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Design and development of a parametrically excited nonlinear energy harvester

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Abstract

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Keywords

nonlinear, design, energy, development, harvester, parametrically, excited

Disciplines

Engineering | Science and Technology Studies

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Design and development of a parametrically excited nonlinear energy harvester

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Abstract

An energy harvester has been designed, fabricated and tested based on the nonlinear dynamical response of a parametrically excited clamped-clamped beam with a central point-mass; magnets have been used as the central point-mass which pass through a coil when parametrically excited. Experiments have been conducted for the energy harvester when the system is excited (i) harmonically near the primary resonance; (ii) harmonically near the principal parametric resonance; (iii) by means of a non-smooth periodic excitation. An electrodynamic shaker was used to parametrically excite the system and the corresponding displacement of the magnet and output voltages of the coil were measured. It has been shown that the system displays linear behaviour at the primary resonance; however, at the principal parametric resonance, the motion characteristic of the magnet substantially changed displaying a strong softening-type nonlinearity. Theoretical simulations have also been conducted in order to verify the experimental results; the comparison between theory and experiment were within very good agreement of each other. The energy harvester developed in this paper is capable of harvesting energy close to the primary resonance as well as the principal parametric resonance; the frequency-band has been broadened significantly mainly due to the nonlinear effects as well as the parametric excitation.

Keywords

Energy Harvesting; vibration based; Parametric Excitation; Experiment

1. Introduction

Due to increased demands for energy and current limitations of batteries, a future prospective technology is motion based energy harvesters (MBEHs) that convert kinetic energy into electrical energy [1]; this type of energy harvester has the potential to be used in powering electronic devices in hostile or remote environments—another benefit of MBEH devices is that they reduce pollutants in the environment which are left behind when batteries are disposed.

For the transduction mechanisms used to convert kinetic energy into electrical power; these can be grouped into *three* main categories. The first category uses piezoelectric conversion, which converts mechanical strain into electrical energy; for instance, Adhikari et al. [2] numerically investigated motion based energy harvesting under broadband excitation using a stack configuration of piezoelectric energy harvesters—expressions were derived for the non-dimensional time constant, electromechanical coupling coefficient and viscous damping factor. Renno et al. [3] investigated into the optimised power that can be harvested using a piezoelectric converter; using the Karush-Kuhn-Tucker technique, the power can be substantially enhanced by using an optimal inductor in the load circuit. Fan et al. [4] numerically and experimentally investigated the performance of a bi-directional nonlinear piezoelectric energy harvester; to achieve a nonlinear frequency-voltage curve two embedded magnets were used inducing a nonlinear stiffness—results showed that the bi-directional energy harvester was more effective than its linear counterpart. Fan et al. [5] designed a piezoelectric based energy harvester that could effectively harvest energy from sway and bi-directional motions; using a cantilever beam, frame and roller a frequency-up conversion mechanism was also achieved—results showed that the output voltage could be enhanced by increasing the sway frequency. Guan et al. [6] recently developed a piezoelectric energy harvester for rotational motion; a theoretical model was developed for the power output and experimental results showed good agreement with the theory. The *second* category of ambient motion energy harvesting transduction mechanisms are electrostatic conversion, where capacitive plates fluctuate inducing a voltage; for example, Bu et al. [7] fabricated a non-resonant wideband micro-energy harvester using electrostatic conversion—to boost low frequency excitations, a parallel structure was used to double the output voltage. The *third* mechanism that can be used to convert ambient kinetic energy into electrical power is electromagnetic induction (EMI), where the relative motion between a magnet and coil generates a backward electromotive force (EMF). For instance, Sardini et al. [8] developed a low frequency energy harvester using polymeric material, due to their low Young's modulus; a theoretical and experimental investigation was conducted—results showed that a small increase in bandwidth could be achieved with this design. Marin et al. [9] fabricated a linear electromagnetic energy harvester with a four Hertz bandwidth using four cantilever beams with closely related natural frequencies. Ooi et al. [10] designed a novel wideband electromagnetic energy harvester using dual resonating cantilever beams; a numerical model was done using MATLAB-Simulink; dual energy harvested peaks were observed resulting in a slight increase in the devices bandwidth. Recently, Siddique [11] presented a comprehensive review of micro-power generators using electromagnetic and piezoelectric energy harvesters; a significant review of recent motion based energy harvesters were compared to each other.

A major drawback with conventional MBEHs is the effective operating frequency range which energy can be harvested; in practical applications, where the excitation frequency is changing or varying with time, precisely matching the natural frequency of a device is crucial for operation and can be achieved with tuning techniques, however, the power required to tune a device will never result in greater energy harvested [12]. There are two main classes for MBEH devices based on the core element being used; the first class employs *linear* resonators and the second class uses *nonlinear* resonators.

