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A note on joint determination of the rotation cycle time and number of shipments for a multi-item EPQ model with a random defective rate

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SUMMARY

This study proposes a two-phase, algebraic approach to resolve the rotation cycle time and number of shipments for a multi-item, economic production quantity (EPQ) model with a random defective rate. A conventional method for solving a multi-item EPQ model is to use differential calculus and Hessian matrix equations on system cost function to prove convexity first and then derive an optimal production-shipment policy [1], whereas the proposed approach obtains the optimal operating policy in a simplified, two-phase algebraic way, using neither differential calculus nor Hessian matrix equations. The simplification enables managing practitioners to effectively determine real-life, vendor-buyer integrated multi-item EPQ systems.

KEY WORDS: *optimization, production-shipment policy, multi-item EPQ model, vendor-buyer integrated system, algebraic approach.*

1. INTRODUCTION

The most economical production lot-size decision that minimizes the total production-inventory costs was determined by Taft [2] using mathematical techniques. This so-called original economic production quantity (EPQ) model considers production lot sizing for a single product with perfect production process and a continuous issuing policy for finished products. However, in real-life, vendor-buyer integrated systems, the following practical factors are commonly considered: (1) an inevitable random defective rate in a given production run rather than a perfect production; (2) a discontinuous end products issuing policy opposed to a continuous inventory distribution policy; and (3) a multi-product production arrangement in preference to the single product production plan. In past decades, several studies have been carried out to address the imperfect quality production systems. Shih [3] examined inventory problems where the proportion of defective products in an accepted batch is a random variable with known probability distribution and derived the optimal solutions to amended systems. Schwaller [4] considered an economic order quantity (EOQ) model with fixed and variable inspection costs, and removal of a known portion of defective items in incoming lots. Hariga and Ben-Daya [5] considered an EPQ problem with imperfect processes, wherein the time to shift from the in-control state to the out-of-control state to be flexible, and both the distribution-based and distribution-free bounds on the optimal system costs, were examined. Rahim and Ben-Daya [6] studied the simultaneous effects of a deteriorating product and deteriorating production processes on the EPQ decision, inspection schedules, and economic design of control charts. Maddah, et al. [7] presented the lot-size decision considering a Markov production process and scrap items. Various additional aspects of imperfect quality and unreliable issues in production systems may be found elsewhere [8-13].

During past decades, different aspects of discontinuous, multi-delivery, vendor-buyer systems have been extensively studied. Goyal [14] presented a method to solve a single supplier-single customer vendor-buyer integrated problem. Sarker and Parija [15] studied production-shipment systems in which raw materials are procured from suppliers and processed for conversion to finished products. A model was proposed for simultaneously determining an optimal ordering policy for raw materials and an optimal manufacturing batch size. Thomas and Hackman [16] examined a supply chain problem wherein a distributor faces price-sensitive demand and has the opportunity to contractually commit to a delivery quantity at regular intervals over a finite horizon in exchange for a per-unit cost reduction for units acquired via committed delivery. A simulation-based approximation was used to develop models yielding closed-form solutions for an optimal order quantity and resell price for the distributor under normally distributed demand. For other various aspects of supply chains management issues, one may refer to [17-23].

In real-life production environments, in order to maximize machine utilization production planners often schedule m products to be manufactured in turn on a single machine rather than a single product, as assumed by an EPQ model. Dixon and Silver [24] derived the optimal lot-sizes for a single work center, multi-product manufacturing system. The period-by-period requirements for each product are assumed to be known constants out to the end of some common time horizon. There is a fixed setup cost, separate linear production and holding costs, different production rates associated with each product. Their objective was to determine lot-sizes so that: (1) costs are minimized, (2) no backlogging occurs, and (3) capacity is not exceeded. Leachman and Gascon [25] proposed a heuristic scheduling policy for multi-product, single-machine production systems with stochastic and time-varying demands. The policy can be applied time period by time period to the decisions concerning which items to produce in what quantities during the next time period. Sambasivan and Schmidt [26] developed a heuristic procedure for solving multi-item, capacitated lot sizing problems with inter-plant transfers. They employed a smoothing routine to remove capacity violations, and carried out extensive experimentation to compare their heuristic (implemented on IBM 3090 mainframe using FORTRAN) solution procedures and LLNDO. Chiu et al. [1] determined an optimal common production cycle time and an optimal number of shipments for a multi-item EPQ model with a random defective rate. They used mathematical modelling, Hessian matrix equations [27] and differential calculus to derive an optimal production-shipment policy.

