Modelling Flexible Thrust Performance for Trajectory Prediction Applications in ATM

Ismael Matamoros   Xavier Prats
Technical University of Catalonia (UPC)
Castelldefels, Spain

Javier López Leonés   Enrique Casado
Miguel Vilaplana
Boeing Research & Technology Europe (BR&TE)
Madrid, Spain

Vincent Mouillet   Angela Nuic   Laurent Cavadini
EUROCONTROL Experimental Centre (EEC)
Brétigny-sur-Orge, France

Abstract—**Reduced thrust operations are of widespread use nowadays due to their inherit benefits for engine conservation. Therefore, in order to enable realistic simulation of air traffic management (ATM) scenarios for purposes such as noise and emissions assessment, a model for reduced thrust is required. This paper proposes a methodology for modelling flexible thrust by combining an assumed temperature (AT) polynomial model identified from manufacturer take-off performance data and public thrust models taken from typical ATM performance databases. The advantage of the proposed AT model is that it only depends on the take-off conditions —runway length, airport altitude, temperature, wind, etc. The results derived from this methodology were compared to simulation data obtained from manufacturer's take-off performance tools and databases. This comparison revealed that the polynomial model provides AT estimations with sufficient accuracy for their use in ATM simulation. The Base of Aircraft Data (BADA) and the Aircraft Noise and Performance (ANP) database were chosen as representative of aircraft performance models commonly used in ATM simulation. It was observed that there is no significant degradation of the overall accuracy of their thrust models when using AT, while there is a correct capture of the corresponding thrust reduction.**

I. INTRODUCTION

The evaluation of the feasibility and benefits of new air traffic management (ATM) initiatives often relies on simulation tools, in which trajectory predictors (TP) play a key role. Trajectory prediction is also the cornerstone of the evolution of the ATM system towards a new paradigm based on the use of decision support tools (DTS) to assist air traffic control. This new concept is a key enabler of trajectory based operations (TBO), which allow airspace users to collaborate with ATM services providers to execute their operations.

As defined in [1], trajectory prediction is the process to estimate the future trajectory of an aircraft through calculation by using mathematical models of the different implied components such as aircraft, meteorology or ATM systems. A TP is a tool that implements this functionality. The quality of trajectory predictions is closely tied to the accuracy of the aircraft performance models (APM) although other factors also affect the quality of the predictions such as intent modelling or weather forecasts.

The most important elements of APM include aircraft power plant (thrust and fuel consumption essentially) and aerodynamic drag modelling. Recent works have focused on the derivation of thrust models using different approaches, such as in [2]. In [3] and [4], propulsive models are developed using genetic algorithms and neural networks. References [5] and [6] explain how to model airplane fuel consumption in terminal areas to support trajectory prediction in ATM simulation for environmental and operational decision-making.

One of the most widespread APM databases for ATM research and assessment purposes is the Base of Aircraft Data (BADA), whose features and capabilities are presented in [7] and [8]. BADA has been used in many cases for ATM research applications, such as the trajectory computation infrastructure in [9] and EUROCONTROL's large scale real-time simulation platform ESCAPE (EUROCONTROL Simulation Capability and Platform for Experimentation). Also many operational applications make use of BADA, such as the NASA's center terminal radar approach control (TRACON) automation system (CTAS), [10]. Another widespread source of performance data is the Aircraft Noise and Performance (ANP) database, whose primary purpose is supporting aircraft noise assessments. The ANP database is an online data resource accompanying the European Civil Aviation Conference (ECAC) Doc 29, 3rd Edition [11], and ICAO Doc 9911 guidance documents on airport noise contour modelling.

Although the aforementioned models and methodologies provide a solid framework to model conventional take-off operations, none of them provides a validated and generic methodology to model take-offs with flexible thrust, i.e. executed at less than full thrust power to reduce engine wear and noise. In order to produce useful ATM simulations that keep fidelity with real operational scenarios, it is necessary to model flexible thrust, in particular for an accurate estimation of noise, fuel consumption and resulting emissions during take-off.

In [11] a very simple approach to compute flexible thrust was proposed. This formula was derived from Flight Data Recorder (FDR) data analyses that showed a correlation between thrust reduction and the ratio of the actual take-off weight to the regulated take-off weight. This method was proven inaccurate in [12], where the results obtained with the previous relationship were compared to the thrust reduction data generated with official manufacturer's take-off performance software. The shortcomings of such a model leave a gap in thrust modelling for take-off as no validated and generic model for reduced thrust is available nowadays.

This paper proposes a methodology for modelling flexible thrust with the assumed temperature (AT) method by combining an AT polynomial model identified from manufacturer take-off performance data and a thrust model taken from typical ATM performance databases. For that end, two of the most commonly used APMs in ATM simulation, i.e. BADA and ANP, are used to validate the results by comparing them with manufacturer data.