For the *first* class, i.e. the *linear* energy harvesters, the maximum energy harvested is achieved when the excitation frequency matches the primary natural frequency of the core element in the MBEH device. The literature regarding this class is quite large. For instance, William and Yates [13] first investigated the use of external resonating devices for powering micro electrical mechanical systems (MEMS); this work was further theoretically extended by Mitcheson et al. [14] for three different damping based resonators. Stephen [15] theoretically investigated the potential of linear MBEH devices for direct mass and base excitations, including the coupling between mechanical and electrical domains. Shahruz [16] developed a multimodal array based on transversely excited cantilever beams with different geometries and tip masses; the results showed that the combination of these linear energy harvesters could achieve a larger bandwidth, however, the required circuitry was more complex. Tang and Zuo [17] theoretically analysed dual mass linear resonators to widen the bandwidth of MBEH devices; dual mass devices could have two local optimums which can further increase the bandwidth of the device. Erturk and Inman [18] further investigated transversely excited linear cantilever beams with piezoelectric bimorph layers, using Euler-Bernoulli beam theory for use as an energy harvester. Leland and Wright [19] designed a linear MBEH device that could be tuned with compressive axial preloading further extending the operating bandwidth of the energy harvester; other tuneable linear resonators based on preloading mechanisms for energy harvesters have also been developed for example in [20].

The *second* class of MBEHs can be divided into *two sub-classes*, there are: energy harvesters whose core elements are either *transversely* excited or *parametrically* excited; for the transversely excited system, there is a large volume of available literature. For instance, Mann and Simms [21] fabricated a nonlinear energy harvester based on the transverse oscillation of a levitated magnet; a perturbation technique known as the method of multiple scales was employed to derive the theoretical frequency response curve—experimental and theoretical results were within good agreement. Maiorca et al. [22] and Liu et al. [23] fabricated MBEH devices utilising mechanical stoppers in which both devices showed nonlinear energy extraction near the primary resonance. Sebald et al. [24] analysed the effects of magnetically induced nonlinearities in transversely excited cantilever beams with piezoelectric layers experimentally. In general, bi-stable energy harvesters have a broader bandwidth [25] and the energy harvested is not influenced under white noise [26].

The literature on the *second* sub-group (of the second class), which are *nonlinear* energy harvesters based on *parametric* excitations of their core elements, is not extensive; for example, Abdelkefi et al. [27] theoretically investigated a parametrically excited cantilever beam for energy harvesting purposes using the Galerkin discretisation and the method of multiple scales; the modelling also includes geometric, inertial and piezoelectric nonlinearities. Daqaq et al. [28] investigated the applicability of parametric energy harvesting with large emphasis on the theoretical model using a perturbation technique; experiments were also conducted on a cantilevered beam with a tip mass—this system showed a *weak* softening-type nonlinearity in the vicinity of the principal parametric resonance.

In this paper, for the first time, an energy harvester has been developed and tested experimentally based on the nonlinear dynamical behaviour of a *parametrically excited beam carrying a point-mass* as the core element subject to a magnetic field. The experiments showed that the fabricated device has an *extended bandwidth* for effectively harvesting kinetic energy; in particular, at the principal parametric resonance, the device displays a *strong* softening-type nonlinearity at higher frequencies which is used to further maximise the frequency band-width and hence kinetic energy harvested from the device—the device harvests energy at *both* the primary and principal-parametric resonances. The energy harvester designed based on *parametric excitation* is shown to *harvest energy* over *larger frequency bands* due to the qualitative and quantitative changes in the *nonlinear dynamical behaviour* of the core element. The paper has been organised as follows: a description of the system including the fabricated device, background theory and experimental procedure are developed in Section 2; the experimentally obtained results for the fabricated energy harvester are acquired and discussed in detail in Section 3. Theoretical verifications are provided in Section 4. Section 5 ends with concluding remarks.

2. System description and experimental procedure

This section describes the specifications of the energy harvester fabricated using a parametrically excited beam carrying a concentrated mass as the core element as well as the experimental setup and data recording and analysing system.

2.1 System description

The system shown in Fig.1 is the core element of an energy harvester with a parametrically excited beam carrying a point-mass as the core element; one of the clamps is fixed while the other is a moveable support in the longitudinal direction. An aluminium beam with length L, width b and thickness h has dimensions of 160, 12, 0.6mm, respectively; a magnet has been attached to either side of the centre of the beam with a net weight of 0.0196 kg (m) (as the concentrated mass). A coil was used as the transduction method to convert the dynamic motion into the electrical energy through electromagnetic induction (EMI); the coil consisted of 600 tightly wound turns and had an internal resistance of 2.7 Ohms. The open-circuit voltage was measured during the experiments when the shaker was exciting the energy harvester (see Figure 2); the moveable support was achieved by using linear bearings.

