Conjecture: A Possible $nn\Lambda$ Resonance

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Abstract. We address the question of whether there might exist a resonance in the $nn\Lambda$ system, using a rank one separable potential formulation of the Hamiltonian. We explore the eigenvalues of the kernel of the Faddeev equation in the complex energy plane using contour rotation to allow us to analytically continue the kernel onto the second energy sheet. We follow the largest eigenvalue as the $n\Lambda$ potentials are scaled and the $nn\Lambda$ continuum is turned into a resonance and then into a bound state of the system.

1 Introduction

Recent experiments have suggested that a bound state of the $nn\Lambda$ system exists. [1] Several theoretical analyses demonstrate that such a bound state can not exist. [2–4] The question we wish to address is: Could we have a three-body resonance in the $nn\Lambda$ system even though all the interactions are predominantly *s*-waves? We consider the $nn\Lambda$ system with the pairwise interactions being rank one separable potentials that fit effective range parameters of the nn system, and those predicted by the Nijmegen model D [5] one-boson exchange potential for the $n\Lambda$ system. The use of rank one separable potentials allows us to easily analytically continue the Faddeev equations into the second complex energy plane in search of resonance poles by examining the eigenvalue spectrum of the kernel of the Faddeev equations as we did previously for Λ-*d* scattering [6].

2 Background

We have only limited data regarding $p\Lambda$ scattering, but we have no data regarding $n\Lambda$ scattering, because there are no free neutron targets. Λ hypernuclei provide only weak constraints. The hypertriton $^3_\Lambda$H is barely bound, being one of the largest halo nuclei known. $[B_\Lambda(^3_\Lambda$H$) = 0.13 \pm 0.05$ MeV]. The A=4 isodoublet $^4_\Lambda$H-$^4_\Lambda$He exhibits significant Charge Symmetry Breaking, some 2-3 times that seen in the ^{3}H-^{3}He isodoublet. The uncertainty in the extracted parameters from the sparse $p\Lambda$ scattering data imply a wide range of variation is possible in the $n\Lambda$ interaction.

The HypHI collaboration reported evidence for a bound $nn\Lambda$ system $^3_\Lambda$n [1]. They observed both 2-body and 3-body decay modes. Such a bound state would provide a strong constraint on the $n\Lambda$ interaction, because the nn interaction is well known. Such a bound state could be observed directly

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in a ^{3}H(e,e′K$^+$)$^3_\Lambda$n experiment at JLab, although a weakly bound system would imply a rather small cross section.

However, the existence of a $^3_\Lambda$n bound state has been strongly questioned. [2–4] Moreover, simple physics suggests that one would not expect a bound state. The hypertriton is barely bound and has as its core a deuteron. A $^3_\Lambda$n bound state would have as its core an unbound di-neutron pair. Could there instead exist a $nn\Lambda$ three-body resonance? If so, then one might still be able to utilize the electro-production reaction (or the HypHI heavy-ion collision) to constrain the $n\Lambda$ interaction.

3 Our Three-Body Model for $nn\Lambda$

For simplicity we utilize pairwise s-wave interactions of rank one separable form

$$V(k, k') = g(k) C g(k') \qquad g(k) = 1/(k^2 + \beta^2),$$

where the nn potential strength C and range β are fitted to the effective range parameters: [7]
$$a_{nn} = \text{-18.9} \pm 0.4 \text{ fm and } r_{nn} = 2.75 \pm 0.11 \text{ fm}$$
and the $n\Lambda$ strength and range are fitted to the Nijmegen model D $n\Lambda$ scattering lengths and effective ranges: [5]
$$a_s = \text{-2.03} \pm 0.32 \text{ fm and } r_s = 3.66 \pm 0.32 \text{ fm},$$
$$a_t = \text{-1.84} \pm 0.10 \text{ fm and } r_t = 3.32 \pm 0.11 \text{ fm}.$$
The use of rank one separable potentials allows us to simply analytically continue onto the second sheet of the energy plane in exploring for three-body resonances. We search for the resonance poles by examining the eigenvalue spectrum of the kernel of the Faddeev equations for the $nn\Lambda$ system. We used a similar technique to explore Λ-d scattering 20 years ago. [6]

As stated above, we analytically continue the Faddeev equations onto the second energy sheet. For a three-body system containing two identical Fermions interacting via Yamaguchi pairwise potentials, the homogeneous integral equation is of the form

$$\lambda_n(E) \phi_{n,k_\alpha}(q, E) = \sum_{k_\beta} \int_0^\infty dq' \; K^{JT}_{k_\alpha, k_\beta}(q, q'; E) \; \phi_{n;k_\beta}(q', E), \tag{1}$$

where the kernel of the integral equation is given by

$$K^{JT}_{k_\alpha, k_\beta}(q, q'; E) = Z^{JT}_{k_\alpha, k_\beta}(q, q'; E) \; \tau_{k_\beta}[E - \epsilon_\beta(q')] \; q'^2. \tag{2}$$

We analytically continue onto the second Riemann energy sheet by utilizing the transformation

$$q \to q \, e^{-i\theta} \qquad q' \to q' \, e^{-i\theta} \qquad \text{with} \qquad \theta > 0. \tag{3}$$

One limitation on the rotation angle θ is imposed by singularities of the kernel; the Born amplitude $Z^{JT}_{k_\alpha, k_\beta}$ requires that $\theta < \frac{\pi}{2}$, which gives us the region $\Im(E) < 0$ on the second Riemann sheet. The other source of singularity is the quasi-particle propagator $\tau_{k_\beta}[E - \epsilon_\beta(q')]$, but because there are no two-body bound states, this does not limit the rotation.