II. REDUCED THRUST OPERATIONS

The required thrust for take-off depends on several factors, such as runway length, take-off weight (TOW), wind conditions, runway elevation and slope, or outside air temperature (OAT). In the majority of take-offs the full-rated thrust[1] is typically higher than the thrust strictly required for a safe take-off [12]. Thus, in those situations it would be possible to take-off with a thrust level lower than this maximum. This procedure is known as reduced thrust —or flexible thrust— operations, which have the advantage of extending the engines' life usage while reducing engine maintenance costs.

Thrust reduction in take-off can be accomplished by either using a manufacturer-provided engine reduced rate —or derate— or using the assumed temperature method, where the thrust reduction is achieved by selecting the rated thrust for a temperature that is higher than the OAT. This is the most common method for take-off thrust reduction.

A. The Assumed Temperature Method

In order to protect the engine from damage and excessive wear and deterioration, the engine manufacturer provides an engine rating, which limits the maximum certified thrust that an engine can provide under certain conditions. This thrust limit is a consequence of the operational limitations imposed by the engine's combustion inlet pressure, the turbine inlet temperature and the fan rotation speed. The combination of such limitations provides a maximum allowable thrust that depends on the air temperature, and contains a pressure-limited and a temperature-limited zones.

A maximum thrust rating —full-rate— is typically established by the manufacturer so that the aforementioned thrust limitations are fulfilled and a safety margin is kept. This situation is depicted in Fig. 1. The thrust rating contains a flat-rated and a temperature-rated area. In the latter, the maximum allowable thrust decreases with the OAT. The temperature at which the temperature-dependent rating starts is the kink temperature T_{K}.

In reduced thrust operations, the minimum thrust required for a safe take-off $T_{\min}$ is computed based on the take-off conditions. Then, it may be possible to find a certain temperature within the temp-rated zone so that the maximum thrust provided by the thrust rating corresponds to $T_{\min}$. If that is the case, this temperature is introduced in the flight management system (FMS) as a *fictitious* OAT —or assumed temperature (AT). As a result, the engine control system will establish a functioning regime for the engine as if the air intake temperature was AT. Nevertheless, since the assumed OAT is higher than the real OAT, the resulting thrust provided by the engine for that regime will be higher than the thrust required $T_{\min}$, providing an extra safety margin.

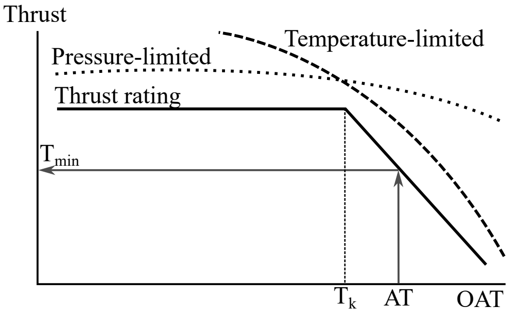

Fig. 1. Thrust limitations due to external temperature and inlet pressure.

Following this method, it is possible to perform a take-off using the minimum thrust required for the operation. Note that the maximum allowable AT depends on the take-off distance available (TODA), and other variables such as wind conditions, runway slope and contamination, flaps/slats configuration used, obstacles, etc.

B. Benefits of Reduced Thrust Operations

The usage of reduced thrust operations is a widespread practice in the majority of current take-off operations in civil aviation. In 2009 the proportion of engine maintenance to direct maintenance cost was 43% for the 40 maintenance cost task force (MCTF) participating airlines [13]. As engine maintenance material cost (MMC) is a significant part of the total maintenance expenses, using methods to optimize the aircraft engines' life usage has become a common practice for today's airlines. As exposed in [14], a reduction in the engines Exhaust Gas Temperature (EGT) can noticeably reduce engine deterioration and life usage, leading to savings in MMC of about 25% for a reduction of 7% of the EGT, and up to a 40% for an 18% EGT reduction [15]. These results encourage airlines to operate engines at the minimum thrust level required for the safe operation of the aircraft. Reduced thrust operations are amongst the most widespread solutions.

III. MODEL DEFINITION

In order to provide a complete framework to model flexible thrust operations using AT, two mathematical models are required. In first place, the AT that is used under given take-off conditions needs to be modelled. In second place, it is required a thrust model that captures the effect of the AT to compute reduced thrust. In this paper, a polynomial AT model identified from manufacturer take-off performance data is proposed. Then, the BADA and ANP-based thrust models

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[1] The full-rated is the maximum thrust that the engines are capable of producing for a certified take-off thrust under the existing conditions of temperature and pressure altitude.

are validated to provide realistic thrust reductions when the obtained AT is used to compute thrust.