Various different aspects of multi-item production planning and optimization have been extensively investigated [28-33].

Grubbström and Erdem [34] proposed an algebraic approach to the EOQ problem with backlogging. Without reference to the first- or second-order derivatives, they algebraically derived a solution to the problem of an optimal order quantity t. Similar methods have also been applied to various different EPQ-based models and supply chains systems [35-37]. This paper extends such an algebraic approach to a vendor-buyer integrated multi-item EPQ model with random defective rate [1] and demonstrates that the optimal policy for such a specific model can be derived without derivatives.

2. MODELLING AND SOLUTION PROCEDURE

A two-phase algebraic solution procedure is proposed in this study to determine the optimal rotation cycle time and optimal number of shipments for a multi-item EPQ model with random scrap [1]. It has been assumed that a manufacturing system produces *m* products in turn on a single machine in order to maximize machine utilization. All products made are screened and the unit inspection cost is included in the unit production cost C_i. During the manufacturing process of a product *i* (where *i* = 1, 2, … *m*), an x_i portion of scrap items is randomly produced at a rate d_i with a disposal cost C_{Si} per scrap item. No shortages are allowed in the proposed system, that is, a constant production rate P_i for a product *i*, must satisfy $(P_i\text{-}d_i\text{-}\lambda_i)>0$, where $d_i = x_iP_i$ and λ_i denotes an annual demand rate for the product *i*. Further, the proposed multi-item, production-shipment integrated EPQ model assumes a multi-delivery policy. The finished items for each product *i* are delivered to customers when the entire production lot is quality assured at the end of the production of each product *i*. That is during a delivery time t_{2i}, a fixed quantity *n* installments of a finished batch are delivered at a fixed interval of time to the customer. Figure 1 shows the on-hand inventory of perfect quality items of the product *i* at producer's side in the proposed multi-item EPQ system with random scrap and under a common cycle policy. The on-hand inventory of product *i* at customers' side in the proposed multi-item EPQ system is illustrated in Figure 2.

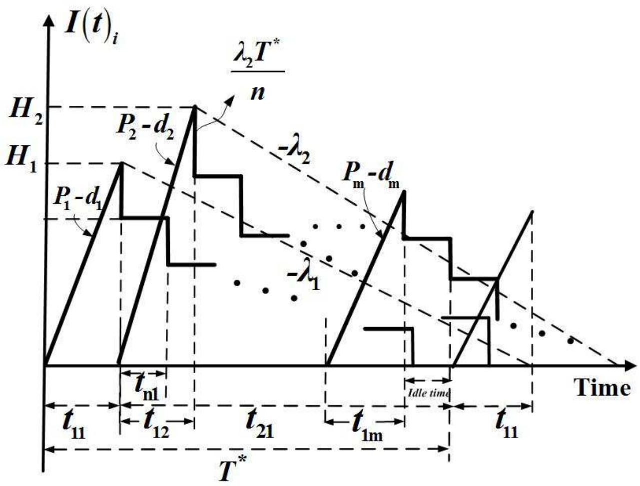

Fig. 1 *On-hand inventory of perfect quality items of the product i at producer's side in the proposed multi-item EPQ system with random scrap and under a common cycle policy [1]*

