Assessment of the feasible CTA windows for efficient spacing with energy-neutral CDO

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Abstract—**Continuous descent operations (CDO) with controlled times of arrival (CTA) at one or several metering fixes could enable environmentally friendly procedures at the same time that terminal airspace capacity is not compromised. This paper focuses on CTA updates once the descent has been already initiated, assessing the feasible CTA window (and associated fuel consumption) of CDO requiring neither thrust nor speed-brake usage along the whole descent (i.e. energy modulation through elevator control is used to achieve different times of arrival at the metering fixes). A multiphase optimal control problem is formulated and solved by means of numerical methods. The minimum and maximum times of arrival at the initial approach fix (IAF) and final approach point (FAP) of an hypothetical scenario are computed for an Airbus A320 descent and starting from a wide range of initial conditions. Results show CTA windows up to 4 minutes at the IAF and 70 seconds at the FAP. It has been also found that the feasible CTA window is affected by many factors, such as a previous CTA or the position of the top of descent. Moreover, minimum fuel trajectories almost correspond to those trajectories that minimise the time of arrival at the metering fix for the given initial condition.**

I. Introduction

With the awareness of global warming and the rising of fuel prices, reducing the environmental footprint of aviation has become one of the main concerns of the different aviation stakeholders. Continuous descent operations (CDO) have demonstrated and proven success in the reduction of emissions, fuel consumption and noise nuisance in terminal maneuvering areas (TMA) [1]. In order to get the maximum benefits of CDO, aircraft should descent with the engines at idle from the top of descent (TOD) down to the stabilisation point, where the aircraft is configured and ready for landing. Due to the difficulty for air traffic controllers (ATC) to accurately predict the vertical profile of these descent trajectories, larger separation margins are required and in the majority of TMA these procedures are limited to off-peak hours, where traffic demand is lower.

An approach to enable CDO while maintaining capacity is to assign to each aircraft a controlled time of arrival (CTA) at a metering fix to safely merge incoming traffic [2]. Several studies [3]–[6] have dealt with the assignment of CTA while the aircraft is still in cruise, well before starting the descent. For instance, [3]–[5] computed the feasible CTA window at a metering fix allowing the aircraft to adjust the TOD position and the descent speed profile. With similar purposes, Ref. [6] also enabled the addition and omission of waypoints to stretch or reduce the flight path length in addition to the adjustment of the TOD position when necessary.

In the future trajectory-based air traffic management (ATM) paradigm, we could envisage ATC updating the CTA once the descent has been initiated, and even assigning a CTA to more than one fix along the route. In such cases (and aiming at minimising the environmental footprint), aircraft should be capable to keep on the CDO without requiring neither additional thrust nor speed-brakes usage, taking advantage of energy modulation to adjust the speed profile by means of elevator control only [7], [8]. The robustness of CDO trajectories in the face of late changes to the CTA during the descent was assessed in [9]. Aiming to minimise the impact on the optimality of the CDO, only elevator control was permitted to adjust the time at which the metering fix is passed, assuming a rather simple Mach/calibrated airspeed (CAS) profile and allowing a single (and instantaneous) modification to the scheduled speed. Furthermore, this assessment was performed for few initial conditions and the employed method could not ensure optimality of the minimum and maximum arrival times.

Another limitation of the previous works is that none of them took into account the remaining descent between the metering fix and the runway threshold. Adjusting the speed profile to minimise or maximise flight time may result in a change to the altitude at which the fix is passed. If the energy of the aircraft at this fix is too low, additional thrust would be needed after overflying it. Similarly, if the energy is too high it would be required to use speed brakes and/or to deploy high-lifting devices or the landing gear earlier. Furthermore, all of them used the BADA v3.x performance model, which has already shown some limitations for accurate trajectory prediction in TMA [10], [11]. More sophisticated aerodynamic drag and engine models are needed in order to compute realistic descent profiles and obtain accurate fuel consumption and flight time figures.

In this paper, we consider a scenario in which the ATC notifies a CTA update once the aircraft has started the descent. The earliest and latest trajectories at a hypothetical initial approach fix (IAF) and final approach point (FAP) are computed for a wide range of initial conditions aiming at quantifying the CTA window as a function of the altitude and distance to the runway threshold. These trajectories are restricted in such a way that neither thrust nor speed-brakes usage is allowed during the descent. Only energy modulation, by means of elevator control, is left to modify the speed profile and meet the CTA. Moreover, the whole descent is subject to this optimisation in order to ensure a completely idle descent without speed brakes down to the stabilisation point. Finally, accurate aircraft performance data derived from Airbus Performance Engineering Program (PEP) are used to model drag, engine thrust and fuel flow.

