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ANALYSIS OF THE BUSINESS CYCLE – THE MARKOV-SWITCHING APPROACH

JEL classification: C63, C82, E37

Abstract

One of the most effective methods of modeling the current and prediction of the future economic situation is the analysis of the business cycles. Some of the analysis are based on the business tendency surveys. Respondents' answers to the questions from the surveys are found to be correlated with the official assessment of the economic situation and some of them seem to be completely inaccurate. Nonetheless the gathered data could be used to identify turning points of the business cycle in Poland. In the article Markov-switching models and their usefulness to the analysis of the macroeconomic time series are explored. Turning points from the presented algorithm were compared with the dating given by the OECD. Time delay between the reference and the calculated time series varied depending on the considered historical time interval. It is however sometimes possible to determine the parameters that allow getting the leading indicator. This knowledge could be used by policy-makers to react upon the upcoming crisis on time.

Key words: business cycle turning point detection, Markov-switching model, Viterbi path

1. INTRODUCTION

There are many methods of estimation the economic situation. Knowing historical, current and predicting the direction of fluctuations in the economy allows to countering the anticipated bad effects for the country. For this purpose usually some kind of economic indicators are used. In general they are based on two categories of data: qualitative and quantitative. They serve as an input to analytic methods starting from simple averaging and ending in complex econometric models. Markov-switching time series models have played the prominent role in the analysis of business cycle for decades. In this paper those models were combined with the Viterbi algorithm to get the approximation of the business cycle turing points. Based on the results of the business tendency survey conducted monthly by the Research Institute for Economic Development in Warsaw School of Economics the empirical experiments show that calculated approximations could be far from optimal.

The aim of the paper is to show the way to verify and possibly to improve the accuracy of state estimation. The idea is to take into account not only input time series for the particular moment of time, but also the Viterbi paths computed for the previous periods. Consistency between results for few succeeding months is considered as a successful verification of the latest states, whereas big differences suggest that new data can be a reflection of the recent changes of the economic climate. The proposed solution is to change the length of the input time series and at the same time the beginning of the studied period.

2. MARKOV-SWITCHING MODELS IN THE ANALYSIS OF THE BUSINESS CYCLE

Over time, number of methods of determining business cycle turning points were developed. In all methods some kind of leading indicator is used. It provides information about the current and the future state of the business cycle. The wide range of econometric methods should be mentioned. Many model-based methods (Cleveland 1972, Bell 1984, Wildi and Schips 2005) rely on ARIMA or state-space model-representations of the data generating process (DGP) often used with the filters, usually Hodrick-Prescott (1997) or Christiano-Fitzgerald (2003). Also using a logistic regression has been yielded quite satisfactory results (Lamy 1997, Birchenhall et al. 1999, Chin et al. 2000, Sensier et al. 2004). Finally there is a group of spectral methods based on frequency filtering by using for example Fourier transform (Addo et al. 2012).

In the paper an alternative approach based on Markov-switching model (MS) was chosen. History of using MS models in the analysis of the business cycle is as long as the concept of model itself (Hamilton 1989). These models are used mainly to determine the rates of growth and business cycle turning points. Due to the possibility of choosing a form of an observable and unobservable component, a huge variety of types of models are used and researched (Hamilton 1994, Koskinen and Oeller 2004).

In this paper the Markov-switching model is used. In the remaining part of the paper so-called “switching” refers to the parameters of a normal distribution. To be more exact we focus on hidden Markov model (HMM), which was described in the literature in the 60s of the previous century (for comprehensive description see Cappé et al. 2005), that is before the first articles by Hamilton. Let us give a brief introduction to the switching-Markov theory, including the basic definition and notation used in the rest of the paper. Let $\{X_k, Y_k\}_{k\geq 0}$ be a discrete stochastic process satisfying the following conditions:

- The unobservable process $\{X_t\}_{t\geq 0}$ is the homogenous Markov chain (MC) with the finite state space *S*.
- Conditionally on the process $\{X_t\}_{t\geq 0}$ the observations $\{Y_t\}_{t\geq 0}$ are independent, and for each t the conditional distribution of Y_t depends on X_t only.

When Y_t has univariate or multivariate Gaussian distribution, which is a common case in macroeconomic application, we say about normal HMM.

