NON-DOMINATED SORTING GRAVITATIONAL SEARCH ALGORITHM FOR MULTI-OBJECTIVE OPTIMIZATION **OF POWER TRANSFORMER DESIGN**

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1 Introduction

A transformer is a static electric device that consists of one or more windings, with or without a magnetic core, for introducing mutual coupling between electric circuits.

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systems. Due to market globalization, power transformer manufacturers are facing an increasingly competitive environment that mandates the adoption of design strategies yielding better performance at lower mass and losses. Multi-objective Optimization Problems (MOPs) consist of several competing and incommensurable objective functions. Recently, as a search optimization technique inspired by nature, evolutionary algorithms have been broadly applied to solve MOPs. In this paper, a power Transformer Design (TD) methodology using Non-dominated Sorting Gravitational Search Algorithm (NSGSA) is proposed. Results are obtained and presented for NSGSA approach. The obtained results for the study case are compared with those results obtained when using other multi objective optimization algorithms which are Novel Gamma Differential Evolution (NGDE) Algorithm, Chaotic Multi-Objective Algorithm (CMOA), and Multi-Objective Harmony Search (MOHS) algorithm. From the analysis of the obtained results, it has been concluded that NSGSA algorithm provides the most optimum solution and the best results in terms of normalized arithmetic mean value of two objective functions using NSGSA to the TD optimization.

Transformers are extensively used in electric power systems to transfer power by electromagnetic induction between circuits at the same frequency, usually with changed values of voltage and current. Transformers are one of the primary components for transmission and distribution system [1].

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Transformer design, in terms of construction, rated power and voltage level, depends mainly on the range of application. Therefore, transformer design is a complex task in which power engineers have to ensure that compatibility with the imposed specifications is met, while keeping manufacturing costs low. Moreover, the design methodology may vary significantly according to the transformer type and its operating frequency (ranging between 50 and 60 Hz). Many alterations in the design may be introduced according to the constructional characteristics of the core, cooling method and type of magnetic material used [2, 3].

the overview of research papers in From Transformer Design (TD), efforts are focused on the prediction of specific transformer characteristics, adopted techniques for transformer design optimization, transformer post-design performance, modeling and recent trends in transformer technology. In a nutshell, TD optimization problems remain an active area [4]. Based on the objective functions used, TD optimization can be achieved through the minimization of no-load losses [5, 6] or load losses [7], maximization of efficiency [8-12], minimization of cost [8, 9] or mass [12], or maximization of rated power [12].

Optimum design of transformer was presented in using Non-Linear Programming [13] (NLP) technique, while Geometric Programming (GP) was used for the minimization of total mass of transformer [12]. Mixed Integer Programming (MIP) in combination with Finite Element Method (FEM) [14], MIP in combination with Branch Bound Algorithm (BBA) [15], Bacterial Foraging Algorithm (BFA) [8] and Simulated Annealing (SA) technique [16] have been all adopted for the minimization of main material cost of transformer. BBA tailored to a Mixed Integer Non-Linear Programming (MINLP) [17], numerical field analysis technique in combination with Boundary Element Method (BEM) [18] and multiple algorithm based hybrid approach [19] have addressed the minimization of Total Life Time Cost (TLTC) of transformer. Scatter Search (SS) algorithm and Genetic Algorithm (GA) [20] are applied for minimization of the cost of transformer. Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [21] is applied for the minimization of purchase cost, TLTC, total mass and losses. Further techniques were used such as Gradient Search Technique Particle (GRT) [22], Swarm

Optimization (PSO) algorithm [23] and Time Variant Particle Swarm Optimization (TV-PSO) algorithm [24]. For multi-objective optimization of TD, application Novel Gamma Differential Evolution (NGDE) approach [25], Chaotic Evolutionary Algorithm (CEA) [26], and Harmony Search Algorithm (HSA) [27] are used.

Evolutionary algorithms are relatively new, but very powerful techniques used to find solutions to many real-world search and optimization problems. Many of these problems have multiple objectives, which lead to the need to obtain a set of optimal solutions, known as effective solutions. It has been found that using evolutionary algorithms is a highly effective way of finding multiple effective solutions in a single simulation run [28].

In engineering design problems, computational models are often used to describe the complex behaviors of physical systems and optimal solutions are sought with respect to some performance criteria. Hence, multi-objective optimization becomes useful in obtaining a set of optimal solutions so that the designer can select the best choice.