2.2 Theoretical background

A parametrically excited perfect system displays a zero-response throughout the frequency space, the non-zero response emerges from the zero-response in the vicinity of the principal parametric resonance; however, an initial threshold is required to activate a parametric resonance—a characteristic parametric equation is known as the Mathieu equation given by

$$\ddot{x} + \alpha x^3 + \left[\omega - 2q\cos(2t)\right]x = 0, \tag{1}$$

where x is the displacement field, ω is the natural frequency, q is the forcing amplitude, α is the nonlinear stiffness coefficient and t is the time. Understanding Eq. (1) from a design perspectives means that energy can be harvested effectively from an imperfect system at two frequencies that are relatively close to each other (depending on system parameters); furthermore, the nonlinear stiffness term can be designed for using geometric extensibility at the centreline of the core element resulting in nonlinear behaviour and hence nonlinear frequency-response curves which will be further used to extend the operating bandwidth of the fabricated energy harvester. For the energy harvester shown in Fig.1, ambient kinetic energy is harvested using EMI; as the magnet passes through the coil an induced back electromotive force is generated in the coil resulting in the conversion of kinetic energy into electrical energy. The electromotive force generated is proportional to the rate of change in the magnetic flux given by [29]

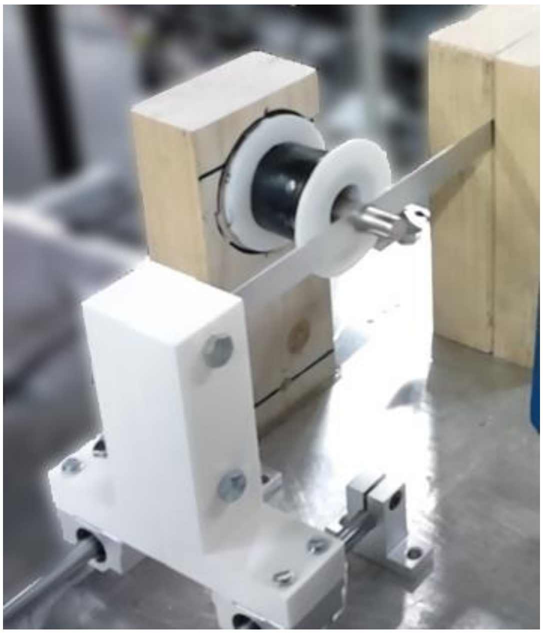
$$V_{EMF} = -\frac{d\phi}{dt}, \tag{2}$$

where V_{EMF} is the back electromotive force and ϕ is the magnetic field flux density. For a uniform coil, the back EMF is proportional to the number of turns by [30]

$$V_{EMF} = -N\frac{d\phi}{dt}, \tag{3}$$

where N is the number of coil turns.

(a)



(b)