A. The Assumed Temperature Model

The exact AT that is used in a particular take-off is in essence not a deterministic variable, since the operator can choose any temperature within the allowed range. However, reduced thrust operations aim to optimize the TODA by using the minimum thrust setting that allows a safe take-off, i.e. a thrust level so that the take-off distance (TOD) equals the TODA. Consequently, it can be assumed that an operator will always use the maximum AT possible, so that the minimum thrust T_{min} is used and TOD=TODA. This maximum AT can be modelled as a deterministic function of the take-off conditions, and will be hereinafter referred to as simply AT.

The take-off requirements basically consider the capability of the aircraft to avoid ground-based obstacles by achieving a minimum certified rate of climb in case of engine failure. These limitations define the maximum thrust reduction that can be applied so that a safe take-off can be performed, and therefore determine the AT. The factors affecting such limitations are:

- Airframe configuration: High lift devices (flaps/slats) have an effect on the aircraft lift, and consequently on the induced drag. When flaps angle is increased, the stall speed is reduced and so is the required take-off speed. Additionally, the increased drag increases the required thrust, limiting the allowable thrust reduction obtainable with the AT method. Flap deployment reduces the TOD.
- Take-off weight: The TOW determines the required lift to overcome the weight force. Higher TOW results in an increased lift-off speed, which leads to an increased TOD. Consequently, higher TOW reduces the applicability of the AT method.
- Runway elevation: Thrust and lift are proportional to air density, which is determined by the atmospheric pressure, OAT and humidity at the considered elevation. At higher elevations, atmospheric pressure and air density are lower, causing a reduction in the engine capability to produce thrust. Hence the applicability of the AT method is more restrictive at higher airport elevations.
- Outside air temperature: The efficiency of jet engines depends on the OAT. For higher OAT the air density drops, resulting in a reduction in the thrust produced by the engine and consequently in an increased TOD.
- Wind: Lift and drag directly depend on airspeed, whereas the TOD depends on ground speed. Headwind reduces ground speed, decreasing the TOD.
- Runway length: Greater thrust reduction can be applied to longer runways, since they imply a higher TODA.
- Runway slope: A positive slope produces degradation in the acceleration phase. In this way, the AT would be lower in order to obtain more thrust than in negative slope. Higher speed is required for take-off, increasing the TOD.
- Obstacles around the airport: TOD is limited since a certain rate of climb and take-off path angle are needed to ensure a safe margin over obstacles. This affects the thrust required and therefore the maximum AT that can be used in the operation.
- Runway contamination: Runway surface contamination conditions impact the grip of the landing gear's tires, increasing the TOD. According to the Federal Aviation Administration (FAA) regulations in [16], operating with reduced thrust is prohibited under such conditions.

In this paper, the AT is modelled as a polynomial function of the take-off conditions identified from manufacturer take-off performance data. An initial version of this kind of model was presented in [12] and [17] by Boeing Research & Technology Europe (BR&TE). In [18], new features were added to the model to improve and test its accuracy.

The proposed AT model specification provides, for each flaps configuration i, a constant temperature $AT^{(i)}_{\text{max}}$ for the flat-rated zone and a polynomial function $AT^{(i)}_{\text{temp}}$ for the temperature-rated zone

$$AT^{(i)}_{\text{temp}} = f(h, w, l, m), \tag{1}$$

where h, w, l and m are respectively the runway geopotential pressure altitude, wind, runway length and TOW. Runway slope is typically very small and its effect was neglected for the sake of model simplicity. $f(h, w, l, m)$ is a polynomial function that can be defined with different orders. Obstacles depend on the particular runway environment where the take-off operation takes place, therefore it was also not included in the model. The results presented in [12] showed that the OAT has no effect on the calculation of the maximum AT. Since reduced thrust operations are forbidden in contaminated runways [16], runway contamination is not considered as an input to the model. The AT is computed as the minimum between the temperature-rated and the flat-rated values:

$$AT^{(i)} = \min\{AT^{(i)}_{\text{max}}, AT^{(i)}_{\text{temp}}\}, \tag{2}$$

B. The Thrust Model

Typically, trajectory predictors for ATM research use three-degrees-of-freedom (3DoF) approaches to model aircraft performance. Such models simplify aircraft dynamics to three translations, neglecting angular rates and moments, which have faster dynamics. This simplification dramatically reduces the complexity of the model while providing sufficient accuracy for ATM simulations. Ref. [19] provides an excellent review on the main APM and data sources commonly used for ATM applications. From the models therein mentioned, the thrust models of BADA and ECAC Doc. 29 / ICAO Doc 9911 guidance documents (with ANP data) were observed to be especially interesting for the purposes of this study.

1) The BADA Thrust Model: BADA is an APM specifically intended for ATM applications created and maintained by EUROCONTROL in cooperation with aircraft manufacturers and operating airlines. It is a total energy model (TEM) based on a 3DoF kinetic point-mass approach. The version BADA 3 provides close to 100% coverage of aircraft types in the ECAC area, whereas the newly developed BADA 4 offers a coverage of about 70% [20]. Since BADA is a *de facto* standard in ATM trajectory prediction and simulation, its use for flexible thrust modelling may ease the applicability of the model in many modern trajectory predictors. The BADA thrust model used in this paper corresponds to BADA 4 [21].

