
An energy flow study of a double-deck tunnel under quasi-static and harmonic excitations

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Abstract

This paper presents a comparison between the vibration energy flow radiated by a double-deck tunnel and the one radiated by a simple tunnel when both are excited by constant or by harmonic moving loads. For both cases, the radiated energy is computed using a three-dimensional semi-analytical model of the system. The total energy radiated upwards is presented for a wide range of load speeds, when a constant moving load is considered, and for a wide range of excitation frequencies, when the excitation is a harmonic moving load. Significant differences have been obtained, first, for constant loads moving at very high speeds and, second, for harmonic loads moving at typical speeds for underground trains.

Keywords: Underground vibrations, Double-deck tunnel, Moving loads, Energy flow.

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1. Introduction

The rapid increase in the number of underground traffic infrastructures in heavily populated areas has motivated the design of new types of tunnels, such

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as double-deck tunnels. However, despite being constructed in many important cities worldwide, the vibration impact of subway traffic circulating along a double-deck tunnel has not yet been properly studied.

One of the most well-established models for predicting railway-induced vibrations in tunnels is the Pipe-in-Pipe (PiP) model presented by Forrest and Hunt [1, 2], a three-dimensional (3D) semi-analytical track-tunnel-soil model that represents the tunnel-soil system as as an infinite thin cylindrical shell perfectly coupled to a viscoelastic full-space. The model was later extended by Hussein and Hunt [3], who added a new floating-slab track model to the tunnel-soil system. The same authors also used the model to develop a power flow method to evaluate the response of underground railway structures excited by infinite multi-point moving loads [4]. More recently, Clot et al. [5] performed a comparison between the power flows radiated by a double-deck tunnel and by a simple tunnel in plane-strain conditions, finding significant differences between both. The results obtained in their work, however, do not consider the propagation of waves in the tunnel axial direction, ignoring how the dynamics of the interior floor may influence this propagation.

The aim of this paper is to extend the results presented in [5] by studying the effect that the load speed has in the comparison between the responses of double-deck and simple tunnels. In order to perform this study, the soil response to static and harmonic moving loads is computed using the 3D double-deck tunnel model recently presented in [6]. The comparison is performed by considering the energy of vibration radiated upwards by both tunnels.

2. Analytical formulation

The proposed double-deck tunnel model is presented in Fig. 1. The tunnel structure is modelled as an infinite thin cylindrical shell of constant thickness h_t and constant mean radius r_t divided into two equal parts by an interior floor,

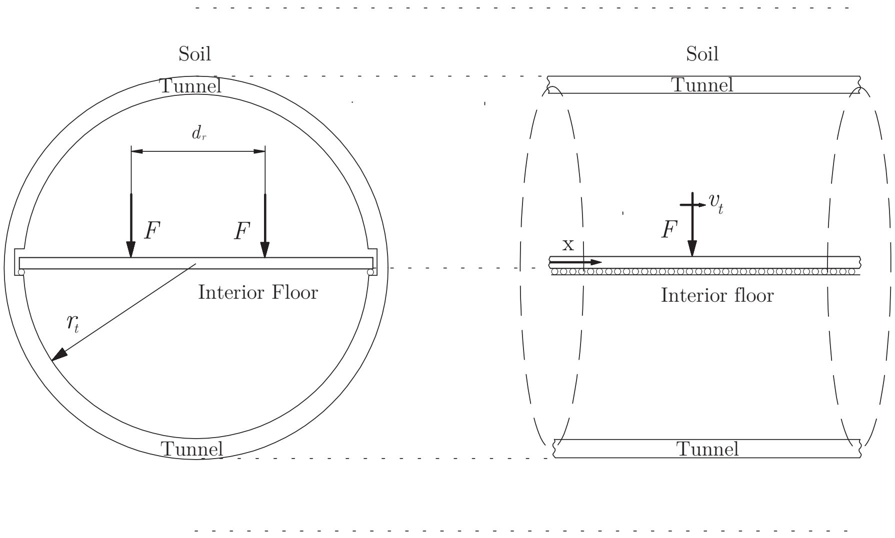

Fig. 1: Representation of the double-deck circular tunnel model.

