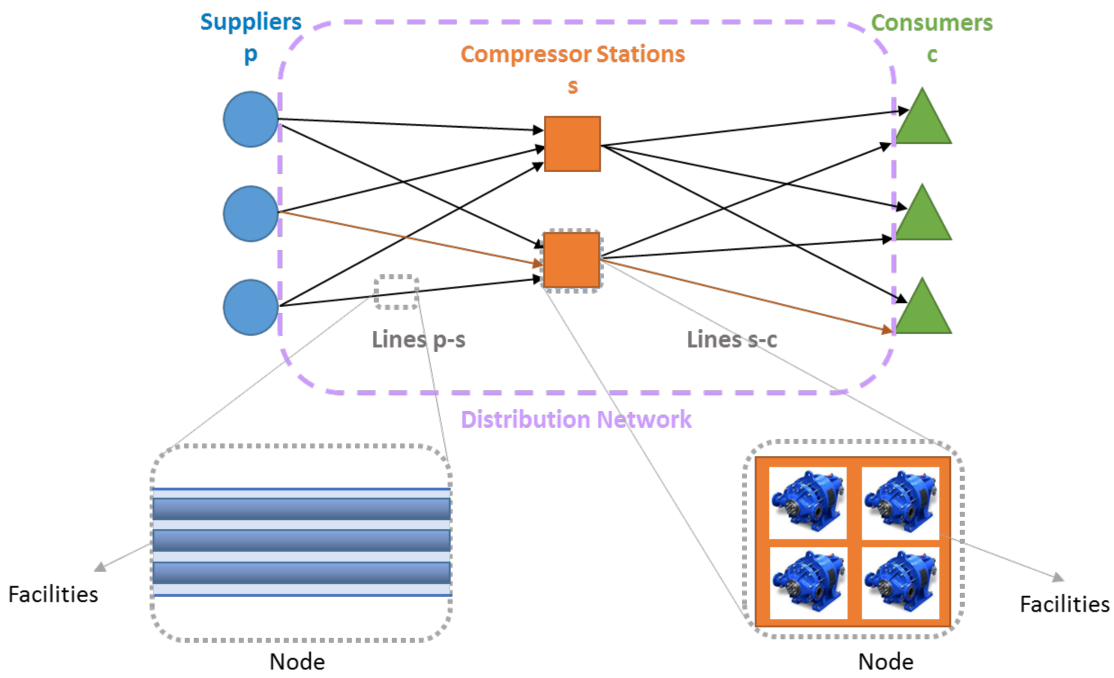

Fig. 4. Scheme of the distribution network.

Given data is:

- The existing configuration of the network.
- The potential location for new substations.
- Distances between the nodes of the network.
- The time horizon, which is discretized in years.
- Parameters describing the decaying performance.
- Complete technical and economic data.

Hence, the optimal natural gas distribution network (piping and substations) taking into account the evolution of the equipment performance over time has to be determined.

3. Methods

This section includes the optimization software selection and the definition of the procedure used to obtain results.

3.1. Optimization software selection

To select the most appropriate software to address the problem, different software available in UPC or the research group CEPIMA are compared.

In optimization software, one must distinguish between solvers and modeling tools. The first ones are algorithms that solve the optimization problem. Their inputs and outputs are matrices and their direct use requires a lot of effort, if no secondary software is used. For this purpose, modeling tools address the connection between users and solvers, often through a modeling language, that defines the problem structure.

To solve easy problems, with only one or two constraints, some generally used software are Microsoft Office Excel or Matlab. Excel has Excel Solver, a tool that minimizes or maximizes an objective function subject to some constraints. While equations are defined in worksheet cells, equalities and inequalities have to be defined in the Solver's interface. This makes it easy to use, but is not suitable for large optimization problems. Matlab has an optimization toolbox with a similar use.

On the other hand, modeling tools with a modeling language are suitable to large optimization problems. GAMS (General Algebraic Modeling System) was one of the firsts to appear and is still the most used one. The language is extended both in academic and industry uses and has a wide range of resources available. AIMMS (Advanced Integrated Multidimensional Modeling Software) is newer and has some useful extra features, such as a graphical application to build the model. However, it is not as extended as GAMS, what means that less resources are available when looking for support.

Table 1 shows a brief summary of the pros and cons of the aforementioned software.

Table 1. Software comparison.

Software	Advantages	Disadvantages
Excel Solver	Easy to use	Has no modeling language Only for small problems
Matlab		Has no modeling language Only for small problems
GAMS	Has a modeling language Widely used in industry One of the firsts modeling tools (has a community of expert users) Simple setup	Has no user-friendly interface
AIMMS	Has a modeling language User-friendly interface GAMS compatibility mode	Not very extended use (only support from an online forum)

Finally, GAMS is selected as the most suitable optimization software. It allows the solution of large optimization problems and offers more information resources. In addition, thanks to the compatibility mode implemented in AIMMS, GAMS problems may easily be translated to AIMMS language.

3.2. GAMS

As stated in its website (www.gams.com) the General Algebraic Modeling System (GAMS) is "a high-level modeling system for mathematical programming and optimization". It consists of a language compiler and a set of integrated high-performance solvers. GAMS is specifically designed for complex, large scale optimizations (e.g. linear, nonlinear and mixed integer optimization problems).

A brief summary of the main solvers commonly used for the different kind of problems is shown in Table 2.

Table 2. Solvers in GAMS.

LP / MILP	NLP	MINLP	Global
CPLEX	CONOPT	DICOPT	BARON
MOSEK	MINOS	SBB	GLOMIQO

3.3. Procedure to obtain results

Fig. 5 represents the steps that have to be followed in order to optimize and get the results. Blue steps are thought on paper, green steps are implemented in Microsoft Office Excel and orange steps in GAMS.

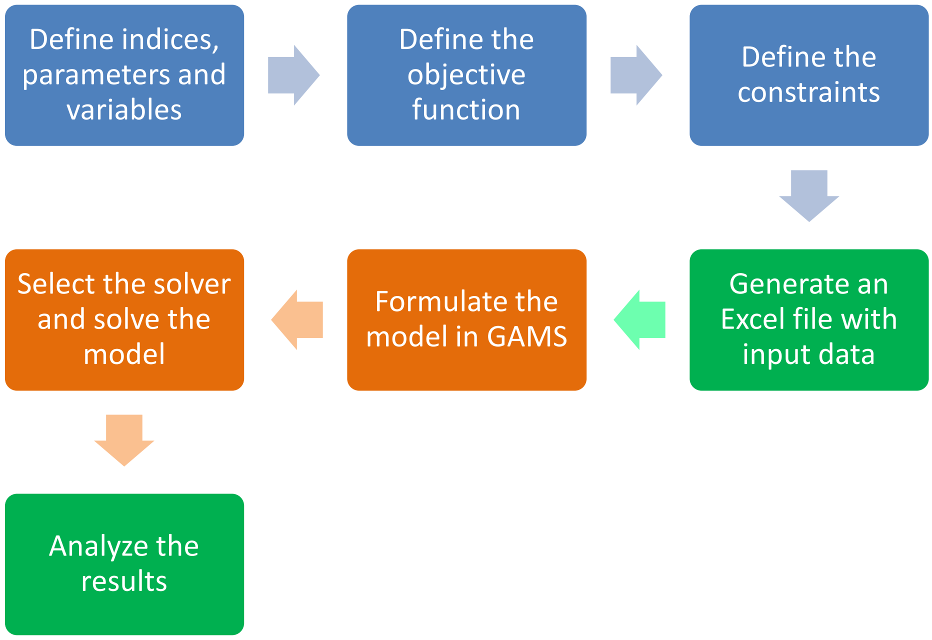

Fig. 5. Flowchart for the solution procedure.

4. Mathematical formulation

The problem is formulated as a MINLP in which continuous variables model capacities and binary variables the structural decisions for the network and its parts (i.e., facilities). It has been based on a typical chemical supply chain (Guillén-Gosálbez and Grossmann, 2009).

4.1. Sets / Indices

Four sets will be defined in GAMS as Table 3: one for each node of the network and one for the time period.

Table 3. Sets definition.

Set	Description
p =	Producer / Source
s =	Substation
c =	Customer
n =	Time period

Those sets correspond to the indices of the model and will appear as subscripts in parameters and variables.

4.2. Parameters

Parameters are the known data required to solve the problem. In this case, they include given data, such as the existing network characteristics, and estimated trends, such as factors to build the function for to represent the decaying performance.

Parameters for balances constraints are defines in Table 4, existing network parameters are shown in Table 5, decaying performance coefficients in Table 6 and cost coefficients in Table 7.

Table 4. Parameters for balances constraints.

Parameter	Definition
D_{cn}=	Demand at customer *c* in time period *n* [m^3/h or kVA]
L_{ps}^{in}=	Distance in line between source *p* and substation *s* [km]
L_{sc}^{out}=	Distance in line between substation *s* and customer *c* [km]
$\overline{PPUB}_{pn}$=	Upper limit on the supply for source *p* in time period *n* [m^3/h or kVA]
$\overline{PUB}^{sub}$=	Upper bound on the capacity at a facility in a substation [m^3/h or kVA]
$\overline{PUB}^{in}$=	Upper bound on the capacity at a facility in a line source-substation [m^3/h or kVA]
$\overline{PUB}^{out}$=	Upper bound on the capacity at a facility in a line substation-customer [m^3/h or kVA]
$\underline{PLB}^{sub}$=	Lower bound on the capacity at a facility in a substation [m^3/h or kVA]
$\underline{PLB}^{in}$=	Lower bound on the capacity at a facility in a line source- substation [m^3/h or kVA]
$\underline{PLB}^{out}$=	Lower bound on the capacity at a facility in a line substation-customer [m^3/h or kVA]

Table 5. Parameters for the existing network

Parameter	Description
$P0_{s}^{sub}$=	Initial capacity at substation *s* [m^3/h or kVA]
$P0_{ps}^{in}$=	Initial capacity at line between source *p* and substation *s* [m^3/h or kVA]
$P0_{sc}^{out}$=	Initial capacity at line between source *p* and substation *s* [m^3/h or kVA]
$S0_{s}^{pon}$=	1 if the substation *s* exists, 0 otherwise
$LPS0_{ps}^{pon}$=	1 if line between source *p* and substation *s* exists, 0 otherwise
$LSC0_{sc}^{pon}$=	1 if line between substation *s* and customer *c* exists, 0 otherwise

$Ain0_{s}^{sub}$=	Initial age of substation *s* [year]
$Ain0_{ps}^{in}$=	Initial age of line between source *p* and substation *s* [year]
$Ain0_{sc}^{out}$=	Initial age of line between substation *s* and customer *c* [year]

Table 6. Decaying performance coefficients.

Parameter	**Description**
C^{sub}=	Multiplier for the exponential equation for substations
k^{sub}=	Exponential term for the decaying performance equation for substations
$PR^{sub\infty}$=	Independent term for the exponential equation for substations
C^{in}=	Multiplier for the exponential equation for lines source-substation
k^{in}=	Exponential term for the decaying performance equation for lines source-substation
$PR^{in\infty}$=	Independent term for the exponential equation for lines source-substation
C^{out}=	Multiplier for the exponential equation for substation-consumer
k^{out}=	Exponential term for the decaying performance equation for substation-consumer
$PR^{out\infty}$=	Independent term for the exponential equation for lines substation-consumer

Table 7. Cost factors.