BADA 4 provides thrust models for turbofan, turboprop and piston engines. However, the AT methodology only applies to turbofan engines, therefore only turbofans were considered in this study. The thrust contribution from all engines is modelled in the form:

$$TF = \delta \, W_{\text{mref}} \, C_{\text{T}}, \tag{3}$$

where TF is the thrust force, δ the atmospheric pressure normalized by the atmospheric pressure at sea level according to the international standard atmosphere (ISA) conditions [22] or pressure ratio, W_{mref} the weight force when the aircraft mass equals the reference mass provided in BADA and C_{T} the thrust coefficient. C_{T} has different formulations for idle or non-idle ratings. For take-off, only non-idle ratings are considered, the expression for C_{T} being

$$C_{\text{T}} = \sum_{i=0}^{5} \delta_{\text{T}}^{i} \left(\sum_{j=1}^{6} a_{6i+j} \, M^{j-1} \right), \tag{4}$$

where δ_{T} is the throttle parameter, M the Mach number and $\{a_1, ...a_{36}\}$ the identification coefficients for the specific aircraft model given in the BADA database.

The throttle parameter δ_{T} represents the throttle lever position, which is the control input for the engine and defines the thrust rating as a function of the OAT, δ and M. Consequently, δ_{T} is defined by a flat-rated and a temp-rated functions, respectively $\delta_{\text{T,flat}}$ and $\delta_{\text{T,temp}}$, separated by a kink point. Such a point is not defined as a temperature, but as a temperature deviation from ISA conditions $\Delta T_{\text{ISA,k}}$. When ΔT_{ISA} is lower than $\Delta T_{\text{ISA,k}}$ the engine behaviour is limited by the internal pressure and δ_{T} equals $\delta_{\text{T,flat}}$. Otherwise, the engine operates in the temperature-rated area and δ_{T} equals $\delta_{\text{T,temp}}$:

$$\delta_T = \begin{cases} \delta_{\text{T,flat}} & \text{if } \Delta T_{\text{ISA}} \leq \Delta T_{\text{ISA,k}} \\ \delta_{\text{T,temp}} & \text{if } \Delta T_{\text{ISA}} > \Delta T_{\text{ISA,k}}. \end{cases} \tag{5}$$

The expressions for $\delta_{\text{T,flat}}$ and $\delta_{\text{T,temp}}$ are given by (6) and (7), where θ_{T} is the total temperature ratio and $\{b_1, ...b_{36}\}$, $\{c_1, ...c_{45}\}$ are the identification coefficients for the specific aircraft model given by the BADA database. θ_{T} is given by (8), where T_{T} is the total temperature, T_0 the temperature at sea level in ISA conditions, γ the adiabatic index of air and T the static air temperature in Kelvin.

$$\delta_{\text{T,flat}} = \sum_{i=0}^{5} \delta^{i} \left(\sum_{j=1}^{6} b_{6i+j} \, M^{j-1} \right) \tag{6}$$

$$\begin{aligned} \delta_{\text{T,temp}} = \sum_{i=1}^{5} c_i M^{i-1} + \sum_{j=1}^{4} \theta_T^{j} \left(\sum_{i=0}^{4} c_{5(j-1)+(i+1)+5} M^{i} \right) \\ + \sum_{j=1}^{4} \delta^{j} \left(\sum_{i=0}^{4} c_{5(j-1)+(i+1)+25} M^{i} \right) \end{aligned} \tag{7}$$

$$\theta_T = \frac{T_T}{T_0}; \quad T_T = \left(1 + \frac{\gamma - 1}{2} M^2 \right) T \tag{8}$$

2) The ANP-based Thrust Model: The ANP database provides noise and performance data for specific aircraft (airframe-engine) types, which are used in conjunction with the calculation method described in the ECAC Doc.29 and ICAO Doc 9911 guidance documents to compute noise contours around civil airports. These documents provide a performance modelling method to calculate the aircraft trajectory from a given flight procedure, along with the engine thrust, which is further used to characterize the noise source state.

The method includes in particular a thrust model, which uses engine coefficients available in the ANP database. This reference data is especially reliable, since it is, in most cases, supplied by aircraft manufacturers, in accordance with a specific ANP Data Request Form developed and maintained within ICAO. The ANP-based thrust model provided in the ECAC and ICAO guidance documents was therefore considered as a possibility to be applied for flexible thrust modelling.

