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Conservation of ζ with radiative corrections from heavy field

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Abstract. In this paper, we address a possible impact of radiative corrections from a heavy scalar field χ on the curvature perturbation ζ. Integrating out χ, we derive the effective action for ζ, which includes the loop corrections of the heavy field χ. When the mass of χ is much larger than the Hubble scale H, the loop corrections of χ only yield a local contribution to the effective action and hence the effective action simply gives an action for ζ in a single field model, where, as is widely known, ζ is conserved in time after the Hubble crossing time. Meanwhile, when the mass of χ is comparable to H, the loop corrections of χ can give a non-local contribution to the effective action. Because of the non-local contribution from χ, in general, ζ may not be conserved, even if the classical background trajectory is determined only by the evolution of the inflaton. In this paper, we derive the condition that ζ is conserved in time in the presence of the radiative corrections from χ. Namely, we show that when the dilatation invariance, which is a part of the diffeomorphism invariance, is preserved at the quantum level, the loop corrections of the massive field χ do not disturb the constant evolution of ζ at super Hubble scales. In this discussion, we show the Ward-Takahashi identity for the dilatation invariance, which yields a consistency relation for the correlation functions of the massive field χ.

Contents

1 Introduction

Inflation provides us with a natural experimental instrument to explore the high energy physics. Measurements of the temperature anisotropies and polarization of the cosmic microwave background can constrain the Hubble parameter H at the time when the fluctuation was generated. The current data puts an upper bound on H at around 10^{14} GeV [1, 2], which is much higher than the accessible energy scale in particle accelerators. The precise measurements of the primordial perturbations generated during inflation may place a constraint on the theory of high energy physics independently of the particle experiments.

In string theory, compactification of the extra dimensions typically yields a number of scalar fields, which may have masses bigger than the Hubble parameter during inflation. Investigating a possible imprint of these massive fields might allow us to explore the high energy physics behind. While one field model is consistent with the current data [1], there is still room to include a contamination of such massive fields, which act as isocurvature modes. If such a massive field has a mass much bigger than the Hubble scale, integrating out the

massive field only gives local contributions to the effective action for the inflaton (relevant works can be found, e.g., in refs. [3, 4]). In such a case, since we are ignorant of the high energy theory, it is impossible to disentangle the radiative corrections of the massive field. However, if one of the isocurvature modes has a mass of order H, the radiative correction may yield a distinctive non-local contribution.

Chen and Wang studied an impact of a massive field on the primordial curvature perturbation ζ in ref. [5] (see also ref. [6]). In their setup, the inflaton has a non-minimal coupling with the massive field, which yields the cross-correlation between them. As emphasized in ref. [7], where a more extensive analysis, including higher spin fields, was done, the massive field leaves more direct information in the squeezed configuration of the correlation functions, which has a soft external leg, than in other configurations. The massive scalar field with $0 < m/H \leq 3/2$ decays as η^{Δ_-} with

$$\Delta \equiv \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{m^2}{H^2}} \tag{1.1}$$

at large scales. Then, as was computed in refs. [6–8] the contribution of the massive field to the squeezed bi-spectrum, when the shorter mode k crosses the Hubble scale, is given by

$$\langle \zeta_q \zeta_k \zeta_k \rangle \propto P_\zeta(q) P_\zeta(k) (q/k)^\Delta , \qquad (q/k \ll 1) , \tag{1.2}$$

where $P_\zeta(k)$ is the power spectrum of ζ. Notice that $(q/k)^\Delta$ encodes the evolution between the Hubble crossing time for the mode k and the one for q. For $m > 3H/2$, the massive field oscillates, while decaying, as $\eta^{\tilde{\Delta}_\pm}$ with

$$\tilde{\Delta}_\pm \equiv \frac{3}{2} \pm i\sqrt{\frac{m^2}{H^2} - \frac{9}{4}}, \tag{1.3}$$

which gives the same momentum dependence as in eq. (1.2) except that Δ_- is replaced with $\tilde{\Delta}_\pm$.

When the curvature perturbation stops evolving after the Hubble crossing time of the shorter mode k, eq. (1.2) gives the squeezed bispectrum at the end of inflation. As far as the massive fields do not contribute to the background evolution (an example where a massive field modulates the background evolution was studied, e.g., in refs. [10–13]) and the tree level contribution is concerned, the curvature perturbation is conserved after all modes cross the Hubble scale [14–19].[1] In this case, eq. (1.2) indeed gives the bi-spectrum at the end of inflation [5, 7, 9]. The argument in ref. [7] is based on the symmetry of the de Sitter spacetime. Therefore, one may speculate that the loop correction of the massive field may also be still given by eq. (1.2), while the scaling dimension Δ will no longer be given as in eq. (1.1) nor eq. (1.3). In this generalization, a non-trivial point may be in showing the conservation of ζ after the Hubble crossing. In refs. [24, 25], the conservation of ζ was addressed, including the loop correction of ζ in the setup of single field inflation.

In this paper, we address the conservation of the curvature perturbation ζ which is affected by loop corrections of a heavy field χ, assuming that the heavy field does not contribute to the classical background trajectory. The constant non-decaying mode of ζ is called the adiabatic mode. To compute the evolution of the curvature perturbation, we integrate

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[1]The conservation of ζ in such a setup also can be understood as a direct consequence of the δN formalism [20–23].

out the heavy field and derive the effective action for ζ. If the mass of the heavy field M is much bigger than the Hubble scale, the loop corrections of χ only yields local terms in the effective action and then following the argument in the single field case, we can show the conservation of ζ at large scales. On the other hand, if the mass M is not large enough compared to the Hubble scale, the loop corrections of χ can give non-local contributions to the effective action. The presence of the non-local contribution can yield a qualitative difference from single field models.

In single field models of inflation, the conservation of the curvature perturbation at large scales is implemented by the dilatation invariance $\boldsymbol{x} \to e^s \boldsymbol{x}$ with a constant parameter s, which changes $\zeta(t, \boldsymbol{x})$ to $\zeta(t, e^{-s}\boldsymbol{x}) - s$. The dilatation invariance is one of the gauge transformations and hence classically it should be preserved for a diffeomorphism invariant theory. However, when we quantize the system, the dilatation invariance is not always preserved, particularly when we allow an arbitrary initial quantum state [26, 27]. When the dilatation invariance is preserved, a part of the IR divergences is canceled out [26–36]. (In order to eliminate all the IR divergences, we also need to preserve the invariance under other gauge transformations. For a detailed explanation, see, e.g., ref. [37].) In refs. [36, 38], it was shown that when we choose the Euclidean vacuum, a.k.a., the adiabatic vacuum or the Bunch-Davies vacuum in the de Sitter limit, there exists a set of quantities which is free from the IR divergences.

In quantum field theory, a symmetry implies a corresponding identity, the so-called Ward-Takahashi (WT) identity. For one field model of inflation, the dilatation invariance yields the consistency relation, which relates the $(n + 1)$-point function of ζ with one soft external leg to the n-point function of ζ [39–41]. The consistency relation is indeed the WT identity for the dilatation invariance [41]. The consistency relation was first shown for the bi-spectrum in the squeezed limit by Maldacena in ref. [39] and it was extended to more general single field models in ref. [40]. The consistency relation for the arbitrary n-point function was derived in ref. [41]. In a single field inflation with diffeomorphism invariance, when the initial state is the Euclidean vacuum and the background trajectory is on attractor, the consistency relation generally holds. When one of these assumptions is not fulfilled, the consistency relation can be violated [42–44].

Various extensions of the consistency relation have been attempted so far. In [40, 45, 46], the squeezed bi-spectrum was computed in a non slow-roll setup and in refs. [47, 48], sub-leading contributions for the consistency relation were computed. (See also refs. [49–52].) The consistency relation can also be obtained from the reparametrization invariance of the wave function of the universe [53–55]. The use of the wave function is also motivated by the holographic description of inflation [39, 55–59]. In refs. [60, 61], the consistency relation was derived by solving the Callan-Symanzik equation in the dual boundary theory (see also ref. [62]). A possible gauge issue for the consistency relation was discussed in refs. [63–65].

In this paper, we derive the consistency relation for the heavy field χ from the requirement of the dilatation invariance. When the dilatation invariance is preserved at the quantum level, we obtain the corresponding WT identity. The WT identity for the dilatation invariance yields the consistency relation which relates the $(n+1)$-point function of the n χs and one soft curvature perturbation ζ to the n-point function of the χ field. The derivation of the consistency relation also applies in the presence of the loop corrections of the heavy field. Using the effective action for ζ, obtained by integrating out χ, we show that when the consistency relation for χ holds, the curvature perturbation ζ is conserved at the super Hubble scales.

This paper is organized as follows. In section 2, we review the conservation of ζ in single field models of inflation, emphasizing the crucial role of the dilatation invariance. In section 3, after we describe our setup of the problem, we introduce the effective action for ζ by integrating out the heavy field in the in-in (or closed time path) formalism. In section 4, we derive the consistency relation for the heavy field from the Ward-Takahashi identity for the dilatation invariance. In section 5, using the consistency relation, derived in section 4, we show the conservation of ζ in the presence of the loop corrections of the heavy field. In section 6, we briefly discuss the renormalization of the heavy field. Finally, in section 7, we conclude.

2 Conservation of ζ and dilatation invariance in single field inflation

In single field models of inflation, it is known that the curvature perturbation is conserved in the large scale limit. In this section, we show that the conservation of ζ is a direct consequence of the dilatation invariance.

