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Technical Research Report

Study on *k*-Shortest Paths with Behavioral Impedance Domain from the Intermodal Public Transportation System Perspective

LSI-03-32-R

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June 11, 2003

Abstract

Behavioral impedance domain consists of a theory on route planning for pedestrians, within which constraint management is considered. The goal of this technical research report is to present the *k*-shortest path model using the behavioral impedance approach. After the mathematical model building, optimization problem and resolution problem by a behavioral impedance algorithm, it is discussed how behavioral impedance cost function is embedded in the *k*-shortest path model. From the pedestrian's route planning perspective, the behavioral impedance cost function could be used to calculate best subjective paths in the objective way.

Keywords: *k*-Shortest Paths, Behavioral Impedance Cost Function, Pedestrian

Contents

1. Introduction

Nowadays, although modeling of transportation networks is a well-known topic and there are several projects developed in this field, the application of these models to specific cases is a complex and hard task. This task becomes more difficult, when the network system is integrated by two or more modes of public transportation (e.g. bus, walk, underground, etc.), and the route planning analysis is carried out from an individual's perspective. Moreover, political interests delimit research and projects.

In this research, designing of a inter(and/or multi)modal network model for urban public transportation from a pedestrian's behavioral perspective. A "pedestrian" is defined as someone who needs to go from an origin-point to a destination-point using a public transportation network. In this model, decision-making problems can be solved using dynamic information of the system and considering pedestrian behavior.

First of all, it is important to define the "multimodality" and "intermodality" terms. Sometimes, authors use both multimodal and intermodal terms to refer to the same concept, but there is a difference. According to Meyer (1993, p. 6), "multimodal planning provides the general context within which intermodal planning occurs. Multimodal planning is focused on system choices, whereas intermodal planning emphasizes the most efficient way of moving from point to point through the system. From an analysis and evaluation perspective, multimodal planning means adopting a generic, non-mode-specific approach to problem definition and problem solving. In intermodal planning, key interactions between modes, including not only transfers but also the policy and service interactions between alternative modes, are identified". Therefore, intermodal planning is the approach used to apply Behavioral Impedance (BI) domain to the study of public transportation system from the pedestrian's perspective.

Several authors research on (*k*-)shortest paths techniques in transportation route planning (Li and Kurt, 2000; Hershberger et al., 2003). However, few researches on pedestrian's behavior related to route planning in inter(or multi)modal transportation network are found in specialized literature (Ahern, 2001; Pereira and Pérez-Vidal, 2001). In order to contribute with decrease of this lack, this report provides a mathematical model to solve the *k*-shortest paths problem (KSPP) considering BI domain proposed by Pereira et al. (2001, 2002a, 2002b, 2002c). Furthermore, a heuristic is presented as an agent that facilitates the adoption of BI mathematical model with respect to KSPP.