The ANP-based thrust model consists of a formula to compute the net thrust available for a specified thrust rating, and another one for the net thrust when the thrust-setting parameter (EPR or N1) is set to a particular value. Since flexible thrust is used during take-off operations, where the engine is set to take-off and go-around (TOGA) thrust rating, only the former formula was considered for this study. The net thrust for a thrust rating is given by

$$TF = n \, \delta \left(E + F \, V_{\text{CAS}} + G_A \, h + G_B \, h^2 + H \, T \right), \tag{9}$$

TF being the total net thrust in pound-force, n the number of engines, δ the pressure ratio, V_{CAS} the Calibrated Airspeed (CAS) in knots, h the geopotential pressure altitude in feet and T a temperature input in Celsius degrees. E, F, G_{A}, G_{B} and H are identification coefficients given by the ANP database for a given aircraft model.

ANP provides two sets of identification coefficients per aircraft model: the low-temp and high-temp coefficients, used in the flat-rated and temp-rated areas, respectively. Note that, for the flat-rated area, $H = 0$. Although T_{K} is not provided explicitly, it can be computed as the intersection between the flat-rated and the temp-rated areas with (10), where the subscripts $(.)_{\text{L}}$ and $(.)_{\text{H}}$ refer to the low-temp and high-temp coefficients, respectively.

$$\begin{aligned} T_{\text{K}} = \frac{1}{H_{\text{H}}} [(E_{\text{L}} - E_{\text{H}}) + V_{\text{CAS}}(F_{\text{L}} - F_{\text{H}}) + h(F_{\text{A,L}} - F_{\text{A,H}}) \\ + h^2 (G_{\text{B,L}} - G_{\text{B,H}})] \end{aligned} \tag{10}$$

TABLE I
TAKE-OFF CONDITIONS FOR THE GENERATION OF REFERENCE AT DATA

Parameter	***Values***	
Wind (kt)	-15:5:40	
Elevation (ft)	0:100:5000	
Runway length (ft)	8000:500:15000	
TOW (1000 kg)	B737	02:05.5
	B757	80:5:134
	B777	200:10:378
Flaps (deg)	B737	1, 5, 10, 15, 25
	B757	5, 15, 20
	B777	5, 15, 20

IV. MODEL VALIDATION

Once the mathematical framework needed to model all the relevant aspects of reduced thrust operations was defined, the validity and accuracy of the selected AT and thrust models was determined through validation. The AT model was tested to provide reliable and accurate results by comparison with thrust performance data provided by BR&TE. The BADA and ANP-based thrust models were tested to provide realistic thrust reduction when the AT obtained with the previous model was used as a temperature input. This validation was supported with climb out performance data and simulation software provided by BR&TE.

A. Validation of the Assumed Temperature Model

The validation of the AT model was carried out for two narrow-body and one wide-body typical Boeing aircraft. Boeing's Standard Take-off Analysis Software (STAS) was used to generate a set of reference AT data. STAS is the official Boeing's take-off performance tool, and provides take-off performance tables from which the AT was derived for the combinations of take-off conditions listed in Table I. Parameter identification was carried out by the Minimum Mean Square Error (MMSE) method to identify the polynomial coefficients and $AT_{\max}^{(i)}$ in (1) and (2) for the three aircraft models. The AT identification data was obtained from STAS as stated above. Polynomials of first, second and third order were identified for each aircraft model. Finally, such polynomials were used to compute the AT for the combinations of parameters in Table I. The accuracy of the model for different polynomial orders was assessed by comparison of the polynomial results to the reference AT data obtained from STAS. The results of this validation will be presented in section V.

B. Validation of the Thrust Models

In this paper the BADA and ANP-based thrust models were tested to provide a realistic reduced thrust when an AT is used a temperature input to the models.

1) Using Assumed Temperature with BADA: To compute flexible thrust with the AT method, T was substituted by the AT in (8). Note that, in such cases, the *physical* parameters depending on the OAT —i.e. M in (4)— can still be computed with the actual OAT, so that the AT only has an effect on the throttle level and does not affect the expressions where physical phenomena are modelled. This fact ensures that, whereas thrust is reduced by limiting the value of δ_{T}, the actual OAT is still taken into account to capture, for instance, the effect of air density and temperature on the thrust produced by the engine.

2) Using Assumed Temperature with the ANP-based Thrust Model: Reduced thrust with ANP data was computed by substituting T by the AT in (9). Unlike BADA, the ANP-based thrust model contains a single temperature input. Consequently, the substitution of T by the AT results in an added error, since not only the throttle level limitation is affected by the increased temperature, but also the physical process of thrust production is considered to happen at OAT=AT —which is not the case in real operations.

3) Experimental Setup: For the validation of the thrust models, 56,700 different take-off procedures were simulated with the Boeing Climbout Program (BCOP), the standardized software used by Boeing and customer airlines for take-off and approach studies in terminal area. BCOP uses the standard low speed Boeing performance databases, which are based upon flight test data and use a Boeing's private specification. This software allows the use of AT for the simulation of flexible thrust procedures.