2.1 Single field inflation

For illustrative purpose, we start our discussion by considering a single scalar field with the standard kinetic term, whose action is given by

$$S = \frac{1}{2} \int \sqrt{-g} \left[R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) \right] \mathrm{d}^4 x \,. \tag{2.1}$$

In this paper, we set the gravitational constant $\kappa^2 \equiv 8\pi G$ to 1. Using the ADM form of the line element:

$$\mathrm{d}s^2 = -N^2 \mathrm{d}t^2 + h_{ij}(\mathrm{d}x^i + N^i \mathrm{d}t)(\mathrm{d}x^j + N^j \mathrm{d}t) \,, \tag{2.2}$$

where we introduced the lapse function N, the shift vector N^i, and the spatial metric h_{ij}, we can express the action (2.1) as

$$
\begin{aligned}
S = \frac{1}{2} \int \sqrt{h} \Big[& N \,^s R - 2NV(\phi) + N(\kappa_{ij}\kappa^{ij} - \kappa^2) \\
& + \frac{1}{N}(\dot{\phi} - N^i \partial_i \phi)^2 - N h^{ij} \partial_i \phi \partial_j \phi \Big] \mathrm{d}^4 x \,,
\end{aligned}
\tag{2.3}
$$

where $\,^s R$ is the three-dimensional Ricci scalar, and κ_{ij} and κ are the extrinsic curvature and its trace, defined by

$$\kappa_{ij} = \frac{1}{2N}(\dot{h}_{ij} - D_i N_j - D_j N_i) \,, \qquad \kappa = h^{ij}\kappa_{ij} \,. \tag{2.4}$$

Here, the spatial indices $i, j, \cdots$ are raised or lowered by the spatial metric h_{ij}, and D_i denotes the covariant differentiation associated with h_{ij}. Taking the variation of the action with respect to N and N^i, which are the Lagrange multipliers, we obtain the Hamiltonian and momentum constraint equations as

$$^s R - 2V - (\kappa^{ij}\kappa_{ij} - \kappa^2) - N^{-2}(\dot{\phi} - N^i \partial_i \phi)^2 - h^{ij}\partial_i \phi \partial_j \phi = 0 \,, \tag{2.5}$$

$$D_j(\kappa^j{}_i - \delta^j{}_i \kappa) - N^{-1}\partial_i \phi \,(\dot{\phi} - N^j \partial_j \phi) = 0 \,. \tag{2.6}$$

We determine the time slicing, employing the uniform field gauge:

$$\delta\phi = 0 \,. \tag{2.7}$$

We express the spatial metric h_{ij} as

$$h_{ij} = e^{2(\rho+\zeta)} \left[e^{\delta\gamma}\right]_{ij} \,, \tag{2.8}$$

where the background scale factor is expressed as $a \equiv e^\rho$ and $\delta\gamma_{ij}$ is set to traceless. As spatial gauge conditions, we impose

$$\partial^i \delta\gamma_{ij} = 0 \,. \tag{2.9}$$

With this gauge choice, the constraint equations are given by

$${}^s R - 2V - (\kappa^{ij}\kappa_{ij} - \kappa^2) - N^{-2}\dot{\phi}^2 = 0 \,, \tag{2.10}$$

$$D_j(\kappa^j{}_i - \delta^j{}_i\kappa) = 0 \,. \tag{2.11}$$

Inserting N and N_i, which are expressed in terms of ζ by solving these constraint equations, into the action (2.3), we can derive the action for ζ [39, 66].

2.2 Dilatation invariance

The transverse condition imposed on $\delta\gamma_{ij}$ is non-local and hence to determine the coordinates, we need to employ boundary conditions. For example, at linear order in perturbation, the tensor perturbation transforms under the spatial coordinate transformation $x^i \to \delta\tilde{x}^i = x^i + \delta x^i$ as

$$\delta\tilde{\gamma}_{ij}(x) = \delta\gamma_{ij}(x) - 2\left(\partial_{(i}\delta x_{j)} - \frac{1}{3}\delta_{ij}\partial^k\delta x_k\right) \,. \tag{2.12}$$

The transverse condition on $\delta\gamma_{ij}$ gives

$$\partial^2 \delta x^i = -\frac{1}{3}\partial^i\partial_j\delta x^j \,, \tag{2.13}$$

which does not determine δx^i uniquely without specifying boundary conditions to solve eq. (2.13). For the scalar mode, all spatial coordinate transformations $\delta x^i = \partial^i\delta x$ which satisfy $\partial^2\partial^i\delta x = 0$ still keep the transverse condition after the transformations. When we consider a compact support on each time slicing, we find an infinite way to impose the boundary conditions in solving $\partial^2\partial^i\delta x = 0$. We analyzed these *gauge* degrees of freedom in detail in refs. [26, 27], where we used the italic font for "gauge" to discriminate the *gauge* transformations defined within the compact support from those in the infinite spatial support. (See ref. [41] for a more recent work.) So far, we kept the tensor perturbation $\delta\gamma_{ij}$ for the illustrative purpose, but in the rest of this paper, we neglect it.

Among these transformations, the important one for ζ is the dilatation $\delta x^i = sx^i$, which is extended to $x^i \to x^i_s = e^s x^i$ at non-linear orders. Here, s is a constant parameter.[2] Under

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[2]In order to evaluate an observable quantity, one may want to compute a quantity which is solely determined by the causal history of the universe. When we solve the evolution of the fluctuation in the causally connected region to us, $\mathcal{O}$, an influence of the causally disconnected region can appear as a boundary condition on the causal boundary. We can show that changing the boundary condition is equivalent to changing the spatial coordinates in $\mathcal{O}$. One of the coordinate transformations is the dilatation with a time dependent $s(t)$ [26, 27]. Requesting the diffeomorphism invariance in the causally connected region is crucial to show the absence of the infrared (IR) divergence. While the IR issue is related to the current study, such a time dependent $s(t)$ is not needed in this paper. Therefore, we do not consider the change of the boundary condition and assume that the parameter s is time independent.

this transformation, the spatial line element is recast into

$$\frac{\mathrm{d}l_d^2}{a^2(t)} = e^{2\zeta(t,\,\boldsymbol{x})}\mathrm{d}\boldsymbol{x}^2 = e^{2\zeta_s(t,\,\boldsymbol{x}_s)}\mathrm{d}\boldsymbol{x}_s^2 = e^{2\{\zeta_s(t,\,e^s\boldsymbol{x})+s\}}\mathrm{d}\boldsymbol{x}^2\,, \tag{2.14}$$

and then the curvature perturbation changes to

$$\zeta_s(t,\,\boldsymbol{x}) = \zeta(t,\,e^{-s}\boldsymbol{x}) - s\,. \tag{2.15}$$

This is a purely geometrical argument, and hence this transformation law also should apply to multi-field models.

Preserving the dilatation invariance is crucial to ensure the infrared (IR) regularity of the curvature perturbation. In refs. [26, 27], we introduced the spatial Ricci scalar evaluated in the geodesic normal coordinates as a *gauge* invariant quantity. Since the contribution from the IR modes, which give rise to the IR divergence, can be eliminated by performing the corresponding *gauge* transformation, the IR divergence also can be removed from the *gauge* invariant quantity.

By construction, the spatial Ricci scalar evaluated in the spatial geodesic normal coordinates is *gauge* invariant and it serves a conceptually clear example of the *gauge* invariant quantity. Nevertheless, using the geodesic normal coordinates can alter the UV behaviour [67, 68]. For a practical use, we may use the smeared geodesic coordinates $\boldsymbol{x}_g(t)$ given by [35, 36, 38]

$$\boldsymbol{x}_g(t) \equiv e^{g\bar{\zeta}(t)}\boldsymbol{x}\,, \tag{2.16}$$

where $g\bar{\zeta}$ is the averaged ζ at a compact support on each time slicing, given by

$$g\bar{\zeta}(t) = \frac{\int \mathrm{d}^3\boldsymbol{x}_g W_t(\boldsymbol{x}_g)\zeta(t,\,e^{-g\bar{\zeta}}\boldsymbol{x}_g)}{\int \mathrm{d}^3\boldsymbol{x}_g W_t(\boldsymbol{x}_g)}\,, \tag{2.17}$$

where $W_t(\boldsymbol{x})$ is a window function which vanishes outside the compact support on the time constant slicing. The spatial Ricci scalar evaluated at $\boldsymbol{x}_g$ is not invariant under all the *gauge* transformations, but it is invariant under the dilatation.

2.3 Conservation of ζ in single field inflation

In single field inflation, solving the Hamiltonian and momentum constraint equations, we can eliminate N and N_i and write down the action only in terms of ζ. Since the action for any diffeomorphism invariant theory remains invariant under the dilatation, the action for ζ should take the following form:

$$S[\zeta] = \int \mathrm{d}t\,\mathrm{d}^3\boldsymbol{x}\,e^{3(\rho+\zeta)}\mathcal{L}_\zeta\Big[\partial_t\zeta,\,e^{-(\rho+\zeta)}\partial_i\zeta\Big]\,, \tag{2.18}$$

where the Lagrangian density $\mathcal{L}_\zeta$ includes ζ only in the form $\partial_t\zeta$ or $e^{-(\rho+\zeta)}\partial_i\zeta$. (A detailed explanation can be found in ref. [35].)

To address the conservation of ζ in the large scale limit, we neglect the terms which include ζ with the spatial derivative. Then, the action for ζ, written in the form (2.18), is given by

$$S[\zeta] \approx \int \mathrm{d}t\,\mathrm{d}^3\boldsymbol{x}\,e^{3(\rho+\zeta)}\varepsilon\left[\dot{\zeta}^2 + \sum_{n=3}^{\infty}\frac{2}{n}f_n(t)\dot{\zeta}^n\right]\,, \tag{2.19}$$

where we schematically wrote the non-linear terms which include $\dot{\zeta}$. Here, the time dependent function $f_n(t)$ is expressed only in terms of the background quantities such as $\dot{\rho}$ and the slow-roll parameters. Varying the action with respect to ζ, we obtain the equation of motion in the large scale limit as

$$\partial_t \left[e^{3(\rho+\zeta)} \varepsilon \left\{ \dot{\zeta} + \sum_{n=3}^{\infty} f_n(t) \dot{\zeta}^{n-1} \right\} \right] \approx 0 . \tag{2.20}$$

This equation motion has the anticipated constant solution in time as the non-decaying mode. I.e., if $\zeta(x) = F(x)$ is a solution of eq. (2.20), $F(x) + C$ with a constant shift should also satisfy the equation. Therefore, when the deviation from the constant mode decays as it occurs in the large scale limit of the standard setup, ζ should be conserved at large scales. The relation between the shift symmetry and the conservation of ζ was pointed out also in Horndeski's theory [69].