The problem in application of HMM models, is to estimate the state of unobservable MC at a fixed time $n \leq T$ knowing the realization of observable variables in the same period of time. It allows to identify the phases of the business cycle. The usually exploit solutions of the problem (Hamilton 1994) are the smoothed probabilities

$$s_t(i) = P(X_t = i | Y_1 = y_1, Y_2 = y_2, \ldots, Y_T = y_T) \tag{1}$$

or the filtered probabilities

$$f_t(i) = P(X_t = i | Y_1 = y_1, Y_2 = y_2, \ldots, Y_n = y_n), \tag{2}$$

where $i \in S$. On the basis of those probabilities the estimation of the state of the hidden Markov chain at the moment t is done (Chauvet and Hamilton 2005, Harding and Pagan 2002). In the simplest case as the state the value $\underset{i}{\operatorname{argmax}}\, w_t(i)$ or $\underset{i}{\operatorname{argmax}}\, f_t(i)$ is assumed. The states on the path of MC are estimated locally, step be step. Each time the state with the highest probability is chosen. Such approach, especially in case of multispace HMM, could be ineffective. A global decoding is possible, when instead of a single point of time the whole period covered by the analysis is taken under consideration. This most likely path of MC $(x_1^*, x_2^*, \ldots, x_T^*) \in S^T$ is called the Viterbi path and is defined as

$$P(X_1 = x_1^*, X_2 = x_2^*, \ldots, X_T = x_T^* | Y_1 = y_1, Y_2 = y_2, \ldots, Y_T = y_T) = \tag{3}$$
$$\max_{(x_1, x_2, \ldots, x_T) \in S^T} \{P(X_1 = x_1, X_2 = x_2, \ldots, X_T = x_T | Y_1 = y_1, Y_2 = y_2, \ldots, Y_T = y_T)\}$$

Elements of the Viterbi path are calculated using the Viterbi algorithm (Viterbi 1967), and to estimate the parameters of the HMM the Baum-Welch algorithm (Baum et al. 1970) has been used. The character of both algorithms is deterministic, but the results of Baum-Welch method strongly depend on the initial values and can be far from optimal. In order to increase the chances of finding the optimal solution, the calculation are repeated many times for the same set of data and different initial values. Summarizing, each hidden Markov model thus is defined by the following parameters (Bernardelli 2013):

- k – number of states,
- set of n symbols (alphabet),
- initial probabilities for every state (k parameters),
- transition matrix P, that is matrix of probabilities of transitions between two states (k^2 parameters),
- parameters (means and variances) of normal distribution defining probability of emission of symbol in each state ($2kn$ parameters).

In the paper hidden Markov models with the state space consist of two, three and four states are considered. In case of 2-state HMM the goal is to defragment the time series into two types of periods: these associated with relatively good conditions and those, which are connected rather with the worse situation. To conduct such classification we consider normal HMM with state space of the form $S=\{0,1\}$. The following condition must be satisfied by an observable component Y_t that corresponds to economic time series being under the analysis:

$$Y_n|_{X_n=0} \sim N(\mu_0, \sigma_0) \quad \text{and} \quad Y_n|_{X_n=1} \sim N(\mu_1, \sigma_1). \tag{4}$$

We assume that $\mu_0 < \mu_1$. State denoted by 0 corresponds to those points of time, in which the situation in the country is considered as deteriorating, whereas state 1 is associated with an improvement of economic condition. The most likely path of MC that reflects changes in economic climate in the scale of two states may be sometimes too poor to describe the dynamic changes or could be seen as too simplistic.

To extract periods difficult to classify in 0-1 scale, which could be treated as the announcement of changes, we exteded state space by adding the intermediate state ½ to the state space. This state should correspond to uncertain periods in economic development. The meaning of states 0 and 1 is the same as in the previously described 2-state model. An extended 3-state model is defined as follows:

$$Y_n|_{X_n=i} \sim N(\mu_i, \sigma_i) \tag{5}$$

for $i=0, ½,1$, where $\mu_0 < \mu_{\frac{1}{2}} < \mu_1$. We add an important restriction to the model, which reflects gradual changes in economy. Namely, we assume that $p(0,1) = p(1,0) = 0$. It means, that the path between two outermost states must always lead through the middle state.

To enrich the analysis by allowing more precise classification, the third class of models were introduced in the research. For this purpose we extend the state space of a Markov chain to the four-level scale $S = \{0, ⅓, ⅔, 1\}$. This allow to distinguish periods clearly good (state 1), worse but still positive (state ⅔), moderately bad (state ⅓) and definitely bad (state 0). It is assumed, that the conditions (5) are fulfilled for $i \in S$, where $\mu_0 < \mu_{\frac{1}{3}} < \mu_{\frac{2}{3}} < \mu_1$. Analogously to the 3-state case, we assume that transitions only between adjacent states are possible, that is

$$p(0,1) = p(1,0) = p\left(0,\tfrac{2}{3}\right) = p\left(\tfrac{2}{3},0\right) = p\left(\tfrac{1}{3},1\right) = p\left(1,\tfrac{1}{3}\right) = 0. \qquad (6)$$

Of course theoretically the state space could be extended to more then four states. However this cause problems with the state interpretation. What is even more important, is the fact that the more numerous state space the more problems with the estimation of the model parameters and the longer computation time (see next section). Models with more than two states combined with the Viterbi path are rather uncommon in the macroeconomic literature. They were first presented in the article (Bernardelli and Dędys, 2012), with some improvements in (Bernardelli 2014). In those articles the focus was on finding the optimal Viterbi path by taking into account the historical data to the specified point of time. In this paper an extra feature is added to the procedure, that is the assessment of the states in the path from the viewpoint of not only the input data, but also the Viterbi paths for the historical points of time. The idea behind this approach is based on the observation that the states established for longer period of time should not be changed when new data are available. When historically justified states differs a lot in the new calculated HMM, it probably means that the new data give completely fresh perspective for the changes in business climate. In that case the reliability of the results can be questionable.