which is represented as a thin strip plate of constant thickness h_p and constant width L_p. It is assumed that the edges of the interior floor are simply supported on the tunnel walls and that the surrounding soil is a linear homogeneous viscoelastic full-space. Two equal vertical point loads separated a distance d_r are applied on the interior floor. The loads, which are assumed to be situated at the same distance from the centre of the interior floor, are moving along the tunnel at a constant speed v_t. The geometry and the mechanical parameters of the system are assumed to be invariant in the tunnel axial direction, which is considered as the x-direction.

The soil response to a load applied on the interior floor is obtained by coupling the interior floor subsystem with the tunnel-soil subsystem in the wavenumber-frequency domain. An upper case letter with an upper bar has been used to identify that a variable is expressed in this domain. The coupling procedure is only outlined in the following paragraphs but the interested reader can find more details in [6] and [7].

For coupling the interior floor to the tunnel-soil system it is assumed that, at the tunnel-floor joint positions, the vertical displacement of the floor is equal to the tangential displacement of the tunnel interior surface. These displacements are expressed in terms of the strip plate transfer functions and of the PiP transfer functions, respectively. The PiP transfer functions are obtained using the formulation presented in [3], which extends the original PiP formulation [1] to the case where the applied loads are antisymmetric. The analytical expressions of the used transfer functions can be found in [6].

With the considered floor-tunnel coupling conditions, the coupling forces can be expressed in terms of the external forces $\bar{F}$, of the strip plate transfer functions and of the PiP transfer functions. Once the coupling forces are determined, the soil displacement field $\bar{\mathbf{U}}$ and the soil stress field $\bar{\mathbf{T}}$ are finally obtained by using additional PiP transfer functions. The resulting expressions can be compactly written as

$$\bar{\mathbf{U}}_i = \mathbf{H}_{u,i}\bar{F}, \qquad \bar{\mathbf{T}}_i = \mathbf{H}_{\tau,i}\bar{F}, \tag{1}$$

where $\mathbf{H}_{u,i}$ and $\mathbf{H}_{\tau,i}$ are, respectively, the double-deck tunnel transfer functions of the displacement field and of the stress field at a position i of the soil due to the two point loads $\bar{F}$ applied on the interior floor.

For the case of a simple tunnel, the point loads $\bar{F}$ are applied at the interior surface of the tunnel. As in the double-deck tunnel case, the loads are separated a distance d_r and are situated at the same distance from the tunnel invert. In this case, the soil displacement and stress fields can be directly expressed using Eq. (1) by just replacing the double-deck tunnel transfer functions with the simple tunnel ones.

In order to compare the vibration impact that both tunnels have on nearby building foundations, the energy of vibration radiated upwards by them is cal-

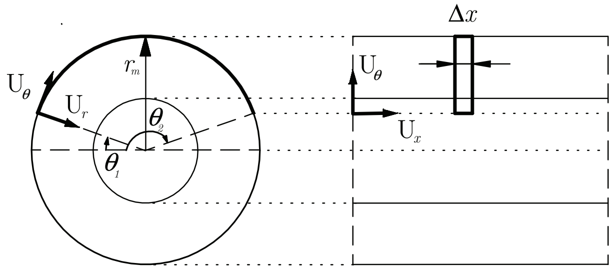

Fig. 2: Chosen integration surface for the energy flow calculation.

culated [4]. The considered surface of integration is the cylindrical strip of length $r_m(\theta_2 - \theta_1)$ and of width Δx represented in Fig. 2. Assuming that Δx is very small and that the cylindrical strip is centered at $x_m = 0$ m, the total energy flow E is given by

$$E = r_m \ \Delta x \int\limits_{\theta_1}^{\theta_2} \int\limits_{-\infty}^{\infty} \mathbf{v}(0,\theta,t) \cdot \boldsymbol{\tau}(0,\theta,t) \mathrm{d}t \mathrm{d}\theta. \tag{2}$$

where $\mathbf{v}$ and $\boldsymbol{\tau}$ are the velocity of vibration and stress fields, respectively, caused by two unitary harmonic moving point loads.