Parameter	**Description**
α_{n}^{in}=	Variable investment term for facility installed in a line source-substation in period *n* [€/(km·(m^3/h or kVA))]
α_{n}^{out}=	Variable investment term for facility installed in a line substation-customer installed in period *n* [€/(km·(m^3/h or kVA))]
β_{n}^{in}=	Fixed investment term for facility installed in a line source-substation in period *n* [€/km]

β_n^{out}=	Fixed investment term for facility installed in a line substation-customer in period *n* [€/km]
γ_n^{in}=	Fixed investment term for a line source-substation start-up in period *n* [€/km]
γ_n^{out}=	Fixed investment term for line substation-customer start-up in period *n* [€/km]
δ_n=	Variable investment term for a facility installed in a substation in period *n* [€/(m^3/h or kVA)]
ε_n=	Fixed investment term for a facility installed in a substation in period *n* [€]
ζ_n=	Fixed investment term for a substation start-up in period *n* [€]
η_n^{in}=	Fixed term for a facility in line source-substation dismantled in period *n* [€]
η_n^{out}=	Fixed term for a facility in line substation–customer dismantled in period *n* [€]
θ_n=	Fixed term for a facility in substation dismantled in period *n* [€]
ι_n^{in} =	Factor for maintenance cost in a substation
ι_n^{out} =	Factor for maintenance cost in a line source-substation
κ=	Factor for maintenance cost in a line substation-customer

4.3. Variables

The variables of the problem include both decision variables and the other elements that can take different values in the model, which in GAMS must be defined as variables.

Variables are divided into tables: Table 8 for continuous variables for capacities, Table 9 for continuous variables for capacities and Table 10 for age and decaying performance variables.

Table 8. Continuous variables for capacities

Variable	Description
P^{sub}_{sn}=	Capacity at substation *s* in time period *n* [m^3/h or kVA]
PA^{sub}_{sn}=	Expansion at substation *s* in time period *n* [m^3/h or kVA]
$PC^{sub}_{snn'}$=	Expansion at substation *s* in period n dismantled in period *n'* [m^3/h or kVA]
P^{in}_{psn}=	Capacity at line between source *p* and substation *s* in period *n* [m^3/h or kVA]
PA^{in}_{psn}=	Expansion at line between *p* and *s* in time period *n* [m^3/h or kVA]
$PC^{in}_{psnn'}$=	Expansion at line between *p* and *s* in period *n* dismantled in period *n'* [m^3/h or kVA]
P^{out}_{scn}=	Capacity at line between substation *s* and customer *c* in period *n* [m^3/h or kVA]
PA^{out}_{scn}=	Expansion at line between substation *s* and customer *c* in period *n* [m^3/h or kVA]
$PC^{out}_{scnn'}$=	Expansion at line between *s* and *c* in period *n* dismantled in period *n'* [m^3/h or kVA]
$PC0^{sub}_{sn}$=	Existing transformation center *s* that is dismantled in time period *n* [m^3/h or kVA]
$PC0^{in}_{psn}$=	Existing line between *p* and *s* that is dismantled in time period *n* [m^3/h or kVA]
$PC0^{out}_{scn}$=	Existing line between *s* and *c* that is dismantled in time period *n* [m^3/h or kVA]

Table 9. Binary variables for set-up and dismantling.

Variable	Description
S^{pon}_{sn}=	1 if transformation center *s* is set up in time period *n*, 0 otherwise
$S^{poff}_{snn'}$=	1 if transformation center *s* is set up in period *n* that is dismantled in period *n'*, 0 otherwise
LPS^{pon}_{psn}=	1 if line between source *p* and transformation center *s* is set up in period *n*, 0 otherwise

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$LPS_{psnn'}^{poff}$=	1 if line between *p* and *s* set up in time period *n* is dismantled in period *n'*, 0 otherwise
LSC_{scn}^{pon}=	1 if line between transformation center *s* and consumer *c* is set up in time period *n*, 0 otherwise
$LSC_{scnn'}^{poff}$=	1 if line between *s* and *c* set up in time period *n* is dismantled in period *n'*, 0 otherwise
X_{sn}^{pon}=	1 if facility in transformation center *s* is set up in time period *n*, 0 otherwise
$X_{snn'}^{poff}$=	1 if facility in transformation center *s* set up in time period *n* is dismantled in period *n'*, 0 otherwise
Y_{psn}^{pon}=	1 if facility in line between *p* and *s* is set up in time period *n*, 0 otherwise
$Y_{psnn'}^{poff}$=	1 if facility in line between *p* and *s* set up in time period *n* is dismantled in period *n'*, 0 otherwise
Z_{scn}^{pon}=	1 if facility in line between *s* and *c* is set up in time period *n*, 0 otherwise
$Z_{scnn'}^{poff}$=	1 if facility n line between *s* and *c* set up in time period *n* is dismantled in period *n'*, 0 otherwise
$X0_{sn}^{poff}$=	1 if existing facility in transformation center *s* is dismantled in time period *n*, 0 otherwise
$Y0_{psn}^{poff}$=	1 if existing facility in line between *p* and *s* is dismantled in *n*, 0 otherwise
$Z0_{scn}^{poff}$=	1 if existing facility in line between *s* and *c* is dismantled in *n*, 0 otherwise

Table 10. Variables for age and decaying performance.

Variable	Description
$A_{snn'}^{sub}$=	Age in period *n'* at substation *s* set up in period *n* [year]
$A_{psnn'}^{in}$=	Age in period *n'* at line between *p* and *s* set up in period *n* [year]
$A_{scnn'}^{out}$=	Age in period *n'* at line between *s* and *c* set up in period *n* [year]
$A0_{sn}^{sub}$=	Age in period *n* at existing substation *s* [year]

$\boldsymbol{A0_{psn}^{in}}$=	Age in period *n* at existing line between *p* and *s* [year]
$\boldsymbol{A0_{scn}^{out}}$=	Age in period *n* at existing line between *s* and *c* [year]
$\boldsymbol{PR_{snn'}^{sub}}$=	Efficiency in period *n'* at substation *s* set up in period *n*
$\boldsymbol{PR_{psnn'}^{in}}$=	Efficiency in period *n'* at line between *p* and *s* set up in period *n*
$\boldsymbol{PR_{scnn'}^{out}}$=	Efficiency in period *n'* at line between *s* and *c* set up in period *n*
$\boldsymbol{PR0_{sn}^{sub}}$=	Efficiency in period *n* at existing *s*
$\boldsymbol{PR0_{psn}^{in}}$=	Efficiency in period *n* at existing line between *p* and *s*
$\boldsymbol{PR0_{scn}^{out}}$=	Efficiency in period *n* at existing line between *s* and *c*

4.4. Objective function

The objective function is the total cost including fixed and variable cost for facilities and stations ($\alpha_n^{in}, \alpha_n^{out}, \beta_n^{in}, \beta_n^{out}, \gamma_n^{in}, \gamma_n^{out}, \delta_n, \varepsilon_n, \zeta_n$), their maintenance ($\iota_n^{in}, \iota_n^{out}, \kappa_n$) and the cost related to dismantling facilities ($\eta_n^{in}, \eta_n^{out}, \theta_n$). It can be calculated as in Eq. (1) has to be minimized in order to optimize the problem.

$$
\begin{aligned}
TCost = {} & \textstyle\sum_n (\sum_p \sum_s (PA_{psn}^{in} \cdot L_{ps}^{in} \cdot \alpha_n^{in}) + \sum_s \sum_c (PA_{scn}^{out} \cdot L_{sc}^{out} \cdot \alpha_n^{out}) \\
& + \textstyle\sum_p \sum_s (Y_{psn}^{pon} \cdot L_{ps}^{in} \cdot \beta_n^{in}) + \sum_s \sum_c (Z_{scn}^{pon} \cdot L_{sc}^{out} \cdot \beta_n^{out}) \\
& + \textstyle\sum_p \sum_s (LPS_{psn}^{pon} \cdot \gamma_n^{in}) + \sum_s \sum_c (LSC_{scn}^{pon} \cdot \gamma_n^{out}) + \sum_s \left(PA_{sn}^{sub} \cdot \delta_n\right) \\
& + \textstyle\sum_s (X_{sn}^{pon} \cdot \varepsilon_n) + \sum_s (S_{sn}^{pon} \cdot \zeta_n) + \sum_s \sum_c ((Z0_{scn}^{poff} + \sum_{n'} Z_{scn'n}^{poff}) \cdot \eta_n^{out}) \\
& + \textstyle\sum_s ((X0_{sn}^{poff} + \sum_{n'} X_{sn'n}^{poff}) \cdot \theta_n)) + MC_n^{in} + MC_n^{out} + MC_n^{sub})
\end{aligned}
\tag{1}
$$

Equations (2-4) detail the calculations for maintenance costs, which consider the installation cost multiplied by a factor.

$$
\begin{aligned}
MC_n^{in} = {} & ((\textstyle\sum_p \sum_s \sum n' | n' \leq n((PA_{psn'}^{in} - \sum_{n''|n \leq n'' \leq n'} PC_{psn'n''}^{in}) \cdot L_{ps}^{in} \cdot \alpha_n^{in} \\
& + (Y_{psn'}^{pon} - \textstyle\sum_{n''|n \leq n'' \leq n'} Y_{psn'n''}^{poff}) \cdot L_{ps}^{in} \cdot \beta_n^{in})) \\
& + (\textstyle\sum_p \sum_s ((P0_{ps}^{in} - \sum_{n'|n' \leq n} PC0_{psn'}^{in}) \cdot L_{ps}^{in} \cdot \alpha_n^{in} \\
& + (LPS_{ps}^{pon} - \textstyle\sum_{n'|n' < n} Y0_{psn'}^{poff}) \cdot L_{ps}^{in} \cdot \beta_n^{in})) \cdot \iota_n^{in}
\end{aligned}
\tag{2}
$$

$$MC_n^{out} = ((\sum_s \sum_c \sum_{n'|n' \le n}((PA_{scn'}^{out} - \sum_{n''|n \le n'' \le n'} PC_{scn'n''}^{out}) \cdot L_{sc}^{out} \cdot \alpha_n^{out}$$
$$+(Z_{scn'}^{pon} - \sum_{n''|n \le n'' \le n'} Z_{scn'n''}^{poff}) \cdot L_{sc}^{out} \cdot \beta_n^{out}))$$
$$+(\sum_s \sum_c ((P0_{sc}^{out} - \sum_{n'|n' \le n} PC0_{scn'}^{out}) \cdot L_{sc}^{out} \cdot \alpha_n^{out}$$
$$+(LSC_{sc}^{pon} - \sum_{n'|n' \le n} Z0_{scn'}^{poff}) \cdot L_{sc}^{out} \cdot \beta_n^{out})) \cdot \iota_n^{out} \quad (3)$$

$$MC_n^{sub} = ((\sum_s \sum_{n'|n' \le n}((PA_{sn'}^{sub} - \sum_{n''|n \le n'' \le n'} PC_{sn'n''}^{sub}) \cdot \delta_n$$
$$+(X_{sn'}^{pon} - \sum_{n''|n \le n'' \le n'} X_{sn'n''}^{poff}) \cdot \zeta_n)) + (\sum_s ((P0_s^{sub} - \sum_{n'|n' \le n} PC0_{sn'}^{sub}) \cdot \delta_n$$
$$+(S_s^{pon} - \sum_{n'|n' \le n} X0_{sn'}^{poff}) \cdot \zeta_n)) \cdot \kappa_n \quad (4)$$

4.5. Equations and constraints

All the equations and constraints required to model the problem are gathered in this chapter.

4.5.1. Resource balances

Balances for capacity of resource (capacity of flow, power, etc.) must be satisfied in each substation (Eqs. 5, 6).

$$\sum_p P_{psn}^{in} \le P_{sn}^{sub} \quad \forall s, n \quad (5)$$

$$\sum_c P_{scn}^{out} \ge P_{sn}^{sub} \quad \forall s, n \quad (6)$$

Eq. 5 presents the balance between lines from sources to substations (represented in variable P_{psn}^{in}) and substations (P_{sn}^{sub}) while Eq. 6 represents the balance between substations and outlet lines, the ones from substations to customers (P_{scn}^{out}). Note that these equations are posed as inequalities rather than as equalities, as otherwise the model would unnecessarily be forced to equalize aging of facilities up and downstream of each station.