4 Results of the Eigenvalue Search

Let us consider the specific example in which we utilize the (^{1}S$_0$) nn and the ^{1}S$_0$ and ^{3}S$_1$ $n\Lambda$ potentials defined in the previous section. We searched in the complex energy plane for the largest eigenvalue

of the kernel $\lambda(E) = 1$. We found a pole at:

$$E = -0.154 - 0.753\,i\;\text{MeV} \quad \text{with eigenvalue} \quad \lambda(E) = 1.0000 + 0.0001\,i\,.$$

Because $\mathfrak{R}(E) < 0$, this pole does not correspond to a resonance.

This pole actually lies below the breakup threshold. Because the pole lies just below the threshold, we may ask how easy it might be to convert the pole into a resonance or even a bound state. We scale the strength of the $^1S_0$ and $^3S_1$ $n\Lambda$ potentials by the same factor s. We follow the path of the pole as it turns into a "resonance" and then into a bound state. See Figure 1. We observe that a change in strength of the order of 15% produces a resonance above the three-body breakup threshold. A change of about 40% produces a $nn\Lambda$ bound state.
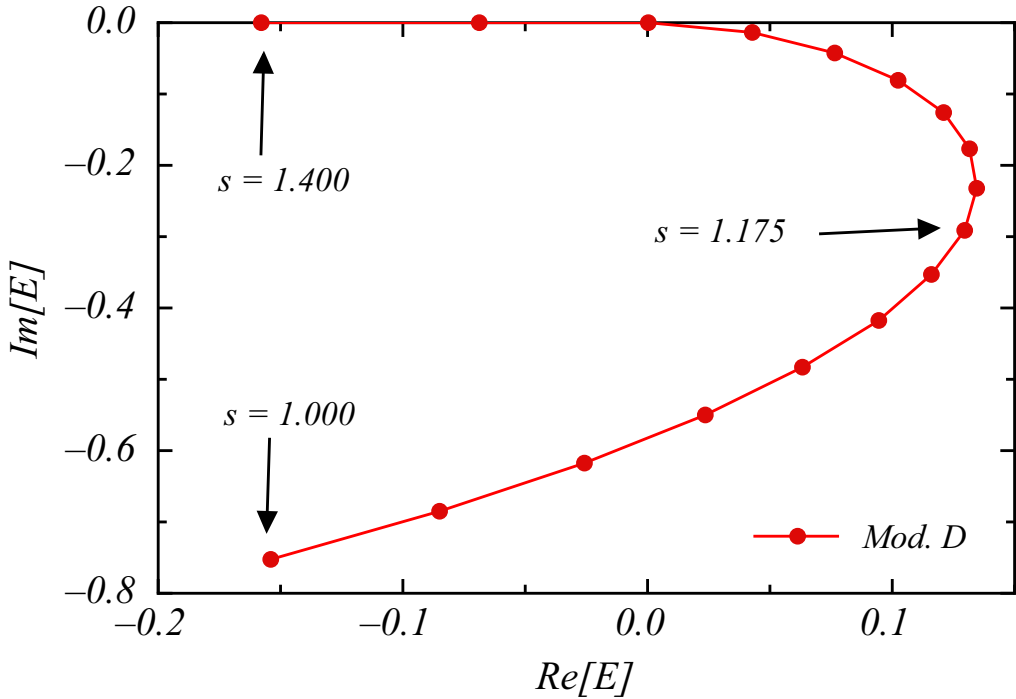
Figure 1. Trajectory of the pole as a function of s for the potentials defined in Section 3.

In other words we follow the trajectory of the "resonance" pole as a function of the strength s of the $n\Lambda$ interactions as s is increased from a value of 1.0 in increments (Δs) of 0.025. We find a sub-threshold resonance at values of $s = 1.000$ up to $s = 1.050$. For $s = 1.075$ up to $s = 1.350$ we obtain a resonance; in particular we obtain a resonance with $E = 0.1295 - 0.2915\,i$ MeV at $s = 1.175$. As s is further increased, we obtain a bound state with energy $E = -0.069$ MeV at $s = 1.375$ and $E = -0.158$ MeV for $s = 1.400$. Thus, one can see for this particular model that an $n\Lambda$ potential whose parameters lie within the uncertainty of the observed low energy $p\Lambda$ scattering parameters could easily produce a resonance in the $nn\Lambda$ system. Therefore, a $^3H(e,e'K^+)^3_\Lambda n$ experiment at JLab could possibly provide a significant constraint upon the $n\Lambda$ low energy scattering parameters.

Acknowledgements

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