3. DATA AND METHOD OF ESTIMATION

As an input the data from the business tendency surveys in industry conducted monthly by the Research Institute for Economic Development in Warsaw School of Economics (RIED) were used. Respondents answer to eight questions about current and eight questions about future situation in enterprise (due to a respondent's knowledge and prediction). Respondents are chosen from the set of all enterprises, including microenterprises. The number of correctly filled questionnaires oscillates around 400. The composition of the sample is more or less constant each month. More precisely approximately 80-90% of entities are the same, the rest are changeable. In econometric methods the non-panel character of the sample must be taken into consideration. In switching Markov models however there are no assumptions concerning this issue. It confirms the advantages of using HMM.

The questionnaires are available on the websites of the RIED[1]. Respondents are answering about the situation in current and the following month (respondent's prediction for the next 3-4 months). There are three possible reply options: increase, decrease or no change. The questions are as follows:

Question 1 – level of production

[1] http://kolegia.sgh.waw.pl/pl/KAE/struktura/IRG/koniunktura/Strony/metody.aspx [accessed: 10 May, 2015]

Question 2 – level of orders
Question 3 – level of export orders
Question 4 – stocks of finished goods
Question 5 – prices of goods produced
Question 6 – level of employment
Question 7 – financial standing
Question 8 – general economy situation

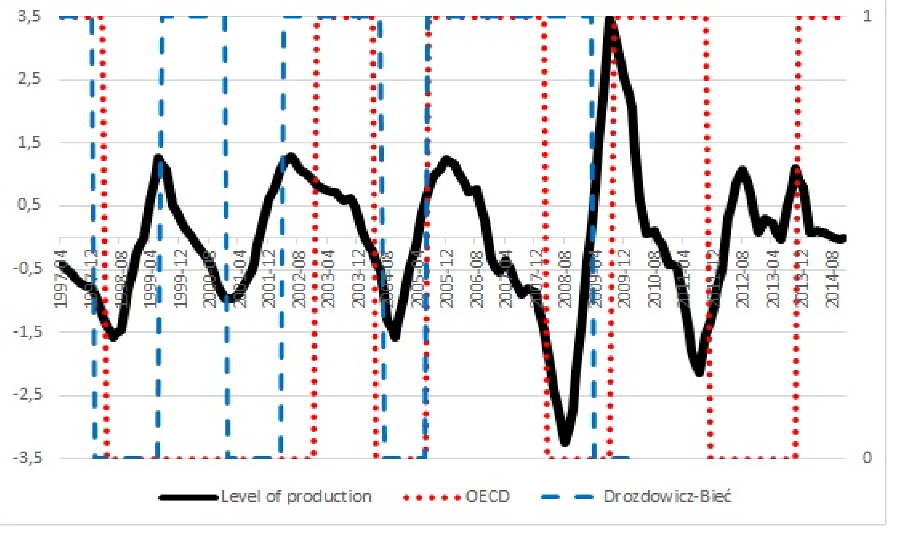

Figure 1. Time series decomposition for the question about level of production (April 1997-November 2014) against the reference time series.

Source: own calculations.

For the calculations data from April 1997 to November 2014 were taken. Only the questions connected with the assessment of the current situation in the enterprise were considered. The input time series were preprocessed by cleaning from seasonal and random fluctuation. For the time series decomposition the procedure STL from the R package was used. STL procedure is an implementation of an algorithm based on local weighted regression method called “loess” (Cleveland 1990). Identification of business cycle turning points is not an easy task, so there are discrepancies between available sources. One of the reference times series is the dating of business cycle turning points given by the Organisation for Economic Co-operation and Development (OECD). For Poland there are also available (only till the first quarter of 2010) datings by M. Drozdowicz-Bieć (Drozdowicz 2008). In the Figure 1 the decomposed time series for the question about level of production (question 1) are presented against the OECD and Drozdowicz-Bieć reference time series.