In addition, the demand of each customer *c*, D_{cn}, must be satisfied in each time period *n* (Eq. 7).

$$\sum_s P_{scn}^{out} \ge D_{cn} \quad \forall c, n \quad (7)$$

The maximum supply for each producer, $\overline{PPUB_{pn}}$, should not be exceeded, as shown in Eq. 8.

$$\sum_s P_{psn}^{in} \le \overline{PPUB_{pn}} \quad \forall p, n \quad (8)$$

4.5.2. Capacity constraints of substations

The capacity of compressor station *s* in a given time period *n* is calculated by accounting for the different capacity expansions and facility dismantling over the previous time periods, as well as their corresponding performance coefficient:

$$P_{sn}^{sub} = \left(P0_{s}^{sub} - \sum_{n' \leq n} PC0_{sn'}^{sub}\right) \cdot PR0_{sn}^{sub} + \sum_{n' \leq n}\left(\left(PA_{sn'}^{sub} - \sum_{n'' | n' \leq n'' \leq n} PC_{sn'n''}^{sub}\right) \cdot PR_{sn'n}^{sub}\right) \forall s, n \tag{9}$$

Here, $P0_{s}^{sub}$ is a parameter denoting the capacity of the original facilities in node *s* (if any); $PC0_{sn'}^{sub}$ is a continuous variable accounting for the initial capacity in location *s* and which is dismantled in time period *n'*; $PA_{sn'}^{sub}$ is a continuous variable denoting the capacity expansion performed in each of the previous time periods *n'* in location *s*; and $PC_{sn'n''}^{sub}$ is a continuous variable accounting for the capacity of that expansion that is dismantled in time period *n''*. The continuous variables $PR0_{sn}^{sub}$ and $PR_{sn'n}^{sub}$ (performance coefficients) are bounded between 1 and a lower value. Note that this formulation allows a different performance for each part of the installation (i.e., original vs expanded), thus providing an accurate representation of equipment aging.

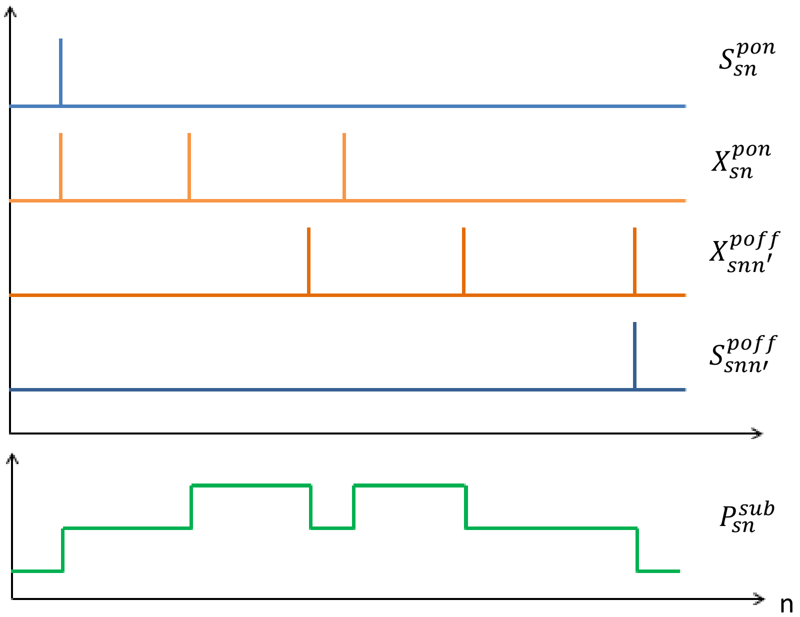

Fig. 6. Continuous and binary variables scheme.

Variables in section 4.3 must follow a structure resembling Fig. 6.

Eqs. (10-14) represent general constraints for capacities that relate continuous variables for capacities with their binary counterpart. X_{sn}^{pon} is a binary variable indicating whether the expansion of a facility is performed in node *s* in time period *n*, while $X0_{sn}^{poff}$ is a binary variable denoting whether the original facility of node *s* is dismantled in time period *n* and $X_{snn'}^{poff}$ is a binary variable that is 1 when the capacity expansion performed in time period *n* is dismantled in time period *n*'. PA_{sn}^{sub}, $P0_{s}^{sub}$, $PC_{snn'}^{sub}$ are the homologous continuous variables, respectively.

$$\underline{PLB^{sub}} \cdot \left(X_{sn}^{pon} - X_{snn'}^{poff}\right) \leq PA_{sn}^{sub} - PC_{snn'}^{sub} \leq \overline{PUB^{sub}} \cdot \left(X_{sn}^{pon} - X_{snn'}^{poff}\right) \quad \forall s,n,n' | \, n' \geq n \tag{10}$$

$$\underline{PLB^{sub}} \cdot X_{snn'}^{poff} \leq PC_{snn'}^{sub} \leq \overline{PUB^{sub}} \cdot X_{snn'}^{poff} \quad \forall s,n,n' | \; n' \geq n \tag{11}$$

$$X_{snn'}^{poff} \leq X_{sn}^{pon} \quad \forall s,n,n' | \; n' > n \tag{12}$$

$$P0_{s}^{sub} \cdot X0_{sn}^{poff} \leq PC0_{sn}^{sub} \leq P0_{s}^{sub} \cdot X0_{sn}^{poff} \quad \forall s,n \tag{13}$$

$$\sum_{n} X0_{sn}^{poff} \leq 1 \quad \forall s \tag{14}$$

S_{sn}^{pon} is a binary variable denoting the set-up of a substation and $S_{sn'n''}^{poff}$ represents its final dismantling. They are related with the binary variables associated with the facilities through Eqs. (15-18).

$$S_{sn}^{pon} \leq X_{sn}^{pon} \quad \forall s,n \tag{15}$$

$$S_{snn'}^{poff} \leq X_{snn'}^{poff} + X0_{sn'}^{poff} \quad \forall s,n,n' \tag{16}$$

$$S0_{s}^{pon} + \sum_{n' \leq n} S_{sn'}^{pon} - 1 \leq \sum_{n' \leq n} \sum_{n'' | n' \leq n'' \leq n} S_{sn'n''}^{poff} \leq S0_{s}^{pon} + \sum_{n' \leq n} S_{sn'}^{pon} \quad \forall s,n \tag{17}$$

$$\underline{PLB^{sub}} \cdot \left(S0_{s}^{pon} + \sum_{n' \leq n} \left(S_{sn'}^{pon} - \sum_{n'' | n' \leq n'' \leq n} S_{sn'n''}^{poff}\right)\right) \leq P_{sn}^{sub} \leq \overline{PUB^{sub}} \cdot \left(S0_{s}^{pon} + \sum_{n' \leq n} \left(S_{sn'}^{pon} - \sum_{n'' | n' \leq n'' \leq n} S_{sn'n''}^{poff}\right)\right) \quad \forall s,n \tag{18}$$

4.5.3. Physical decaying performance of substations

The physical performance of the facilities is assumed to follow an exponential decay as shown in Eqs. (19, 20). This exponential function is the main cause of the non-linearity of the problem. In these equations, $PR0_{sn}^{sub}$ indicates the performance in period *n* of existing installation in location *s*, whereas $PR_{snn'}^{sub}$ denotes the performance in period *n'* of the capacity expansion performed in period *n*.

$$PR_{snn'}^{sub} = C^{sub} \cdot \exp(-k^{sub} \cdot A_{snn'}^{sub}) + PR^{sub\infty} \qquad \forall s, n, n' \geq n \tag{19}$$

$$PR0_{sn}^{sub} = \left(C^{sub} \cdot \exp\left(-k^{sub} \cdot A0_{sn}^{sub}\right) + PR^{sub\infty}\right) + (1 - \left(C^{sub} \cdot \exp\left(-k^{sub} \cdot A0_{sn}^{sub}\right) + PR^{sub\infty}\right)) \qquad \forall s, n \tag{20}$$

These equations include performance parameters ($C^{sub}, k^{sub}, PR^{sub\infty}$) and the antiquity of each facility. In particular, $A_{snn'}^{sub}$ is a continuous variable denoting the antiquity in time period *n'* of the facility expansion performed in time period *n* in location *s* (which is calculated via Eq. (21)), whereas $A0_{sn}^{sub}$is a continuous variable denoting the antiquity in period *n* of the facility originally existing in location *s* (calculated as in Eq. (22)).

$$A_{snn'}^{sub} = X_{sn}^{pon}\left((n'-n) - \textstyle\sum_{n''|n\leq n''\leq n'}\left(X_{snn''}^{poff}(n'-n'')\right)\right) \quad \forall s, n, n' \geq n \tag{21}$$

$$A0_{sn}^{sub} = S0_{s}^{pon}\left(A0_{s}^{pon} + (n) - \textstyle\sum_{n'|n\leq n'}\left(X0_{sn'}^{poff}(n-n')\right)\right) \quad \forall s, n \tag{22}$$

Note that Eq. (22) requires the use of parameter $A0_{s}^{pon}$ denoting the antiquity at the start of the operation of the existing facility installed in location *s* and a corrective term. The aim of the later is to allow that the performance of the installation at time 0 is equal to 1 (i.e., the actual value) but decays at appropriate pace considering the real antiquity.

4.5.4. Economic decaying performance of substations

Economic decaying performance is applied by redefining parameters for maintenance cost ($\iota_n^{in}, \iota_n^{out}, \kappa_n$) as variables depending on equipment age or performance ($A_{snn'}^{sub}$ or $PR_{snn'}^{sub}$). A term depending on one of those variables is added to the already existing fixed factors that are multiplying the installation cost. Some examples are presented in Eqs. (23, 24).

$$\kappa_n^{dp} = (1 + (A0_{psn}^{in} - 1)/C)) \cdot \kappa_n \tag{23}$$

$$\kappa_n^{dp} = (2 - PR_{snn'}^{sub}) \cdot \kappa_n \tag{24}$$

Here, κ_n^{dp} is the redefined variable that will modify maintenance cost and C is a constant that models the slope of the decay.

4.5.5. Piping equations

Eqs. (9-24) are also applied to lines source-substation and substation-consumer. They are reported by the implicit equations (25-28).

$$h^{in}\begin{pmatrix} P_{psn}^{in}, PA_{psn}^{in}, PC_{psnn'}^{in}, PI_{psn}^{in}, LPS_{psn}^{pon}, \\ LPS_{psnn'}^{poff}, Y_{psn}^{pon} Y_{psnn'}^{poff}, Y0_{psn}^{poff}, PC0_{psn}^{in}, \\ A_{psnn'}^{in}, A0_{psn}^{in}, PR_{psnn'}^{in}, PR0_{psn}^{in} \end{pmatrix} = 0 \tag{25}$$

$$g^{in}\begin{pmatrix} P_{psn}^{in}, PA_{psn}^{in}, PC_{psnn'}^{in}, PI_{psn}^{in}, LPS_{psn}^{pon}, \\ LPS_{psnn'}^{poff}, Y_{psn}^{pon}, Y_{psnn'}^{poff}, Y0_{psn}^{poff}, PC0_{psn}^{in}, \\ A_{psnn'}^{in}, A0_{psn}^{in}, PR_{psnn'}^{in}, PR0_{psn}^{in} \end{pmatrix} \leq 0 \tag{26}$$

$$h^{out}\begin{pmatrix} P_{scn}^{out}, PA_{scn}^{out}, PC_{scnn'}^{out}, PI_{scn}^{out}, LSC_{scn}^{pon}, \\ LSC_{scnn'}^{poff}, Z_{scn}^{pon}, Z_{scnn'}^{poff}, Z0_{scn}^{poff}, PC0_{scn}^{out}, \\ A_{scnn'}^{out}, A0_{scn}^{out}, PR_{scnn'}^{out}, PR0_{scn}^{out} \end{pmatrix} = 0 \tag{27}$$

$$g^{out}\begin{pmatrix} P_{psn}^{in}, PA_{psn}^{in}, PC_{psnn'}^{in}, PI_{psn}^{in}, LPS_{psn}^{pon}, \\ LPS_{psnn'}^{poff}, Y_{psn}^{pon}, Y_{psnn'}^{poff}, Y0_{psn}^{poff}, PC0_{psn}^{in}, \\ A_{psnn'}^{in}, A0_{psn}^{in}, PR_{psnn'}^{in}, PR0_{psn}^{in} \end{pmatrix} \leq 0 \tag{28}$$

4.6. Model formulation

After defining all the required equations, the resulting MINLP model can be formally posed as follows:

$$\begin{aligned} &DPmodel \quad min\, TCost \\ &\quad s.t. Eqs. (1-28) \end{aligned} \tag{29}$$

The model is formulated in GAMS following the structure proposed by GAMS User's Guide (2015), which has been summarized in Table 11.