Geographic information systems for transportation (GIS-T) generate several research areas. Intermodal route planning area characterizes one of these research areas and represents the approach presented in this report. In this context, intermodal route planning is associated to two elements: the multi-criterion decision making process and the theory of Behavioral Impedance Domain. Using the natural relationship between the two elements, it is identified as an emerging problem that is solved by the application of BI mathematical model using the *k*-shortest, fast and best path algorithms (Figure 1).
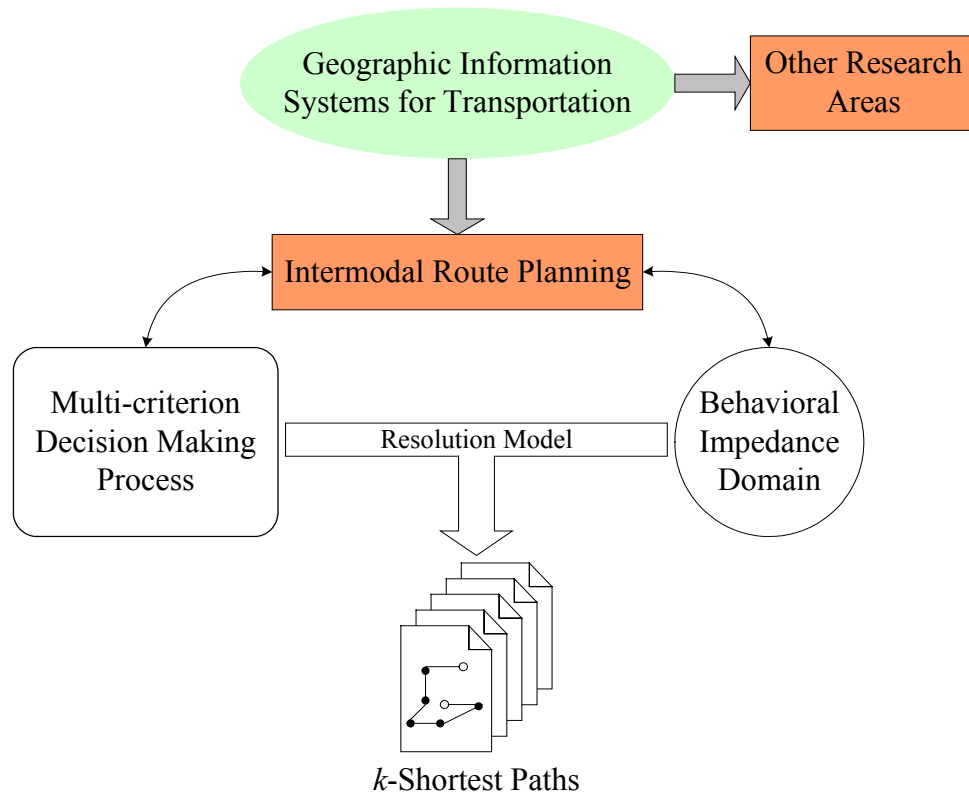
Figure 1. Application of BI mathematical model using the *k*-shortest path algorithms.

The structure of the technical research report is as follows. Section 2 presents the problem definition and formulation. In this section the network assumption is defined, BI model is presented and the optimization problem is formulated. Section 3 presents the resolution of the problem presented in the previous one establishing a heuristic used to solve KSPP optimization problem. Section 4 provides the conclusions and points out future research activities.

2. Problem Definition and Formulation

Route planning from the pedestrian's perspective is a Pereira et al. (2001,2002a, 2002b)'s proposal, which regards BI domain. Within this context, the problem is defined as a search of an optimal path from two given point: origin and destination ones. Several objects of this problem such as minimal travel time and distance, and pedestrian's behavior (e.g. comfort, safety, price, modal change, etc.) are taken into account in BI cost function. Therefore, all behavioral constraints imposed by pedestrian and transportation system can be used as parameters to improve the travel cost.

2.1. Network Assumption

In this research, layers, connections among the layers and neighborhoods related to the different nodes characterize the network used to solve the optimization problem. The layers can be view as sub-networks, where the nodes are of the same type (e.g. stations of bus). Each layer represents a different transportation mode that, according to the present proposal, can be ordered as public transportation (e.g. bus and railway modes) and the non-motorized transportation (e.g. sidewalk and bicycle). The connections among the different layers are well known as transfer links in the specialized literature.

The transfer links allow modeling the changes between two different transportation modes. Finally, neighborhood definition is used to model the network modality and intermodality.

The transportation network consists of nodes (i.e. stations and stops), regular (i.e. one connection between two nodes of the same layer) and transfer (i.e. one connection between two different layers) links. A layer is a sub-network that represents the different transportation modes. A new element taken into account in the BI model is the neighborhood, which is defined as a geographic area where there is a connection between two different nodes. This geographic area is limited by maximal tolerance of distance established by attractiveness and space-time accessibility.

Using the transportation network elements and neighborhood definition presented above as a starting point, it is depicted in Table 1, the network key elements used in the identification of basic characteristics considered during the formulation process of BI model.

Table 1. Considered key elements of transportation network.