3 Effective action for ζ with loop corrections of heavy field

Next, we consider a two-field model with one inflaton and one heavy field. The latter field does not contribute to the classical background evolution. Following the Feynman and Vernon's method [70], in this section, we compute the effective action for the curvature perturbation with loop corrections of the massive field.

3.1 Two field model

In this paper, we consider a light scalar field ϕ and a massive scalar field χ whose action is given by

$$S = \frac{1}{2} \int \sqrt{-g} \left[R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - 2V(\phi, \chi) \right] \mathrm{d}^{d+1} x , \tag{3.1}$$

where $V(\phi, \chi)$ is a potential for the scalar fields:

$$V(\phi, \chi) = V_{ph}(\phi) + V_{ch}(\phi, \chi) , \tag{3.2}$$

with

$$V_{ch}(\phi, \chi) \equiv \frac{1}{2} M^2(\phi) \chi^2 + \frac{\lambda}{4!} \chi^4 . \tag{3.3}$$

We decomposed the potential $V(\phi, \chi)$ into the χ independent part V_{ph} and the rest V_{ch}. When the mass of χ field, M, depends on the inflaton ϕ, this model includes the direct interaction between ϕ and χ, e.g., $\phi^2 \chi^2$, which was addressed in refs. [71, 72]. We assume that the mass of the inflaton m is much smaller than the Hubble parameter H as $m \ll H$, while the mass of the field χ is bigger than H as $M > H$. Then, the classical background evolution is determined solely by the inflaton ϕ and (the linear perturbation of) χ becomes the pure isocurvature perturbation. In this paper, we only allow the renormalizable interactions for χ, while $V_{ph}(\phi)$ may include non-renormalizable interactions. To perform the dimensional regularization, we consider a general $(d+1)$ dimensional spacetime. An extension to include non-renormalizable interactions for χ is straightforward as long as a finite order of loop corrections is concerned.

As in the single field case, we determine the time slicing, imposing

$$\delta\phi = 0 \,. \tag{3.4}$$

For our later use, we discriminate the part which explicitly depends on χ from the rest as

$$S = S_{ad}[N, N_i, \zeta] + S_\chi[N, N_i, \zeta, \chi] \,, \tag{3.5}$$

where S_{ad} agrees with the action in single field models, given in eq. (2.3), and S_χ is given by

$$S_\chi = \frac{1}{2} \int e^{d(\rho+\zeta)} \left[\frac{1}{N} (\dot{\chi} - e^{-2(\rho+\zeta)} \delta^{ij} N_i \partial_j \chi)^2 \right.$$
$$\left. - N e^{-2(\rho+\zeta)} (\partial_i \chi)^2 - 2NV(\phi, \chi) \right] \mathrm{d}t \, \mathrm{d}^d \boldsymbol{x} \,. \tag{3.6}$$

Even if there is no direct interaction between ϕ and χ, gravity yields the non-linear interaction between ζ and the heavy field χ.

3.2 Effective action

Since the mass of the field χ is much bigger than the Hubble scale H, it is natural to set the background value of χ to 0. Then, the field χ does not contribute to the classical background evolution. Meanwhile, because of the interaction between ζ and χ, the quantum fluctuation of χ can affect the evolution of the field ζ. In order to compute the evolution of ζ under the influence of χ, we compute the Feynman and Vernon's influence functional [70, 73, 74], which can be obtained by integrating out the field χ, in the closed time path (or the in-in) formalism.

As in the single field case, the action S also includes the lapse function and the shift vector, which can be removed by solving the Hamiltonian and momentum constraint equations. These constraint equations are also modified due to the quantum fluctuation of the heavy field χ.

Using the closed time path, the n-point function of the curvature perturbation is given by

$$\langle \Psi | T \zeta(x_1) \cdots \zeta(x_n) | \Psi \rangle$$
$$= \frac{\int D\zeta_+ \int D\chi_+ \int D\zeta_- \int D\chi_- \, \zeta_+(x_1) \cdots \zeta_+(x_n) \, e^{iS[\delta g_+, \chi_+] - iS[\delta g_-, \chi_-]}}{\int D\zeta_+ \int D\chi_+ \int D\zeta_- \int D\chi_- \, e^{iS[\delta g_+, \chi_+] - iS[\delta g_-, \chi_-]}} \,, \tag{3.7}$$

where we used an abbreviation $\delta g = (\delta N, N_i, \zeta)$. In the closed time path, we double the fields: δg_+ and χ_+ denote the fields integrated from the past infinity to the time t and δg_- and χ_- denote the fields integrated from the time t to the past infinity. Here, T denotes the time ordering. Inserting $\zeta_-(x)$ into the path integral in the numerator, we can compute the n-point function ordered in the anti-time ordering. Since N and N_i are not independent variables, we perform the path integral only regarding ζ and χ.

Introducing the effective action S_{eff} as

$$iS_{\mathrm{eff}}[\zeta_+, \zeta_-] \equiv \ln \left[\int D\chi_+ \int D\chi_- \, e^{iS[\delta g_+, \chi_+] - iS[\delta g_-, \chi_-]} \right] \,, \tag{3.8}$$

we rewrite the n-point function for ζ as

$$\langle \Psi | T \zeta(x_1) \cdots \zeta(x_n) | \Psi \rangle = \frac{\int D\zeta_+ \int D\zeta_- \, \zeta_+(x_1) \cdots \zeta_+(x_n) \, e^{iS_{\mathrm{eff}}[\delta g_+, \delta g_-]}}{\int D\zeta_+ \int D\zeta_- \, e^{iS_{\mathrm{eff}}[\delta g_+, \delta g_-]}} \,. \tag{3.9}$$

By inserting the action S into eq. (3.8), the effective action is recast into

$$S_{\text{eff}}[\delta g_+, \delta g_-] = S_{ad}[\delta g_+] - S_{ad}[\delta g_-] + S'_{\text{eff}}[\delta g_+, \delta g_-] \,, \tag{3.10}$$

where S'_{eff} is the so-called influence functional, given by

$$iS'_{\text{eff}}[\delta g_+, \delta g_-] \equiv \ln\left[\int D\chi_+ \int D\chi_- \, e^{iS_\chi[\delta g_+, \chi_+] - iS_\chi[\delta g_-, \chi_-]}\right] \,, \tag{3.11}$$

where we factorized $S_{ad}[\delta g_\pm]$ which commute with the path integral over $\chi_\pm$. The effective action $S_{\text{eff}}[\delta g_+, \delta g_-]$ describes the evolution of the curvature perturbation affected by the quantum fluctuation of the heavy field χ.

3.3 Computing the effective action

Performing the path integrals about χ_+ and χ_-, we can compute the effective action $S'_{\text{eff}}[\delta g_+, \delta g_-]$. Expanding S'_{eff} in terms of $\delta g = (\delta N, N_i, \zeta)$, we obtain

$$iS'_{\text{eff}}[\delta g_+, \delta g_-] \equiv \sum_{n=0}^{\infty} iS'_{\text{eff}(n)}[\delta g_+, \delta g_-] \,, \tag{3.12}$$

where $S'_{\text{eff}(n)}$ denotes the terms which include n δg_αs, given by

$$iS'_{\text{eff}(n)}[\delta g_+, \delta g_-] = \frac{1}{n!} \sum_{\alpha_1 = \pm} \cdots \sum_{\alpha_n = \pm} \int \mathrm{d}^{d+1}x_1 \cdots \int \mathrm{d}^{d+1}x_n$$
$$\times \, \delta g_{\alpha_1}(x_1) \cdots \delta g_{\alpha_n}(x_n) W^{(n)}_{\delta g_{\alpha_1} \cdots \delta g_{\alpha_n}}(x_1, \cdots, x_n) \,, \tag{3.13}$$

with

$$W^{(n)}_{\delta g_{\alpha_1} \cdots \delta g_{\alpha_n}}(x_1, \cdots, x_n) \equiv \left. \frac{\delta^n iS'_{\text{eff}}[\zeta_+, \zeta_-]}{\delta g_{\alpha_1}(x_1) \cdots \delta g_{\alpha_n}(x_n)} \right|_{\delta g_\pm = 0} \,. \tag{3.14}$$

In eq. (3.13), each δg_{α_m} with $m = 1, \cdots, n$ should add up δN_{α_m}, N_{i,α_m}, and ζ_{α_m}. Here and hereafter, for notational brevity, we omit the summation symbol over δg unless necessary. Using eq. (3.11), we can express the variation of S'_{eff} with respect to $\delta g_\pm$ by using the propagators for χ. Notice that the shift symmetry is not manifest in this series expansion.

The linear term in the effective action is given by

$$iS'_{\text{eff}(1)} = \sum_{\alpha = \pm} \int \mathrm{d}^{d+1}x \, \delta g_\alpha(x) W^{(1)}_{\delta g_\alpha}(x) \,, \tag{3.15}$$

where $W^{(1)}_{\delta g_\alpha}$ is given by the expectation value as

$$W^{(1)}_{\delta g_+}(x) = -W^{(1)}_{\delta g_-}(x) = \left\langle \left. \frac{\delta iS_\chi}{\delta g(x)} \right|_{\delta g = 0} \right\rangle \,. \tag{3.16}$$

Next, we compute the quadratic terms in S'_{eff}. Taking the second variation of S'_{eff} with respect to δg_+, we obtain

$$W^{(2)}_{\delta g_+ \delta \tilde{g}_+}(x_1, x_2) = i^2 \left\langle \left. \frac{\delta S_\chi[\delta g_+, \chi_+]}{\delta g_+(x_1)} \right|_{\delta g_+ = 0} \left. \frac{\delta S_\chi[\delta g_+, \chi_+]}{\delta \tilde{g}_+(x_2)} \right|_{\delta g_+ = 0} \right\rangle_\pm$$
$$+ i\delta(x_1 - x_2) \left\langle \left. \frac{\delta^2 S_\chi[\zeta_+, \chi_+]}{\delta g_+(x_1) \delta \tilde{g}_+(x_1)} \right|_{\delta g_+ = 0} \right\rangle_\pm \,, \tag{3.17}$$

where δg and $\delta \tilde{g}$ are either δN, N_i, or ζ, and they can be different metric perturbations. Here, we introduced the expectation value:

$$\langle \mathcal{O}[\chi_+, \chi_-]\rangle_{\pm} \equiv \frac{\int D\chi_+ \int D\chi_- \, \mathcal{O}[\chi_+, \chi_-] e^{iS_\chi[0, \chi_+] - iS_\chi[0, \chi_-]}}{\int D\chi_+ \int D\chi_- \, e^{iS_\chi[0, \chi_+] - iS_\chi[0, \chi_-]}} \, . \tag{3.18}$$