The soil velocity of vibration and stress fields caused by a unitary harmonic moving load $p(x,t) = \cos(\tilde{\omega}t)\delta(x - v_t t)$, where $\tilde{\omega}$ is the excitation frequency, are obtained by first transforming it to the wavenumber-frequency domain, then obtaining the transformed responses using Eq. (1) and, finally, transforming these responses to the space-time domain. The resulting expressions at $x = 0$ m are

$$\mathbf{v}(0,t) = \frac{1}{(2\pi)^2 v_t} \mathrm{Re} \left[\int_{-\infty}^{\infty} \mathrm{i}\omega \mathbf{H}_u \left(\frac{\omega - \tilde{\omega}}{v_t}, \omega \right) \mathrm{e}^{\mathrm{i}\omega t} \mathrm{d}\omega \right] \tag{3}$$

and

$$\boldsymbol{\tau}(0,t) = \frac{1}{(2\pi)^2 v_t}\mathrm{Re}\left[\int_{-\infty}^{\infty} \mathbf{H}_\tau\left(\frac{\omega-\tilde{\omega}}{v_t},\omega\right)\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}\omega\right], \tag{4}$$

where $\mathbf{H}_u$ and $\mathbf{H}_\tau$ are the transfer functions defined in Eq. (1). Once both fields are known, the energy flow radiated across the considered surface is obtained computing Eq. (2).

3. Results and discussion

This section presents the results obtained in the comparison of the energy flows radiated by a simple and by a double-deck tunnel for quasi-static and for a dynamic excitations. The considered mechanical properties for the interior floor, the tunnel and the soil are presented in Tables 1 and 2. Material damping is introduced assuming complex-valued Young modulus, in the case of the tunnel parts, and complex-valued Lamé parameters, in the case of the soil. Both external loads have an amplitude of 0.5 N and are separated a distance $d_r = 1.8$ m. More details regarding the computation of both tunnel models can be found in [6].

3.1. Quasi-static excitation

The external excitation considered in this section is two constant moving point loads. The velocity of vibration and stress fields caused by constant moving loads can be obtained using $\tilde{\omega} = 0$ in Eqs. (3) and (4).

The results presented in Fig. 3 have been obtained computing Eq. (2) at $r_m = 10$ m, for v_t between 10 and 250 m/s and considering a typical Tertiary soil (a) and a soft Quaternary soil (b). The energy radiated upwards has been taken into account defining $\theta_1 = 0$ and $\theta_2 = \pi$ rad, with an angular resolution of $\Delta\theta = \pi/60$ rad and with a space resolution $\Delta x = 1$ m.