Basic characteristics	Considered key elements	
Transportation modes	Public transportation	Bus
		Railway (i.e. Train, Subway, Metro)
		Cable Railway (i.e. Cable-car)
	Non-motorized transportation	Sidewalk
		Bicycle
Route characteristics	Fixed	
	Departure point	
	Arrival point	
	Schedule	
Neighbourhood	Attractiveness	
	Space-time accessibility	

Figure 2 shows a graphical representation of layer overlapping and the key elements of transportation network presented in Table 1.
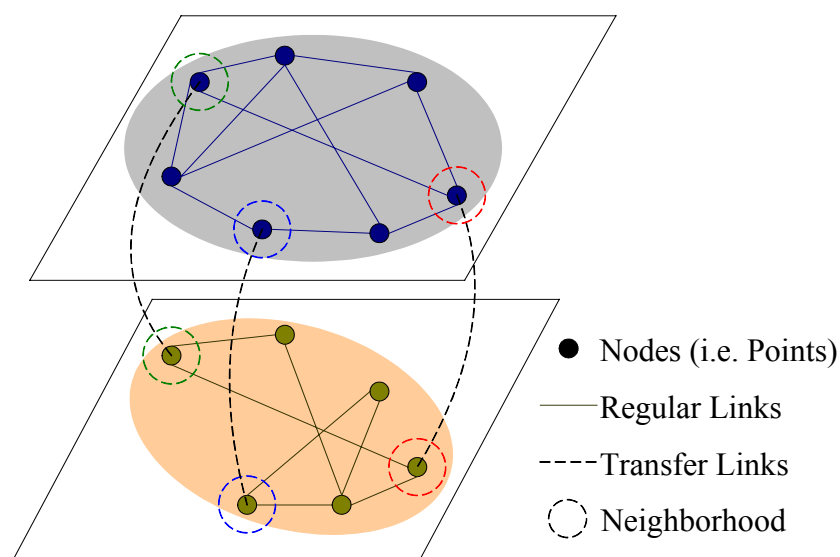

Figure 2. Network assumption: Layer overlapping of the transportation modes.

Before to survey the mathematical formulation of BI model, some concepts are presented in order to define the transportation network. Let $G = (N, A)$ be a connected graph, where N is the set of nodes and A is the set of arcs.

Transportation network G can be separate into natural components. Each one of these components defines a sub-network, which is composed of similar nodes and regular links. As commented, a sub-network is represented by different transportation modes (e.g. bus, subway, etc.), and it is called layer. There are other elements that allow the connection between two layers. These elements are the transfer links (Figure 2).

For instance, the bus layer is a sub-network represented by a sub-graph $G_1 = (N_1, A_1)$ (Figure 3.a) and subway layer is a sub-network represented by a sub-graph $G_2 = (N_2, A_2)$ (Figure 3.b).

a) $G_1 = (N_1, A_1)$

(e.g. Bus layer)
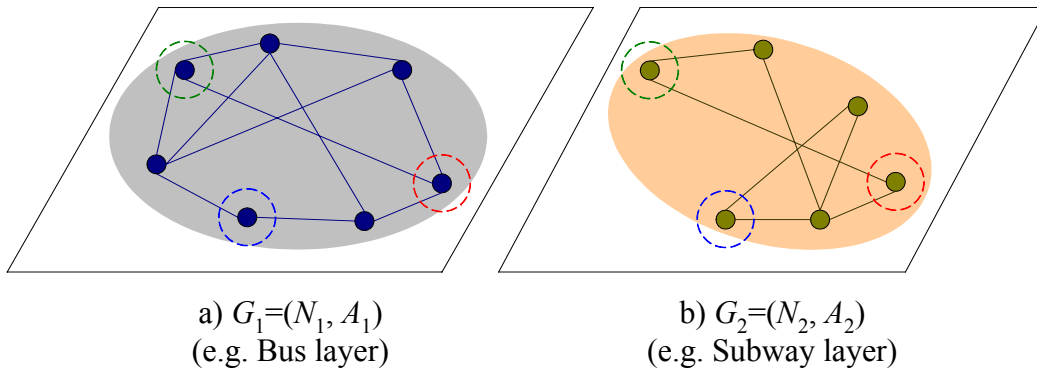
b) $G_2 = (N_2, A_2)$

(e.g. Subway layer)

Figure 3. Layers: Transportation modes.