Based on the precomputed respondents answers to the questions from the business tendency surveys as well as the theory of switching-Markov models, the attempt to identify the turning points of the business cycle turning points for Poland was made. Described in the previous section the Baum-Welch algorithm was used. Unfortunately to get the the reliable results it is necessary to use Monte Carlo simulations. The more unstable calculations, depend usually on the number of states, the more simulations are needed. In the research the initial values were chosen randomly using independent and identically distributed draws from the univariate distribution. The number of draws used for parameters estimation of the time series being under study, varied between 500 and 5 000. The number of repetitions depends on the number of HMM states and the numerical stability of computations. Of course more Monte carlo simulations means the longer time of computations. Parallelization of calculations is possible (Bernardelli 2014). The real challenge is to define the proper optimization criteria, that allow to choose the best (according to those criteria) model. The best estimates of parameters of models are usually chosen by taking into account the following indicators:

- Akaike's information criterion (AIC),
- Bayesian information criterion (BIC),
- the log likelihood value,
- frequency of obtaining certain solution of the Baum-Welch algorithm.

The procedure designed to the identification of business cycle turning points is described in (Bernardelli 2015). The procedure uses historical data to a specific point of time in order to get the most reliable representation of the hidden Markov chain. Whole effort in this case is put to get realistic representation of the Viterbi path till that moment. It is easy to imagine such situation, in which Viterbi paths for a current and a previous month are not consistent due to adding the newest data to the model. Meanwhile, under the premise of gradual changes in the economy it should be not possible. What’s more in this case the prognostic usefulness of MS models seems to be questionable.

The idea for improving the accuracy of state estimation is to take into account not only input time series, but also the previously calculated Viterbi path. In case of the successfully verified consistency between results for two succeeding months it is understandable to assume that historical paths, which were stable for some time, should be considered as the correct one. The only thing is changed is the new data, that make parameters of two models and connected with them states on Viterbi path different. The solution worth considering is the change the time interval by deleting the oldest data from months, for which the result have been accepted as known. By comparing the Viterbi paths for various ends of the time interval it can be easier to make the decision about local stability or instability of the economic climate.

4. EMPIRICAL ANALYSIS

The research covers the period from April 1997 to November 2014. As a reference time series mainly the business cycle turning points dated by OECD were used. Due to space limitations there were presented results only for the questions 1 (level of production) and 6 (level of employment). The empirical analysis consists of two parts. In the first part the Viterbi paths were calculated for each month starting from January 2013 to November 2014. To be more precise, the procedure described in the second section was applied to the input data, that cover the period from April 1997 to the particular month from the considered time interval. The calculated paths were compared to each other and to the reference time series.

In Figure 2 the result of applying 2-state HMM to the time series of balances computed for the question about level of production (current situation) is presented. The Viterbi paths for the months from the ends of the interval (January 2013 and November 2014) were compared with the OECD reference time series. Paths for each month were almost identical. They differs only in few points, wherein the time shift is within the range of the 1-2 months. Furthermore each Viterbi path seems to detect all the turning points with an extra peak in relation to the OECD. Although this peak is consistent with the dating given by (Drozdowicz 2008). It is worth to notice that it look like the turning points are caught earlier than the OECD indicator.

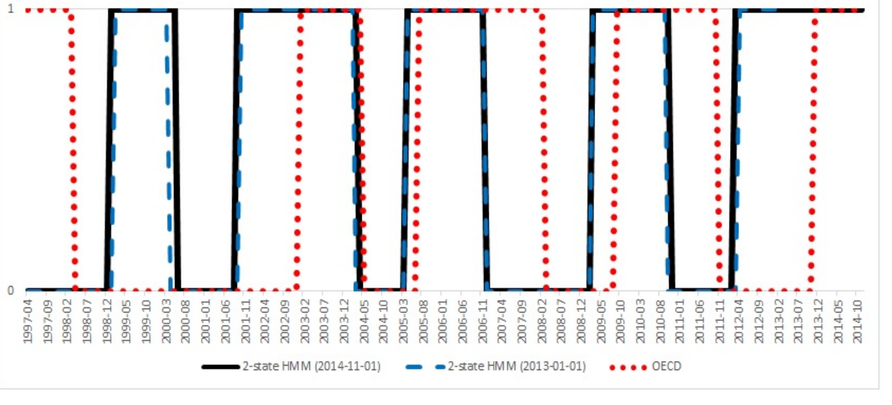

Figure 2. Comparison OECD reference time series with the Viterbi path for 2-state HMM for the question about level of production in the period April 1997-November 2014 and January 2013-November 2014.
Source: own calculations.