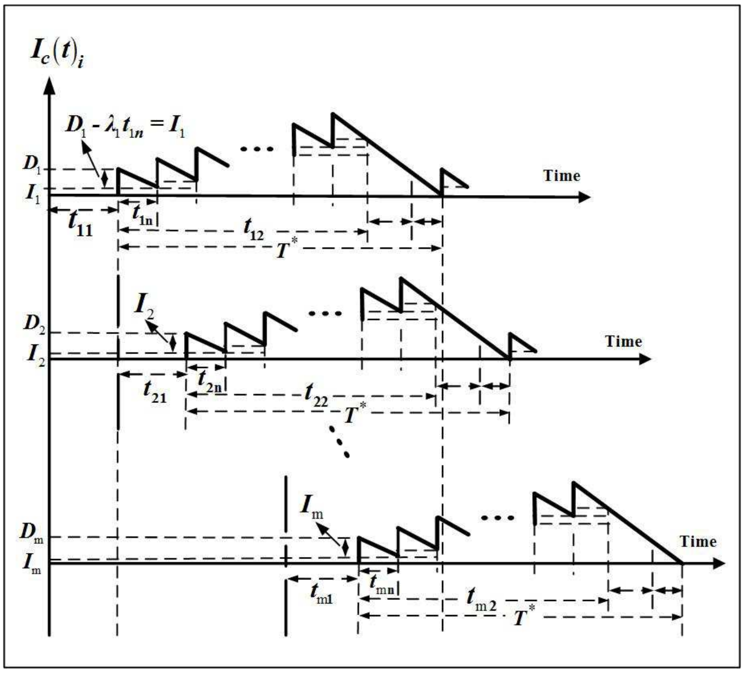

Fig. 2 *On-hand inventory of product i at customers' side in the proposed multi-item EPQ system with random scrap and under common cycle policy [1]*

Notation for cost parameters used in this study includes a production setup cost K_i for a product i, the producer's unit holding cost h_i for a product i, a fixed delivery cost K_{1i} per shipment and the unit shipping cost C_{Ti} for a product i, the customers' unit holding cost h_{2i} for a product i. Other notation used is listed below:

Q_i = production lot size per cycle for a product i,

T = common production cycle time - decision variable,

t_{1i} = production uptime for a product i,

H_i = maximum level of on-hand inventory in units for the product i when the production process ends,

I_i = number of left over items for a product i per delivery after the depletion during t_{ni},

D_i = number of finished items (fixed quantity) for a product i distributed to customer per delivery,

t_{ni} = a fixed interval of time between each installment of finished products delivered during t_{2i}, for a product i,

n = number of fixed quantity installments of the finished lot to be delivered to customers in each cycle - the other decision variable,

$I(t)_i$ = on-hand inventory of perfect quality items for a product i at time t,

$I_c(t)_I$ = on-hand inventory of perfect items for a product i kept at the customer's end at time t,

$TC(Q_i,n)$ = total production-inventory-delivery costs per cycle for a product i in the proposed system,

$E[TCU(Q_i,n)]$ = expected production-inventory-delivery costs per unit time for m products in the proposed system,

$E[TCU(T,n)]$ = expected production-inventory-delivery costs per unit time for m products in the proposed system, using rotation production cycle time as the decision variable.

It can be seen that the total production-inventory-delivery cost per cycle of m products, $TC(Q_i,n)$ consists of the production setup cost, variable production cost, disposal cost, holding cost during production uptime t_{1i} and during the delivery time t_2, fixed and variable delivery cost, and customers' holding costs for a product i. Therefore, $TC(Q_i,n)$ can be expressed as:

$$\sum_{i=1}^{m} TC(Q_i,n) = \sum_{i=1}^{m} \left\{ \begin{array}{l} K_i + C_i Q_i + C_{Si} x_i Q_i + h_i \left[t_{1i} \left(\frac{H_i}{2} + \frac{x_i Q_i}{2} \right) + \frac{n-1}{2n} (H_i t_{2i}) \right] + nK_{1i} + \\ + C_{Ti} Q_i (1-x_i) + h_{2i} \left[\frac{n(D_i - I_i) t_{ni}}{2} + \frac{n(n+1)}{2} I_i t_{ni} + \frac{n I_i t_{1i}}{2} \right] \end{array} \right\} \quad (1)$$

Since scrap rate during the production process is random, to take such a randomness of x into account, we use the expected values of x in our cost analysis. From Figure 1, it can be seen that applying the common production cycle policy, the expected cycle length for any product i is:

$$E[T] = Q_i (1 - E[x_i]) \frac{1}{\lambda_i} \quad (2)$$

Then, from Figures 1 and 2, by substituting all basic variables in Eq. (1) and applying the renewal reward theorem [1], the expected system cost $E[TCU(T,n)]$ can be obtained as follows:

$$E\left[TCU(T,n)\right] = E\left[\sum_{i=1}^{m} TC(Q_i,n)\right] \frac{1}{E[T]} =$$