Table 11.Structure of GAMS formulation.

Inputs in GAMS	
Sets	Declaration Assignment of members
Parameters	Declaration

	Assignment of values
Variables	Declaration Assignment of type
Assignment of bounds (optional)	
Equations	Declaration Definition
Model and Solve statements	
Display statement (optional)	

Assigning bounds to the variables of the model is not compulsory in GAMS. However, it delimits the search space so that the algorithm only considers reasonable values, thus reducing the computational time required to solve the model. In this case, due to the dimensions of the problem, it is essential to define sensible bounds to the variables.

In addition, non-linear models require the definition of an initialization. This means that a feasible solution must be used as a starting point to drive the search algorithm towards the feasible region.

Due to the number of variables is considerable, solutions are not gathered from GAMS directly with the display statement. Instead, the flow of data is canalized through Excel as illustrated in Fig. 7.

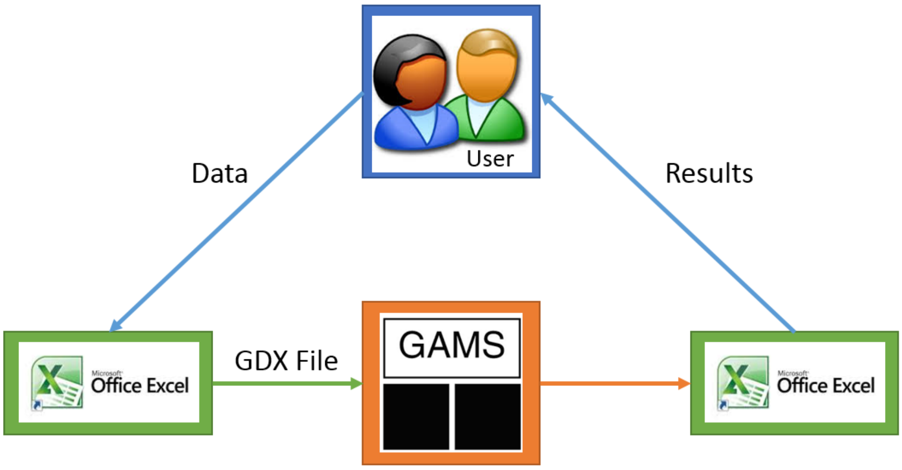

Fig. 7. Data flow between the user, Excel and GAMS.

The user introduces parameters in an Excel file. Then, the GAMS file is prepared with some instructions to read this data through a GDX. Some programming is added to the GAMS file, so that, after solving the problem, it writes the results in al Excel file. This connections make it easier for the user to introduce and gather data from GAMS.

5. Case studies

The capabilities of the proposed model are illustrated in two case studies. The physical decaying performance model is tested in a natural gas distribution network and the economical decaying performance approach is tried out in an electricity distribution network.

5.1. Natural gas distribution network

This case study consists of a simplified version of the Spanish natural gas distribution network. This includes 4 natural gas suppliers, 4 compressor stations (substations) and 7 consumers as presented in Fig. 8. In addition, 3 potential locations for new compressor stations are considered.

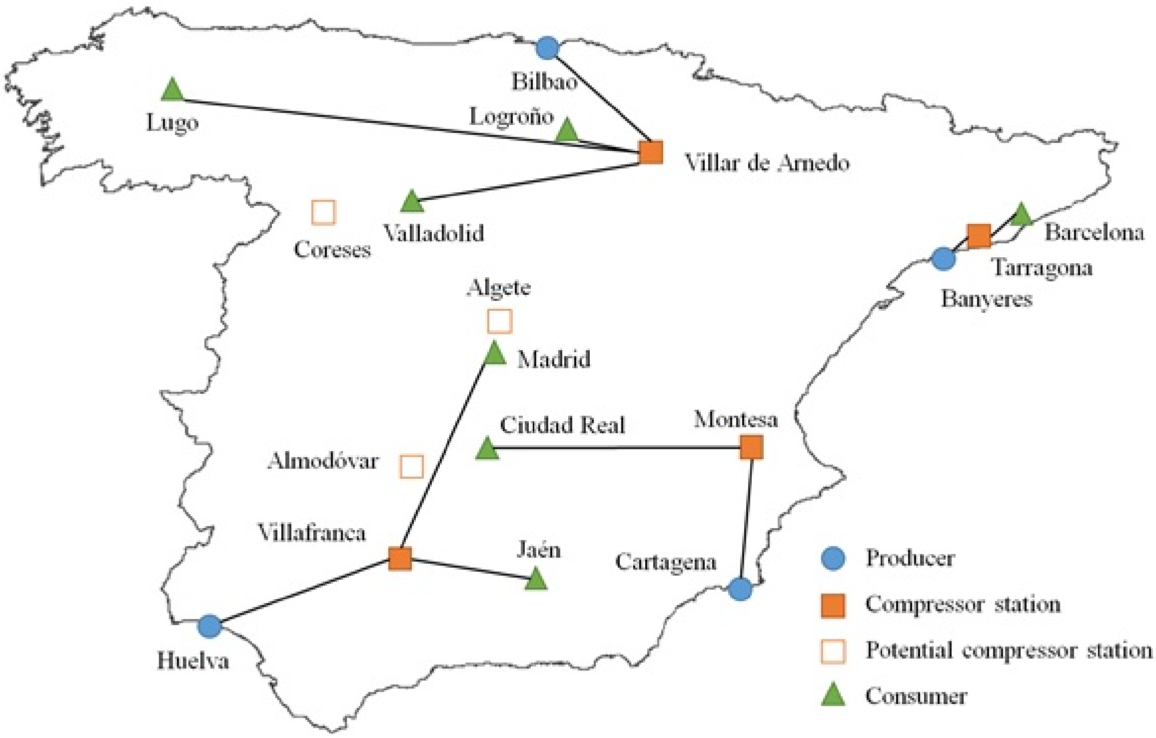

Fig. 8. Scheme of the natural gas distribution network.

The time horizon comprises 10 years with a 5% annual increase in the demand. The investment costs are drawn from official data (ACER, 2015). The decaying performance applied to the different facilities is estimated to tend asymptotically to a 70% of the initial value.

Values for parameters may be consulted in Annex.

5.2. Electricity distribution network

The second case study is based on a reduced real problem of a distribution company in Spain (Fig. 9). A transformation center is considered as the only electricity source that must supply electricity to 9 consumer nodes through 9 transformation centers (TC1 to TC9). No potential locations for transformation centers are available.

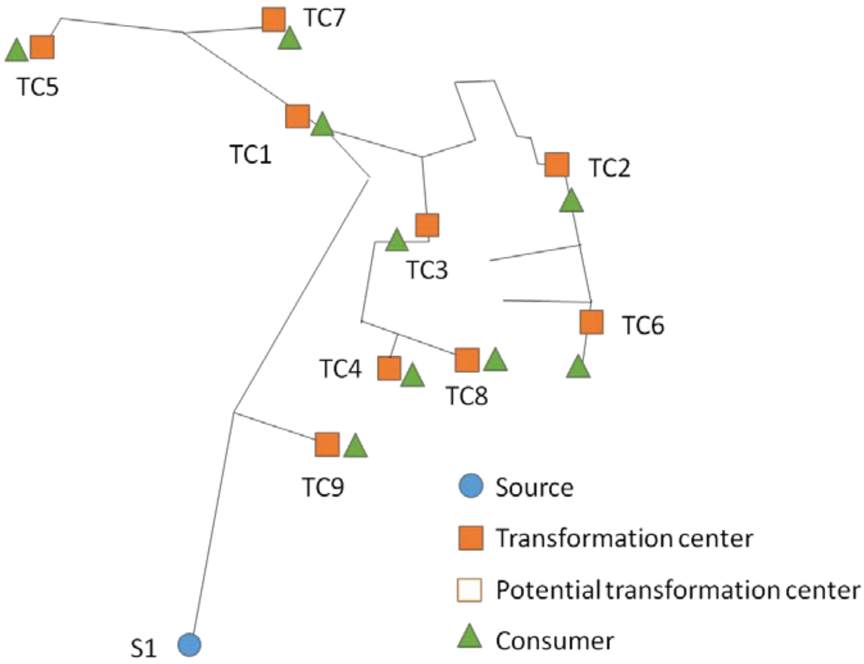

Fig. 9. Scheme of the natural gas distribution network.

In the calculation process, the apparent power has been used as the demand to be supplied to consumers. Its value has been derived from the contracted power and power factor associated to each consumption point.

As in case study 1, the time horizon analyzed comprises 10 years with a 5% annual increase in the demand.

The investment costs have been calculated based on the recommendations by the National Commission of Markets and Competence (2014).

Values for parameters may be consulted in Annex.

6. Results and discussion

In this chapter, results for the optimization of both case studies are analyzed.

The proposed models have been coded in GAMS 24.4.6 and solved with different solvers depending on the linearity of the model.

6.1. Natural gas distribution network

The procedure followed to obtain and analyze the results for this case study includes the comparison of different linear and non-linear optimization models.

First, a MILP model without considering decaying performance (source of non-linearity) is solved with CPLEX 12.6.2.0. The result can be rapidly obtained and the linear program ensures achieving the optimal solution. However, not considering decaying performance may lead to infeasible solutions.

Then, a simulator, programmed in GAMS but without being solved (as it is not an optimization), applies a decaying performance to the solution obtained with the linear program in step 1. This allows validating the feasibility of the linear solution under a real operation with decaying performance and testing the sensitivity of the model to the decaying performance.

Finally, the MINLP model considering decaying performance is solved with DICOPT. It is a time-consuming solution but robust and feasible in terms of considering the decaying performance in planning.

6.1.1. Initialization

The starting point given to the solver algorithm is a key point to find the solution of the optimization, especially in non-linear problems, which are more difficult to solve. The initialization guides the solver to a feasible solution.

However, the best way to choose a starting point is not fixed. It consists on making an estimation of the solution, which is not easy predictable. To analyze the sensitivity of the solution to the starting point, three extremely different strategies are tested:

- *Approach 1*: Every year one may build the necessary expansions to satisfy first year's demand. The expansions are performed in existing facilities and are proportional to their initial capacity.

- *Approach 2*: In year 1, substation *Banyeres* is expanded until its maximum capacity. Then, in year six, those expansions are closed and reopened in *Villafranca*.

- *Approach 3*: No starting point is given to the model.

This test is performed on the MILP because it reaches a feasible solution easier. The selection of the best initialization method is the comparison of the time to reach a value of the optimality gap (value that indicates how far the algorithm's solution is from the optimal result) under a 35%.

The time spent on each one are summarized in Table 12.

Table 12. Time spent in solving the MILP until an optimality gap under a 35%.

Approach 1	Approach 2	Approach 3
1429 s	1467 s	2973 s

Results show that initialization plays a crucial role in finding a solution faster. When the model is not initialized, it takes twice the time to reach to the specified point. When having to choose a way to build the starting point, no significant differences can be found, so any of the two approaches will imply a similar time to reach the solution.

Some detailed data of approaches 1 and 2 is given below.