Each node has a neighborhood itself associated. There are two kinds of neighborhood. On the one hand, modal neighborhood (N_{modal}) only allows the connection between nodes of the same layer. On the other hand, intermodal neighborhood ($N_{intermodal}$), by definition, has all characteristics of the modal neighborhood, but also it allows the connection between different layers.

Regarding that each node has an associated neighborhood, which can be modal or intermodal one. If the neighborhood is $N_{intermodal}$, then the neighborhood is N_{modal}. This is true since it is possible the connections among nodes of the same layer. In general, N_{modal} is necessarily not $N_{intermodal}$, since N_{modal} condition not guarantees the connection with other layers. This can be summarized in the following expression:

$$N_{intermodal} \Rightarrow N_{modal}, \text{ but } N_{modal} \nRightarrow N_{intermodal}$$

Therefore, network assumption presented in Figure 2 is the union of the all layers that represent the transportation modes, the transfer links and the neighborhoods (e.g. $N = N_1 \bigcup N_2, A = A_1 \bigcup A_2 \bigcup A_{TL}$, where A_{TL} is set of transfer links).

2.2. BI Mathematical Model

BI Mathematical Model has been initially presented by Pereira et al. (2002c) and refined by Pereira et al. (2003).

Assuming a transportation network is a directly connected graph $G = (N, A)$, let N be the set of nodes (i.e. nodal points that correspond to the origin and destination points of the pedestrian's path, and intersection points) and let A be the set of arcs, let $n = card(N)$ and $m = card(A)$. Moreover, it is assumed G is connected. Let P be the set of all paths in G. For all $Cam_j \in P$ there are $1 \le s \le m$ such that

$$Cam_j = (Link_{j_1}, .., Link_{j_s}) \tag{1}$$

where $Link_{j_i} \in A$ for all $i \in \{1, .., s\}$.

Using the definition presented in Equation 1, the cost function for a path j in a moment t taking into account the BI weigh table is expressed as:

$$f_{Cost}\left(Cam_j\right)_t = \sum_{i=1}^{s}\left(C_{Link_{j_i}}\right)_t \cdot \left[1 + f\left(BI_{Link_{j_i}}\right)_t\right] \tag{2}$$

where $f_{Cost}\left(Cam_j\right)_t$ represents the cost function of the path Cam_j in a moment t, $C_{Link_{j_i}}$ represents the cost (i.e. a cost measure) in a arc $Link_{j_i}$ in a moment t.

As shown in Equation 2, the travel cost is calculated in a multiplicative fashion between the sum of the default costs of each arc $Link_{j_i}$ ($C_{Link_{j_i}}$) (i.e. distance or time cost) and the BI calculated value imposed to the path Cam_j in a moment t is represented by $\left[1 + f\left(BI_{Link_{j_i}}\right)_t\right]$; the resultant products are summed for all s arcs of the path Cam_j. In this way, there are two premises with respect to the BI function for a arc $Link_{j_i}$ of a path Cam_j in a moment t:

1. If there is behavioral impedance defined by pedestrian then $f\left(BI_{Link_{j_i}}\right)_t \to 1$

2. But if there is not behavioral impedance, $f\left(BI_{Link_{j_i}}\right)_t = 0$

Assuming the BI values are saved in a table composed of $k+1$ attributes (i.e. the quantity of BI conditions plus an identifier attribute), the follows notations are defined:

K':   It is the set of all BI attribute values predefined by the transportation system through the BI table. Let $K' = \{hs_1, hs_2, .., hs_k\}$, where hs_i represents each attribute of the BI table, for all $i \in \{1, ..., k\}$;

U':   It is the set of BI attribute values defined by the user. Let $U' = \{hu_1, hu_2, .., hu_q\}$, where hu_i represents each attribute of the BI table, for all $i \in \{1, ..., q\}$;