Since the action $S_\chi[\delta g_+, \chi_+]$ includes only local terms, the variation of $S_\chi[\delta g_+, \chi_+]$ with respect to $\delta g_+(x_1)$ and $\delta \tilde{g}_+(x_2)$ yields the delta function $\delta(x_1 - x_2)$ in eq. (3.17). Similarly, the second variation of S'_{eff} with respect to δg_- is given by

$$W^{(2)}_{\delta g_- \delta \tilde{g}_-}(x_1, x_2) = i^2 \left\langle \left. \frac{\delta S_\chi[\delta g_-, \chi_-]}{\delta g_-(x_1)} \right|_{\delta g_-=0} \left. \frac{\delta S_\chi[\delta g_-, \chi_-]}{\delta \tilde{g}_-(x_2)} \right|_{\delta g_-=0} \right\rangle_{\pm}$$

$$- i\delta(x_1 - x_2) \left\langle \left. \frac{\delta^2 S_\chi[\delta g_-, \chi_-]}{\delta g_-(x_1)\delta \tilde{g}_-(x_1)} \right|_{\delta g_-=0} \right\rangle_{\pm} \, . \tag{3.19}$$

Taking the derivative with respect to both δg_+ and δg_-, we obtain

$$W^{(2)}_{\delta g_+ \delta \tilde{g}_-}(x_1, x_2) = -i^2 \left\langle \left. \frac{\delta S_\chi[\delta g_+, \chi_+]}{\delta g_+(x_1)} \right|_{\delta g_+=0} \left. \frac{\delta S_\chi[\delta g_-, \chi_-]}{\delta \tilde{g}_-(x_2)} \right|_{\delta g_-=0} \right\rangle_{\pm} , \tag{3.20}$$

and

$$W^{(2)}_{\delta g_- \delta \tilde{g}_+}(x_1, x_2) = -i^2 \left\langle \left. \frac{\delta S_\chi[\delta g_-, \chi_-]}{\delta g_-(x_1)} \right|_{\delta g_-=0} \left. \frac{\delta S_\chi[\delta g_+, \chi_+]}{\delta \tilde{g}_+(x_2)} \right|_{\delta g_+=0} \right\rangle_{\pm} . \tag{3.21}$$

These functions $W^{(2)}_{\delta g_{\alpha_1} \delta \tilde{g}_{\alpha_2}}(x_1, x_2)$ can be expanded by the propagators of χ for $\lambda = 0$, i.e., the time-ordered (Feynman) propagator:

$$G_F(x_1, x_2) \equiv \frac{\int D\chi_+ \, \chi_+(x_1)\chi_+(x_2) \, e^{iS_{\chi,0}[\chi_+]}}{\int D\chi_+ \, e^{iS_{\chi,0}[\chi_+]}} \, , \tag{3.22}$$

the anti-time ordered (Dyson) propagator:

$$G_D(x_1, x_2) \equiv \frac{\int D\chi_- \, \chi_-(x_1)\chi_-(x_2) \, e^{-iS_{\chi,0}[\chi_-]}}{\int D\chi_- \, e^{iS_{\chi,0}[\chi_-]}} \, , \tag{3.23}$$

and the Wightman functions:

$$G^+(x_1, x_2) \equiv \frac{\int D\chi_+ \int D\chi_- \, \chi_-(x_1)\chi_+(x_2) e^{iS_{\chi,0}[\chi_+] - iS_{\chi,0}[\chi_-]}}{\int D\chi_+ \int D\chi_- \, e^{iS_{\chi,0}[\chi_+] - iS_{\chi,0}[\chi_-]}} \, ,$$

$$G^-(x_1, x_2) \equiv \frac{\int D\chi_+ \int D\chi_- \, \chi_+(x_1)\chi_-(x_2) e^{iS_{\chi,0}[\chi_+] - iS_{\chi,0}[\chi_-]}}{\int D\chi_+ \int D\chi_- \, e^{iS_{\chi,0}[\chi_+] - iS_{\chi,0}[\chi_-]}} \, . \tag{3.24}$$

Here, $S_{\chi,0}[\chi]$ denotes the action given by

$$S_{\chi,0}[\chi] = \frac{1}{2} \int e^{d\rho} \left[\dot{\chi}^2 - e^{-2\rho}(\partial_i \chi)^2 - M^2(\phi)\chi^2 \right] dt \, d^d \boldsymbol{x} \, . \tag{3.25}$$

Recall that these propagators are mutually related as

$$G^-(x_1, x_2) = G^{+\,*}(x_1, x_2) \, , \tag{3.26}$$

$$G_F(x_1, x_2) = \theta(t_1 - t_2)G^+(x_1, x_2) + \theta(t_2 - t_1)G^-(x_1, x_2) \, , \tag{3.27}$$

$$G_D(x_1, x_2) = \theta(t_1 - t_2)G^-(x_1, x_2) + \theta(t_2 - t_1)G^+(x_1, x_2) \, , \tag{3.28}$$

where θ is the Heaviside function.

3.4 Propagators for χ

In this subsection, solving the mode function for the heavy field χ, we compute the propagators introduced in the previous subsection. At the linear order of χ, the equation of motion is given by

$$\ddot{\chi}_k + d\dot{\rho}\dot{\chi}_k + \{M^2(\phi) + (ke^{-\rho})^2\}\chi_k = 0 \,, \tag{3.29}$$

where χ_k is the Fourier mode of χ. Changing the variable from χ_k to $X_k(t) = e^{\frac{d-1}{2}\rho(t)}\chi_k(t)$ and using the conformal time η, the mode equation (3.29) is recast into

$$X_k'' + \Omega_k^2(\eta)\, X_k = 0 \,, \tag{3.30}$$

where the dash denotes the derivative with respect to the conformal time η and the time dependent frequency Ω_k is given by

$$\Omega_k^2(\eta) = k^2 + (Me^\rho)^2 - \rho'' - \rho'^2 \,. \tag{3.31}$$

Using W_k which satisfies

$$W_k^2 = \Omega_k^2 + \frac{3}{4}\left(\frac{W_k'}{W_k}\right)^2 - \frac{1}{2}\frac{W_k''}{W_k} \,, \tag{3.32}$$

the mode equation (3.30) can be solved as

$$X_k(\eta) = \frac{1}{\sqrt{2W_k}}e^{-i\int^\eta \mathrm{d}\eta' W_k(\eta')} \,. \tag{3.33}$$

Using the mode function χ_k, we quantize the non self-interacting heavy field as follows

$$\chi(x) = \int \frac{\mathrm{d}^d \boldsymbol{k}}{(2\pi)^{d/2}}e^{i\boldsymbol{k}\cdot\boldsymbol{x}}a_{\boldsymbol{k}}\chi_k(\eta) + (\text{h.c.}) \,, \tag{3.34}$$

where $a_{\boldsymbol{k}}$ denotes the annihilation operator. With this expansion, the Wightman function $G^+(x_1,\, x_2)$ is given by

$$G^+(x_1,\, x_2) = \int \frac{\mathrm{d}^d \boldsymbol{k}}{(2\pi)^d}e^{i\boldsymbol{k}\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)}\chi_k(\eta_1)\chi_k^*(\eta_2) \,. \tag{3.35}$$

Once the mode function $\chi_k(\eta)$ is given, using eqs. (3.26)–(3.28) and (3.35), we can compute all the propagators which appear in the expansion (3.17).

4 Ward-Takahashi identity from the dilatation invariance

As was discussed in section 2.2, the action for the single field model preserves the invariance under the dilatation, which changes $\zeta(t,\, \boldsymbol{x})$ to $\zeta(t,\, e^{-s}\boldsymbol{x}) - s$. This invariance is preserved classically also for multi-field models of inflation, since it is a part of the spatial diffeomorphism. In a quantum field theory, it is known that a symmetry leads to a corresponding Ward-Takahashi (WT) identity. When the dilatation invariance is also preserved at the quantum level, we obtain the WT identity.[3]

---

[3]The dilatation is one of the *gauge* transformations. The *gauge* invariance would be preserved for an arbitrary quantum state, if we could use a *gauge* invariant variable for the canonical quantization. However, since we used the curvature perturbation ζ, which in fact varies under the dilatation transformation, for the canonical quantization, the dilatation invariance is not automatically guaranteed. (A more detailed discussion on this point can be found in refs. [35, 36].)

In section 4.1, we discuss the WT identity from the dilatation invariance of the correlators of χ. In single field models of inflation, it was shown that the WT identity of the dilatation invariance yields the consistency relation which relates the $(n+1)$-point function of ζ with one soft external leg to n-point function of ζ [39–41]. Likewise, in section 4.2, we find that the WT identity derived in section 4.1 gives the consistency relation which relates the $(n+1)$-point cross-correlation with n χs and one soft ζ to the n-point auto-correlation of χ.

4.1 Ward-Takahashi identity

In single field models, an invariant quantity regarding the dilatation was constructed in refs. [26, 27] by using the smeared geodesic normal coordinates, defined in eq. (2.16). Using $\boldsymbol{x}_g \equiv \boldsymbol{x}_g(t_f)$, evaluated at the end of inflation $t = t_f$, we define

$$^g\chi(t, \boldsymbol{x}_g) = \chi(t, \boldsymbol{x}) = \chi(t, e^{-^g\bar{\zeta}}\boldsymbol{x}_g), \qquad (4.1)$$

which is invariant under the dilatation with the constant parameter s. Here, $^g\bar{\zeta} \equiv {}^g\bar{\zeta}(t_f)$.