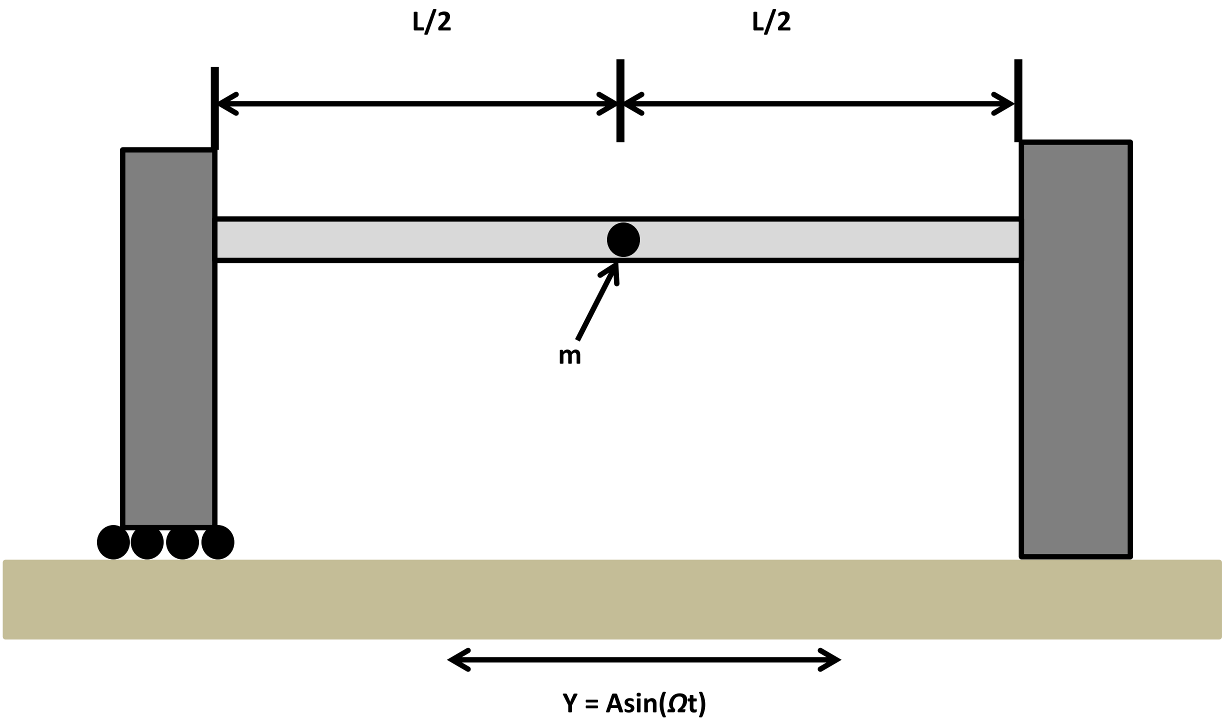


Figure 1: (a) A parametrically excited clamped-clamped beam carrying a concentrated mass as the core element of the energy harvester; (b) Schematic of (a)

2.3 Experimental procedure

Figure 2 shows the experimental setup and the connection between system components, for measuring and acquiring experimental data for analysis. The beam was excited using an electrodynamic shaker (VTS, VC 100-8); the corresponding displacement of the central magnet was measured using a laser displacement sensor (Microepsilon opto NCDT1700ILD1700-50) and the output voltage of the coil was measured through a data acquisition board (DAQ National instruments PCI-6221). The closed loop control system was achieved using an accelerometer (CA YD106) connected to the electrodynamic shaker to control the base excitation amplitude; the output voltage of the DAQ board was enlarged through a power amplifier (Sinocera YE5871).

The system was excited in the horizontal plane and the resulting motion of the magnets was 90^0 (perpendicular) to the base excitation; a swept sine signal in the form of $Y = A\sin(\Omega t)$ (with A and Ω being the acceleration amplitude and frequency) using LABVIEW software was implemented to test the energy harvester.