II. Background

In the recent years, several research has focused on the use of energy principles to perform accurate, time-constrained, engine-idle descents to reduce the environmental impact of aviation [7], [8], [12]. The idea behind time and energy management is to exchange altitude for speed and vice versa to gain or lose time and energy through elevator control. Following this process, deviations from the plan (including CTA updates) are corrected without the need for additional thrust or speed-brakes usage, leading to what is called an *energy-neutral* trajectory. The earliest and latest energy-neutral trajectories at a metering fix can be computed by solving a trajectory optimisation problem, where the time of arrival at the fix is minimised or maximised at the same time different operational constraints are fulfilled (and namely the impossibility to use thrust or speed brakes during the descent).

Trajectory optimisation requires the definition of a mathematical model describing the aircraft dynamics along with a model for certain atmospheric variables. Section II-A shows these equations of motion, while section II-B provides a detailed description of the time and energy management concept. Finally, in Section II-C the formulation of the optimisation problem for an aircraft trajectory is presented.

A. Aircraft point-mass model

In this paper, the dynamics of the aircraft are expressed by the following set of non-linear differential equations, assuming a point-mass representation of the aircraft reduced to what is commonly called a gamma-command model (where continuous vertical equilibrium is assumed):

$$\begin{aligned} \frac{\mathrm{d}v}{\mathrm{d}t} &= \dot{v} = \frac{T-D}{m} - g\sin\gamma \\ \frac{\mathrm{d}s}{\mathrm{d}t} &= \dot{s} = v\cos\gamma \\ \frac{\mathrm{d}h}{\mathrm{d}t} &= \dot{h} = v\sin\gamma \end{aligned} \tag{1}$$

where the state vector $\boldsymbol{x} = [v, s, h]$ is formed respectively, by the true airspeed (TAS), the distance to go and the altitude of the aircraft; T is the total thrust; D is the aerodynamic drag and g is the gravity acceleration. Since neither additional thrust nor speed-brakes usage is permitted, the control vector is composed by the aerodynamic flight path angle only (i.e. $\boldsymbol{u} = [\gamma]$). Note that the variations in mass m are neglected in the dynamic model because the fuel consumption during an idle descent is a very small fraction of the total mass [1]. In spite of that, the fuel flow FF is also computed to determine the amount of fuel consumption.

Regarding the atmosphere, the International Standard Atmosphere [13] model is considered, which defines the density ρ, pressure p and temperature τ magnitudes as functions of h. Moreover, this study assumes wind calm conditions. Finally, it should be noted that operational constraints are usually given in terms of the Mach number or CAS, which both can be computed as a function of the TAS and the atmospheric magnitudes.

B. Time and energy management

The total energy E_t of an aircraft is the sum of its kinetic energy E_k and potential energy E_p:

$$E_t = E_k + E_p = \frac{1}{2}mv^2 + mgh. \tag{2}$$

The energy rate can be obtained by differentiating Eqn. (2):

$$\dot{E}_t = mv\dot{v} + mg\dot{h}. \tag{3}$$

By combining Eqns. (3) and (1) the total energy rate can be expressed in terms of the forces acting upon the aircraft:

$$\dot{E}_t = v\,(T - D)\,. \tag{4}$$

According to Eqn. (4) the total energy of an aircraft can be increased by applying thrust and decreased by increasing drag. In addition, the law of conservation of energy states that potential energy can be exchanged for kinetic energy and vice versa through energy modulation. It is well known that thrust and speed-brakes are the most effective means of increase and decrease the total energy of the aircraft, whereas elevator control provides an effective mean to modulate energy.

During descent (and in order to reduce the fuel consumption and noise nuisance) the aircraft engines are set to idle thrust and the speed-brakes usage is minimised. In such conditions, it is still possible to trade altitude for acceleration and vice versa using the elevator control only, adjusting the airspeed profile as desired by means of energy modulation. Namely, if the aircraft requires a higher velocity, it could loose altitude instead of applying additional thrust. Alternatively, it could reduce speed by pitching up instead of using drag devices.

C. Optimal control problem formulation

The optimisation of an aircraft trajectory can be formulated as a multi-phase constrained optimal control problem, in which it is desired to determine the controls of a system such that a given cost function J is maximised or minimised while satisfying a set of constraints. For some simple problems, the solution can be obtained analytically from the necessary and sufficient conditions of optimality [14]. However, finding solutions when strong nonlinear functions or interior point constraints appear in the definition of the model (as in the problem discussed herein) is not a straightforward task. In such cases, it is necessary to employ numerical methods [15].

Numerical methods for solving optimal control problems can be divided into two major classes: indirect methods and direct methods. In this paper, the latter have been used because they can easily cope with inequality constraints, among many other advantages. Such direct methods transform the original continuous (and thus infinite) optimal control problem into a (discrete and finite) nonlinear programming (NLP) optimisation problem. The time histories of control and state variables are discretised at a set of collocation points, being the system of ordinary differential equations (1) approximated by some continuous function (such as polynomials) over each collocation step. The values of these discretised variables, along with the non-time dependent parameters, become the unknowns of the new finite variable problem, which can be formulated as a NLP problem and solved by standard NLP solvers.