Non-dominated Sorting Gravitational Search Algorithm (NSGSA) is a non-conventional optimization technique with high global searching capability as it utilizes the non-dominated sorting concept to update the gravitational acceleration of the agents. The NSGSA approach applied to solve multi-objective optimization TD problems aims to minimize the total mass and losses of power transformer. The proposed optimization algorithm was applied to the design of a dry-type, and shell core for single power transformer. The simulation results show advantages of using the NSGSA approach and its applicability to TD.

This paper is organized as follows: Section II presents the design of power transformer; Section III presents an overview of NSGSA algorithm (objective functions and constraints); Section IV presents the simulation results and finally the conclusions of the paper are presented in Section V.

2 Fundamentals of transformer design

To verify using the non-dominated sorting optimization algorithm in electromagnetic modeling, the classical TD problem has been selected since electrical transformers play a key role in AC systems.

The optimization method used here aims to obtain the best configuration of a single-phase transformer trying to optimize its electrical and magnetic losses and geometrical characteristics. The parameters of the transformer under study are shown in the Appendix. Figure 1 shows the geometry of the transformer with the dimensions of the core, primary (N_1) and secondary (N_2) windings.



Figure 1. Transversal transformer cutaway: dimensions of the core, primary (N₁) and secondary (N₂) windings.

The transformer must present a maximum efficiency with a minimum mass, including windings mass. Thereby, the optimization procedure must find an optimum core dimension and a number of turns that satisfy both the constraints and the operational requirements. In the following text, a classical methodology of transformers design is presented. Some transformer parameters are related to the spatial dimensions defined as follows [25, 26]:

1) Central leg core area (central limb transversal area):

$$A_c = 2.c.t, \qquad (1)$$

where, *t* is the deepness of the transformer.

2) Effective central leg core area:

$$A_e = K_f A_c, \tag{2}$$

where, K_f is the core stacking factor.

3) Mean length per turn (copper winding length):

$$MLT = 4.c + 2.t + \pi b_w. \tag{3}$$

4) Window area (frontal window area where the windings will be fitted):

$$W_a = b_w \cdot h_w \cdot \tag{4}$$

5) Winding volume:

$$V_w = W_a.MLT . (5)$$

6) Core volume:

$$V_{c} = A_{c} \cdot \left[2.h_{w} + 2b_{w} + 4c \right].$$
(6)

First, the primary and secondary currents are obtained using the ratio between power and voltage transformer: I=S/V, with S in VA. The number of turns for primary windings is obtained as follows:

$$N_{1} = \frac{V_{1}}{\sqrt{2}.\pi.K_{f}.f.B.A_{c}},$$
(7)

where, *B* is the magnetic flux density in *T*. The number of turns of secondary winding is given by the voltage ratio: $N_2 = N_1 (V_2/V_1)$.

The efficiency is obtained by calculating the transformer core losses (including hysteresis and eddy currents) and the copper losses (Joule losses) in the windings. The core losses are given by:

$$p_c = K_c \cdot B^{\alpha} \cdot f^{\beta} , \qquad (8)$$

where, p_c is the ferromagnetic losses in W/kg, B is the magnetic flux density in T and f is the frequency in Hz. The coefficients K_c , α , and β are constants characteristics for each core magnetic material. The total core losses in the transformer volume are given by:

$$P_c = \rho_c . K_f . V_c . K_c . B^{\alpha} . f^{\beta}, \qquad (9)$$

where, P_c is the power losses in W and ρ_c is the core density. The primary and the secondary currents are obtained by the ratio between power and voltages of the transformer: I = S/V, with S in VA.

Once the current densities d (in A/mm²) has been defined, the conductors section areas are calculated

for primary and secondary windings as $A_{cd} = I/d$, in mm².

Once the number of turns of the primary and secondary windings and the conductor section areas have been obtained, it is possible to calculate the copper-occupied area W_{cu} . For a transformer with a primary winding and secondary windings, the area occupied by the copper is given by:

$$W_{cu} = N_1 A_{cd1} + N_2 A_{cd2}, (10)$$

where, A_{cd1} and A_{cd2} are the primary and the secondary conductor section areas, respectively. The copper mass, while taking into account the primary and the secondary windings, is given by:

$$m_{cu} = \rho_{cu} W_{cu} LMT. \qquad (11)$$

For a copper density of 8890 kg/m³ and a resistivity of $2.3 \times 10^{-8} \Omega$ m at 75 °C, the copper losses can be calculated approximately by:

$$P_{cu} = 2.43 d^2 . m_{cu} , \qquad (12)$$

where, P_{cu} is the copper losses in *Watt*, and m_{cu} are the copper mass in kg, and *d* is the current density in A/mm² (d = 3 A/mm² for power under 500 VA). Thus, the total losses are the sum of core and copper losses:

$$P_T = P_c + P_{cu} = \rho_c K_f N_c K_c B^{\alpha} . f^{\beta} + 2.43.d^2 . m_{cu} .$$
 (13)