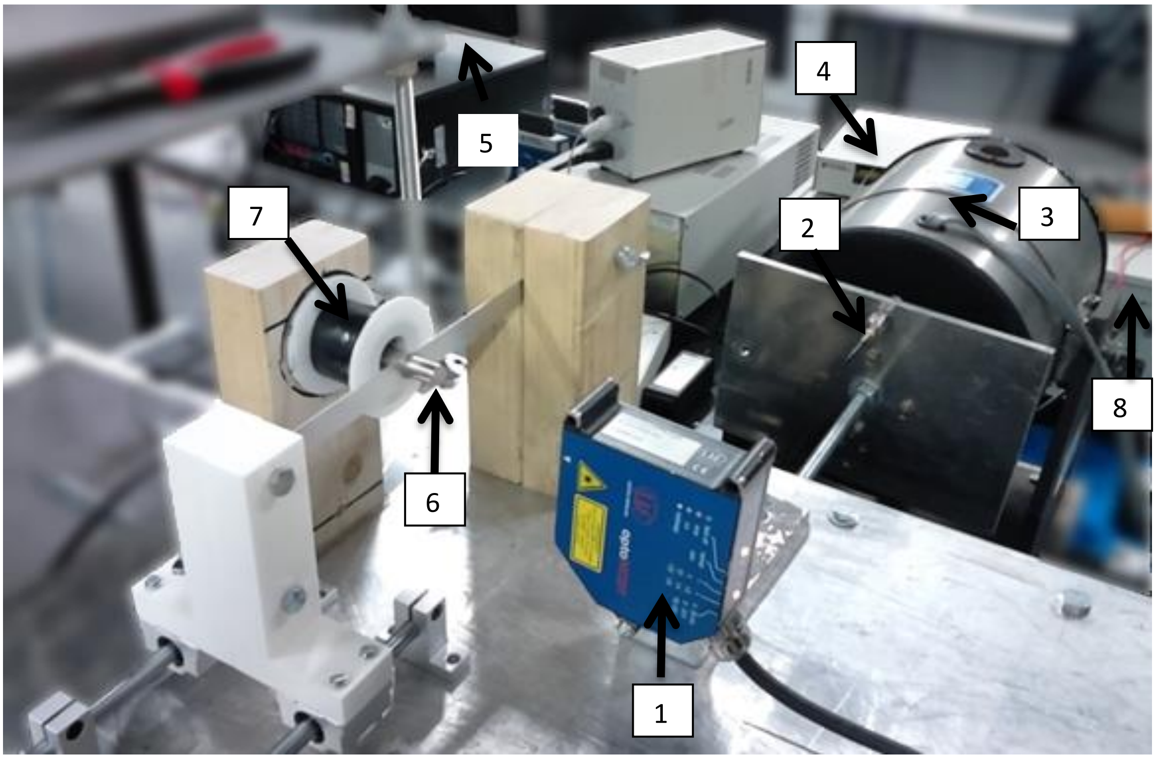


Figure 2: Photograph of the experimental setup (1. Laser Displacement Sensor; 2. Accelerometer; 3. Electrodynamic shaker; 4. DAQ board; 5. Computer; 6. Magnet; 7. Coil; 8. Power amplifier)

3. Experimental results for energy harvested

The fabricated energy harvester using a beam carrying a central mass as the core element was experimentally tested when the system was excited (i) harmonically near the primary resonance; (ii) harmonically near the principal resonance; (iii) near the principal resonance with a non-smooth periodic function. Experiments (i-iii) were used to *qualitatively* and *quantitatively* investigate the *nonlinear dynamical response* of the *parametric resonator* under different acceleration amplitudes to determine the MBEH devices *broader operating range*. A non-dimensional frequency has been introduced as the excitation frequency divided by the experimentally obtained fundamental primary linear resonance of the energy harvester (Ω/ω_1) and at the principal parametric resonance ($\Omega/2\omega_1$); similarly, a non-dimensional motion amplitude has been introduced as the parametric motion amplitude divided by the thickness of the beam (Displacement/h). The experimentally obtained results for the displacement and voltage are presented; referring to Fig.2, the positive motion amplitude and voltage occur when the magnet is moving away from the displacement sensor and the negative motion amplitude and voltage occur when the magnet moves toward the displacement sensor. The frequency-response curves, time traces, fast Fourier transform (FFT), phase-plane diagrams and probability density function (PDF), for experiments (ii) and (iii) are experimentally obtained and plotted to describe the *nonlinear dynamical response* and hence the *nonlinear energy harvested*; the frequency-response curve for the open-circuit voltage of the energy harvester are also experimentally acquired and plotted.

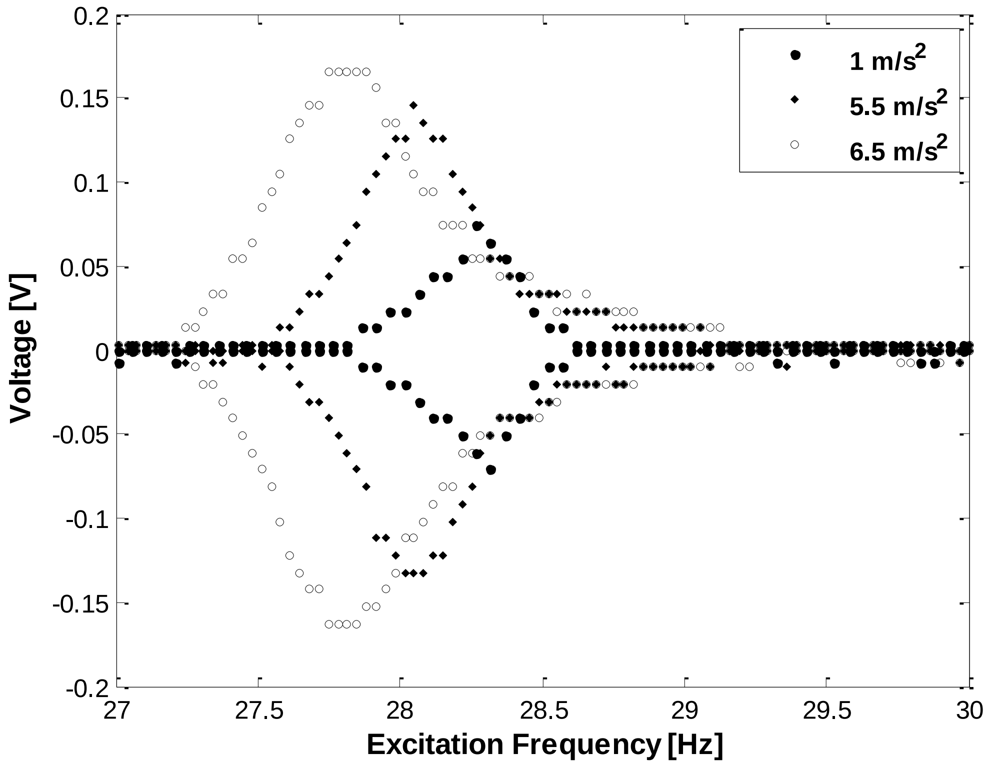
3.1 Energy harvested in the vicinity of the primary resonance

In this section, experiments have been conducted in order to analyse the response of the energy harvester when excited in the vicinity of the *primary resonance* of the core element; results are plotted for both the energy harvested and motion characteristics of the resonator.