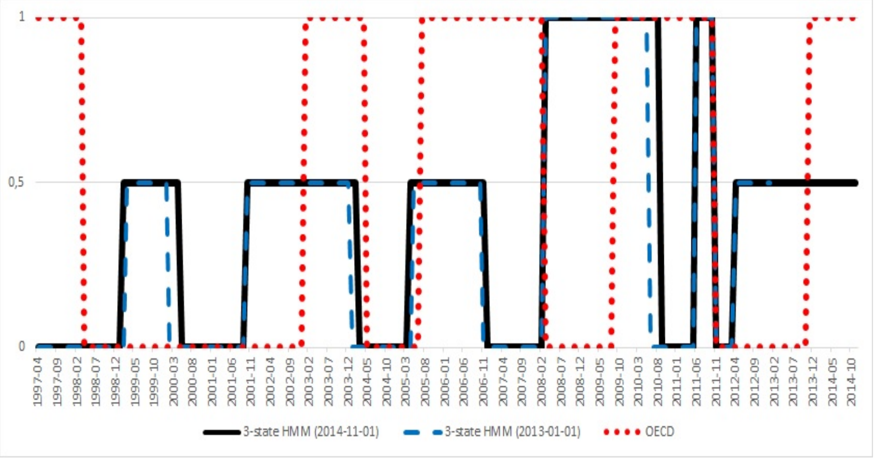

Figure 3. Comparison OECD reference time series with the Viterbi path for 3-state HMM for the question about level of production in the period April 1997-November 2014 and January 2013-November 2014.
Source: own calculations.

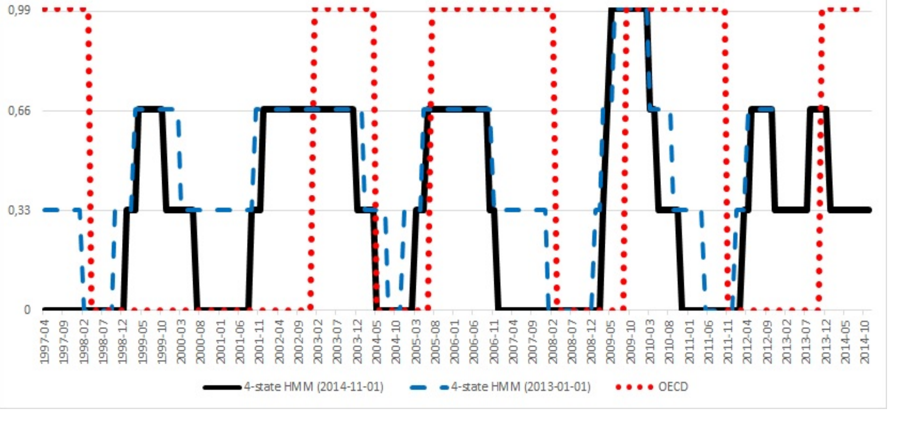

Figure 4. Comparison OECD reference time series with the Viterbi path for 4-state HMM for the question about level of production in the period April 1997-November 2014 and January 2013-November 2014.
Source: own calculations.

Figures 3 and 4 present results for 3-state and 4-state HMM for the question about level of production. Increasing number of states entails greater differences between Viterbi paths from different months. Still during the whole considered period those discrepancies are rather small and the stability of the solutions is visible. From the viewpoint of the comparison to the reference time series, multistate HMM seems to enrich the analysis. Transitions between states become smoother and the signals of peaks and troughs are strengthened. Especially 4-state model seems to give a quite accurate estimate of the economic situation in the country.

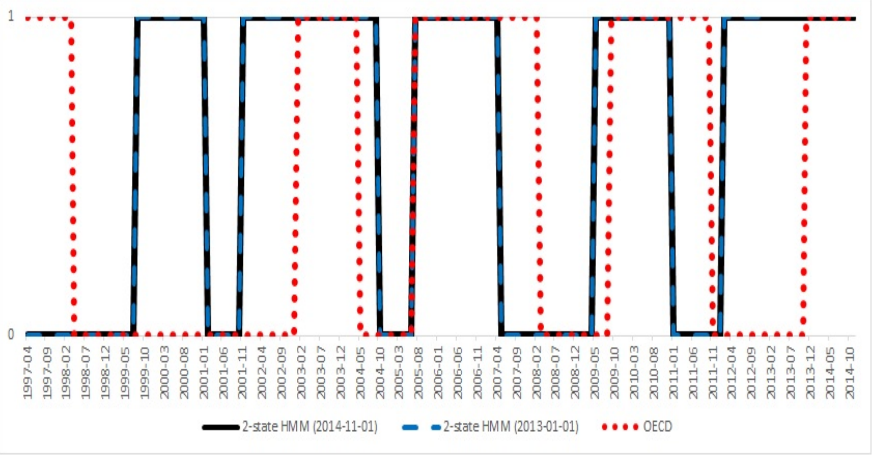

Figure 5. Comparison OECD reference time series with the Viterbi path for 2-state HMM for the question about level of employment in the period April 1997-November 2014 and January 2013-November 2014.
Source: own calculations.