The ferromagnetic core mass is given by:

$$m_c = \rho_c V_c. \tag{14}$$

The transformer mass is calculated as the sum of core and copper mass as:

$$M_T = m_c + m_{cu} = \rho_c V_c + \rho_{cu} W_{cu} LMT$$
. (15)

The multi-objective optimization problem aims to minimize mass (f_1) and the losses (f_2) while ensuring the operational requirements. The design variables are the core dimensions, turns of windings, and currents densities. In this way, the objective functions are [25-27]:

$$f_1 = \min(M_T), \tag{16}$$

$$f_2 = \min(P_T). \tag{17}$$

From the previous equations, it is obvious that the number of variables involved in the design of a transformer is relatively large, even for a simple single-phase structure, which would justify the use of an optimization methodology for the structure.

The leakage reactance, the phenomenon of saturation, costs, and temperature restrictions are all neglected since the inclusion of these phenomena and additional restriction will make the work of the designer more complex and complicated.

To illustrate the magnetic flux density distribution, the 2D finite element method is used. A transversal transformer cutaway with primary and secondary windings is shown in Figure 2. It can be observed that the magnetic flux density distribution is concentrated at central limb with an average value close to 1.15 T [25].



Figure 2. Transversal transformer cutaway: magnetic flux density distribution on the transformer core [25].

3 Non-dominated sorting multi objective gravitational search algorithm

This multi-objective problem is mathematically formulated as follows:

Minimize
$$\left[f_1(X), f_2(X), \dots, f_T(X)\right]^{T}$$
, (18)

Subject to:
$$\begin{cases} \lambda(X) \le 0\\ \xi(X) = 0 \end{cases}$$
 (19)

The overall objective function is the combined function which includes all the different objective functions f_1, f_2, \ldots, f_T etc.; λ and ζ are the inequality and equality constraints, respectively, and *X* is the vector of decision variables.

The solution to the above problem is not unique but a set of Pareto optimal points. A Pareto optimal solution is the best solution vector out of several numbers of solution vectors that could be achieved without disadvantaging other objectives. Each solution set is known as a non-dominated solution set. An array containing all the Pareto optimal solutions of a multi-objective problem is called the Pareto optimal set. Thus, instead of being a unique solution to the problem, the solution of a Multiobjective problem is a possibly infinite set of Pareto points. A Non-dominated Sorting Gravitational Search Algorithm (NSGSA) is a non-conventional optimization technique with high global searching capability and it utilizes the non-dominated sorting concept to update the gravitational acceleration of the agents. In NSGSA, the mass of each agent is calculated according to its fitness value, as follows [29, 30]:

$$m_p(t) = \frac{\operatorname{fit}_p(t) - \operatorname{worst}(t)}{\operatorname{best}(t) - \operatorname{worst}(t)}, \quad p = 1, 2, \dots, n_a, \quad (20)$$

$$M_{p}(t) = \frac{m_{p}(t)}{\sum_{q=1}^{n_{m}} mq(t)}, \quad 0 \le Mp(t) < 1, \quad (21)$$

where, $fit_p(t)$ is the fitness of the p^{th} agent at time t (iteration); $M_p(t)$ is the normalized mass; n_a is the number of agents, while the *worst*(t) and *best*(t) for a minimization problem are defined as:

worst
$$(t) = \max_{q \in \{1, 2, \dots, N\}} \operatorname{fit}_{q}(t),$$
 (22)

$$\operatorname{best}(t) = \min_{q \in \{1, 2, \dots, N\}} \operatorname{fit}_{q}(t).$$
(23)

In the case of a maximization problem, the *worst* (t) and *best* (t) may be written as follows:

worst
$$(t) = \min_{q \in \{1, 2, \dots, N\}} \operatorname{fit}_{q}(t),$$
 (24)

$$\operatorname{best}(t) = \max_{q \in \{1, 2, \dots, N\}} \operatorname{fit}_{q}(t).$$
(25)

The gravitational constant G is initialized at the beginning and will be reduced with time to control the search accuracy.