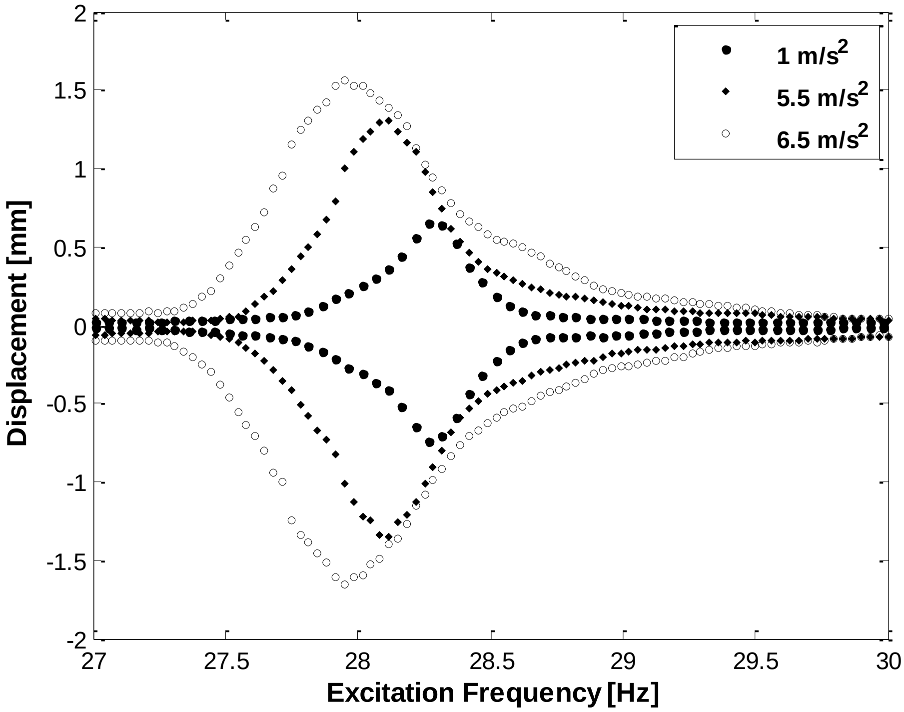
Figures 3(a)–(c) show the open-circuit voltage measured from the coil, the dimensioned displacement of the central mass and the non-dimensioned motion amplitude of the central mass, respectively. It was observed that there was a slight shift in the primary resonance at different excitation amplitudes; this was due to slight initial geometric imperfections in the beam—the corresponding primary resonance frequencies for 1, 5.5, 6.5 ms^{-2} excitation amplitudes were experimentally obtained as 28.27, 28.12 and 27.95 Hz, respectively. When the energy harvester was excited near the primary resonance, a linear response was observed characterised by a gradual incline to a maximum at Ω/ω_1 followed by a gradual and continuous decline; no jump phenomenon was observed. It was also shown in

246 Fig.3 (a) that the open-circuit voltage was generated in the primary resonance region is proportional to the relative displacement of the magnet and the coil.

(a)



(b)



(c)