Several collocation schemes are proposed in the literature, being the trapezoidal collocation method the approach used in this paper. Trapezoidal collocation shows a good trade-off between accuracy and execution time needed to solve highly constrained NLP problems [15]. Further mathematical details on the formulation of optimal control problems for trajectory optimisation applications can be found in [16].

III. ACCURATE AIRCRAFT PERFORMANCE MODEL

Aiming to obtain accurate fuel consumption and flight time figures, a performance model for an Airbus A320 has been developed using accurate data from the manufacturer. Typical aerodynamic and engine data are specified in tabular form, generally obtained as a result of experimental tests. For optimisation solvers to work efficiently, continuity and differentiability for the right hand sides of the model equations are required. Although some of the performance data could be approximated quite well by polynomial functions, this method is prone to oscillation due to the Runge's phenomenon, leading to poor convergence or local minima issues.

In this paper, $\mathbb{C}^2$ is achieved by approximating the performance data by tensor product cubic B-splines as suggested in [15]. The aerodynamic drag is modelled as:

$$D = \frac{1}{2}\rho S v^2 C_D, \tag{5}$$

where C_D is the drag coefficient and S is the wing area. Aerodynamic data from the manufacturer has been taken to approximate C_D by a tensor product cubic B-spline:

$$C_D\left(C_L, M, h\right) = \sum_{i,j,k} c_{i,j,k} B_i(C_L) B_j(M) B_k(h), \tag{6}$$

where B_l (with $l \in \{i,j,k\}$) are B-spline basis functions and $c_{i,j,k}$ are the B-spline control points [17]; C_L is the lift coefficient, which is obtained by assuming continuous vertical equilibrium:

$$C_L = \frac{2mg\cos\gamma}{S\rho v^2}. \tag{7}$$

Following the same methodology, the idle thrust T_{idle} and idle fuel flow FF_{idle} are expressed as a function of h and M:

$$\begin{aligned} T = T_{idle}\left(h, M\right) &= \sum_{i,j} c_{i,j} B_i(h) B_j(M), \\ FF = FF_{idle}\left(h, M\right) &= \sum_{i,j} c_{i,j} B_i(h) B_j(M). \end{aligned} \tag{8}$$

Eqn. (8) is valid for all the descent down to the stabilisation point (typically at 1000 ft). For the remaining descent, T is left free between T_{idle} and the maximum thurst T_{max} and the corrected fuel flow $FF/\delta\sqrt{\theta}$ is computed as a function of the corrected thrust T/δ and M. Both T_{max} and $FF/\delta\sqrt{\theta}$ are also modelled by approximating the manunfacturer engine data with tensor product cubic B-splines.

IV. EARLIEST AND LATEST TRAJECTORY COMPUTATION

The optimisation process presented in this paper is a constrained non-lineal optimal control problem, as described in Section II-C. This study aims at computing minimum and maximum time trajectories at a given metering fix. Accordingly, the cost function is defined as:

$$J = t_{\text{FIX}}; \quad \text{FIX} \in \{\text{IAF},\ \text{FAP}\}, \tag{9}$$

where t_{FIX} is the time of arrival at the associated fix. Constraints on the aircraft dynamics are particularised by the point-mass model given by Eqn. (1) and generic box constraints on certain variables are specified as follows:

$$\gamma_{min} \leqslant \gamma \leqslant 0\,;\ \text{FAS} \leqslant v_{CAS} \leqslant \text{VMO}\,;\ M \leqslant \text{MMO}, \tag{10}$$

where γ_{min} is the minimum flight path angle; FAS is the final approach speed; and VMO and MMO are, respectively, the maximum operational CAS and Mach. The descent is split in different phases, where different path constraints may apply. Table I shows the different phases and the related constraints.

The first phase starts at the initial conditions h_0 and s_0, and ends at FL100. Below this altitude, ATC procedures typically restrict the CAS to 250 kt. Afterwards, the aircraft directs to the IAF, where the approach phase begins. Few nautical miles before the FAP, the aircraft decelerates to green dot speed GD[1] and starts configuring with flaps at constant altitude (still at idle thrust), in order to intercept the instrumental landing system (ILS) glide path at the FAP and at the S speed[2].

When descending on the ILS glide path, the aircraft decelerates at idle thrust and reaches the FAS at 1000 ft with the gear down and in landing configuration. The last phase terminates upon the aircraft reaching 50 ft over the runway threshold at the FAS. It should be noted that nominal flap/slat (and landing gear) transitions are also considered, but not depicted in Table I aiming to keep it simple.