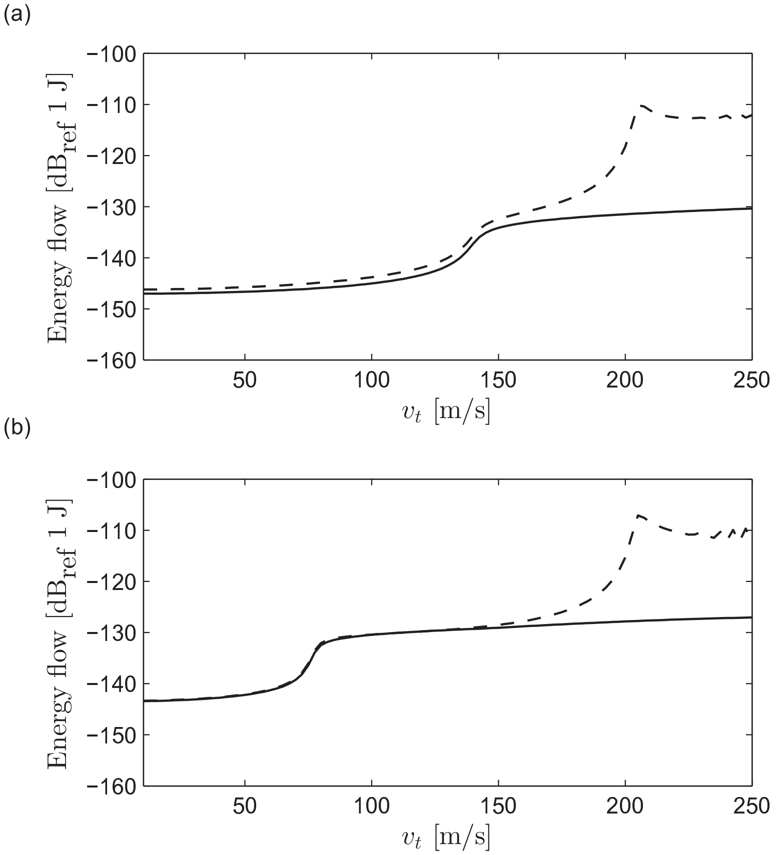

Fig. 3: Total energy radiated upwards by a simple tunnel (solid line) and by a double-deck tunnel (dashed line) for different speeds v_t. Results are presented for (a) a typical Tertiary soil and (b) a soft Quaternary soil.

Floor Parameter	Value	Tunnel Parameter	Value
Length (L_p)	10.9 m	Radius (r_t)	5.65 m
Width (h_p)	0.4 m	Width (h_t)	0.4 m
Young modulus	27.6 GPa	Young modulus	27.6 GPa
Poisson ratio	0.175	Poisson ratio	0.175
Density	3000 kg m^{-3}	Density	3000 kg m^{-3}
Damping ratio	0.02	Damping ratio	0.02

Table 1: Mechanical parameters used to model the interior floor as a thin plate and the tunnel as a thin shell.

Soil Parameter	Tertiary soil	Quaternary soil
Young modulus	100 MPa	30 MPa
Poisson ratio	0.3	0.3
Density	1950 kg m^{-3}	1950 kg m^{-3}
P-wave phase speed	262.74 m s^{-1}	143.91 m s^{-1}
S-wave phase speed	140.44 m s^{-1}	76.92 m s^{-1}
Volumetric damping ratio	0.03	0.03
Deviatoric damping ratio	0.03	0.03

Table 2: Mechanical parameters used to model the soils as elastic continua.

Two different phenomena can be identified in the presented results. The first one, which can be observed for both tunnels, is that a significant increase of the radiated energy is obtained for speeds between 135 and 145 m/s, in the case of the Tertiary soil, and between 70 and 80 m/s, in the case of the Quaternary soil. For both soils these speed values are around their S-wave phase speed (see Table 2). The second phenomena, which is only observed in the case of the double-deck tunnel, is an important increase of the radiated energy when the speed is around 200 m/s. This second increase occurs at the same speed for both types of soils, which indicates that it is mainly caused by the interior floor dynamics. However, for nowadays trains and vehicles circulation speeds, the total energy radiated upwards by both tunnels when they are excited with quasi-static loads is very similar.

3.2. Harmonic excitation

The vibration energy flow radiated upwards when both tunnels are excited by harmonic moving point loads is compared in Fig. 4. The comparison has been performed at $r_m = 10$ m for excitation frequencies $f_e = 2\pi\tilde{\omega}$ between 1 Hz and 80 Hz [8] with increments of 0.5 Hz and for two speeds: $v_t = 15$ m/s (a) and 40 m/s (b).

The main difference between the radiated energy flows is that the double-deck tunnel response presents a significant increase around 5 Hz and around 45 Hz, frequencies that are similar to those obtained in the power flow study presented in [5]. Despite this, there are considerable differences between the ratio of energy flows presented in this work and the ratio of power flows presented there. These differences are especially clear for excitation frequencies between 50 and 80 Hz, where the results presented in Fig. 4 show that the energy flow radiated by the simple tunnel is clearly lower than the one radiated by the double-deck tunnel while the previous power flow study predicted the opposite trend. Therefore, it can be concluded that a two-dimensional study can be used for estimating the resonance frequencies of the floor-tunnel-soil system but is not suitable for quantifying the differences between the energies radiated by both tunnels.