$$= \sum_{i=1}^{m} \left\{ \begin{array}{l} \frac{K_i}{T} + C_i \lambda_i \cdot \frac{1}{[1-E(x_i)]} + C_{Si} \lambda_i \frac{E(x_i)}{[1-E(x_i)]} + C_{Ti} \lambda_i + \frac{nK_{1i}}{T} + \\ + \frac{h_i T \lambda_i}{2} \left[\frac{\lambda_i}{P_i} \frac{E(x_i)}{[1-E(x_i)]} \cdot \frac{1}{[1-E(x_i)]} + 1 - \frac{1}{n} \left(1 - \frac{\lambda_i}{P_i} \frac{1}{[1-E(x_i)]} \right) \right] + \\ + \frac{h_{2i} T \lambda_i}{2} \left[\frac{1}{n} \left(1 - \frac{\lambda_i}{P_i} \cdot \frac{1}{[1-E(x_i)]} \right) + \frac{\lambda_i}{P_i} \cdot \frac{1}{[1-E(x_i)]} \right] \end{array} \right\} \quad (3)$$

Upon derivation of $E[TCU(T,n)]$, we proposed a two-phase algebraic approach instead of a conventional method employing differential calculus (Chiu et al., 2013) to determine the rotation cycle time and an optimal number of shipments for such a specific multi-item EPQ model.

2.1 THE PROPOSED APPROACH

Phase 1: Derivation of Optimal Number of Deliveries n*

It can be seen from Eq. (3) that $E[TCU(T,n)]$ contains two decision variables (T and n), and they are written down in the forms of T^{-1}, nT^{-1}, T and Tn^{-1}, with different coefficients associated to them. Let π_0, π_1, π_2, π_3 and π_4 denote the following:

$$\pi_0 = \sum_{i=1}^{m} \lambda_i \left\{ C_i \frac{1}{[1-E(x_i)]} + C_{Si} \frac{E(x_i)}{[1-E(x_i)]} + C_{Ti} \right\} \quad (4)$$

$$\pi_1 = \sum_{i=1}^{m} K_i \quad (5)$$

$$\pi_2 = \sum_{i=1}^{m} K_{1i} \tag{6}$$

$$\pi_3 = \sum_{i=1}^{m} \frac{\lambda_i}{2} \left\{ h_i + \left[\frac{\lambda_i}{P_i} \cdot \frac{1}{[1-E(x_i)]} \right] \cdot \left[\frac{h_i \cdot E(x_i)}{[1-E(x_i)]} + h_{2i} \right] \right\} \tag{7}$$

$$\pi_4 = \sum_{i=1}^{m} \left(\frac{\lambda_i}{2} \right) \left(1 - \frac{\lambda_i}{P_i} \cdot \frac{1}{[1-E(x_i)]} \right) (h_{2i} - h_i) \tag{8}$$

Then, Eq. (3) becomes:

$$E\left[TCU(T,n)\right] = \pi_0 + \pi_1 \cdot \left(T^{-1}\right) + \pi_2 \cdot \left(nT^{-1}\right) + \pi_3 \cdot (T) + \pi_4 \cdot \left(Tn^{-1}\right) \tag{9}$$

With further rearrangement, Eq. (9) becomes:

$$E\left[TCU(T,n)\right] = \pi_0 + \left(T^{-1}\right)\left[\pi_1 + \pi_3 (T)^2\right] + \left(nT^{-1}\right)\left[\pi_2 + \pi_4 \left(Tn^{-1}\right)^2\right] \tag{10}$$

or:

$$\begin{aligned} E\left[TCU(T,n)\right] &= \pi_0 + \left(T^{-1}\right)\left[\sqrt{\pi_1} - \left(\sqrt{\pi_3} T\right)\right]^2 + \\ &+ \left(nT^{-1}\right)\left[\sqrt{\pi_2} - \left(\sqrt{\pi_4} Tn^{-1}\right)\right]^2 + \\ &+ 2\sqrt{\pi_1 \cdot \pi_3} + 2\sqrt{\pi_2 \cdot \pi_4} \end{aligned} \tag{11}$$