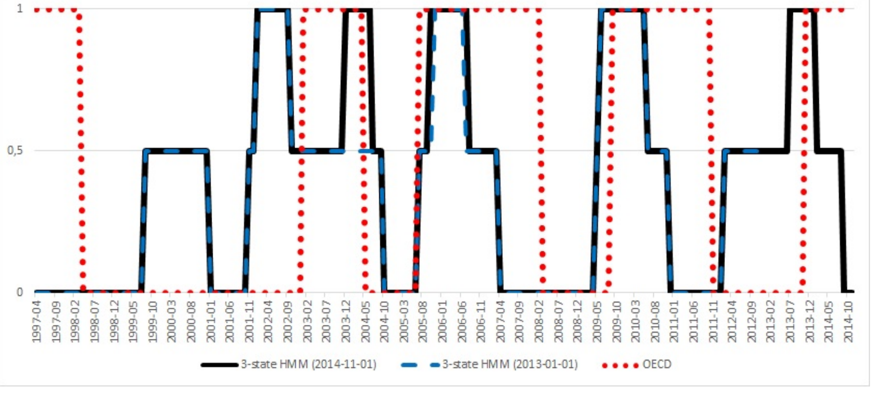

Figure 6. Comparison OECD reference time series with the Viterbi path for 3-state HMM for the question about level of employment in the period April 1997-November 2014 and January 2013-November 2014.
Source: own calculations.

The same observation can be made while analysing the graphs for the question about level of employment (Figures 5-7). Viterbi paths in the entire time interval are highly consistent. In case of the hidden markov model with two states paths are identical. Just as in the case of the question about level of production, all turning points according to the OECD reference time series were identified. One extra peak finds the justification in the datings given by Drozdowicz-Bieć. Turning points seem to be caught in advance.

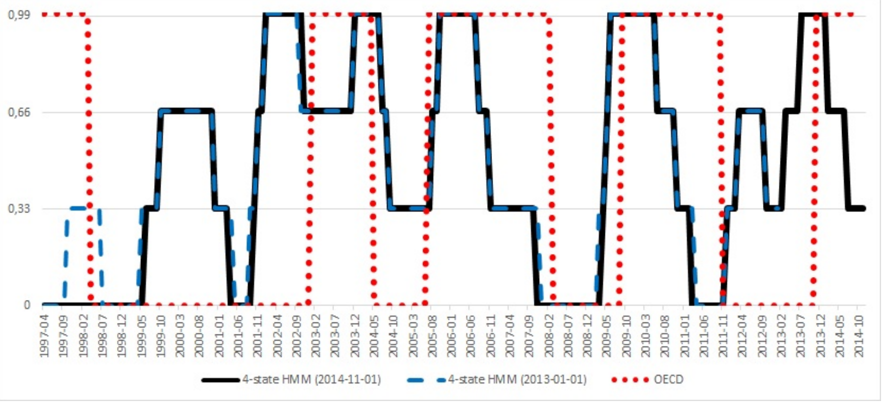

Figure 7. Comparison OECD reference time series with the Viterbi path for 4-state HMM for the question about level of employment in the period April 1997-November 2014 and January 2013-November 2014.
Source: own calculations.

The second part of the empirical analysis was intended to check how changing the length of the input data will influence the shape of the Viterbi path. In the first part of the research the starting point was constant and the ending points were changed. In this part the end is fixed and the begining is changed. The time interval is shorter, but analogously to the results from the first part we expect calculations to be stable and Viterbi paths consistent with each other. In the research the length of the input data was shortened considerably, from April 1997 to March 2001 (four years). All years between were analysed, but due to the readability of graphs, besides the path for the whole period, only two other Viterbi paths were visualized. One is distant for almost half a year and the second was computed for the shortest time interval.

In Figures 8-10 the graphs with computed Viterbi paths for the level of production are presented. In case of 2-state HMM (Figure 8) and 4-state HMM (Figure 10) states are similar. Although for the shortest input time series one peak is missed, it doesn't affect the latest state estimates and results can be successfully recognized as stable are reliable. This is not the case with 3-state HMM (Figure 9). Historical datings differ depending on the length of the input time interval. Based on one, even optimal, Viterbi path it would be hard to precisely specify the states of the business cycle. Fortunately knowing the historical pattern (see the description of the first part of the research) it is enough to decide what are the latest (half a year in the example) states. These are consistent with each other, which solves the problem.