Event constraints for the state variables fix the initial and final conditions of the problem. In this paper, only the initial altitude and distance to go are enforced. Consequently, the

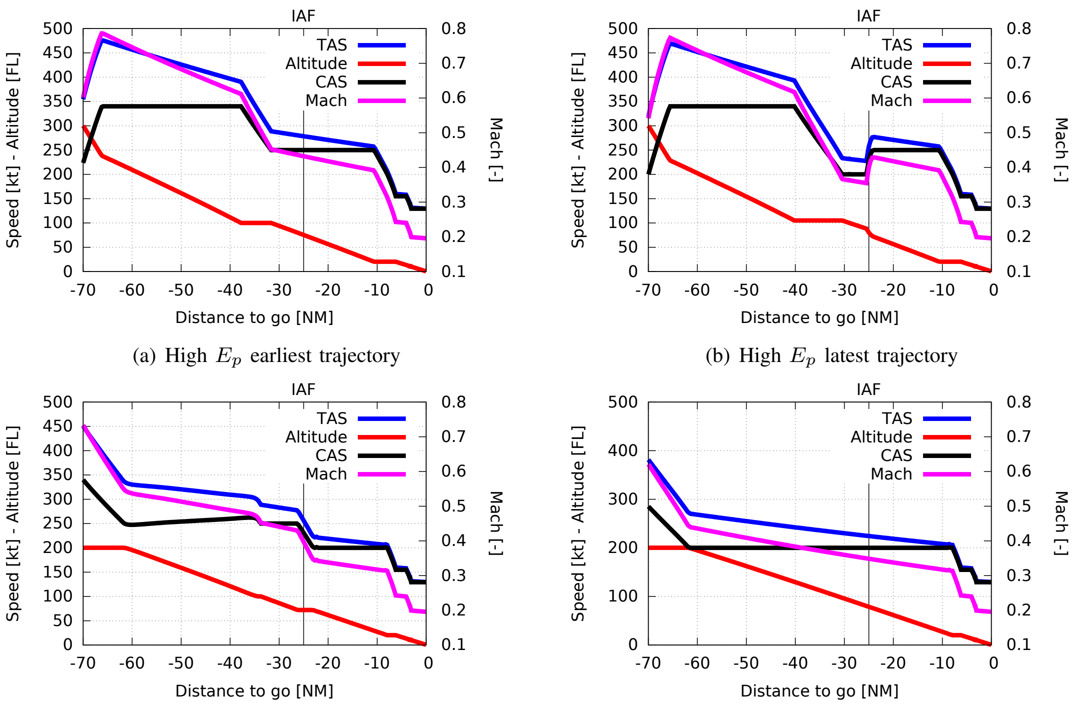
[1] Green dot speed is the minimum operating speed in managed mode and clean configuration, being approximately the best lift-to-drag ratio speed.

[2] S speed is the target speed when the aircraft is in configuration 1.

TABLE I
DEFINITION OF THE FLIGHT PHASES AND THEIR CONSTRAINTS

Phase	Path Constraints	Event constraints
Descent above FL100	$\text{GD} \leq v_{CAS}(t)$	$h(t_0) = h_0;\ s(t_0) = s_0$
Descent below FL100	$\text{GD} \leq v_{CAS}(t) \leq 250\,\text{kt}$	$h(t_0) = 10000\,\text{ft}$
Approach	$\text{GD} \leq v_{CAS}(t) \leq 250\,\text{kt}$	$s(t_0) = s_{\text{IAF}}$
Leveled Deceleration	$h(t) = 2000\,\text{ft};\ \text{S} \leq v_{CAS}(t)$	$v_{CAS}(t_0) = \text{GD};\ s(t_0) = s_{\text{FAP}} - 2\,\text{NM}$
Deceleration on glide path	$\gamma(t) = -3°$	$v_{CAS}(t_0) = \text{S};\ s(t_0) = s_{\text{FAP}}$
Stabilised on glide path*	$\gamma(t) = -3°;\ v_{CAS}(t) = \text{FAS}$	$h(t_0) = 1000\,\text{ft};\ h(t_f) = 50\,\text{ft};\ s(t_f) = 0$

*Additional thurst is allowed in order to maintaint the final approach speed at the ILS glide path.



(a) High E_p earliest trajectory

(b) High E_p latest trajectory

(c) Low E_p earliest trajectory

(d) Low E_p latest trajectory

Fig. 1. Examples of trajectories

initial airspeed is left free, assuming that the transition from any airspeed to that chosen by the optimiser would be quick enough in practice by using speed on elevator. The final states of the optimisation problem are fixed at the moment the aircraft reaches 50 ft over the runway threshold at the FAS.