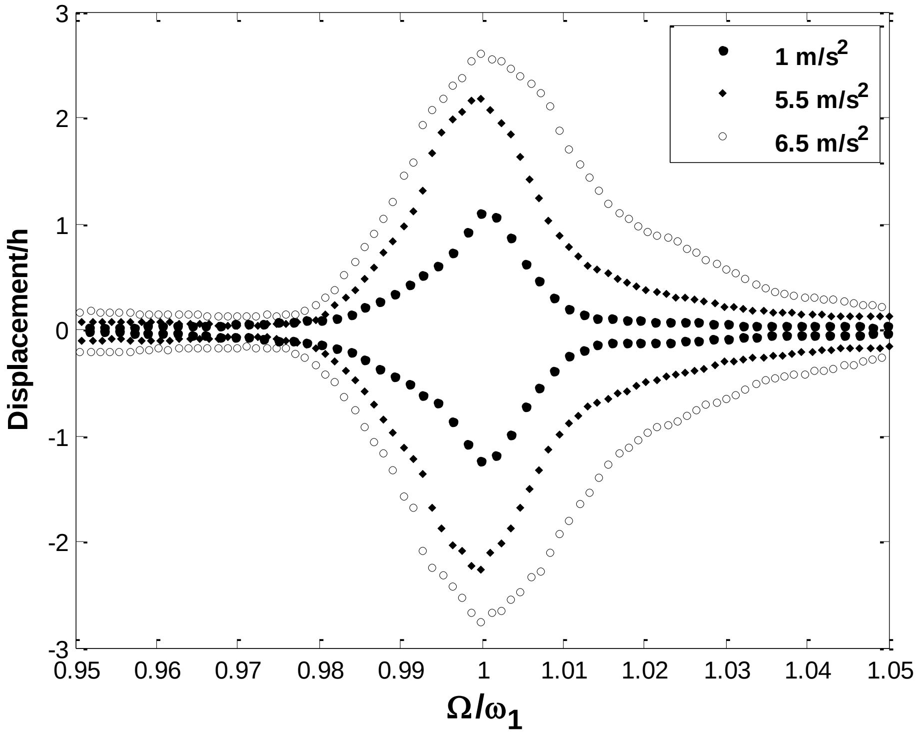


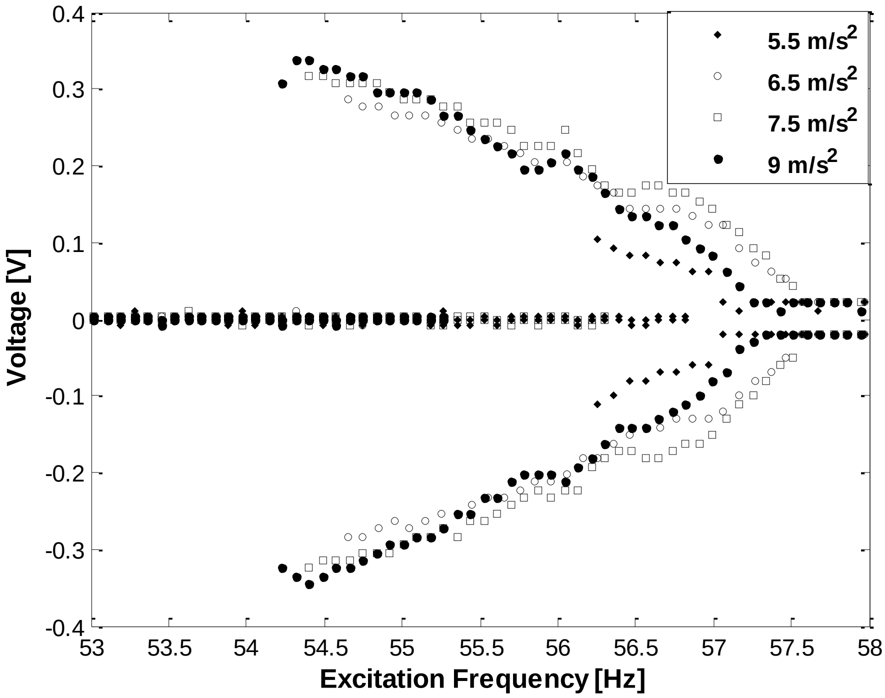
Figure 3: Frequency-response curves for the energy harvester in the vicinity of the primary resonance (a) Open-circuit voltage generated; (b) Dimensional displacement; (c) Non-dimensional displacement

3.2 Energy harvested in the vicinity of the principal parametric resonance

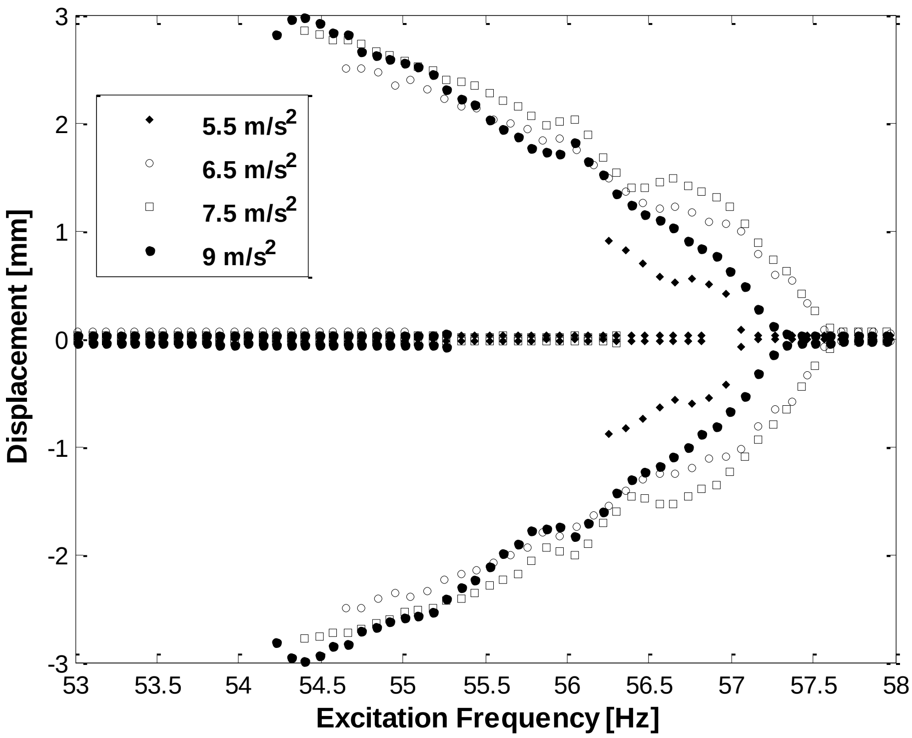
In this section, the nonlinear energy harvested from the energy harvester is experimentally obtained when the nonlinear core element was excited in the vicinity of the *principal parametric resonance*; the experimentally obtained results were plotted in Fig. 4 and are discussed in the following.