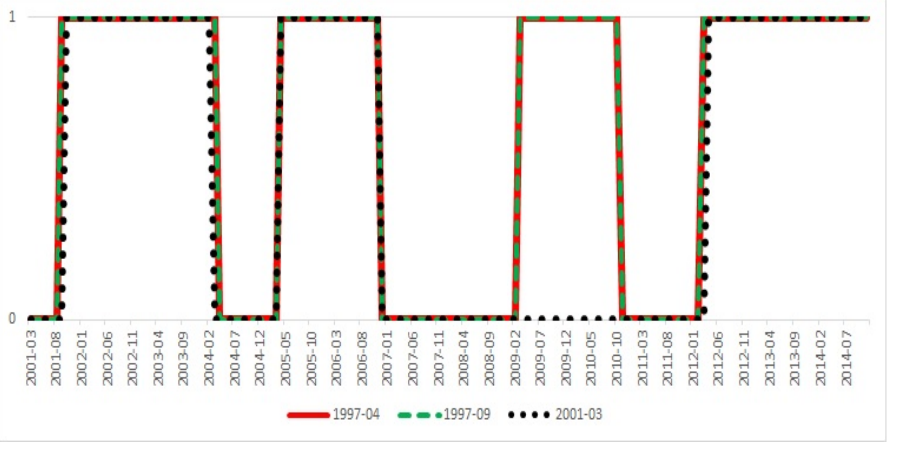

Figure 8. Viterbi path for 2-state HMM for the question about level of production in different periods: April 1997-November 2014, September 1997-November 2014 and January 2001-November 2014.

Source: own calculations.

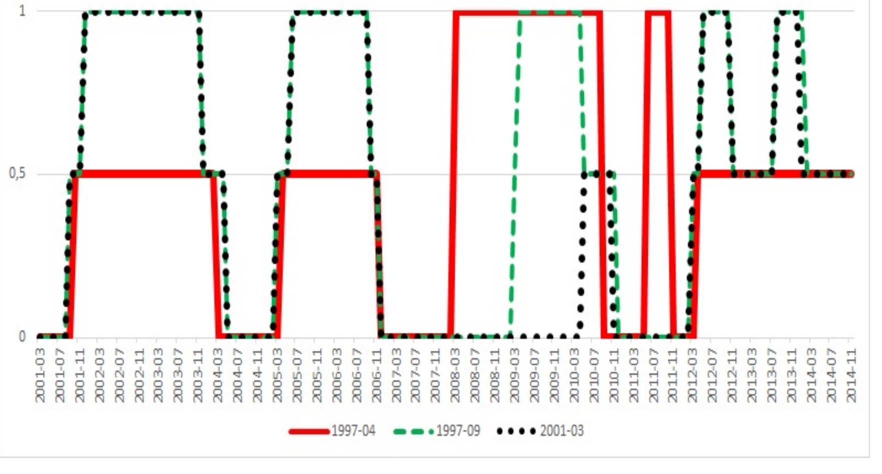

Figure 9. Viterbi path for 3-state HMM for the question about level of production in different periods: April 1997-November 2014, September 1997-November 2014 and January 2001-November 2014.

Source: own calculations.

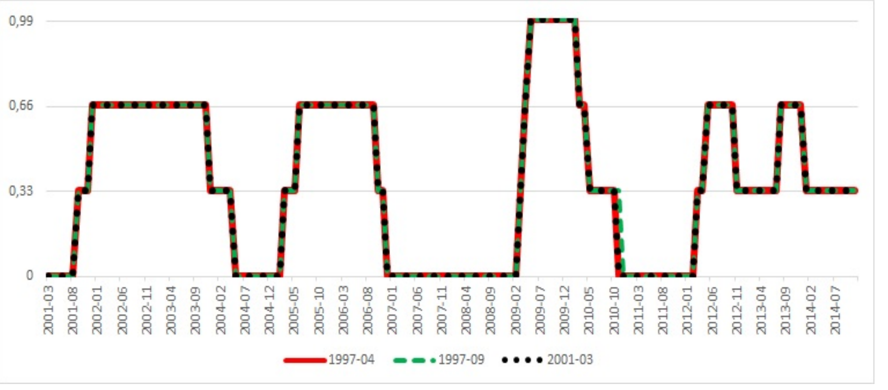

Figure 10. Viterbi path for 4-state HMM for the question about level of production in different periods: April 1997-November 2014, September 1997-November 2014 and January 2001-November 2014.
Source: own calculations.

In Figures 11-13 there are Viterbi paths of the hidden Markov chains for the question about level of employment with two, three and four states respectively. In some parts the differences between calculated states are quite visible. In most cases they correspond with old historical data, which should be established by the moment, for which the computations were done. In case of three states (Figure 12) and four states (Figure 4) similarity in the whole period was surprisingly good. The latest states in case of two states (Figure 11) may be potentially problematic. Depending on the time period the last state is completly different. It is connected with the low sensitivity of 2-state HMM. Multistate models give more information and richer description.