When the dilatation invariance is also preserved for the quantum system, the correlation functions of $^g\chi(t, \boldsymbol{x}_g)$ should be invariant under the dilatation as

$$\langle \chi_{\alpha_1}(t_1, e^{-^g\bar{\zeta}}\boldsymbol{x}_{g1}) \cdots \chi_{\alpha_n}(t_n, e^{-^g\bar{\zeta}}\boldsymbol{x}_{gn})\rangle_\pm \big|_{\delta g}$$
$$= \langle \chi_{\alpha_1}(t_1, e^{-^g\bar{\zeta}+s}\boldsymbol{x}_{g1}) \cdots \chi_{\alpha_n}(t_n, e^{-^g\bar{\zeta}+s}\boldsymbol{x}_{gn})\rangle_\pm \big|_{\delta g_s} \qquad (4.2)$$

with $\alpha_i = \pm$. Here, δg_s denotes the metric perturbations after the dilatation. Under the dilatation, δN and N_i change as

$$\delta N_s(t, \boldsymbol{x}) = \delta N(t, e^{-s}\boldsymbol{x}), \qquad (4.3)$$
$$N_{i,s}(t, \boldsymbol{x}) = e^{-s}N_i(t, e^{-s}\boldsymbol{x}), \qquad (4.4)$$

and ζ changes as in eq. (2.15), and then $^g\bar{\zeta}$ changes to $^g\bar{\zeta}_s = {}^g\bar{\zeta} - s$. Equation (4.2) holds only when the quantum state also preserves the dilatation invariance. This is the WT identity for the dilatation invariance. At $\mathcal{O}(s)$, setting $\delta g = 0$, the WT identity yields

$$\sum_{i=1}^n \boldsymbol{x}_i \cdot \partial_{\boldsymbol{x}_i} \langle \chi_{\alpha_1}(x_1) \cdots \chi_{\alpha_n}(x_n)\rangle_\pm - \int d^{d+1}x \left\langle \chi_{\alpha_1}(x_1) \cdots \chi_{\alpha_n}(x_n) \frac{\delta iS_\chi[\delta g_+, \chi_+]}{\delta\zeta_+(x)}\bigg|_{\delta g_+=0} \right\rangle_\pm$$

$$+ \int d^{d+1}x \left\langle \chi_{\alpha_1}(x_1) \cdots \chi_{\alpha_n}(x_n) \frac{\delta iS_\chi[\delta g_-, \chi_-]}{\delta\zeta_-(x)}\bigg|_{\delta g_-=0} \right\rangle_\pm = 0. \qquad (4.5)$$

Since the changes of δN and N_i under the dilatation are linear in N and N_i and their derivatives, they vanish after setting δg to 0.

Using the WT identity (4.5) with $x_1 = \cdots = x_p \equiv x$ and $x_{p+1} = \cdots = x_n \equiv x'$, we obtain

$$(\boldsymbol{x} \cdot \partial_{\boldsymbol{x}} + \boldsymbol{x}' \cdot \partial_{\boldsymbol{x}'})\langle \chi_\alpha^p(x)\chi_\alpha^{n-p}(x')\rangle_\pm - \int d^{d+1}y \left\langle \chi_\alpha^p(x)\chi_\alpha^{n-p}(x') \frac{\delta iS_\chi[\delta g_+, \chi_+]}{\delta\zeta_+(y)}\bigg|_{\delta g_+=0} \right\rangle_\pm$$

$$+ \int d^{d+1}y \left\langle \chi_\alpha^p(x)\chi_\alpha^{n-p}(x') \frac{\delta iS_\chi[\delta g_-, \chi_-]}{\delta\zeta_-(y)}\bigg|_{\delta g_-=0} \right\rangle_\pm = 0, \qquad (4.6)$$

where $\alpha = \pm$. In the next section, using these identities, we show the conservation of ζ, including the loop corrections of the heavy field.

4.2 Consistency relation (soft theorem)

In single field models of inflation, it is known that the WT identity for the dilatation invariance gives the consistency relation. The consistency relation for ζ is an example of the soft theorem, which was first shown for the soft graviton scattering by Weinberg [75]. Recently, Weinberg's soft theorem was recaptured by Strominger et al. and was shown to be equivalent to a Ward-Takahashi identity in an asymptotically flat spacetime [76–78].

Here, we show that the WT identity (4.5) also gives a consistency relation in multi-field models. Performing the Fourier transformation of the WT identity (4.5) evaluated at an equal time t with all α_is chosen as $+$, we obtain

$$
\left(\sum_{i=1}^{n} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} + nd \right) \langle \chi_+(\boldsymbol{k}_1) \cdots \chi_+(\boldsymbol{k}_n) \rangle_{\pm}
$$
$$
- i \int \mathrm{d}^{d+1} y \left\langle \left. \frac{\delta S_\chi[\delta g_-, \chi_-]}{\delta \zeta_-(y)} \right|_{\delta g_-=0} \chi_+(\boldsymbol{k}_1) \cdots \chi_+(\boldsymbol{k}_n) \right\rangle_{\pm}
$$
$$
+ i \int \mathrm{d}^{d+1} y \left\langle \chi_+(\boldsymbol{k}_1) \cdots \chi_+(\boldsymbol{k}_n) \left. \frac{\delta S_\chi[\delta g_+, \chi_+]}{\delta \zeta_+(y)} \right|_{\delta g_+=0} \right\rangle_{\pm} = 0 \,, \tag{4.7}
$$

where we abbreviated t in the argument of χs. The correlator in the first line is simply the in-in n-point function of $\chi(t, \boldsymbol{k})$. The correlator in the second line is given by the product of the Wightman function, the Feynman propagator, and the Dyson propagator, which appear by contracting $\chi_\pm$ with $\chi_\mp$, χ_+ with χ_+, and χ_- with χ_-, respectively. The correlator in the third line is given by the product of the Feynman propagator. The interaction vertices inserted at any time after t are canceled between the terms in the second and third lines. This ensures the causality in the closed time path formalism. Taking into account this cancellation, we can rewrite eq. (4.7) as

$$
\left[\sum_{i=2}^{n} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} + (n-1)d \right] \left\langle \chi\left(-\sum_{j=2}^{n} \boldsymbol{k}_j \right) \chi(\boldsymbol{k}_2) \cdots \chi(\boldsymbol{k}_n) \right\rangle'
$$
$$
= -i \int^t \mathrm{d} t_y \int \mathrm{d}^d \boldsymbol{y} \left\langle \left[\chi(\boldsymbol{k}_1) \cdots \chi(\boldsymbol{k}_n), \left. \frac{\delta S_\chi}{\delta \zeta(y)} \right|_{\zeta=0} \right] \right\rangle' \,, \tag{4.8}
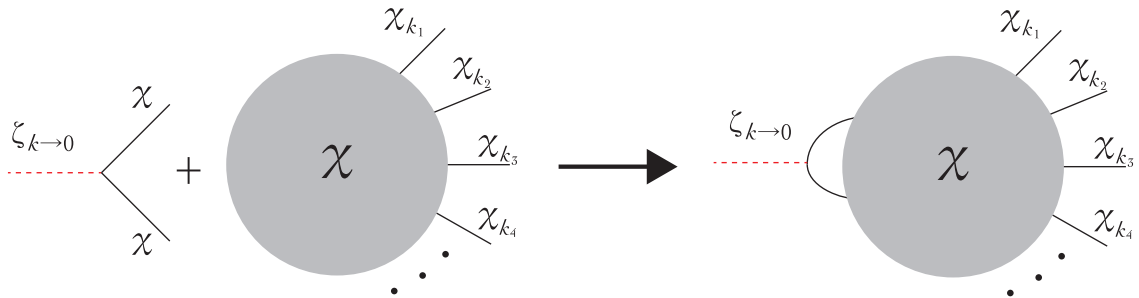$$

where the correlation function with dash denotes the correlation function from which $(2\pi)^d$ and the delta function are removed, e.g.,

$$
\langle \chi(\boldsymbol{k}_1) \cdots \chi(\boldsymbol{k}_n) \rangle \equiv (2\pi)^d \delta(\boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n) \langle \chi(\boldsymbol{k}_1) \cdots \chi(\boldsymbol{k}_n) \rangle' \,. \tag{4.9}
$$

In deriving eq. (4.8), we used

$$
\sum_{i=1}^{n} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} \delta(\boldsymbol{k}_1 + \cdots \boldsymbol{k}_n) \langle \chi(\boldsymbol{k}_1) \cdots \chi(\boldsymbol{k}_n) \rangle'
$$
$$
= \delta(\boldsymbol{k}_1 + \cdots \boldsymbol{k}_n) \left(\sum_{i=2}^{n} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} - d \right) \left\langle \chi\left(-\sum_{j=2}^{n} \boldsymbol{k}_j \right) \chi(\boldsymbol{k}_2) \cdots \chi(\boldsymbol{k}_n) \right\rangle' \,. \tag{4.10}
$$

Figure 1. As an example, we consider a three point interaction vertex in S_χ with two χs and one ζ. Since ζ is removed from the interaction vertex by operating the functional derivative, only two χs remain in the vertex. Contracting these two χs with χ included in the Heisenberg operator $\chi(\boldsymbol{k}_i)$ where $i = 1, \cdots n$, we obtain the diagram in the right of the arrow. The red dotted line represents the amputated ζ.

The correlation function in the second line of eq. (4.8) is the in-in n-point function of $\chi(\boldsymbol{k})$ where the gravitational interaction vertices with n heavy fields χ and one amputated soft curvature perturbation ζ are inserted. (See figure 1.) Then, attaching the external soft propagator of ζ to this correlation function yields the $(n+1)$-point function of n χs and one soft ζ, i.e.,

$$\left[\sum_{i=2}^{n} \boldsymbol{k}_i \cdot \partial_{\boldsymbol{k}_i} + (n-1)d\right] \left\langle \chi\left(-\sum_{j=2}^{n} \boldsymbol{k}_j\right) \chi(\boldsymbol{k}_2) \cdots \chi(\boldsymbol{k}_n) \right\rangle'$$
$$= -\lim_{k \to 0} \frac{\langle \zeta(\boldsymbol{k}) \chi(\boldsymbol{k}_1) \cdots \chi(\boldsymbol{k}_n) \rangle'}{P_\zeta(k)}, \tag{4.11}$$

where $P_\zeta(k)$ is the power spectrum of the free ζ.[4] This is the consistency relation for the heavy field χ. The correlation function in the second line contains only one gravitational interaction vertex with ζ, but it can contain more than one self interaction vertexes for the heavy field χ. This is one example of the soft theorem.