The take-off trajectories were defined and coded with the Standard Computerized Airplane Performance (SCAP) specification for climb out, which defines the interface requirements for manufacturer-provided performance modules to be implemented in the airline user environment. MATLAB was used to generate SCAP files and launch BCOP. Each take-off procedure represents one particular airframe and engine combination for a set of given take-off conditions (i.e. weight, runway length and elevation, wind, OAT, AT and flap setting).

The take-off trajectories were defined in several segments. The thrust setting used throughout all the operation was the maximum take-off thrust (MTKF) or TOGA with flexible thrust reduction. The take-off segment encompassed from brakes release until reaching 35ft over the runway threshold at the safety take-off speed of V2 plus 10 kt. This segment was followed by a constant CAS climb until gear retraction, a constant CAS climb until the acceleration height of 400 ft, and finally a constant altitude acceleration with flap retraction until achieving clean configuration. Subsequent climb segments are typically operated at maximum continuous thrust (MCT) and are not of interest for this study because thrust reduction does not apply anymore. The values of the take-off parameters varied between simulations are listed in Table II. All trajectories were simulated using the B737's APM from BCOP.

The obtained trajectories contained a set of thrust values corresponding to different combinations of atmospheric and flight conditions with different AT settings. These data were used as inputs for the BADA and ANP-based thrust models to compute reduced thrust values for different flight conditions and ATs. The results were compared to the reference thrust values of the BCOP trajectories to assess the validity and accuracy of each thrust model when using AT.

The objective of this study was to observe the error added

TABLE II

PARAMETERS VARIED IN THE SIMULATED TAKE-OFF TRAJECTORIES

Take-off parameter	***Values***
Flaps (deg)	1, 5, 10, 15, 25
Elevation (ft)	0, 1500, 3000, 4500
Runway length (ft)	9000, 12000, 15000
TOW (1000 kg)	50, 65, 75
OAT (°C)	-20:5:50
AT (°C)	-20:5:80

to the model as a consequence of using AT. BADA and ANP already have a certain error associated to the thrust model itself. In order to isolate the error introduced by the AT, the difference between the model and the reference data when AT=OAT —i.e. when no thrust reduction is applied— was subtracted from the absolute error of all data points.

V. RESULTS

This section presents the results obtained from the validations of the AT polynomial model and the use of AT to compute thrust with BADA and ANP-based thrust models.

A. Results of the Assumed Temperature Model Validation

AT polynomials of first, second and third order —with 5, 15 and 35 coefficients respectively— were identified and evaluated for each aircraft model and flaps configuration. Fig. 2 shows an example of the AT estimation for a first order polynomial and fixed flaps, wind and TOW conditions. Note that the maximum AT has a flat region that is accounted for in the AT polynomial model with the value $AT^{(i)}_{\text{max}}$. This flat region has an unique value for the dependencies of AT with runway length, wind and TOW, but has different values depending on the airfield elevation (see Fig. 2b). As a consequence, there are different possible choices for the value of $AT^{(i)}_{\text{max}}$.

In this paper different $AT^{(i)}_{\text{max}}$ values were considered separately: the lowest and highest values, and the value that applied to the widest range of elevations (i.e. 65°C in Fig. 2b). Different model accuracies were observed for different choices of $AT^{(i)}_{\text{max}}$. However, no single option was observed to simultaneously provide the best accuracy for all aircraft models and polynomial orders. Since the $AT^{(i)}_{\text{max}}$ offering the best accuracy could not be determined *a priori*, the solution for its choice was to compute the overall accuracy for all possible $AT^{(i)}_{\text{max}}$ and choose the value that provides the best fitting.

The root mean square error (RMSE) between the AT model and the STAS data was observed to decrease for higher-order polynomials and higher flap angles. The RMSE obtained for first-order polynomials was within the range of 6 to 11 °C, for second-order of 5 to 8 °C and of 3 to 6 °C for third-order polynomials. Since STAS provides temperature values with a resolution of 5 °C, the observed accuracy turns out to be acceptable in the three cases. It must be noted, however, that the number of coefficients of the model grows exponentially with the order of the polynomial, increasing its complexity. Whereas the improvement in accuracy from first to second order polynomials is noticeable, increasing the order from second to third order leads to a very slight improvement in accuracy, which falls below the resolution of the reference data and therefore can be considered as irrelevant.

Second-order polynomials are suggested as a fair trade-off between complexity and accuracy. The RMSE obtained for second-order polynomials of three aircraft models considered for different flaps configurations is shown in Table III.