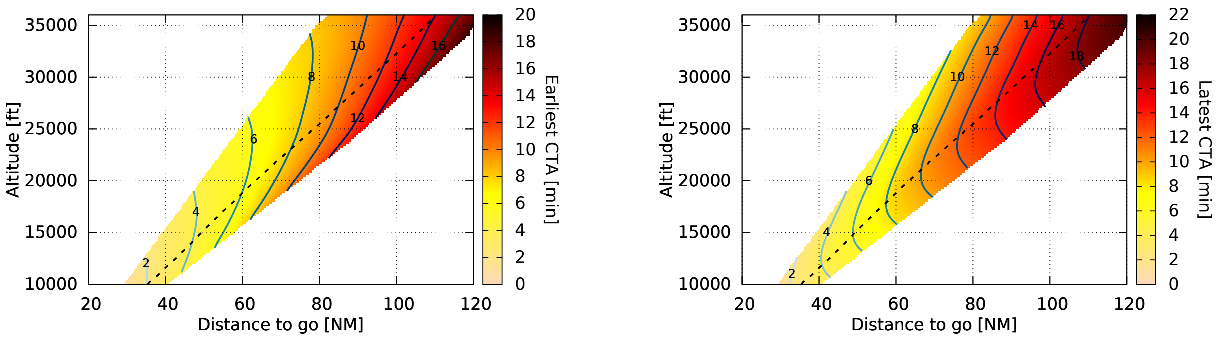
Fig. 1 shows four examples of optimal trajectories computed with the aforementioned model for an aircraft located at 70 NM from the runway threshold.

Figs. 1(a) and 1(b) show, respectively, the earliest and latest trajectories at the IAF for an aircraft having excess of potential energy. In both cases, the aircraft aims to increase the aerodynamic drag in order to release enough potential energy so that the final condition is satisfied. Initially, energy is modulated exchanging altitude by airspeed at the maximum descent gradient until VMO is reached. Thereafter, the descent is performed at this speed down to FL100, where the aircraft levels off (maintaining thrust idle) in order to decelerate as quick as possible. Below this altitude, the earliest and latest trajectories keep, respectively, the maximum and minimum allowed CAS until the IAF position is reached (depicted in Fig. 1 as a vertical black line). What happens after the IAF has no impact on the cost function. Yet, all the constraints in Table I must be satisfied.

By contrast, Figs. 1(c) and 1(d) show, respectively, the earliest and latest trajectories for an aircraft lacking of potential energy. The former starts at the maximum allowed speed (MMO is the limiting speed constraint in this case) in order to spare the initia energy for the remainder of the descent. The latter selects the initial speed corresponding to the minimum needed energy to satisfy the final condition gliding at GD.

V. Numerical results

This section presents the results obtained after computing the earliest and latest trajectories at two typical metering fixes:



(a) Earliest CTAs

(b) Latest CTAs

Fig. 2. Earliest and latest CTAs at the IAF

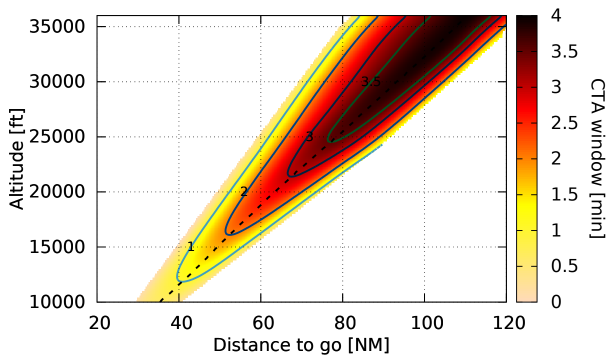


Fig. 3. CTA window at the IAF

TABLE II
PARAMETERS USED IN THE SIMULATIONS

	Parameter	Value
Aircraft	γ_{min}	$-15°$
	GD	200 kt
	S	155 kt
	FAS	129 kt
	VMO	340 kt
	MMO	0.80
Scenario	s_{IAF}	25 NM
	s_{FAP}	6 NM

A. Experimental setup

The following scenario has been considered in this paper: an aircraft attempts to perform an energy-neutral CDO and somewhere in the descent (before reaching the IAF) the ATC notifies a CTA update at the IAF. Thereafter, below FL100, the ATC assigns a CTA at the FAP. The goal is to quantify the feasible CTA window at these two metering fixes as a function of the aircraft altitude and distance to go.

Results have been obtained using an Airbus A320 (a typical twin-engine, narrow-body, transport aircraft) and considering an aircraft mass corresponding to 90% of the maximum landing mass. Table II wraps up the different aircraft and scenario parameters used in the simulations.

The earliest and latest trajectories at the IAF and FAP have been computed for several initial conditions (corresponding to the moment at which the CTA update is notified). For the former, initial altitudes between FL100 and FL360 and distances to go ranging from 25 NM to 120 NM have been considered. For the FAP, initial altitudes range between 3000 ft and FL100 and distances from 6 NM to 35 NM.