- *Approach 1*

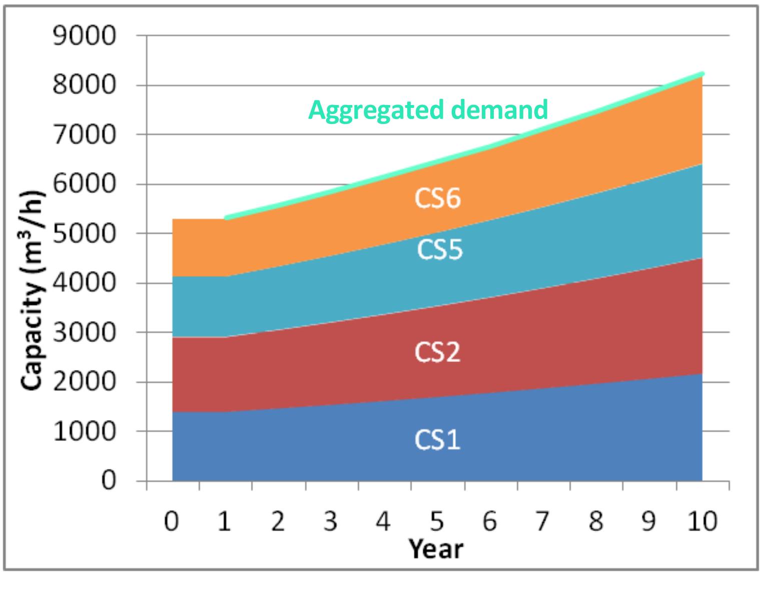

Fig. 10. Aggregated capacity of each CS in the approach 1 of the initialization.

Fig. 10 depicts $\sum_c P_{scn}^{out}$ for this approach.

Fig. 11 shows the annual evolution of the disaggregated costs. No dismantling is performed, so the related costs do not appear. Even though there are installation costs for compressor stations, they appear to be insignificant comparing to line installation or maintenance costs. While line investment keeps constant, due to having to face a 5% increase in demand each year, maintenance costs increase with time, as capacity is expanded every year. Total cost ascends to 10239 million euros.

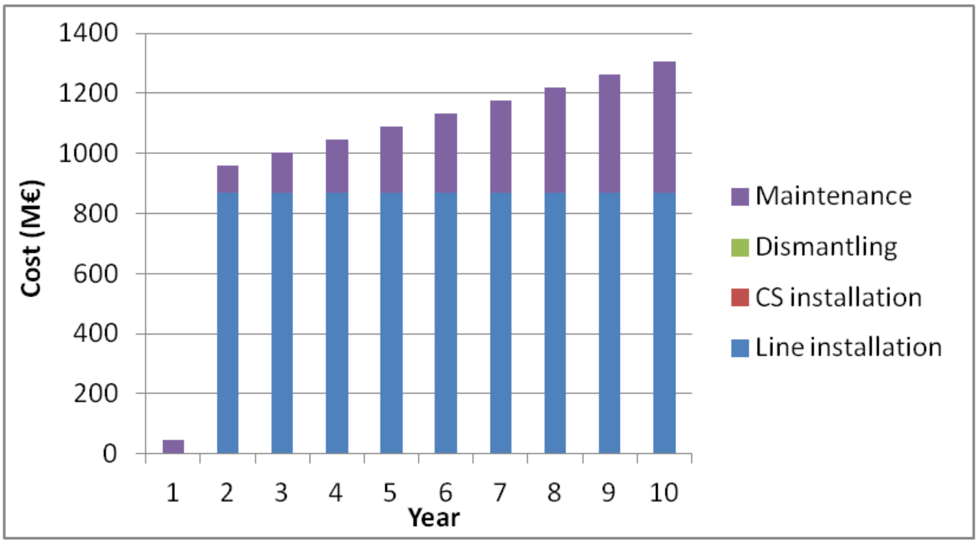

Fig. 11. Costs evolution in the approach 1 of the initialization.

- *Approach 2*

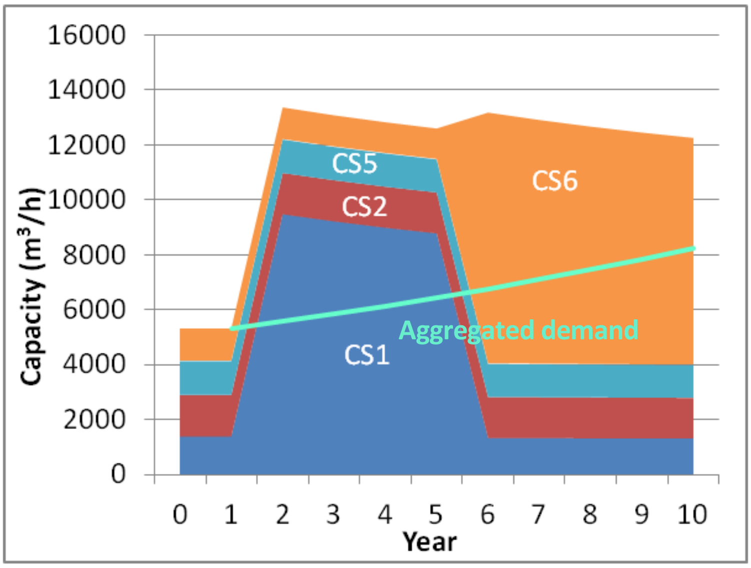

Fig. 12. Aggregated capacity of each TC in the approach 2 of the initialization.

Fig. 12 shows the aggregated capacity of each compressor station in approach 2, where *Banyeres* and *Villafranca* compressor stations are expanded to their maximum capacity in years 1 and 6, respectively. In year 6, *Banyeres* expansion is dismantled too.

In this approach, total cost ascends to 6691 million euros. However, as actions are performed in years 1 and 6, their cost is higher than in the others, where only maintenance cost applies (Fig. 13).

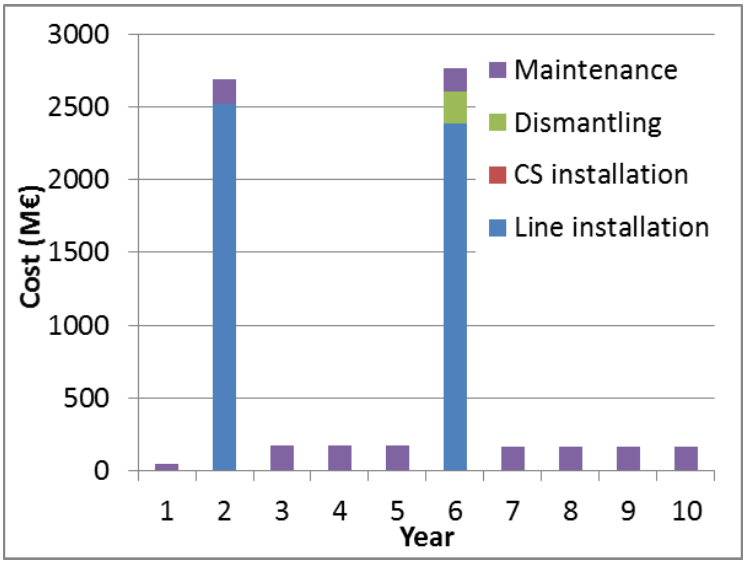

Fig. 13. Costs evolution in the approach 2 of the initialization.

6.1.2. MILP

The MILP model, without considering decaying performance, is solved. It features 49985 equations, 30331 continuous variables and 19320 binary variables, and has been solved with CPLEX 12.6.2.0 providing an optimal solution with a *TCost* of 1212 million €.

Fig. 14 depicts the aggregated capacity of each compressor station in the optimal solution. Main decisions include:

- Expansion of existing facilities CS1, CS2 and CS6 in years 1 and 2.
- Buildup of new facilities CS3 in year 1.
- Dismantling of CS5 and of existing facilities that are renewed.

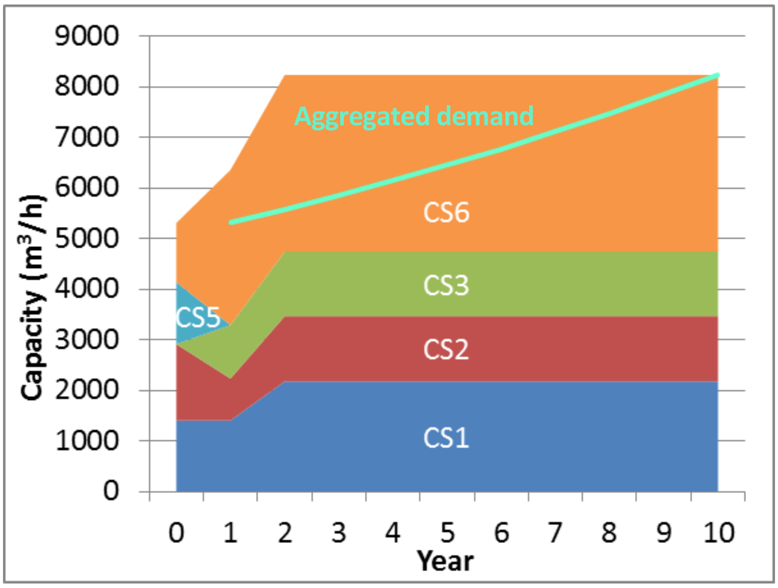

Fig. 14. Aggregated capacity of each CS in the MILP.

The cost evolution (Fig. 15) shows that dismantling of old substations is done in years 1 and 2. The renewal of the facilities dismantled and the expansions required to adapt the network to demand increases are also made in years 1 and 2. This entails that the 70% of the total investment is done in those years.

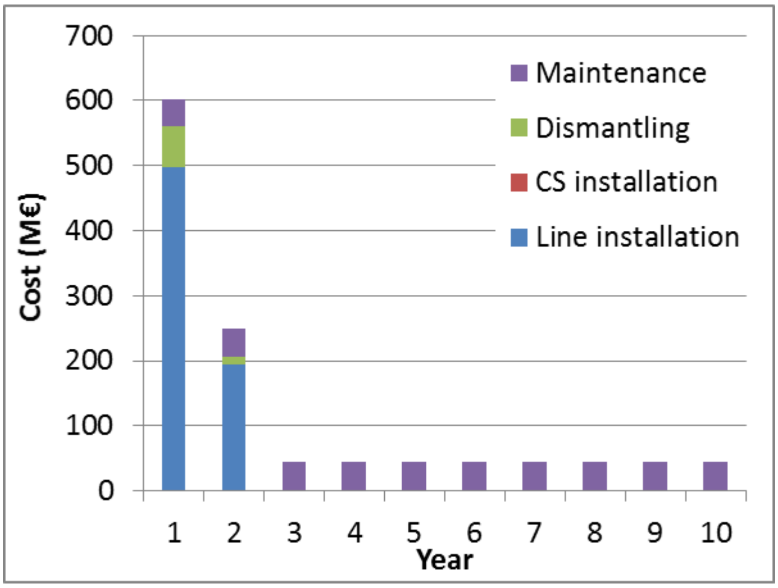

Fig. 15. Costs evolution in the MILP.

6.1.3. MILP + DP Simulation

To check the feasibility of the MILP solution and the sensitivity of the model to the decaying performance, the decisions made in the model go through the simulation where decaying performance is applied.

As demonstrated in Fig. 16, when the solution without considering decaying performance faces real operation, the sizing turns out to be infeasible, as a 20% of the demand could not be covered.

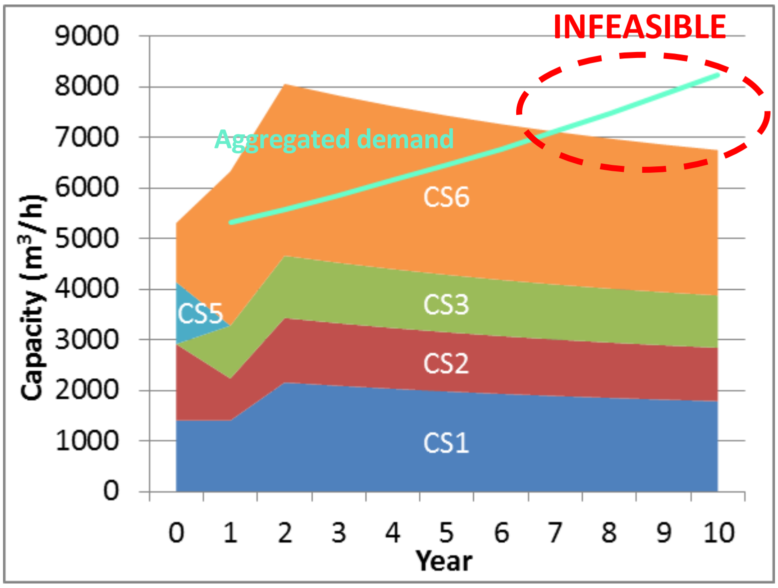

Fig. 16. Aggregated capacity of each CS in the MILP + DP simulation.