TABLE III

RMSE OF THE AT MODEL FOR DIFFERENT AIRCRAFT AND AIRFRAME CONFIGURATIONS

Aircraft	**RMSE (°C) – Second-order polynomial, 15 coef.**				
	Flaps 1	**Flaps 5**	**Flaps 10**	**Flaps 15**	**Flaps 25**
B737	7.90	6.81	5.73	5.31	5.12
Aircraft	**Flaps 5**		**Flaps 15**	**Flaps 20**	
B757	7.58		7.54	6.77	
B777	8.52		7.84	7.68	

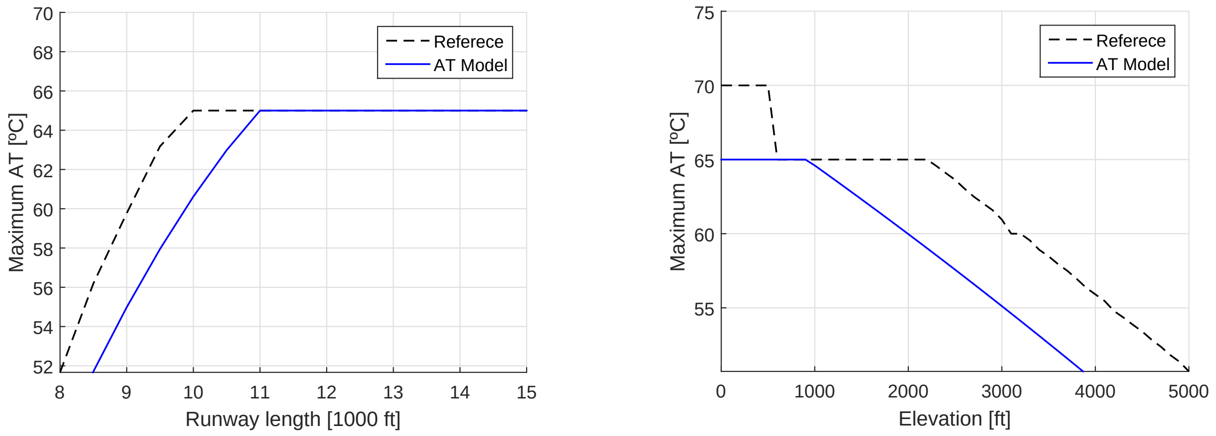
B. Results of the Thrust Models Validation

The validation of the BADA and ANP models was based on the computation of thrust and thrust reduction as a function of AT for different conditions of airspeed, OAT and pressure altitude obtained from the climb out simulations. The net thrust was computed using ANP and BADA formulas discussed in section III-B. The thrust reduction was defined as the percentage reduction of net thrust achieved with a given AT relative to the baseline or full-rated thrust under the same conditions when no AT was applied. The baseline thrust was taken from BCOP for a take-off with AT=OAT. The thrust reduction for a given set of conditions of airspeed, OAT and pressure altitude (n) was defined as

$$TR^{(n)} = \frac{T^{(n)}_{\text{BCOP}}(OAT) - T^{(n)}(AT)}{T^{(n)}_{\text{BCOP}}(OAT)} \cdot 100 \qquad (11)$$

where $TR^{(n)}$ is the thrust reduction for the given conditions, $T^{(n)}(AT)$ the net thrust obtained from either BADA, ANP or BCOP for a given AT, and $T^{(n)}_{\text{BCOP}}(OAT)$ the baseline thrust obtained from BCOP for AT=OAT.

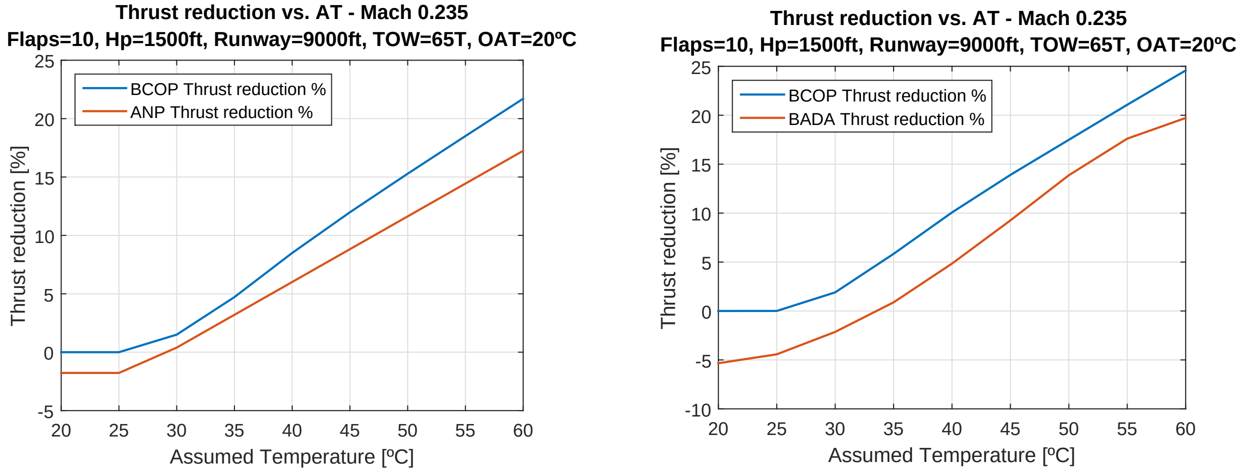
During the validation process, the BADA 4 polynomial formula for the maximum take-off regime provided very high thrust values when used at speeds lower than Mach 0.235 and close to 0. The reason is that the parameter identification of the current BADA thrust model uses a set of flight data in airborne conditions, this means above Mach 0.235, and therefore the model is not valid out of this range. As a consequence, BADA was only validated using BCOP data above Mach 0.235 to determine the validity of the model for thrust reduction using AT, but it is worth pointing out that for the model to be applicable to the full take-off phase —from brakes release at the runway threshold up to the change to maximum climb thrust regime—, the range of parameter identification data needs to be extended towards Mach 0 to also capture runway acceleration.