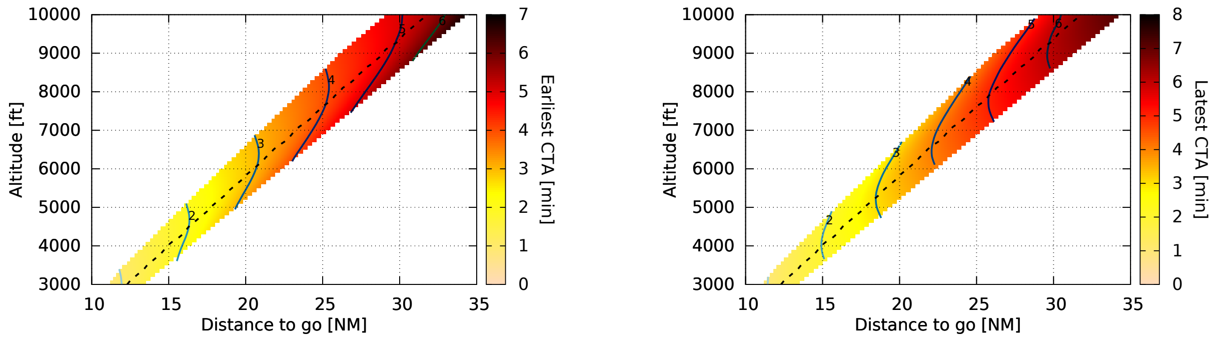
It should be noted that, for each initial condition, there exist several energy-neutral trajectories requiring neither additional thrust nor speed-brakes usage while satisfying all the constraints. Only one of them, however, is optimal in terms of fuel consumption. In order to quantify the extra fuel consumption that earliest and latest trajectories would entail, the minimum fuel trajectory has been also computed for each initial condition.

Results have been obtained using CONOPT as NLP solver, bundled into the GAMS software suite.

B. Results

Fig. 2(a) and 2(b) show, respectively, the earliest and latest times at the IAF as a function of the initial altitude and distance to go of the aircraft. The black dashed line represents the trajectory that maximises the CTA window (i.e. the difference between earliest and latest times of arrival at the metering fix) throughout the descent (assuming an instantaneous change of the initial speed). An aircraft starting the descent in the white region would need to apply additional thrust or to deploy speed-brakes to satisfy the final condition.

As expected, for a given altitude, the closer (resp. farthest) to the metering fix, the sooner (resp. later) the aircraft would be able to reach it. For a given distance, at the highest feasible altitudes the aerodynamic drag needs to be increased in order to release potential energy. Since speed-brakes usage is restricted aiming to reduce noise nuisance, the best practice is to accelerate by means of energy modulation, thus penalising the latest trajectories (see Fig. 1(b)).



(a) Earliest CTAs

(b) Latest CTAs

Fig. 4. Earliest and latest CTAs at the FAP

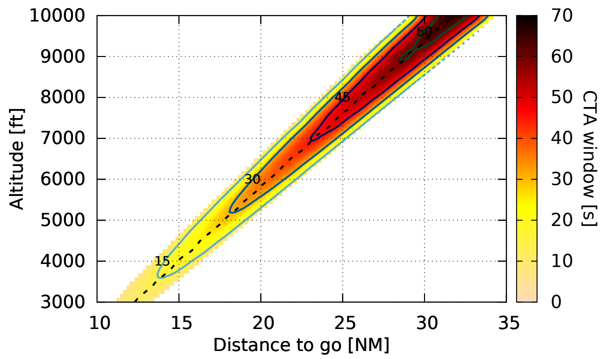


Fig. 5. CTA window at the FAP

As the altitude decreases, the potential energy needed to be released in order to satisfy the final condition reduces. Eventually, no acceleration is longer required, resulting in the maximum achievable latest time. Below this altitude, the initial kinetic energy needs to be increased in order to have enough total energy to reach the runway threshold at the required altitude and airspeed. In such case, the higher initial speed penalises the latest trajectory, as shown in Fig. 1(d).

Fig. 3 shows the CTA window at the IAF as a function of the aircraft position when the CTA is updated during the descent.

According to Figs. 2 and 3, there exist only a feasible region of initial distances and altitudes such that an aircraft could satisfy the final condition without requiring neither additional thrust nor drag devices. If the aircraft is close to the lower border of this feasible region, the potential energy needs to be sustained by means of a gentler flight path angle and almost no margin is left to increase the aircraft speed using energy modulation. As the aircraft position approaches the trajectory of maximum CTA window, it has more freedom either to accelerate or decelerate by means of energy modulation and the CTA window increases until it reaches its maximum. However, if the aircraft is close to the upper border of the feasible region, potential energy needs to be exchanged by kinetic energy by descending with a steeper gradient and almost no margin is left to decrease the aircraft airspeed.

The maximum time window (approximately $4\,\mathrm{min}$) is reached at the higher altitude (FL360) at $114\,\mathrm{NM}$ and it reduces almost $2.4\,\mathrm{s}$ per NM or $7.8\,\mathrm{s}$ per $1000\,\mathrm{ft}$.