4. Conclusions

This paper presents a study of the vibration energy radiated by a double-deck tunnel when it is excited by constant or harmonic moving point loads and compares this energy to the one obtained when a simple tunnel is considered.

For the case of a quasi-static excitation, the total energy radiated upwards by both tunnels has been compared for a wide range of speeds and two important increases in this energy have been observed. While the first increase, which occurs around the S-wave phase speed of the soil, has been obtained for both

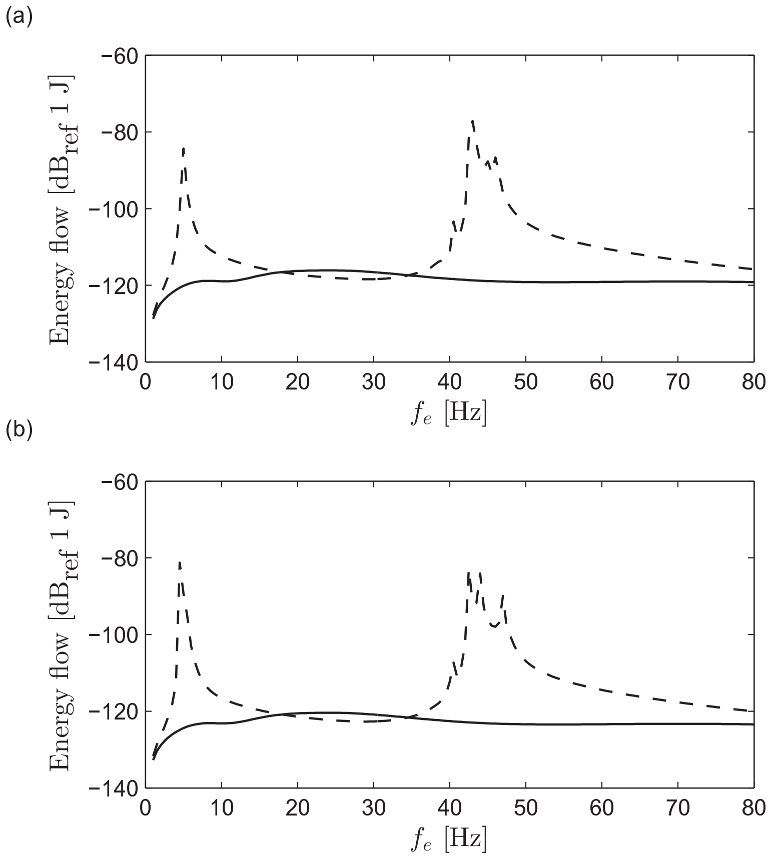

Fig. 4: Total energy radiated upwards by a simple tunnel (solid line) and by a double-deck tunnel (dashed line) for different excitation frequencies. Results are presented at $r_m = 10$ m for two loads speeds: 15 m/s (a) and 40 m/s (b).

tunnels, the other, which is not affected by the type of soil considered, has only been found for the case of the double-deck tunnel. However, for nowadays circulating speeds in tunnels, the response of both systems to a quasi-static excitation is very similar.

For the case of a dynamic excitation, the total energy radiated upwards by both tunnels has been calculated for a wide range of excitation frequencies. The results show that, while smooth variations of this energy are observed in the simple tunnel response, sharp peaks are found in the double-deck tunnel case. Therefore, significant differences have been found between the energy radiated by both tunnels for the whole range of frequencies studied.

The similarities and differences between the energy flow results shown in this work and the power flow ones presented in [5] have been also discussed. It is concluded that, despite that the power flow study could estimate the resonance frequencies of the floor-tunnel-soil system, it is necessary to take into account the motion of the load along the track and the 3D nature of the problem for studying the effect of the quasi-static excitation and for quantifying the amount of energy radiated by both tunnels for the whole range of frequencies of interest.

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