(a) AT vs. Runway length

(b) AT vs. Airfield elevation

Fig. 2. First order polynomial AT model results for runway length 1100ft, elevation 900ft, no wind, flaps 10 and TOW 65 t.



(a) Thrust reduction for BCOP and ANP

(b) [Thrust reduction for BCOP and ANP

Fig. 3. Thrust reduction for the B737 at Mach 0.235 and flaps 10.

Fig. 3 shows the obtained thrust reduction for a specific set of take-off conditions at a given speed as a function of the AT. Note that both models have a certain error for AT=OAT. This error is the error of the model, and cannot be associated to the use of AT. As a consequence, for AT=OAT the thrust obtained from both ANP and BADA is higher than the thrust from BCOP, resulting in a negative thrust reduction associated to the error of the model up to a given point. As the value of AT increases, thrust reduction builds up until positive reductions are achieved for ATs of 30°C for ANP and 35°C for BADA at these particular conditions. The overall effect of the error of the model is an offset of the TR-AT curve with respect to BCOP. This means that the model overestimates emissions, noise, etc., leading to a conservative margin.

The RMSE in thrust and thrust reduction for both models and the reference BCOP data was assessed for the whole range of the data set (see Table II). In every data point, the error of the model was subtracted from the total error to differentiate between the error associated to the use of AT and the error of the thrust model itself. The resulting absolute and relative errors in thrust are listed in Table IV. The absolute errors in thrust reduction are listed in Table V. The AT column of such tables indicates the RMSE of the difference between the total observed error and the model error.

Note that when AT is used in the ANP thrust equation no temperature input is left for the OAT, so the physical process of thrust generation is considered to occur at AT. Since AT is always higher than the OAT, a higher temperature is considered for the physical process, which means lower air density and therefore less thrust. This leads to a negative error associated to the use of AT —the modelled thrust is lower than the real one— that compensates for the positive error of the ANP model, resulting in a reduction of the total error. This effect can be observed in the region from 25 to 35 °C in Fig. 3a.

For both models it can be observed that the AT and model errors compensate for each other in some regions. The overall

TABLE IV
THRUST RMSE FOR BADA AND ANP

Thrust RMSE						
APM	**Absolute error [lbf]**			**Relative error [%]**		
	Total	**Model**	**AT**	**Total**	**Model**	**AT**
BADA	873	833	618	4.88	4.10	3.42
ANP	685	495	307	3.16	2.02	1.64

TABLE V
THRUST REDUCTION RMSE FOR BADA AND ANP

Thrust reduction RMSE [%]			
APM	**Total**	**Model**	**AT**
BADA	4.01	4.10	5.80
ANP	2.68	2.02	3.92

error increase in this particular data set due to the introduction of AT is around 0.8% for BADA and 1.1% for ANP in terms of net trust and -0.1% for BADA and 0.7% for ANP in terms of thrust reduction. These results show that reduced thrust using AT can be computed by using the hereby proposed polynomial approximation for AT as a temperature input for the thrust models of BADA and ANP without a significant deterioration on their overall accuracy.

VI. CONCLUSION

This paper proposes a methodology to model flexible thrust with the assumed temperature method by means of an AT polynomial model. Typical ATM performance databases were tested to produce reliable and accurate thrust reductions when the modelled AT is used as a temperature input.

The validation of the AT polynomial model reveals that first to third order polynomials with coefficients identified from manufacturer take-off performance data provide an estimation of the AT used by the operator under certain take-off conditions with sufficient accuracy to be used in ATM simulations. This AT has been validated to provide realistic thrust reduction when used as a temperature input for BADA and ANP-based thrust models without significant degradation of their overall accuracy. For the BADA 4 thrust model to be applicable to the full take-off phase, the range of parameter identification data needs to be extended towards Mach 0 to also capture runway acceleration.

In sight of these promising results, further validation of the AT polynomial identification methodology shall be conducted in the future for a wider range of aircraft models. The use of the methodology proposed in this paper in ATM simulations can enable ATM assessment tools to reproduce current take-off operations with better fidelity, so that more realistic what-if scenarios can be assessed and more reliable data can be made available to support decision-making in the development and deployment of future ATM concepts and technologies.

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