It can be seen that Eq. (11) is minimized, if both the second and the third square terms in the right-hand size of Eq. (11) equal zero. This means:

$$\begin{aligned} &\sqrt{\pi_1} - \left(\sqrt{\pi_3} T\right) = 0 \ \text{and} \\ &\sqrt{\pi_2} - \left(\sqrt{\pi_4} Tn^{-1}\right) = 0 \end{aligned} \tag{12}$$

Thus, we have:

$$T = \frac{\sqrt{\pi_1}}{\sqrt{\pi_3}} \ \text{and} \ n = \frac{\sqrt{\pi_4}}{\sqrt{\pi_2}} (T) \tag{13}$$

Therefore, we obtain:

$$n = \sqrt{\frac{\pi_1 \pi_4}{\pi_2 \pi_3}} \tag{14}$$

Substituting Eqs. (5) to (8) in Eq. (14), and with further derivations, n^* can be obtained as follows:

$$n = \sqrt{\frac{\sum_{i=1}^{m} K_i \left(1 - \frac{\lambda_i}{P_i [1-E(x_i)]} \right) (h_{2i} - h_i)}{\sum_{i=1}^{m} K_{1i} \left[h_i \left(1 + \frac{\lambda_i}{P_i} \frac{E(x_i)}{[1-E(x_i)]^2} \right) + \frac{h_{2i} \lambda_i}{P_i [1-E(x_i)]} \right]}} \tag{15}$$

In real-life, production-shipment applications, the number of deliveries n can only take on integer values. Let n^+ represent the smallest integer greater than, or equal to, n (i.e., the computational result of Eq. (15)), and let n^- be the largest integer smaller than or equal to n. From this it follows that an optimal n^* is neither n^+ nor n^-.

Phase 2: Derivation of Optimal Common Cycle Time T*

We consider that $E[TCU(T,n)]$ contains a single decision variable T (i.e., n is considered a constant). Eq. (9) can be rearranged as:

$$E\left[TCU(T,n)\right]=\pi_0+(\pi_1+\pi_2 n)\left(T^{-1}\right)+\left(\pi_3+\pi_4 n^{-1}\right)(T) \tag{16}$$

Eq. (16) can be rearranged as:

$$E\left[TCU(T,n)\right]=\pi_0+\left(T^{-1}\right)\left[(\pi_1+\pi_2 n)+\left(\pi_3+\pi_4 n^{-1}\right)(T)^2\right] \tag{17}$$

or:

$$E\left[TCU(T,n)\right]=\pi_0+\left(T^{-1}\right)\left\{\left(\sqrt{\pi_1+\pi_2 n}\right)-\left[\sqrt{\pi_3+\pi_4 n^{-1}}\,(T)\right]\right\}^2+ \\ +2\sqrt{\pi_1+\pi_2 n}\cdot\sqrt{\pi_3+\pi_4 n^{-1}} \tag{18}$$

It can be seen that Eq. (18) is minimized, if the second square term in the right-hand side of Eq. (18) equals zero. That is:

$$\left(\sqrt{\pi_1+\pi_2 n}\right)-\left[\sqrt{\pi_3+\pi_4 n^{-1}}\,(T)\right]=0 \tag{19}$$

Therefore, we obtain:

$$T^*=\sqrt{\frac{\pi_1+\pi_2 n}{\pi_3+\pi_4 n^{-1}}} \tag{20}$$

Substituting Eqs. (5) to (8) in Eq. (20), and with further derivations, we have:

$$T^*=\sqrt{\frac{2\sum_{i=1}^{m}(K_i+nK_{1i})}{\sum_{i=1}^{m}\left\{\begin{array}{l}h_i\lambda_i+\left(\dfrac{h_iE(x_i)}{[1-E(x_i)]}+h_{2i}\right)\left[\dfrac{\lambda_i^2}{P_i[1-E(x_i)]}\right]+ \\ +\dfrac{\lambda_i}{n}(h_{2i}-h_i)\left(1-\dfrac{\lambda_i}{P_i[1-E(x_i)]}\right)\end{array}\right\}}} \tag{21}$$

One notes that Eqs. (15) and (21) are identical to those obtained by Chiu et al. [1].

Now, in order to jointly determine an optimal common cycle time and an optimal number of deliveries, we can first apply n^+ or n^- in Eq. (21) to find the T, then insert them in $E[TCU(T,n)]$ (i.e., Eq. (3)) and select the one with the minimal cost.