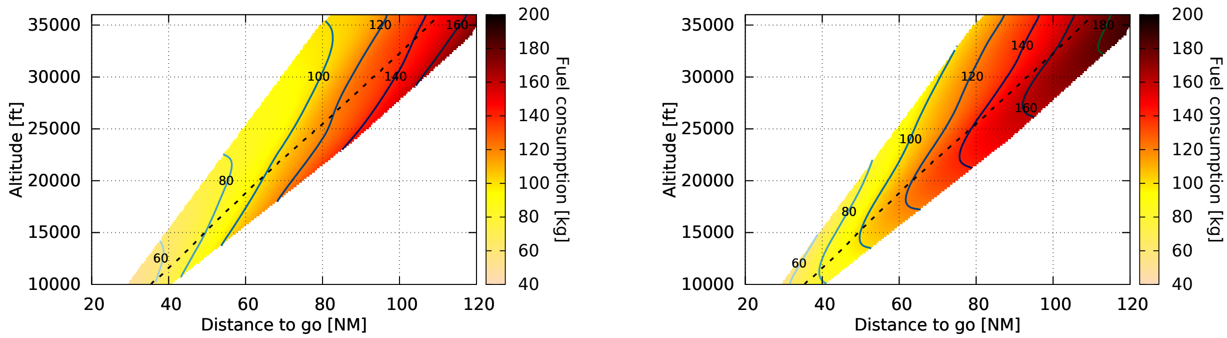
Another conclusion that arises from Fig. 3 is that the CTA window strongly depends on the position of the TOD, which in turn is a function of many factors such as the cost index (CI), the aircraft mass, the scheduled speed profile, the wind field or even a previous CTA assigned in cruise. For instance, depending on the CI values the location of the TOD will be "moved" towards or away the metering fix. Too high or too low CI values will narrow the CTA window while intermediate CI values will place the TOD in such a way that a wider window could be achieved.

The same analysis has been performed to determine the CTA window at the FAP changing the initial conditions subject to study. Fig. 4(a) and 4(b) show, respectively, the earliest and latest times of arrival at the FAP as a function of the initial altitude and distance to go.

Results shown in Fig. 4 are very similar to those observed in Fig. 2. The main difference is the magnitude of the earliest and latest times of arrival, since less time is available to either gain or loose time. In addition, the feasible range of initial conditions is significantly reduced due to the speed limitation at FL100, which restricts the aircraft capacity to descent with a steeper gradient by flying at high speeds. Fig. 5 shows the CTA window at the FAP as a function of the aircraft position when the CTA update is notified.

As expected, the CTA window at the FAP is considerably narrower than at the IAF. The maximum CTA window (approximately $70\,\mathrm{s}$) is reached at FL100 and $32\,\mathrm{NM}$. In this case, it reduces at a rate of $3.2\,\mathrm{s}$ per NM or $8.7\,\mathrm{s}$ per $1000\,\mathrm{ft}$. It should be noted that a previous CTA (for instance at the IAF) will determine the altitude profile and, consequently, the CTA window at the FAP.

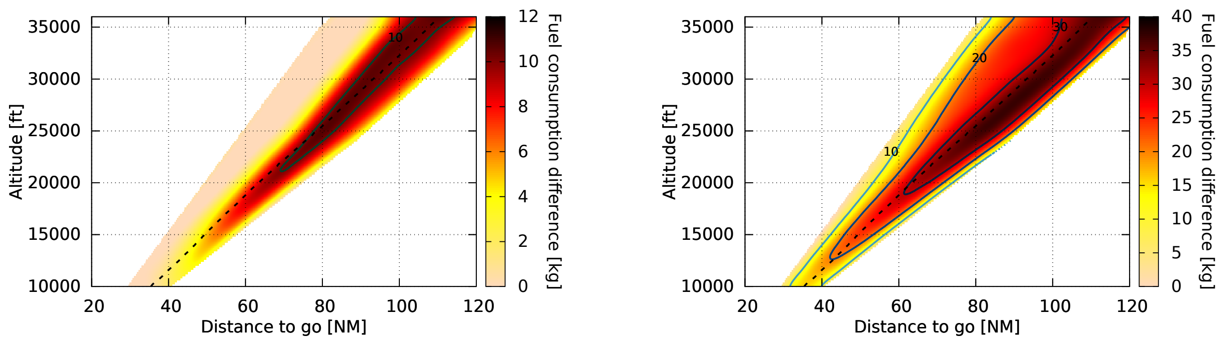
The goal of energy-neutral trajectories is to correct deviations from the plan (including CTA updates) with the minimum



(a) Earliest CTA trajectories

(b) Latest CTA trajectories

Fig. 6. Fuel consumption at the IAF



(a) Earliest CTA trajectories

(b) Latest CTA trajectories

Fig. 7. Extra fuel consumption with respect to the minimum fuel trajectory at the IAF

impact on the optimality of the operation. Nevertheless, even if keeping the engines at idle throughout the descent, the resulting fuel consumption depends on the aircraft speed and altitude profiles. Figs. 6(a) and 6(b) show, respectively, the fuel consumption of the earliest and latest trajectories at the IAF, as a function of the initial position of the aircraft.