In other words, G is a function of the initial value G_0 and time t and therefore can be written as follows:

$$G(t) = G(G_0, t).$$
⁽²⁶⁾

In NSGSA, the force acting on mass *p* due to mass *q* may be represented as:

$$F_{pq}(t) = G(t) \times \left(\frac{M_{aq}(t) + M_{pp}(t)}{R_{pq}(t) + \varepsilon}\right) \times \left(x_q(t) - x_p(t)\right), (27)$$

where, M_{aq} is the active gravitational mass related to agent q; M_{pp} is the passive gravitational mass related to agent p; $G(t) = G(G_0, t)$ is gravitational constant at time t; ε is a constant term whose magnitude is very small; $x_p(t)$ is the position vector of the *pth* agent; $x_q(t)$ is the position vector of the *q*th agent; and $R_{pq}(t)$ is the Euclidian distance between two agents p and q which can be represented as:

$$R_{pq}(t) = \left\| X_{p}(t), X_{q}(t) \right\|_{2}.$$
 (28)

Total gravitational acceleration of the p^{th} agent can be determined as follows:

$$a_{p}(t) = G(t) \sum_{q=1}^{n_{m}} rand_{q} \cdot \frac{M_{q}(t)}{R_{pq}(t) + \varepsilon} (X_{q} - X_{p}), \quad (29)$$

where, $rand_q$ is a uniform random number in the interval [0, 1]. The velocity and position of each agent can be calculated as follows:

$$v_p(t+1) = rand_q \times v_p(t) + a_q(t), \qquad (30)$$

$$x_{p}(t+1) = x_{p}(t) + v_{p}(t+1),$$
 (31)

where, $rand_p$ is another uniform random number in the interval [0, 1]. To reduce the exploitation capability and to enhance the exploration power of the agents, a set of best agents ($K_{best}(t)$) are chosen so that only the $K_{best}(t)$ of the moving agents will attract the others. Therefore, Equation (29) can be modified as follows:

$$a_{p}(t) = G(t) \sum_{q \in K_{hest}(t)}^{n_{m}} rand_{q} \cdot \frac{M_{q}(t)}{R_{pq}(t) + \varepsilon} (X_{q} - X_{p}). \quad (32)$$

For updating the gravitational acceleration of agents, NSGSA uses the same equation as basic GSA. The most important parameter of this equation is the gravitational constant, G(t). In [30], G(t) is suggested as $G_0 \exp(\alpha t / t_{max})$ so that α and G_0 are the function parameters. It is difficult to find a suitable constant G_0 for various test functions. In this paper, the procedure to calculate the value of G_0 is as follows:

$$G_0 = \sigma \max_{d \in \{1, 2, \dots, N\}} (|x_u^d - x_l^d|),$$
(33)

where, σ is a coefficient of search interval parameter of NSGSA. In NSGSA method, the main loop starts after initializing the particle positions and velocities. Then, the external archive is updated based on the Pareto dominance criterion. To obtain a set of solutions that spans the whole Pareto optimal area as homogeneously as possible, a crowding distance is calculated as in [32] as follows:

$$d_{c,p} = \sqrt{\sum_{q=1}^{N} \left(d_{c,p}^{q} \right)^{2}}, \qquad (p = 1, 2, ..., n_{archive}), \qquad (34)$$

$$d_{c,p}^{q} = \operatorname{fit}_{p+1}^{q} - \operatorname{fit}_{p-1}^{q}, \quad (p = 1, 2, ..., n_{\operatorname{archive}}; q = 1, 2, ..., N), \quad (35)$$

$$\hat{o} = \sum_{\substack{p=1\\p \notin ep}}^{n_{\text{archive}}} \frac{\left| d_{c,p} - \overline{d}_{c} \right|}{\left(n_{\text{archive}} - q \right) \overline{d}_{c}} , \qquad (36)$$

where, *ep* is the set of extreme points, the size of which is taken equal to *q*. To get a uniform spread of Pareto-archive, the spread indicator (∂) must be reduced. This has been done by reducing the difference between max ($d_{c,p}$) and min ($d_{c,p}$).

In order to update the list of moving agents for the next flight, m extreme points of the single objective optimal solutions of the external archive and m points located in the least crowded area are inserted to the list. In NSGSA, two mutation operators, called sign and reordering mutation has been added to the original GSA to decrease the chance of trapping in local optima. Moreover, these two mutation operators promote and preserve diversity

within the moving agents. The mutation operator in NSGSA works as follows: the velocity of particle is first mutated by the sign and then by reordering the mutations.