Thus, the function that calculates the BI value from the data defined by the transportation system (i.e. BI table of the database GIS-T) and/or by the user, for each arc $Link_{j_i}$ of a path Cam_j in a moment *t*, it is expressed as:

$$f\left(BI_{Link_{j_i}}\right)_t = \frac{1}{k}\left[\sum_{h\in\left(K'\backslash\Omega\right)}\left(\sigma_{h\,Link_{j_i}}\right)_t + \sum_{h\in U'}\left(\sigma_{h\,Link_{j_i}}\right)_t\right] \qquad (3)$$

where $\displaystyle\sum_{h\in\left(K'\backslash\Omega\right)}\left(\sigma_{h\,Link_{j_i}}\right)_t$ represents the sum of the variables determined by the

transportation system through the BI table, $\displaystyle\sum_{h\in U'}\left(\sigma_{h\,Link_{j_i}}\right)_t$ represents the sum of the

values provided by the users of the transportation system, Ω represents the resultant set of the intersection of the sets K' and U' ($K'\cap U'$), $K'\backslash\Omega$ represent the difference between the sets K' and Ω, and $\sigma_{h\,Link_{j_i}}$ represents the values of the attributes *h* for the arc $Link_{j_i}$ of a path Cam_j.

The BI value, for each arc $Link_{j_i}$ of a path Cam_j in a moment *t*, is calculated through of the arithmetic average of the BI values obtained from BI table. However, if the user changes the predetermined values of the transportation system during the route selection, these values are used in the BI function calculus.

The values of $\sigma_{h\,Link_{j_i}}$ for the BI attribute values that pertain to the BI table (i.e. $h\in K'$ if $\Omega=\varnothing$) are initially determined by data collection tools (e.g. questionnaire and/or interviews) and maintained by the spatial and geographic database management system. The variables that represent these data such as meteorological and topographical data, information about rush hour, schedule system of the transportation lines, dangerous zones, information about transportation network incidents, fare systems and strategic and operating information can be automatically updated by GIS-T.

On the other hand, the values of $\sigma_{h\,Link_{j_i}}$ for the BI attributes defined by the user (i.e. $h\in U'$), are basically determined by the users of the transportation system during the route selection. These values are provided considering a qualitative and subjective analysis from the user's preferences and converted into quantitative values used in the calculus of the BI cost function. Some of these values are personal data (i.e. user's profile), travel reason, user's demand (i.e. tolerance in meters to walk between two points), emergency situations and schedule system associated to professional activities.

As commented above, Equation 3 calculates the BI value of each arc $Link_{j_i}$ of a path Cam_j in a moment *t*. Therefore, the BI value sum of all arcs of the path Cam_j in a moment *t* is represented as:

$$f\left(BI_{Cam_j}\right)_t = \sum_{i=1}^{s} f\left(BI_{Link_{j_i}}\right)_t \qquad (4)$$

2.3. Optimization Problem Formulation

The network model used in the KSPP optimization problem has the different layers, connections among between different layers, and neighborhoods in the different nodes. The "costs" associated to the network are determined by the BI function (Equation 2). Moreover, the method with which the BI values were calculated allows studying the network without taking into account layers and neighborhoods associated to the nodes. In this way, the network can be considered as one big layer. Using the BI cost function presented in Equation 2 as a starting point, the optimization problem is formulated as follows.

Assuming p_o as the origin point and p_d as the destination point, $P_{od} \subseteq P$ is the set of possible paths identified between p_o and p_d in $G=(V,A)$ and $f_{Cost}(P_{od}) = \{ f_{Cost}(p) : p \in P_{od} \}$ denotes the set of the paths' costs, where $f_{Cost}(p)$ represents the cost function of the path $p \in P_{od}$.

Regarding the set of possible paths during the selection of a route in a transportation network, there are three kinds of paths according to the cost criteria defined by the user. The first one is the shortest path in which the distance criterion is fixed. The second one is the fastest path, which takes into account the time criterion. The third one is the best path, which not only takes into account distance and time criteria, but also other criteria defined by users. In this way, the values identified in to BI domain are considered as criteria that will support the best path calculus.