To avoid this problem, it is necessary to run the optimization considering the equations for decaying performance. Thus, the MINLP is solved in the next section.

6.1.4. MINLP

The MINLP model, considering decaying performance, features 60905 equations, 41251 continuous variables and 19320 binary variables, and has been solved with DICOPT obtaining an optimal solution with a *TCost* of 1409 million €.

In this case, all existing compressor stations 1 and 6 are expanded in year 1 (Fig. 17). In addition, potential compressor station 3 is build up.

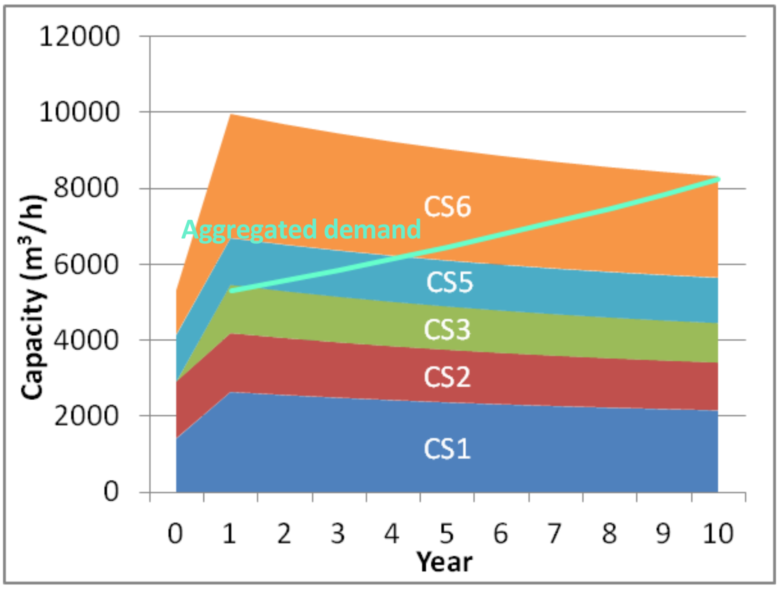

Fig. 17. Aggregated capacity of each CS in the MINLP.

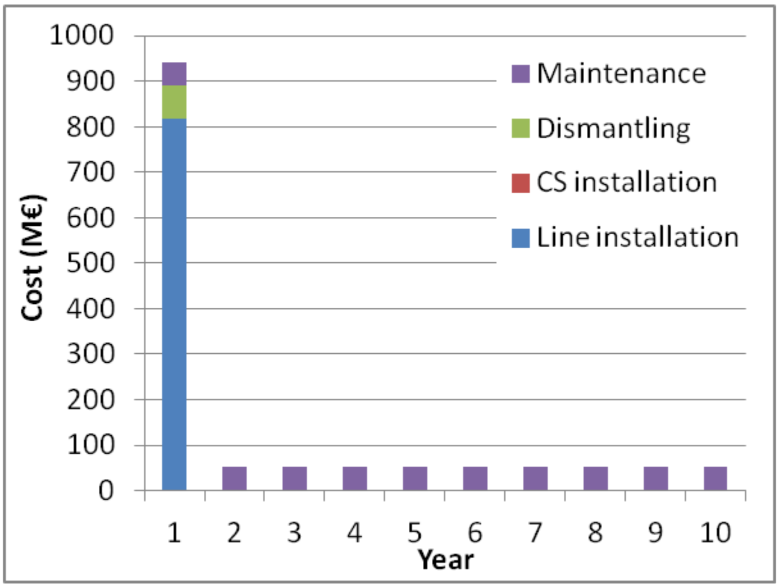

Fig. 18. Costs evolution in the MINLP.

The effect on the costs evolution (Fig. 22) is as expected, a 67% of the total cost is spent in year 1.

6.2. Electricity distribution network

As the electricity distribution case study does not consider physical decaying performance, it is not as common to loose efficiency in electrical equipment, its source of non-linearity comes from the maintenance cost, due to its increase as a function of age.

As the source of non-linearity of this model is in the objective function, the previous strategy cannot be applied. The MINLP has to be directly solved with DICOPT.

6.2.1. Initialization

As demonstrated in section 6.1.1, it is important to specify a feasible starting point to the search algorithm. Thus, as the two proposed approaches gave similar computational times, based on the cost typology, the most naïve estimation is to build every year the necessary to cover the demand of this period (approach 1).

Fig. 19 shows the evolution of the aggregated capacity of each transformation center (TC) to cover the demand. As stated, expansions are performed annually to cover increases in demand.

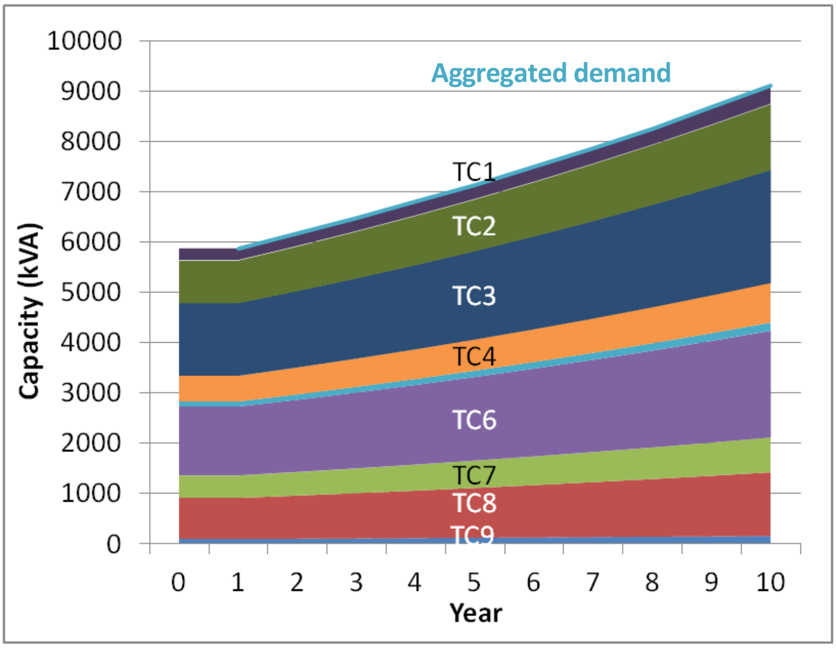

Fig. 19. Aggregated capacity of each transformation center in the initialization.

Fig. 20 depicts the evolution of costs for the initialization. As can be seen, the costs increases annually, mainly because of the effect of maintenance cost, which has to cover older facilities. Total cost reaches 13.69 million €.

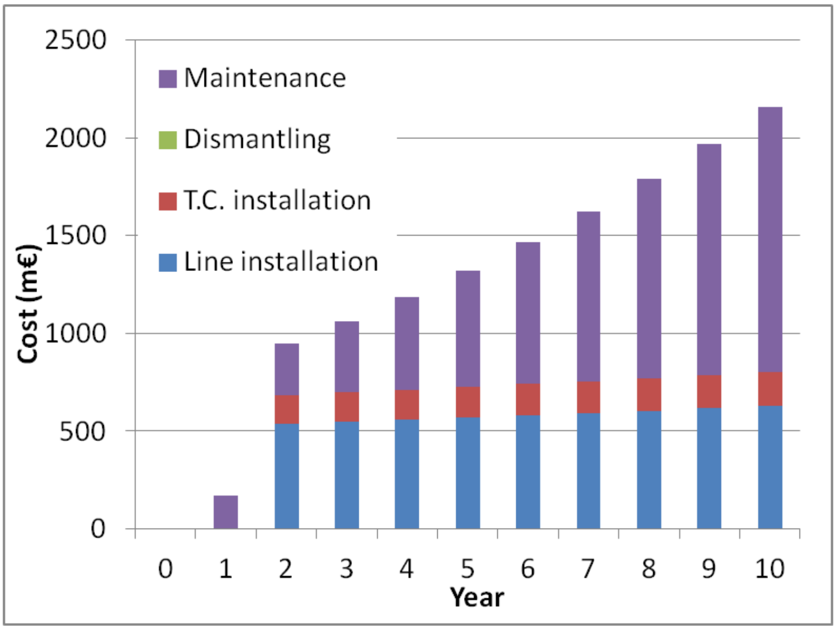

Fig. 20. Costs evolution in the initialization.

6.2.2. MINLP

The MINLP model has been solved considering the increasing maintenance cost as in Eq. (23), what leads to a decaying economic performance, and without considering decaying physical performance (not implementing Eqs. (19, 20)). It features 74225 equations, 42166 continuous variables and 22770 binary variables, and has been solved with DICOPT providing an optimal solution with a *TCost* of 0.60 million €. This represents a decrease of the 96% respect the naive estimation.

Fig. 21 depicts the evolution of $\sum_c P_{scn}^{out}$ for each transformation center *s* in the optimal solution of DP. As can be seen, all the capacity expansions required to meet the demand of the whole time horizon are performed in year 1. This year, transformation centers 3, 5 and 9 have to be expanded and transformation center 5 has to be dismantled.

An analysis of the annual evolution of the disaggregated costs in the optimal solution (Fig. 22), reveals that these expansions must also compensate for the closure of some of the old facilities which are dismantled in the first time period. This is because equipment aging causes an increase in the frequency at which maintenance operations are required in old facilities, thus turning them unprofitable. Note that the fix cost associated to the installation of lines and facilities ($\beta_n^{in}, \beta_n^{out}, \varepsilon_n$) is particularly high.

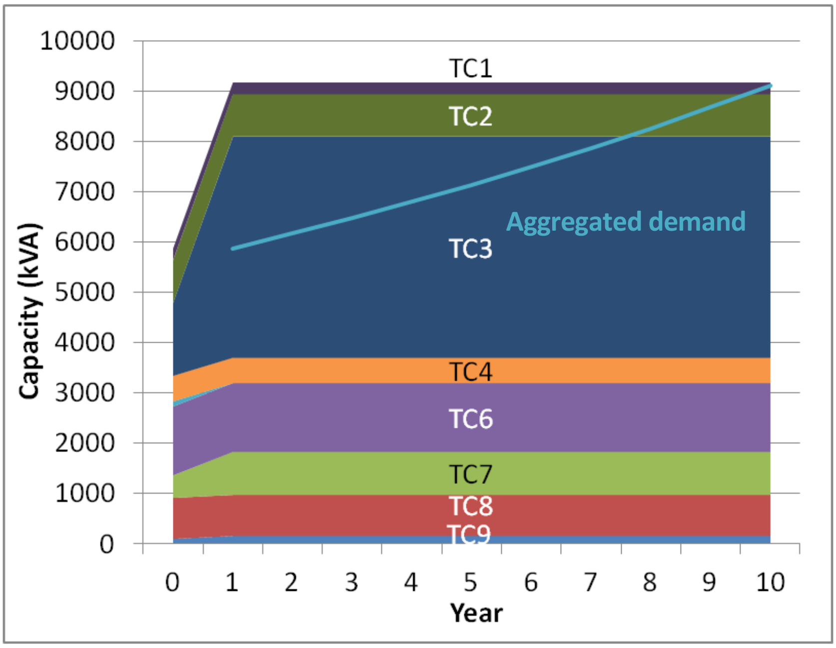

Fig. 21. Aggregated capacity of each TC in the MINLP.