Using the WT identity at $\mathcal{O}(s)$, we derived the consistency relation for the correlation functions with one soft ζ. Using the WT identity at $\mathcal{O}(s^p)$, we can derive the consistency relations with p soft ζs.

4.3 Dilatation invariance and WKB solution

In this subsection, in order to provide an intuitive understanding of the consistency relation (4.8), we consider the case with $\lambda = 0$. We will find that in this case, the condition (4.8) restricts the mode function for the massive field χ and the condition can be satisfied, e.g., for the WKB solution.

---

[4]In rewriting the WT identity (4.8) into the consistency relation (4.11), we implicitly assumed the continuity at $k = 0$. When the curvature perturbation changes in time in the limit $k \to 0$, a discontinuity can appear from the shift vector. This is because the divergence of the shift vector, which is roughly given by $\partial^i N_i \propto \dot{\zeta}$ in this limit, takes a non-vanishing value and N_i becomes discontinuous to keep it finite in the limit $k \to 0$. When the curvature perturbation is conserved as will be discussed in the next section, the continuity at $k = 0$ can be naturally guaranteed. (This issue was briefly discussed in ref. [79].)

Inserting eq. (3.34) into eq. (4.8), we obtain

$$(\boldsymbol{k} \cdot \partial_{\boldsymbol{k}} + d)|\chi_k(t)|^2$$
$$= 2\mathrm{Im}\left[\int^t \mathrm{d}t' e^{d\rho(t')}\chi_k^2(t)\left\{d(\dot{\chi}_k^{*2}(t') - M^2\chi_k^{*2}(t')) - (d-2)\frac{k^2}{e^{2\rho(t')}}\chi_k^{*2}(t')\right\}\right]. \quad (4.12)$$

Integrating by parts and using the mode equation, we obtain

$$\boldsymbol{k} \cdot \partial_{\boldsymbol{k}}|\chi_k(t)|^2 = 4\,\mathrm{Im}\left[\int^t \mathrm{d}t' e^{d\rho(t')}\frac{k^2}{e^{2\rho(t')}}\chi_k^2(t)\chi_k^{*2}(t')\right], \quad (4.13)$$

where $d|\chi_k(t)|^2$ in the left hand side was canceled with the term which appears by operating the time derivative on the Heaviside function $\theta(t - t')$.

We can show that the WKB solution satisfies the WT identity (4.13). In order to show this statement, we rewrite eq. (4.13) as

$$\chi_k(\eta)L_k^*(\eta) + \chi_k^*(\eta)L_k(\eta) = 0, \quad (4.14)$$

introducing

$$L_k(\eta) \equiv k\partial_k\chi_k(\eta) - 2i\chi_k^*(\eta)\int^\eta \mathrm{d}\eta' e^{(d-1)\rho(\eta')}k^2\chi_k^2(\eta')$$
$$+ 2i\chi_k(\eta)\int_{\bar{\eta}}^\eta \mathrm{d}\eta' e^{(d-1)\rho(\eta')}k^2|\chi_k(\eta')|^2 + ik\bar{\eta}\chi_k(\eta). \quad (4.15)$$

The last two terms in $L_k(\eta)$ are canceled between the two terms in eq. (4.14). The time integral of the second term converges by rotating the time path as $\eta \to -\infty(1 + i\epsilon)$ where ϵ is a positive constant. For $L_k^*(\eta)$, the time integral of the second term should be rotated as $\eta \to -\infty(1 - i\epsilon)$. We choose $\bar{\eta}$ at a time in the distant past when the mode function can be well approximated by the leading order WKB solution with $W_k = k$. (To be precise, $\bar{\eta}$ differs for a different wavenumber k.) Since $L_k(\eta)$ satisfies the mode equation for $\chi_k(\eta)$, i.e.,

$$L_k'' + (d-1)\rho'L_k' + \{k^2 + M^2(\phi)e^{2\rho}\}L_k = 0, \quad (4.16)$$

and the initial conditions $L_k(\bar{\eta}) = L_k'(\bar{\eta}) = 0$, $L_k(\eta)$ vanishes all the time. Thus, we find that the WKB solution satisfies eq. (4.14).

For the exact de Sitter space, in the limit $ke^{-\rho} \ll M$ and $H \ll M$, we can easily check that the WKB solution, given by

$$\chi_k(t) \simeq \frac{e^{\frac{d}{2}\rho(t)}}{\sqrt{2M}}\left[1 + \frac{1}{4}\left(\frac{k}{Me^\rho}\right)^2\left(-1 + i\frac{M}{H}\right)\right]e^{-iMt}, \quad (4.17)$$

satisfies eq. (4.13) as

$$\boldsymbol{k} \cdot \partial_{\boldsymbol{k}}|\chi_k(t)|^2 = -e^{(d-1)\rho(t)}\frac{k^2}{2\omega_k^3(t)} \simeq -\frac{e^{-d\rho(t)}}{2M}\left(\frac{k}{Me^\rho}\right)^2\left\{1 - \frac{3}{2}\left(\frac{k}{Me^\rho}\right)^2\right\}. \quad (4.18)$$

5 Conservation of ζ with loop corrections of heavy field

In this section, we show that when the dilatation invariance is preserved, the curvature perturbation ζ is conserved in time at super Hubble scales, including the loop correction of the heavy field χ. For this purpose, first we rewrite the effective action, using the WT identities derived in the previous section. Then, using the obtained effective action, we show the conservation of ζ.

5.1 Effective action with dilatation invariance

As discussed for the single field inflation in section 2.1, the presence of the constant solution is implied by the dilatation invariance. In this subsection, using the WT identity, we rewrite the effective action in such a way that the dilatation invariance becomes manifest.

Taking the variation of S_χ with respect to δg, we can compute $W^{(n)}_{\delta g_{\alpha_1} \cdots \delta g_{\alpha_n}}$ and the effective action. For instance, taking the n-th derivative of S_χ with respect to ζ and setting δg to 0, we obtain

$$\frac{\delta^n S_\chi[\delta g, \chi]}{\delta\zeta(x_1) \cdots \delta\zeta(x_n)}\bigg|_{\delta g=0}$$

$$= \delta(x_1 - x_2) \cdots \delta(x_{n-1} - x_n)\frac{e^{d\rho(t_1)}}{2}$$

$$\times \left[d^n \left(\dot{\chi}^2(x_1) - M^2\chi^2(x_1) - \frac{\lambda}{12}\chi^4(x_1) \right) - (d-2)^n e^{-2\rho(t_1)}(\partial_{\boldsymbol{x}_1}\chi(x_1))^2 \right]. \quad (5.1)$$

In this subsection, we show that when the WT identity for χ, given in eq. (4.6), is fulfilled, the effective action for ζ preserves the dilatation invariance.

To show this, we further rewrite the WT identity (4.6). Operating

$$\int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x)\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t_x - t_y)$$

and performing the integration by parts, we obtain

$$\int \mathrm{d}^d\boldsymbol{x}\langle\chi_\alpha^n(x)\rangle\partial_{\boldsymbol{x}}\{\boldsymbol{x}\delta g_\alpha(x)\} + \int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x) \left\langle \chi_\alpha^n(x)\frac{\delta i S_\chi}{\delta\zeta_+(y)}\bigg|_{\delta g_+=0} \right\rangle$$

$$- \int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x) \left\langle \chi_\alpha^n(x)\frac{\delta i S_\chi}{\delta\zeta_-(y)}\bigg|_{\delta g_-=0} \right\rangle = 0. \quad (5.2)$$

Here, after rewriting $\delta(\boldsymbol{x} - \boldsymbol{y})(\boldsymbol{x} \cdot \partial_{\boldsymbol{x}} + \boldsymbol{y} \cdot \partial_{\boldsymbol{y}})$ as $\delta(\boldsymbol{x} - \boldsymbol{y})(\boldsymbol{y} \cdot \partial_{\boldsymbol{x}} + \boldsymbol{x} \cdot \partial_{\boldsymbol{y}})$, we performed the integration by parts and then we used

$$(\boldsymbol{y} \cdot \partial_{\boldsymbol{x}} + \boldsymbol{x} \cdot \partial_{\boldsymbol{y}})\delta(\boldsymbol{x} - \boldsymbol{y}) = (\partial_{\boldsymbol{x}}\,\boldsymbol{x} - \boldsymbol{x} \cdot \partial_{\boldsymbol{x}})\delta(\boldsymbol{x} - \boldsymbol{y}) = d\delta(\boldsymbol{x} - \boldsymbol{y}).$$