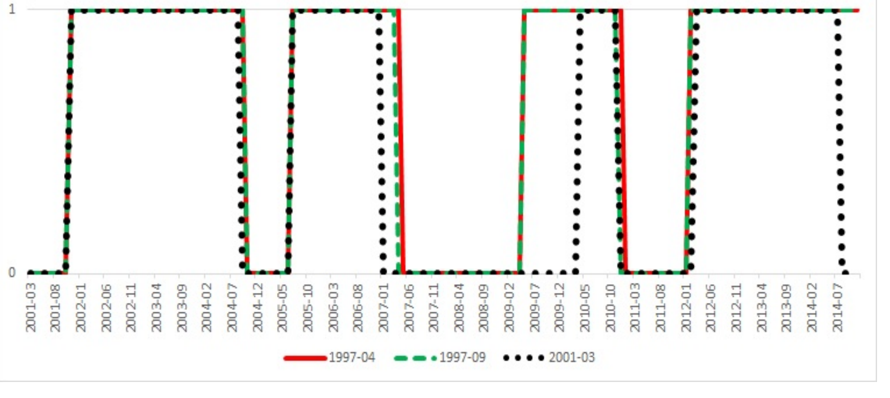

Figure 11. Viterbi path for 2-state HMM for the question about level of employment in different periods: April 1997-November 2014, September 1997-November 2014 and January 2001-November 2014.
Source: own calculations.

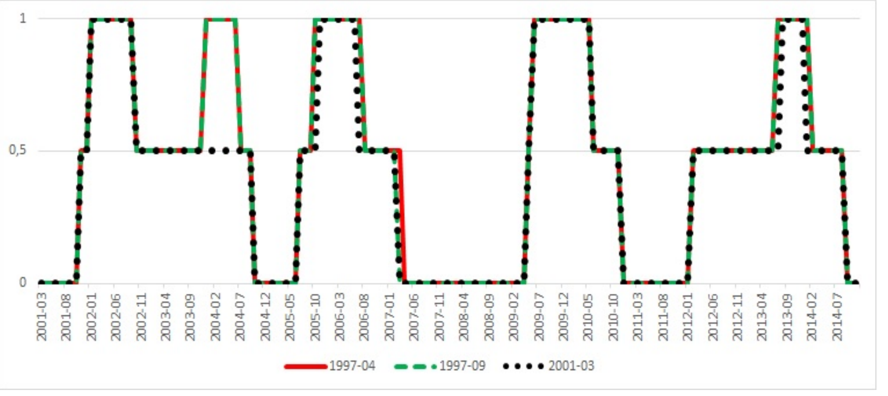

Figure 12. Viterbi path for 3-state HMM for the question about level of employment in different periods: April 1997-November 2014, September 1997-November 2014 and January 2001-November 2014.
Source: own calculations.

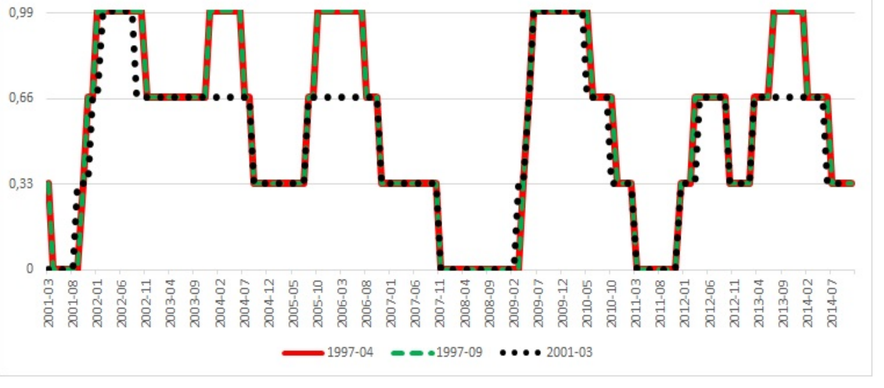

Figure 13. Viterbi path for 4-state HMM for the question about level of employment in different periods: April 1997-November 2014, September 1997-November 2014 and January 2001-November 2014.
Source: own calculations.

5. SUMMARY

This paper introduces the theory of multistate switching Markov models and the concept of the Viterbi path. This methodology is used for identification of turning points of the business cycle in Poland. The aim of the research was to analyze the relationship between Viterbi paths for the different periods of time. A reasonable is to make an assumption that in stable conditions, stages of the economic climate are more or less constant with the limitation to the given time point. Consecutive comparison of the Viterbi paths allows to assess the stability of the economic situation and established with high probability the states on the Viterbi path.

Research hypothesis were verified by computer simulations. As an input data the business tendency survey in industry conducted monthly by the Research Institute for Economic Development in Warsaw School of Economics were used. Based on the previous research (Bernardelli 2013, 2015) and results of experiments it is justified to draw the following conclusions. The described procedure is an efficient method for the turning points identification. It also allows to analyze the current and historical economic situation. It is considered as a powerful alternative for classical econometric methods. Due to the non-deterministic character of the procedure, as well as the high volatility of input data, analysis in limitation only to one moment of time may not give the full information. Proposed in this paper solution based on holistic approach to the problem seems to expand the possibilities and offer more complete information. Testing the compatibility of Viterbi paths for several different periods could be the key to developing the method of predicting turning points based on switching Markov models.

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