3. NUMERICAL EXAMPLE

For the purpose of making the comparison easier for readers, and to verify the results, the same numerical examples, as in Chiu et al. [1], are used in this section. Values of system parameters used in this example are listed as follows:

P_i = annual production rates of five different products are *16000*, *18000*, *20000*, *22000* and *24000*.

λ_i = annual demand rates of five products are *3000*, *3200*, *3400*, *3600* and *3800*, respectively.

x_i = random defective rates for each product which follow the uniform distribution over the intervals of [*0*, *0.10*], [*0*, *0.15*], [*0*, *0.20*], [*0*, *0.25*] and [*0*, *0.30*], respectively.

C_{Si} = unit disposal costs are *$50*, *$55*, *$60*, *$65* and *$70* per scrapped item, respectively.

C_i = unit manufacturing costs are *$80*, *$90*, *$100*, *$110* and *$120*, respectively.

h_i = unit holding costs are *$10*, *$15*, *$20*, *$25* and *$30*, respectively.

K_i = production set up costs are *$16000*, *$18000*, *$20000*, *$22000* and *$24000*, respectively.

h_{2i} = unit holding costs in the customers' end are *$70*, *$75*, *$80*, *$85* and *$90*, respectively.

K_{1i} = the fixed delivery costs per shipment are *$1600*, *$1800*, *$2000*, *$2200* and *$2400*, respectively.

C_{Ti} = unit transportation costs are *$0.5*, *$0.4*, *$0.3*, *$0.2* and *$0.1*, respectively.

From Eq. (15) we found $n=3.63$. Therefore, the optimal number of delivery is either $n^+=4$ or $n^-=3$. Applying them in Eq. (21), we obtain $(T,n^+)=(0.5826,4)$ and $(T,n^-)=(0.5393,3)$. Then, substituting values of (T,n^+) and (T,n^-) in Eq. (3), we have $E[TCU(0.5826,4)]$ = *$2541548* and $E[TCU(0.5393,3)]$ = *$2543001*.

Finally, by selecting the minimum cost among the aforementioned results, we obtain that the optimal common cycle time $T^*=0.5826$, optimal number of deliveries $n^*=4$, and optimal system cost $E[TCU(T^*,n^*)]$ = *$2541548*. These numerical results are identical to that obtained by Chiu et al. [1].

4. CONCLUDING REMARKS

Unlike the conventional method where differential calculus and Hessian matrix equations are employed to prove the convexity of the system cost function, and to derive an optimal production-shipment policy [1], this study proposes a two-phase algebraic approach to determine an optimal common cycle time and number of shipments for the vendor-buyer, integrated multi-item EPQ model with scrap. Such a simplified approach may help practitioners in the field to solve the real-life, multi-item EPQ problem.

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ISTOVREMENO ODREĐIVANJE VREMENA IZMJENE CIKLUSA PROIZVODNJE I BROJA ISPORUKE ZA EPQ MODEL ZA VIŠESTRUKI TIP PROIZVODA SA SLUČAJNOM POJAVOM NESUKLADNOSTI

U radu je predložen algebarski pristup u dvije faze za određivanje vremena izmjene ciklusa proizvodnje i broja isporuke za model ekonomične količine proizvodnje (EPQ) za višestruki tip proizvoda uz slučajnu nesukladnost proizvoda. Tradicionalna metoda za rješavanje EPQ-a za višestruki tip proizvoda jest primjena diferencijalnog računa i Hesse-ove matrične jednadžbe na funkciju troška sustava radi prvotnog dokaza konveksnosti, a zatim i određivanja optimalnog režima proizvodnje i isporuke [1]. U ovom radu, optimalni režim upravljanja se određuje pomoću algebarskog pristupa u dvije faze bez korištenja diferencijalnog računa ni Hesse-ovih matričnih jednadžbi. Navedeni pristup omogućuje voditeljima proizvodnje u praksi jednostavnije određivanje EPQ-a u integriranom sustavu isporučitelj-kupac.

KLJUČNE RIJEČI: *optimizacija, režim proizvodnja-isporuka, EPQ za višestruki tip proizvoda, integrirani sustav isporučitelj-kupac, algebarski pristup.*