Similarly, operating

$$\int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y\delta g_\alpha(x)\delta(\boldsymbol{x} - \boldsymbol{y})\delta(t_x - t_y)\partial_{x^\mu}\partial_{y^\nu},$$

on eq. (3.14) with $n = 2$ and $p = 1$, where $\mu, \nu = 0, i$, we obtain

$$\int \mathrm{d}^d\boldsymbol{x}\langle\dot{\chi}_\alpha^2(x)\rangle\partial_{\boldsymbol{x}}\{\boldsymbol{x}\delta g_\alpha(x)\} + \int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x) \left\langle \dot{\chi}_\alpha^2(x)\frac{\delta i S_\chi}{\delta\zeta_+(y)}\bigg|_{\delta g_+=0} \right\rangle$$

$$- \int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x) \left\langle \dot{\chi}_\alpha^2(x)\frac{\delta i S_\chi}{\delta\zeta_-(y)}\bigg|_{\delta g_-=0} \right\rangle = 0, \quad (5.3)$$

$$\int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x) \left\langle \dot{\chi}_\alpha(x)\partial_i\chi_\alpha(x)\frac{\delta i S_\chi}{\delta\zeta_+(y)}\bigg|_{\delta g_+=0} \right\rangle$$

$$- \int \mathrm{d}^d\boldsymbol{x} \int \mathrm{d}^{d+1}y \, \delta g_\alpha(x) \left\langle \dot{\chi}_\alpha(x)\partial_i\chi_\alpha(x)\frac{\delta i S_\chi}{\delta\zeta_-(y)}\bigg|_{\delta g_-=0} \right\rangle = 0, \quad (5.4)$$

and

$$\int \mathrm{d}^d \boldsymbol{x} \langle \partial_i \chi_\alpha(x) \partial^i \chi_\alpha(x) \rangle \{ \boldsymbol{x} \cdot \partial_{\boldsymbol{x}} + (d-2) \} \delta g_\alpha(x)$$

$$+ \int \mathrm{d}^d \boldsymbol{x} \int \mathrm{d}^{d+1} y \, \delta g_\alpha(x) \left\langle \partial_i \chi_\alpha(x) \partial^i \chi_\alpha(x) \frac{\delta i S_\chi}{\delta \zeta_+(y)} \Big|_{\delta g_+ = 0} \right\rangle$$

$$- \int \mathrm{d}^d \boldsymbol{x} \int \mathrm{d}^{d+1} y \, \delta g_\alpha(x) \left\langle \partial_i \chi_\alpha(x) \partial^i \chi_\alpha(x) \frac{\delta i S_\chi}{\delta \zeta_-(y)} \Big|_{\delta g_- = 0} \right\rangle = 0 \,, \qquad (5.5)$$

where we used $\langle \dot{\chi}(x) \partial_i \chi(x) \rangle = 0$. Recalling the expressions of $W^{(1)}_{\delta g_\alpha}$ and $W^{(2)}_{\delta g_{\alpha_1} \delta g_{\alpha_2}}$, given in section 3.3 and adding eqs. (5.2), (5.3), and (5.5) in such a way that their first terms give $W^{(1)}_{\delta g_\alpha}$, we obtain

$$\int \mathrm{d}^{d+1} x \, \{ \boldsymbol{x} \cdot \partial_{\boldsymbol{x}} \delta g_\pm(x) \} W^{(1)}_{\delta g_\pm}(x)$$

$$+ \int \mathrm{d}^{d+1} x \int \mathrm{d}^{d+1} y \, \delta g_\pm(x) \left\{ W^{(2)}_{\delta g_\pm \zeta_\pm}(x, y) + W^{(2)}_{\delta g_\pm \zeta_\mp}(x, y) \right\} = 0 \,. \qquad (5.6)$$

As is clear from the derivation, eq. (5.6) also holds, even if we replace $\delta g_\pm(x)$ included in each term with an arbitrary function. Therefore, replacing $\delta g_\pm(x)$ with a constant nonzero number, we obtain

$$\int \mathrm{d}^{d+1} x \int \mathrm{d}^{d+1} y \left\{ W^{(2)}_{\delta g_\pm \zeta_\pm}(x, y) + W^{(2)}_{\delta g_\pm \zeta_\mp}(x, y) \right\} = 0 \,. \qquad (5.7)$$

By adding the left hand side of eq. (5.6) multiplied by a constant parameter $-s$ and eq. (5.7) with $\delta g_\pm = \zeta_\pm$ multiplied by $-s^2/2$, the linear and the quadratic terms in the effective action can be given by

$$iS'_{\mathrm{eff}(1)}[\delta g_+, \delta g_-] + iS'_{\mathrm{eff}(2)}[\delta g_+, \delta g_-]$$

$$= \sum_{\alpha=\pm} \int \mathrm{d}^{d+1} x \, \delta g_{\alpha,s}(x) W^{(1)}_{\delta g_\alpha}(x)$$

$$+ \frac{1}{2!} \sum_{\alpha_1, \alpha_2 = \pm} \int \mathrm{d}^{d+1} x_1 \int \mathrm{d}^{d+1} x_2 \, \delta g_{\alpha_1, s}(x_1) \delta \tilde{g}_{\alpha_2, s}(x_2) W^{(2)}_{\delta g_{\alpha_1} \delta \tilde{g}_{\alpha_2}}(x_1, x_2)$$

$$+ \mathcal{O}(\delta g^3) \,, \qquad (5.8)$$

where δg_s are related to δg as given in eqs. (2.15), (4.3), and (4.4). Here, each $\delta g_{i,\alpha}$ ($i = 1, 2$) sums over $\delta N_{\alpha,s}$, $N_{i,\alpha,s}$, and $\zeta_{\alpha,s}$.

We can drop the term with one shift vector, because $W^{(1)}_{N_{i,\alpha}}$, which is proportional to $\langle \dot{\chi} \partial_i \chi \rangle$, vanishes. In deriving eq. (5.8), we used

$$W^{(2)}_{\delta g_{\alpha_1} \delta \tilde{g}_{\alpha_2}}(x_1, x_2) = W^{(2)}_{\delta \tilde{g}_{\alpha_2} \delta g_{\alpha_1}}(x_2, x_1) \,, \qquad (5.9)$$

and $\bar{\zeta}_+ = \bar{\zeta}_-$, which holds since $\zeta_+(t_f, \boldsymbol{x}) = \zeta_-(t_f, \boldsymbol{x})$. The first term in eq. (5.6) changes the argument of the metric perturbations in the linear term of eq. (5.8). We also changed the arguments of the quadratic terms, since the modification appears only in higher orders of δg.

Equation (5.8) shows that with the use of the WT identity, $\delta g_\alpha(x)$ in S'_{eff} can be replaced with $\delta g_{\alpha,\,s}(x)$. Since the rest of the effective action, S_{ad}, is simply the classical action for the single field model, it also should be invariant under this replacement. Therefore, when the WT identity (4.2) holds, the total effective action S_{eff} preserves the invariance under the change of δg_α to $\delta g_{\alpha,s}$.

The effective action (5.8) includes the lapse function and the shift vector. By solving the Hamiltonian and momentum constraint equations, we can express δN_s and $N_{i,s}$ in terms of ζ_s. Using these expressions, we can eliminate δN_s and $N_{i,s}$ in the effective action as in the single field model [39]. Since the constraint equations for δg_s are given by replacing δg with δg_s in the constraint equations for δg, the effective action for ζ_s obtained after eliminating δN_s and $N_{i,s}$ should be given simply by replacing ζ with ζ_s in the effective action expressed only in terms of ζ.

5.2 Conservation of ζ

5.2.1 Tadpole contribution

Before we discuss the conservation, we show that the linear terms in the effective action S_{eff}, which is the tadpole terms, vanish all together. Taking the variation of the effective action with respect to N and N_i, we obtain the constraint equations. The Hamiltonian constraint for the FRW background gives

$$d(d-1)\dot\rho^2 = \dot\phi^2 + \langle\dot\chi^2\rangle + \langle(\partial_p\chi)^2\rangle + 2\langle V(\phi,\,\chi)\rangle\,, \tag{5.10}$$

and at the liner order

$$(d-1)e^{-2\rho}\dot\rho\partial^i N_i + \delta N(2\langle V\rangle + \langle(\partial_p\chi)^2\rangle) - d(d-1)\dot\rho\dot\zeta = 0\,, \tag{5.11}$$

where $\partial^i \equiv \delta^{ij}\partial_j$. By changing the spatial distance, the curvature perturbation ζ can affect on the evolution of the heavy field χ. In $\langle\dot\chi^2\rangle$ and $\langle V(\phi,\,\chi)\rangle$, the dependence on ζ disappears after taking the coincidence limit, while $\langle(\partial_{\boldsymbol{x}}\chi(t,\,\boldsymbol{x}))^2\rangle$ still depends on ζ. Using the proper distance $\boldsymbol{x}_{phys}(t) = ae^\zeta\boldsymbol{x}$, where ζ is absorbed, we introduce the expectation value

$$\langle(\partial_p\chi)^2\rangle \equiv \langle(\partial_{\boldsymbol{x}_{phys}}\chi(t,\,\boldsymbol{x}_{phys}))^2\rangle = \int\frac{\mathrm{d}^d\boldsymbol{k}}{(2\pi)^d}k^2|\chi_k(t)|^2\,, \tag{5.12}$$

which does not depend on ζ. Here, k is the Fourier mode of $\boldsymbol{x}_{phys}$. The scalar part of the momentum constraint gives

$$\partial_i(\dot\rho\delta N - \dot\zeta) - \frac{1}{d-1}N_i\langle(\partial_p\chi)^2\rangle = 0\,, \tag{5.13}$$

where we used $\langle\partial_i\chi\partial_j\chi\rangle \propto \delta_{ij}$. The momentum constraint equation can be solved as

$$\delta N = \frac{\dot\zeta}{\dot\rho} + \frac{1}{(d-1)\dot\rho}\sum_{i=1}^3\langle(\partial_p\chi)^2\rangle\partial^{-2}\partial^i N_i\,. \tag{5.14}$$

We can add a homogeneous solution of the Laplace equation on the right hand side.

The action S_{ad} which is accurate at the linear order of δg is given by

$$\begin{aligned}
S_{ad} \simeq \frac{1}{2}\int\mathrm{d}^{d+1}x N e^{d(\rho+\zeta)}\bigg[&-2W(\phi) + \frac{1}{N^2}\big\{-d(d-1)\dot\rho^2 + \dot\phi^2\big\} \\
&+ 2(d-1)\frac{\dot\rho}{N^2}\big\{-d\dot\zeta + e^{-2(\rho+\zeta)}\partial^i N_i\big\}\bigg]\,,
\end{aligned} \tag{5.15}$$

where, for our purpose, we partially kept the higher order terms in the exponential form. The n-th order effective action $S'_{\text{eff}(n)}$ includes the local terms given by

$$\frac{1}{n!} \int \mathrm{d}^{d+1}x \, \delta g_{1,\,\alpha}(x) \cdots \delta g_{n,\,\alpha}(x) \left\langle \frac{\delta^n i S_\chi[\delta g_\alpha, \, \chi_\alpha]}{\delta g_{1,\alpha}(x) \cdots \delta g_{n,\alpha}} \right\rangle$$

with $\alpha = \pm$. Adding up these local terms for all n, we obtain

$$S'_{\text{eff,local}}[\delta g_+, \, \delta g_-] = \langle S_\chi[\delta g_+, \, \chi_+] \rangle - \langle S_\chi[\delta g_-, \, \chi_-] \rangle \,, \tag{5.16}$$

where the terms which do not depend on δg are canceled between the two terms on the right hand side. Adding these local terms to S_{ad} and using the Hamiltonian constraint, we obtain a concise expression as

$$\begin{aligned} &S_{ad}[\zeta_\alpha] + \langle S_\chi[\zeta_\alpha, \, \chi_\alpha] \rangle \\ &\simeq - \int \mathrm{d}^{d+1}x \, N e^{d(\rho+\zeta_\alpha)} \left[2\langle V(\phi, \, \chi) \rangle + \langle (\partial_p \chi)^2 \rangle \right]. \end{aligned} \tag{5.17}$$