The objective of the single shortest path problem is to determine a path $p^* \in P_{od}$, for which $p_{od}^* \in P_{od} : f_{Cost}(p_{od}^*) \le f_{Cost}(p)$ for all $p \in P_{od}$. If it is required not only to find the first shortest path, but also to determine the second, third and, in general, the kth-shortest path (between a given origin and destination), the shortest path problem can be extended to calculate the group of *k*-shortest paths in a non-decreasing order respect to the objective values provide by f_{Cost}. Thus, the KSPP problem is to determine a set of $P_{od}^k = \{ p_1,..,p_k \} \subseteq P_{od}$ such that:

$$f_{Cost}(p_i) \le f_{Cost}(p_{i+1}), \text{ for any } i \in \{1,..,k-1\};$$
$$f_{Cost}(p_k) \le f_{Cost}(p), \text{ for any } p \in P_{od} \setminus P_{od}^k. \tag{5}$$

3. Heuristic Solution

The high complexity of BI mathematical model adoption entails the use of an heuristic solution, since subjective aspects and dynamic information can change the path in a moment t, which $t \in \left[t_{origin}, t_{destination} \right)$, where t_{origin} is the trip initial time and $t_{destination}$ is the trip final time.

3.1. Big Heuristic

Assuming H a big heuristic composed of $H_1,...,H_n$ (n is an integer positive number)

rules. Each H_i is defined by two non-empty finite sets I_i and O_i, and a function $f_i : I_i \rightarrow O_i$ such as $\{f_{i-1}(I_{i-1})\} \subseteq I_i$ for all $i = 1,...,n$. Each H_i is called the sub-heuristic. I_i and O_i represent the input and output data for each H_i, and the function f_i represents the processing procedure. The relation $\{f_{i-1}(I_{i-1})\} \subseteq I_i$ allows the interconnection between sub-heuristics H_{i-1} and H_i. In this research, it is denoted the heuristic H with $(H_i)_{i=1}^{n}$, and $H_i = (I_i, f_i, O_i)$ for all $i = 1,...,n$. In the following section is described the particular heuristic of BI mathematical model based on big heuristic H.

3.2. Particular Case of BI Mathematical Model

Let $H = (H_i)_{i=1}^{3}$ and $G = (N, A)$. The sets I_i and O_i, and the functions f_i $(i = 1, 2, 3)$ used in the particular case of BI mathematical model are defined in Table 2. The rule's descriptions related to each H_i $(i = 1, 2, 3)$ are also presented.

Table 2. Big Heuristic H's definitions used in BI mathematical model.

H_i	Instances and data	Rule's Description
H_1	$I_1 = N$; $O_1 = \{0, o, d\}$; $f_1 : I_1 \rightarrow O_1$ such as f_1 verifies the following conditions: a) $f_1(I_1) = O_1$; b) $\left\| f_1^{-1}(0) \right\| = \|N\| - 2$.	Locate the origin p_o and destination p_d provide by pedestrian.
H_2	$I_2 = \{G, f_1(I_1)\}$; $O_2 = \{J \subset P_{od} : \|J\| = k\}$; $f_2 : I_2 \rightarrow O_2$, where $f_2(I_2) = P_{od}^{k}$ is a completely ordered set with respect to f_{Cost}.	Calculate the k-shortest paths in G from p_o to p_d optimizing the cost function f_{Cost}.
H_3	$I_3 = \{P_{od}^{k}, s\}$ where s is an integer positive number; $O_3 = P_{od}^{k}$; $f_3 : I_3 \rightarrow O_3$, where $f_3(I_3) = P_{od}^{k} \setminus \{p_{s+1}, .., p_k\}$.	Choose the s-first paths of P_{od}^{k}, $s \leq k$.

Using the definitions presented in Table 2 as a starting point, it is guarantee that Big Heuristic H provides to pedestrian the *k*-shortest paths. Moreover, the pedestrian can define the *s*-first shortest paths.

4. Concluding remarks

Behavioral impedance domain is an innovative proposal. In this report, BI function was modeled to fit to *k*-shortest paths problem, allowing the increase of the precision of best path calculus from the pedestrian's perspective.

Two reasons justify the importance of this research. On the one hand, it is established how is presented the transportation network related to the layers, the

connections among the layers and the neighborhoods of the different nodes. On the other hand, it is defined the Big Heuristic H used to simplify the high complexity of BI mathematical model adoption.

Regarding the Pereira et al. (2003)'s work and the present report, it is proposed a framework towards the resolution of *k*-shortest paths problem based on Behavioral Impedance Domain from the intermodal public transportation system perspective.

Acknowledgements

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