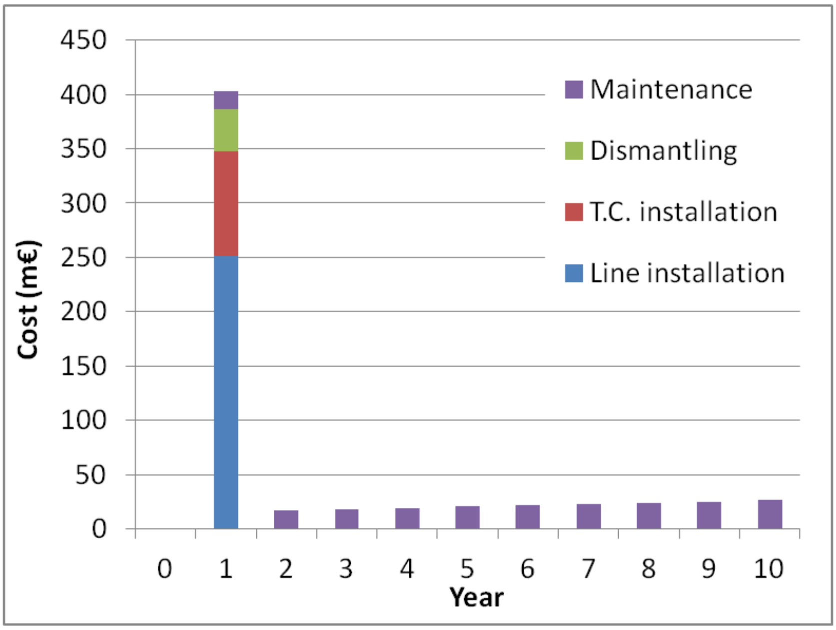

Fig. 22. Costs evolution in the MINLP.

As a result, the optimal solution involves expansions in one time period only (year 1), which reaches a total cost of 0.40 million € (a 66% of the total cost). For the remaining years, only maintenance cost applies.

This solution may not be realistic under real operation. In the next section, a limit in the annual inversion is tested to see the performance of the model under cash restrictions.

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6.2.3. MINLP + Annual inversion limit

In this section, a limit in the annual inversion of 300000 € is applied.

Solutions change as depicted in Fig. 23. In year 1, only transformation center 7 is expanded and TC5 is dismantled. In year 2, transformation center 9 is expanded.

This solution has the associated costs shown in Fig. 24, with a total cost of 0.62 million €. Of those, 0.27 are invested in year 1 and 0.17 in year 2.

Comparing to the solution without the fixed maximum, the cost is only increased by a 3.3%. This means that with a reduced increase in the inversion, the solution is more flexible in terms of cash availability or inversion security.

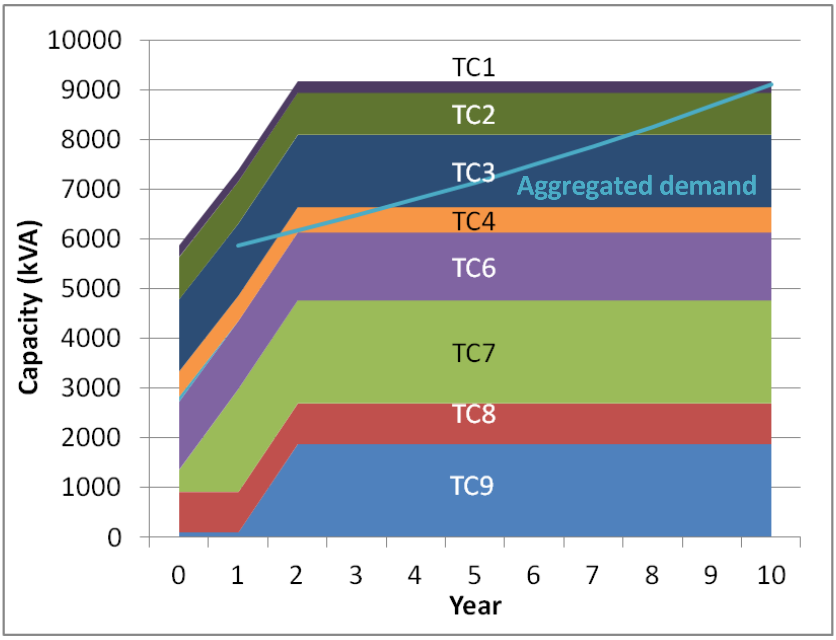

Fig. 23. Aggregated capacity of each TC in the MINLP with annual inversion limit.

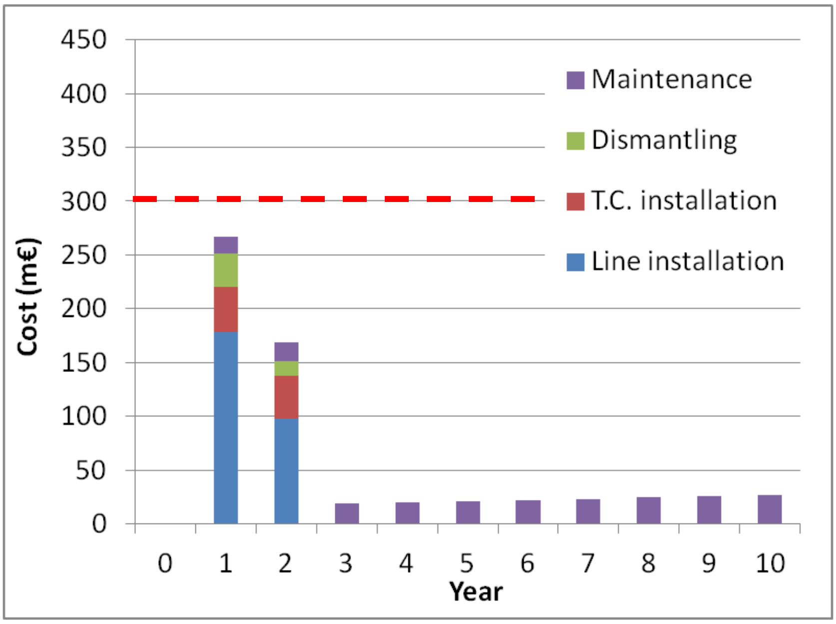

Fig. 24. Evolution of costs in the MINLP with annual inversion limit.

7. Economic evaluation

This section evaluates the economic impact of the master's thesis.

7.1. Costs

7.1.1. Human resources

Human resources costs are detailed in Table 13.

Table 13. Human resources cost.

Position	Dedication (h)	Annual net salary (€)	Annual social security (€)	Annual dedication (h)	Total (€)
Engineer	300	20000.00	6000.00	1700	4588.24
Expert	20	28000.00	8400.00	1700	428.24
		TOTAL (€)			**5016.48**

7.1.2. IT

IT costs include the amortization of the computers used (Table 14) and the inversion in the needed software (Table 15). Computers are used the time dedicated to the project (the same proportion of 300h/1700h in human resources). GAMS license is shared in

Table 14. Equipment cost.

Equipment	Price (€)	Life span (years)	Usage (years)	Total (€)
Computer 1	1000.00	4	0.18	44.12
Computer 2	1000.00	4	0.18	44.12
	TOTAL (€)			**88.24**

Table 15. Software cost.

Software	Cost (€)	Life span (years)	Shared use (person)	Dedication (years)	Total (€)
GAMS	12000.00	5	4	0.18	105.88
		TOTAL (€)			**105.88**

7.1.3. Overhead

Overhead for this project includes generic software acquired by UPC (e.g. Microsoft Office), electricity costs and all the other general expenses derived from the project. Usually, it is accounted as a fixed percentage of the final cost for the project (also considering overhead).

7.1.4. Total cost

Total cost is calculated in Table 16.

Table 16. Total costs.

Type	Cost (€)
Human resources	5016.48
Computers	88.24
Software	105.88
SUBTOTAL (€)	5210.60
Overhead (15%)	919.52
TOTAL (€)	**6130.12**

Conclusions

The conclusions of this master's thesis are listed below:

1. This work presents a novel model for capacity planning of supply chains and distribution networks under decaying performance.

2. Its general approach makes it readily applicable to similar supply chain problems, from chemical chains to electrical distribution networks.

3. Results demonstrate the importance of accurately modeling the decaying performance of the system. Otherwise, equipment sizing will likely become insufficient when facing real operation in systems with physical decaying performance and cost estimation would be unrealistic if the system has economic decaying performance.

4. It is essential to carefully select a feasible starting point, taking into account the features of both the model and the case study. Otherwise, solution time may increase significantly. If a reasonable starting point is provided, the solution algorithm is shown to be not much sensitive to the type of initialization.

5. The procedure to obtain solutions for this model is not unique, it depends on the case study. The best way to obtain results will be different for each model and case study and should be planned beforehand.

6. This work has allowed me to improve my knowledge on optimization and has resulted in two contributions to international congresses. The work done with the natural gas distribution network case study has been presented in the European Symposium of Computer-Aided Chemical Engineering - ESCAPE 26 (Somoza et al., 2016a). The results for the electricity distribution network have been presented in the IEEE International Energy Conference - ENERGYCON 2016 (Somoza et al., 2016b).

Future work

Based on the promising findings, current and future work could address:

- The acceleration of the search algorithm by using global optimization methods. An outer approximation, a method that uses decomposition and relaxation techniques to solve problems faster, is currently being designed.

- The consideration of uncertainty in demand, decaying performance factors and cost parameters. This would imply to move from a deterministic model to a stochastic approach.

- To expand the model with corrective and preventive maintenance, i.e. allowing maintenance to recover part of the efficiency lost with time.

- To include reliability and supply quality considerations in the objective function.

- To test the model with a third case study that considers physical and economic decaying performance at the same time.

Acknowledgement

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Derived publications

SOMOZA, A., POZO, C., GUILLÉN-GOSÁLBEZ, G., GRAELLS, M. *Long-term planning and retrofitting of supply and distribution chains with decaying performance* (Ref. 276). European Symposium on Computer Aided Chemical Engineering (ESCAPE 26). Portorož (Slovenia): June 15th, 2016a.

SOMOZA, A., POZO, C., DE LA HOZ, J., GUILLÉN-GOSÁLBEZ, G., ESPUÑA, A., GRAELLS, M. *An optimization model for the long-term planning of energy distribution networks* (Ref. 424). IEEE International Energy Conference (ENERGYCON 2016). Leuven (Belgium): April 5th, 2016b.

ANNEX

Data for the natural gas distribution network

Table 17. Demand at customer *c* in time period *n* in m^3/h.

Period (n) Market (c)	1	2	3	4	5	6	7	8	9	10
Barcelona	1402.0	1472.1	1545.7	1623.0	1704.1	1789.3	1878.8	1972.8	2071.4	2175.0
Logroño	830.0	871.5	915.1	960.8	1008.9	1059.3	1112.3	1167.9	1226.3	1287.6
Valladolid	354.0	371.7	390.3	409.8	430.3	451.8	474.4	498.1	523.0	549.2
Madrid	398.0	417.9	438.8	460.7	483.8	508.0	533.4	560.0	588.0	617.4
Ciudad Real	1225.0	1286.3	1350.6	1418.1	1489.0	1563.4	1641.6	1723.7	1809.9	1900.4
Jaén	769.0	807.5	847.8	890.2	934.7	981.5	1030.5	1082.1	1136.2	1193.0
Lugo	326.0	342.3	359.4	377.4	396.3	416.1	436.9	458.7	481.7	505.7

Table 18. Maximum capacity at source *p* in time period *n* in m^3/h.

Period (n) Source (p)	1	2	3	4	5	6	7	8	9	10
Tarragona	3992	3992	3992	3992	3992	3992	3992	3992	3992	3992
Cartagena	2929	2929	2929	2929	2929	2929	2929	2929	2929	2929
Huelva	3747	3747	3747	3747	3747	3747	3747	3747	3747	3747
Bilbao	2913	2913	2913	2913	2913	2913	2913	2913	2913	2913

Table 19. Distance in line between source *p* and substation *s* expressed in km.

Com. st. (s) Sources (p)	Banyeres	Villar de Arnedo	Coreses	Algete	Montesa	Villafranca	Almodóvar
Tarragona	50	370	739	537	336	746	685
Cartagena	584	730	688	463	208	427	437
Huelva	1040	952	621	650	695	257	372
Bilbao	536	170	365	377	689	762	680

Table 20. Distance in line between substation *s* and customer *c* expressed in km.

Customer (c) Com. st. (s)	Barcelona	Logroño	Valladolid	Madrid	Ciudad Real	Jaén	Lugo
Banyeres	75	413	610	552	634	738	924
Villar de Arnedo	444	40	286	345	542	665	554
Coreses	755	342	87.9	241	423	571	311
Algete	610	318	207	40.8	247	370	522
Montesa	427	550	565	372	338	393	876
Villafranca	832	691	560	372	170	89.3	870
Almodóvar	735	567	436	248	46.2	174	723

Table 21. Initial value for capacity installed at substation *s* expressed in m^3/h and binary variable and age in years related.