Using the Hamiltonian constraint (5.10) and (5.11),[5] we can rewrite the action which is valid up to the linear order as

$$\begin{aligned} &S_{ad}[\zeta_\alpha] + \langle S_\chi[\zeta_\alpha, \, \chi_\alpha] \rangle \\ &\simeq -(d-1) \int \mathrm{d}^{d+1}x \, \partial_t \left(\dot\rho \, e^{d(\rho+\zeta_\alpha)} \right) + \int \mathrm{d}^{d+1}x \, e^{d(\rho+\zeta_\alpha)} \left\{ (d-1)\ddot\rho + \dot\phi^2 + \langle \dot\chi^2 \rangle \right\}. \end{aligned} \tag{5.18}$$

The first term vanishes as a total derivative. The second term is proportional to

$$(d-1)\ddot\rho = -\dot\phi^2 - \langle \dot\chi^2 \rangle \,, \tag{5.19}$$

which can be verified by using the time derivative of the Friedman equation and the field equations for ϕ and χ, given by

$$\ddot\phi + d\dot\rho\dot\phi + V'_{ph}(\phi) + \langle \chi^2 \rangle M M_\phi = 0 \,, \tag{5.20}$$

$$\ddot\chi_k + d\dot\rho\dot\chi_k + \left(M^2 + \frac{\lambda}{2}\langle \chi^2 \rangle + k^2 e^{-2\rho} \right) \chi_k = 0 \,. \tag{5.21}$$

The tadpole terms contained in the last line cancel with each other and the term which does not depend on ζ is canceled between the action for $+$ and the one for $-$. In this way, using the background equations, we can show that the tadpole contributions all disappear.

As we discussed in the previous subsection, the effective action S_{eff} stays invariant under the replacement of $\zeta_\alpha(x)$ with $\zeta_{\alpha,s}(x)$. Therefore, the tadpole contribution for ζ_s should be given simply by replacing $\zeta(x)$ with $\zeta_s(x)$ in eq. (5.17). When the background equations are satisfied, the terms in the second line of eq. (5.8), which are linear in the metric perturbations, all vanish.

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[5]Here, we implicitly assumed that the boundary term of the spatial infinity is chosen such that $\int \mathrm{d}^d\boldsymbol{x} \, \partial^i N_i = 0$. As far as $\dot\zeta_k$ vanishes in the large scale limit, this boundary condition can be consistently imposed. Notice the dilatation transformation with the constant parameter s does not alter this boundary condition.

5.2.2 Existence of constant solution

Removing the tadpole contribution which vanishes with the use of the background equations, we only consider the quadratic terms about ζ. At the linear level, $\zeta_{\alpha,s}$ simply gets the constant shift as

$$\zeta_{\alpha,\,s}(x) \simeq \zeta_\alpha(x) - s\,. \tag{5.22}$$

Therefore, the symmetry under the change of ζ_α into $\zeta_{\alpha,\,s}$ immediately implies the existence of the constant solution also in the presence of the loop corrections of the heavy field.

In single filed cases, it is well known that only the constant solution survives while the other independent solution simply decays in the late time limit, as far as the background evolution is on an attractor (see, e.g., ref. [80]). Then, the curvature perturbation becomes time independent at super Hubble scales. When we add a quantum correction from a heavy field, in principle, the "decaying" mode can turn into a growing mode. Such a drastic change of the behaviour of perturbation can occur, in case the trajectory sizably deviates from the attractor solution, for instance, owing to an effect of an additional field. In the present context, the classical background evolution is determined only by the inflaton and we assume that the quantum effects of the heavy field always remain to be perturbative. In such cases, the effect of the heavy field does not drive the "decaying" mode to grow in time. Then, the presence of the constant mode implies the conservation of the curvature perturbation in time as well as in the presence of the loop corrections of the heavy field.

6 Renormalization and dilatation invariance

As is common in a non-linear quantum field theory, the effective action for ζ potentially diverges due to UV corrections. In our case, the bare coefficients of the effective action $W^{(n)}_{\delta g_{\alpha_1}\cdots\delta g_{\alpha_n}}$, which are expressed in terms of the correlators for χ, can diverge. The UV divergence should be renormalized by introducing counter terms. Depending on a way to introduce the counter terms, the dilatation invariance might be broken. If it were the case, the WT identity would not hold any more and the renormalized effective action does not preserve the dilatation invariance.

When the counter terms are introduced in such a way that the dilatation invariance is preserved, the WT identity holds also after the UV renormalization. Then, inserting the WT identity into the effective action, which can be renormalized following the standard procedure since the theory (before the gauge fixing) is a local theory, and repeating the same argument as we did for the non-renormalized effective action, we can replace $\zeta_\alpha(x)$ into $\zeta_{\alpha,\,s}(x)$ in the renormalized effective action.

Since only the heavy field is quantized in computing the effective action, the curvature perturbation ζ should be dealt with as a classical external field. We may set the arbitrary constant parameter s to a c-number variable $\bar{\zeta}$ in order to express the effective action in terms of the fluctuation in the local region. Then, the effective action includes the non-local contribution $\bar{\zeta}$. Nonetheless, the renormalization should proceed in the standard way, because the inserted non-local contribution, which is schematically in the following form:

$$0 = (\text{WT identity, which identically vanishes and is local}) \times \bar{\zeta}^n \qquad (n = 1,\,2)$$

is fictitious and does not introduce any non-local interactions.

This aspect may be instructive to speculate on the UV renormalization of an IR regular quantity. Preserving the dilatation invariance is crucial to cancel out the potentially IR divergent contribution. In refs. [26, 27], a quantity which preserves the dilatation invariance was proposed and it contains non-local contributions. Because of that, in refs. [67, 68], it was suggested that the quantity which preserves the dilatation invariance may not be able to be renormalized in the standard way by introducing local counter terms.

In this paper, we presented a handy example where the UV renormalization of the heavy field can be performed simply by introducing local counter terms as well as for a quantity which looks to include a non-local contribution. Here, we only considered the UV renormalization of the heavy field χ. It will be important to see if the UV renormalization of the curvature perturbation also can proceed by introducing local counter terms or not. We leave this issue for a future study.

7 Concluding remarks

String theory predicts the presence of a bunch of massive excitations after reduction to four dimensional spacetime, which may encode, for instance, the information on the structure of the compactification of the extra dimensions. It is important to explore a possible imprint of such massive modes on the curvature perturbation. In this paper, we considered an influence of a heavy scalar field on the curvature perturbation ζ at the super Hubble scales. When the mass of the heavy field χ is of $\mathcal{O}(H)$, it can give non-local radiative corrections to the effective action of ζ, which may provide a distinctive imprint of the heavy field. We showed that the time evolution of ζ at the super Hubble scales is not affected by the loop corrections of the heavy field as far as the dilatation invariance, which is entailed in a covariant theory at the classical level, is preserved. The implies that the constant adiabatic mode exists as well as in the presence of the loop corrections of the heavy field.

For simplicity, we considered one heavy field with the standard canonical kinetic term. However, our argument can be extended in a straightforward manner to a more general model which contains more than one heavy fields with a non-canonical kinetic term.

Our result indicates that in order to leave an imprint of massive fields well after the Hubble crossing, we need to break either of the following conditions:[6]

- The massive fields do not alter the background evolution at the classical level.

- The quantum system preserves the dilatation invariance, which yields the Ward-Takahashi identity.

- The radiative corrections of the massive fields on the curvature perturbation ζ are perturbatively suppressed.

If the last condition does not hold, we need to perform a non-perturbative analysis to compute the radiative corrections of the massive fields.

In this paper, using the WT identity (4.2) for the dilatation invariance at $\mathcal{O}(s)$, we showed that the metric perturbation $\delta g(x)$ in the effective action can be replaced with $\delta g_s(x)$, given in eqs. (2.15), (4.3), and (4.4), keeping up to the quadratic terms. This argument can

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[6]Here, we also assume that the (spatially averaged) background universe is the FRW universe. This excludes, say, the solid inflation case, where the anisotropic pressure does not vanish in the large scale limit [81]. (See also ref. [82].)

be extended to higher orders in δg. Using the WT identity (4.2), we can derive the WT identity which relates $W^{(n)}$ to $W^{(n')}$s with $n' < n$. Adding the WT identity for $W^{(m)}$ with $m \leq n$ (multiplied by some particular constant factors) to the effective action S'_{eff}, we can replace all $\delta g(x)$ with $\delta g_s(x)$ in S'_{eff} up to the n-th order of perturbation. After removing the lapse function and the shift vector, we find that the effective action for the curvature perturbation is invariant under the replacement of $\zeta(x)$ with

$$\zeta_s(x) = \zeta(x) - s - s\boldsymbol{x} \cdot \partial_{\boldsymbol{x}}\zeta(x) + \frac{s^2}{2}(\boldsymbol{x} \cdot \partial_{\boldsymbol{x}})^2\zeta(x) + \mathcal{O}(s^3). \tag{7.1}$$

This implies that the curvature perturbation includes a solution which is given by the s-dependent terms in eq. (7.1), whose first term is the constant adiabatic mode. In order to keep the terms which explicitly depend on $\boldsymbol{x}$ perturbatively small, we need to confine the perturbation within a finite spatial region on each time slicing. For that, we will need to use other residual *gauge* degrees of freedom, which are addressed in refs. [26, 27].

In this paper, we studied a spin 0 scalar field as the heavy field. It will be interesting to extend the discussion to include a field with a more general spin [7]. Our discussion does not rely on the explicit form of the interaction vertices nor the propagator. Therefore, we expect that this extension will be feasible. We leave this study for a future project [83].

Acknowledgments

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