As expected, the total fuel consumption and the time of arrival at the metering fix are strongly correlated.

In Fig. 6 significant differences (in terms of fuel consumption) between the earliest and latest trajectories are observed, even though both are performed with the engines at idle. Interestingly, it has been observed that earliest trajectories are more fuel-efficient than latest trajectories. This affirmation may sound conflicting with the general belief that the higher the aircraft speed, the higher the fuel consumption. If the CTA were updated in cruise, earliest trajectories would select a later TOD, leading to higher fuel consumption due to the larger amount of time spent in the cruise phase. In the scenario discussed herein the aircraft is already descending when the CTA is updated and only energy modulation is allowed to gain or loose time.

The idle fuel flow depends mainly on altitude, being lower at high altitudes. The latest trajectories tend to release as much energy as possible at the beginning of the descent, then flying at low speeds and low altitudes. On the other hand, earliest trajectories attempt to keep the faster speeds at the higher altitudes where the density (and the drag) are lower in order to maintain the total energy level as long as possible, then releasing this energy close to the constrained metering fix. The more fuel-efficient altitudes at which earliest trajectories operate result in lower fuel consumption.

Among all the possible energy-neutral trajectories, there exist only one that minimises fuel consumption for each initial condition. Figs. 7(a) and 7(b) show, respectively, the extra fuel consumption of the earliest and latest trajectories at the IAF with respect to the minimum fuel trajectory.

According to Fig. 7(a) the most fuel-efficient trajectories at the IAF are very close to those of minimum time, since both select similar altitude and speed profiles. Interestingly, the trajectory that maximises the CTA window throughout the descent also entails the maximum extra fuel consumption with respect to the minimum fuel trajectory for the same initial conditions. Consequently, there is a trade-off between maximimising the CTA window (and therefore maximising the probability to achieve the whole descent at thrust idle) and minimising the fuel consumption.

A similar study could be also performed to quantify the extra fuel consumption below FL100, considering the FAP as metering fix. However, fuel consumption differences are not so significant and have not been considered relevant for the completeness of this paper.

VI. Conclusion

An approach to minimise the environmental impact in terminal manoeuvring areas (TMA), whilst maintaining runway capacity is through the introduction of time-based continuous descent operations (CDO), in which the air traffic controllers (ATC) assign each aircraft with a controlled time of arrival (CTA) at one or several metering fixes to safely merge incoming traffic. In order to efficiently assign CTAs, the ATC should know the feasible CTA window for each of the aircraft. This paper quantified this window at the initial approach fix (IAF) and final approach point (FAP) for an Airbus A320 performing a CDO such that the speed profile is only adjusted by means of energy modulation along the whole descent and assuming that the descent has already been initiated.

Results show that CTA windows up to 4 min at the IAF could be achieved for certain initial conditions. However, when considering the FAP as a metering fix the CTA window is significantly reduced since not only the available distance is smaller, but also the 250 kt speed limitation below FL100 significantly restricts the aircraft capacity to modulate energy.

Another important remark that arises from this study is that a previously assigned CTA or even the cost index could have a significant impact on the achievable CTA window.

Furthermore, it has been also observed that minimum time trajectories are very similar to those of minimum fuel, provided that the optimisation takes place once in the descent (the top of descent has been overflown) and neither additional thrust nor drag devices usage are allowed. There is a trade-off between maximising the robustness of a CDO in the face of late changes to the CTA throughout the descent (i.e. maximising the probability to achieve the whole descent at thrust idle) and minimising the fuel consumption.

In order to maximise the potential benefits of time-based CDO in a future trajectory-based air traffic management paradigm, ground systems should have accurate trajectory predictors to provide the ATC with the feasible CTA window for each aircraft in the merging sequence. This will also entail advanced on-board real-time trajectory (re)planning systems, capable to support CTA updates during the descent.

In future works, the absolute CTA window could be computed allowing the aircraft to apply thrust or speed-brakes if required. The extended CTA window and the associated cost (in terms of additional fuel consumption and noise nuisance) could be analysed. Moreover, a sensitivity study on the influence of wind, the aircraft mass and the positions of the metering fixes on fuel consumption and flight time figures is also foreseen. It would be also interesting to compare the energy-neutral CTA window with that achievable by using path lengthening or stretching, even though pilots would probably prefer energy modulation to reduce workload and increase situation awareness.

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