To update the positions of each agent using sign operator, the sign of velocity vectors changes temporally in d^{th} direction with a predefined probability, P_u , as follows [29]:

$$V_{p}^{'d}(t+1) = S_{p}^{d}V_{p}^{d}(t+1), \qquad (37)$$

 $d=1, 2, \ldots, N; p=1, 2, \ldots, n_m,$

$$U_p^d = \begin{cases} -1 & \text{rand} < Pu \\ +1 & \text{Otherwise} \end{cases},$$
(38)

$$X_{p}(t+1) = X_{p}(t) + V'(t+1), \qquad (39)$$

where, $V_{p}^{d}(t+1)$ is the mutated velocity by the sign mutation operator and rand is a uniform random number generated in the interval [0, 1]. Then, the reordering operator is used for updating the position of each agent as follows [28, 29]:

$$V_{p}(t+1) = w(t) + V_{p}(t) + a_{p}(t), \qquad (40)$$

$$V_{p}(t+1) = \operatorname{sign_mutate}(V_{p}(t+1)), \quad (41)$$

$$V_{p}^{"}(t+1) = \text{reordering_mutate}\left(V_{p}^{"}(t+1)\right), \qquad (42)$$

$$X_{p}(t+1) = X_{p}(t) + V_{p}^{"}(t+1), \qquad (43)$$

where, w(t) is the Time Varying Inertia Co-efficient (TVIC) which can be determined by the following equation:

$$w(t) = w_0 \left(w_0 - w_1 \right) \times \left(\frac{t}{t_{\text{max}}} \right) . \tag{44}$$

The basic difference between TVIC taken in basic GSA and in NSGSA is that the TVIC value is considered as a random number in the interval [0, 1], in GSA based approach, whereas in NSGSA, a decreasing inertia is considered instead of using a random number.

The TVIC is taken as a decreasing inertia in this paper for proper exploration and exploitation in the first and the last iterations, respectively.

Having obtained the Pareto optimal set, choosing a best compromise solution is vital in decision making process. In this article, an interactive fuzzy membership approach has been applied to obtain satisfactory and best compromise solution from the non-inferior solution or non-dominated solution set in the multi-objective optimization.

The imprecise or fuzzy goal for each of the objective functions is quantified by defining their corresponding membership functions. The fuzzy sets are defined by Equation (45) and known as membership functions μ_{fi} . These functions represent the degree of membership in certain fuzzy sets in terms of values from 0 to 1. The membership value 0 indicates incompatibility with the sets, while 1 means full compatibility. By taking account of the minimum (f_{min}) and maximum (f_{max}) values of each objective function together with the rate of increase of membership satisfaction, the membership function μ_{fi} is determined in an individual manner. Here, it is assumed that μ_{fi} is a strictly monotonically decreasing and continuous function defined as [30-34]:

$$\mu_{f_{i}} = \begin{cases} -1 & f_{i} \leq f_{i}^{\min} \\ \frac{f_{i}^{\max} - f_{i}}{f_{i}^{\max} - f_{i}^{\min}} & f_{i}^{\min} < f_{i} < f_{i}^{\max} \\ 0 & f_{i} \geq f_{i}^{\max} \end{cases}$$
(45)

Table 1. Different algorithmic steps of NSGSA approach

Step	Operation
Step 1	Search space identification
Step 2	Randomized initialization
Step 3	Fitness evaluation of each particles
Step 4	Update external archive
Step 5	Non-dominated sorting
Step 6	Update the list of moving particles
Step 7	Update the mass of moving particles
Step 8	Update particles acceleration
Step 9	Update particles velocity
Step 10	Update and mutate particles position
Step 11	If the number of iterations exceeds its maximum predetermined bound, then end; or else, repeat Steps 3–10 until the maximum number of iteration is exceeded

The values of the membership functions indicate that how much (in the scale of 0–1) a nondominated solution has satisfied the objective μ_i [33]. The sum of the membership function values (μ_{fi} ; i = 1, 2, ..., N) for all the objectives can be computed in order to measure the effectiveness of each solution in satisfying the objectives. This effectiveness of each non-dominated solution can be rated with respect to all the non-dominated solutions (*M*) by normalizing its values over its total sum as follows [30, 33]:

$$\mu^{k} = \frac{\sum_{i=1}^{N} \mu_{f_{i}}^{k}}{\sum_{k=1}^{M} \sum_{i=1}^{N} \mu_{f_{i}}^{k}}$$
 (46)

The functions μ^k in Equation (46) are treated as a normalized membership function for non-dominated solutions in a fuzzy set and are represented as a fuzzy cardinal priority ranking of the non-dominated solutions. The solution that attains the maximum normalized membership μ^k in the fuzzy set, i.e., max { μ^k ; k = 1, 2, ..., M}, is chosen as the 'best non-dominated' solution.