Compressor station (s)	$P0_s^{sub}$ (m^3/h)	$S0_s^{pon}$	$A0In_s^{sub}$
Banyeres	1402	1	10
Villar de Arnedo	1510	1	20
Coreses	0	0	0
Algete	0	0	0
Montesa	1225	1	15
Villafranca	1167	1	5
Almodóvar	0	0	0

Table 22. Initial value for capacity installed at line between producer *p* and substation *s* expressed in m^3/h and binary variable and age in years related.

	Com.st. (s) Source (p)	Banyeres	Villar de Arnedo	Coreses	Algete	Montesa	Villafranca	Almodóvar
$P0_{ps}^{in}$	**Tarragona**	1402	0	0	0	0	0	0
	Cartagena	0	0	0	0	1225	0	0
	Huelva	0	0	0	0	0	1167	0
	Bilbao	0	1510	0	0	0	0	0
$LPS0_{ps}^{in}$	**Tarragona**	1	0	0	0	0	0	0
	Cartagena	0	0	0	0	1	0	0
	Huelva	0	0	0	0	0	1	0
	Bilbao	0	1	0	0	0	0	0

$A0In_{ps}^{in}$	**Tarragona**	10	0	0	0	0	0	0
	Cartagena	0	0	0	0	15	0	0
	Huelva	0	0	0	0	0	5	0
	Bilbao	0	20	0	0	0	0	0

Table 23. Initial value for capacity installed at line between substation *s* and customer *c* expressed in m^3/h and binary variable and age in years related.

	Customer(c) **Com. st. (s)**	**Barcelona**	**Logroño**	**Valladolid**	**Madrid**	**Ciudad Real**	**Jaén**	**Lugo**
$P0_{sc}^{out}$	**Banyeres**	1402	0	0	0	0	0	0
	Villar de Arnedo	0	830	354	0	0	0	326
	Coreses	0	0	0	0	0	0	0
	Algete	0	0	0	0	0	0	0
	Montesa	0	0	0	0	1225	0	0
	Villafranca	0	0	0	398	0	769	0
	Almodóvar	0	0	0	0	0	0	0
$LSC0_{sc}^{out}$	**Banyeres**	1	0	0	0	0	0	0
	Villar de Arnedo	0	1	1	0	0	0	1
	Coreses	0	0	0	0	0	0	0
	Algete	0	0	0	0	0	0	0
	Montesa	0	0	0	0	1	0	0
	Villafranca	0	0	0	1	0	1	0
	Almodóvar	0	0	0	0	0	0	0
$A0Out_{sc}^{out}$	**Banyeres**	10	0	0	0	0	0	0
	Villar de Arnedo	0	20	20	0	0	0	20
	Coreses	0	0	0	0	0	0	0
	Algete	0	0	0	0	0	0	0
	Montesa	0	0	0	0	15	0	0
	Villafranca	0	0	0	5	0	5	0
	Almodóvar	0	0	0	0	0	0	0

Table 24. Upper and lower bounds for capacity at substations in m^3/h.

Parameter	Value (m^3/h)
$\overline{PUB^{sub}}$	100000
$\overline{PUB^{in}}$	100000
$\overline{PUB^{out}}$	100000
$\underline{PLB^{sub}}$	100
$\underline{PLB^{in}}$	100
$\underline{PLB^{out}}$	100

Table 25. Decaying performance coefficients.

Parameter	Value
c^{sub}	0.29
k^{sub}	0.11
$PR^{sub\infty}$	0.70
c^{in}	0.29
k^{in}	0.11
$PR^{in\infty}$	0.70
c^{out}	0.29
k^{out}	0.11
$PR^{out\infty}$	0.70

Table 26. Cost factors.

Parameter	Value
α_n^{in}	39.97 [€/(km·(m^3/h))]
α_n^{out}	39.97 [€/(km·(m^3/h))]
β_n^{in}	353972.67 [€/km]
β_n^{out}	353972.67 [€/km]
γ_n^{in}	0 [€/km]
γ_n^{out}	0 [€/km]
δ_n	192.78 [€/(m^3/h)]
ε_n	0 [€]
ζ_n	0 [€]

η_n^{in}	0.10 [€]
η_n^{out}	0.10 [€]
θ_n	0.10 [€]
ι_n^{in}	0.05
ι_n^{out}	0.05
κ	0.05

Data for the electricity distribution network

Table 27. Demand at customer *c* in time period *n* in kVA.

Period (n) **Market (c)**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
C1	231.0	242.5	254.6	267.4	280.7	294.8	309.5	325.0	341.2	358.3
C2	845.9	888.2	932.6	979.2	1028.2	1079.6	1133.6	1190.3	1249.8	1312.3
C3	1450.7	1523.2	1599.4	1679.4	1763.3	1851.5	1944.1	2041.3	2143.4	2250.5
C4	509.1	534.6	561.3	589.4	618.8	649.8	682.3	716.4	752.2	789.8
C5	104.0	109.2	114.6	120.3	126.4	132.7	139.3	146.3	153.6	161.3
C6	1366.9	1435.2	1507.0	1582.3	1661.5	1744.5	1831.8	1923.3	2019.5	2120.5
C7	443.5	465.7	489.0	513.4	539.1	566.1	594.4	624.1	655.3	688.1
C8	814.5	855.2	898.0	942.9	990.0	1039.6	1091.5	1146.1	1203.4	1263.6
C9	106.6	111.9	117.5	123.4	129.6	136.1	142.9	150.0	157.5	165.4

Table 28. Maximum capacity at source *p* in time period *n* in kVA.

Period (n) **Source (p)**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
P1	15000	15000	15000	15000	15000	15000	15000	15000	15000	15000

Table 29. Distance in line between source *p* and substation *s* expressed in km.

Com. st. (s) **Sources (p)**	**TC1**	**TC2**	**TC3**	**TC4**	**TC5**	**TC6**	**TC7**	**TC8**	**TC9**
P1	1.49	2.20	1.70	2.47	2.32	2.71	2.17	2.71	0.76

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Table 30. Distance in line between substation *s* and customer *c* expressed in km.

Customer (c) Tr. Cent. (s)	C1	C2	C3	C4	C5	C6	C7	C8	C9
TC1	0.10	1.05	0.47	1.06	1.26	1.50	1.26	1.13	1.16
TC2	1.05	0.10	1.13	1.78	1.33	0.62	1.71	1.61	1.94
TC3	0.47	1.13	0.10	0.64	1.36	1.64	1.30	0.65	1.34
TC4	1.06	1.78	0.64	0.10	2.06	2.28	2.02	0.25	1.92
TC5	1.26	1.33	1.36	2.06	0.10	2.68	0.60	2.22	2.24
TC6	1.50	0.62	1.64	2.28	2.68	0.10	2.67	2.33	2.53
TC7	1.26	1.71	1.30	2.02	0.60	2.67	0.10	2.96	2.10
TC8	1.13	1.61	0.65	0.25	2.22	2.33	2.96	0.10	2.04
TC9	1.16	1.94	1.34	1.92	2.24	2.53	2.10	2.04	0.10

Table 31. Initial value for capacity installed at substation *s* expressed in kVA and binary variable and age in years related.

Transformation center (s)	$P0_s^{sub}$ (m^3/h)	$S0_s^{pon}$	$A0In_s^{sub}$
TC1	230.97	1	8
TC2	845.91	1	7
TC3	1450.71	1	5
TC4	509.12	1	15
TC5	103.96	1	10
TC6	1366.88	1	20
TC7	443.54	1	18
TC8	814.52	1	12
TC9	106.60	1	10

Table 32. Initial value for capacity installed at line between producer *p* and substation *s* expressed in kVA and binary variable and age in years related.

	Tr. Cen. (s) Source (p)	TC1	TC2	TC3	TC4	TC5	TC6	TC7	TC8	TC9
$P0_{ps}^{in}$	**P1**	230.97	845.91	1450.71	509.12	103.96	1366.88	443.54	814.52	106.60
$LPS0_{ps}^{in}$	**P1**	1	1	1	1	1	1	1	1	1
$A0In_{ps}^{in}$	**P1**	8	7	5	15	10	20	18	12	10

Table 33. Initial value for capacity installed at line between substation *s* and customer *c* expressed in kVA and binary variable and age in years related.

	Customer(c) Tr. Cent. (s)	C1	C2	C3	C4	C5	C6	C7	C8	C9
	TC1	230.97	0	0	0	0	0	0	0	0
	TC2	0	845.91	0	0	0	0	0	0	0
	TC3	0	0	1450.71	0	0	0	0	0	0
	TC4	0	0	0	509.12	0	0	0	0	0
$P0_{sc}^{out}$	**TC5**	0	0	0	0	103.96	0	0	0	0
	TC6	0	0	0	0	0	1366.88	0	0	0
	TC7	0	0	0	0	0	0	443.54	0	0
	TC8	0	0	0	0	0	0	0	814.52	0
	TC9	0	0	0	0	0	0	0	0	106.60
	TC1	1	0	0	0	0	0	0	0	0
	TC2	0	1	0	0	0	0	0	0	0
	TC3	0	0	1	0	0	0	0	0	0
	TC4	0	0	0	1	0	0	0	0	0
$LSC0_{sc}^{out}$	**TC5**	0	0	0	0	1	0	0	0	0
	TC6	0	0	0	0	0	1	0	0	0
	TC7	0	0	0	0	0	0	1	0	0
	TC8	0	0	0	0	0	0	0	1	0
	TC9	0	0	0	0	0	0	0	0	1
	TC1	8	0	0	0	0	0	0	0	0
	TC2	0	7	0	0	0	0	0	0	0
	TC3	0	0	5	0	0	0	0	0	0
	TC4	0	0	0	15	0	0	0	0	0
$A0Out_{sc}^{out}$	**TC5**	0	0	0	0	10	0	0	0	0
	TC6	0	0	0	0	0	20	0	0	0
	TC7	0	0	0	0	0	0	18	0	0
	TC8	0	0	0	0	0	0	0	12	0
	TC9	0	0	0	0	0	0	0	0	10

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Table 34. Upper and lower bounds for capacity at substations in kVA.

Parameter	Value (kVA)
$\overline{PUB}^{sub}$	100000
$\overline{PUB}^{in}$	100000
$\overline{PUB}^{out}$	100000
$\underline{PLB}^{sub}$	0
$\underline{PLB}^{in}$	0
$\underline{PLB}^{out}$	0

Table 35. Decaying performance coefficients.

Parameter	Value
c^{sub}	0.29
k^{sub}	0.11
$PR^{sub\infty}$	0.70
c^{in}	0.29
k^{in}	0.11
$PR^{in\infty}$	0.70
c^{out}	0.29
k^{out}	0.11
$PR^{out\infty}$	0.70

Table 36. Cost factors 1.

Parameter	Value
α_n^{in}	0 [€/(km·(kVA))]
α_n^{out}	0 [€/(km·(kVA))]
γ_n^{in}	0 [€/km]
γ_n^{out}	0 [€/km]
ζ_n	0 [€]
η_n^{in}	5052.81 [€]
η_n^{out}	5052.81 [€]
θ_n	3042.62 [€]

ι_n^{in}	0.10 [€]
ι_n^{out}	0.10 [€]
κ	0.23 [€]

Table 37. Cost factors 2.

Parameter **Period (n)**	β_n^{in} **[€/km]**	β_n^{out} **[€/km]**	δ_n **[€/(kVA)]**	ε_n **[€]**
1	27008.00	27008.00	12.53	15579.78
2	27548.16	27548.16	12.78	15891.37
3	28099.12	28099.12	13.04	16209.20
4	28661.11	28661.11	13.30	16533.38
5	29234.33	29234.33	13.56	16864.05
6	29819.01	29819.01	13.84	17201.33
7	30415.39	30415.39	14.11	17545.36
8	31023.70	31023.70	14.39	17896.26
9	31644.18	31644.18	14.68	18254.19
10	32277.06	32277.06	14.98	18619.27

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