The different steps of the implemented NSGSA algorithm are depicted in Table 1 below:

4 Simulation results and discussion

The single-phase transformer was designed following the presented methodology. For this particular case, the stacking factor was chosen to be 10%.

A classical transformer design procedure was also performed to provide a comparative device. Transformer parameters are given in the Appendix. The program has been developed using MATLAB.

To validate the presented approach, two TD objectives were considered: the mass and the losses that are to be minimized simultaneously in order to achieve the most appropriate trade-off between objectives all across the Pareto front. The Pareto front obtained for the TD using NSGSA approach is shown in Fig. 3.



Figure 3. The Pareto-optimal front obtained using NSGSA.

Table 2.	Parameters	and	results	of TD.
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Parameters		Classical [25]	NGDE [25]	<i>CMOA</i> [26]	MOHS [27]	Proposed NSGSA
Optimized Variables	<i>c</i> [cm]	2.50	1.82	2.60	1.71	1.70
	<i>t</i> [cm]	4.00	5.77	4.00	3.99	3.95
	h_w [cm]	7.50	6.07	10.00	10.00	9.98
	b_w [cm]	2.50	2.81	1.50	1.76	1.81
N ₁		231	166	210	319	354
N_2		507	334	423	643	662
f_1 : mass [kg]			4.77	6.11	4.25	4.173
f_2 : losses [W]			0.07	0.07	0.0838	0.067

Table 2 shows the parameters of the single-phase transformer obtained with a classical procedure (not optimized) and the NSGSA approach with normalized arithmetic mean minor.

In spite of the fact that NSGSA approach provides better results than NGDE, CMOA and MOHS, Table 2 shows that NSGSA algorithm offers the best performance and provides the minimum objective function when compared with the other multi objective optimization algorithms.

Simulation results indicate the advantages of the NSGSA approach and its applicability to the TD.

The simulation results in Table 2 shows the effectiveness of the proposed NSGSA algorithm, in terms of the solution quality, and the amount of elements of the Pareto set found by NSGSA.

The result of NSGSA with normalized arithmetic mean minor between the objective functions was $f_1 = 4.173$ kg and $f_2 = 0.067$ W.

Comparing the proposed optimization algorithm with the MOHS approach, the percentage reduction in mass and losses is found to be increased by 1.82%, and 20.04%, respectively.

5 Conclusions

In this paper, NSGSA algorithm is employed for the optimum design of single phase distribution transformer. The work aims at developing a TD that minimizes the objectives (such as the total mass and losses of power transformer) by using proposed multi-objective optimization approach, and by taking into account the constraints imposed by the international standards and transformer specifications.

The validity of the NSGSA algorithm for solving TD optimization problem is illustrated with its application to a 400 VA distribution transformer design, and then by comparing the obtained simulation results with those obtained in literature when using other optimization algorithms, such as NGDE, CMOA, and MOHS algorithms.

Based on the obtained simulation results, NSGSA algorithm in particular proved its superiority in providing the minimum values of the two objective functions (mass and losses).

In this simple transformer model, voltage regulation, power factor and temperature limits were not considered. A more detailed and accurate model will be developed in future papers. Future work will also consider development of the proposed algorithm capable of dealing with more complicated cases such as three phase transmission and distribution transformers for various feasible transformer designs.

6 Appendix

6.1. Power transformer parameters

Shell core, dry-type, single-phase transformer: S = 400 VA, $f_n = 50$ Hz, $V_1 = 110$ V and $V_2 = 220$ V. Minimum efficiency $\eta_{min} = 80$ %, $K_f = 1.1$, $K_c = 0.5 \times 10^{-3}$ and $\alpha = 1.7$, $\beta = 1.9$.

6.2. NSGSA parameters

Archive length $(n_{archive}) = 100$, Agent size $(n_m) = 100$, Reordering mutation probability = 0.4, Sign mutation probability = 0.9, Percent of elitism $(p_{elitism}) = 0.5$, Initial value of inertial coefficient $(W_0) = 0.9$, Final value of inertial coefficient $(W_1) = 0.5$, Coefficient of search interval $(\sigma) = 2.5$.

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