It is shown in Fig. 4 that when the energy harvester is excited in the vicinity of the principal parametric resonance of the core element, the system displays a strong softening-type nonlinearity bifurcated from a near-zero response; the softening behaviour can be explained due to initial geometric imperfections in the fabrication process which is realistic in manufacturing processes. Moreover, Fig. 4 displays a nonlinear dynamical behaviour characterised by having two discontinuous points, theoretically these discontinuous points are limit point and period-doubling bifurcations and experimentally these points occur as jump down and jump up points or sudden growth in the response; there are two responses at a specific excitation frequency which are the result of combing the forward and reverse frequency sweeps. The system shows substantially different energy harvested and motion characteristics compared to conventional *linear* energy harvesters in the literature, thus, requiring a *nonlinear* analysis. It is shown in Fig.4(a) that the nonlinear energy extracted in the vicinity of the principal parametric resonance improves the operating bandwidth of the energy harvester and the maximum energy harvested; it was observed that an 11.5% energy harvesting frequency band was achieved—it was also observed that there was a significant increase in the maximum open-circuit voltage by about 300%, between, 5.5 and 6.5 ms^{-2} base excitation. For the maximum motion amplitude occurring at 54 Hz for 9 ms^{-2} excitation amplitude, the time trace, FFT, phase-plane diagram and PDF are shown in Figs.5(a)–(d), respectively, showing a period-2 motion; however, the system is slightly non-symmetric which is due to the initial geometric imperfection.

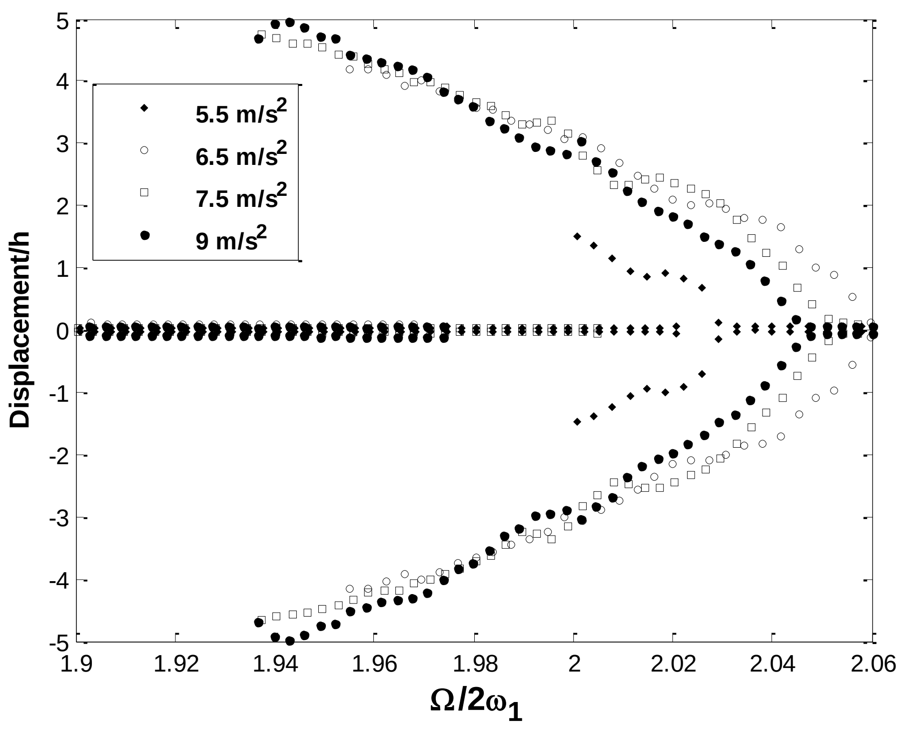
(a)



(b)



(c)



277 Figure 4: Frequency-response curves for the energy harvester in the vicinity of the principal parametric resonance (a) Open-
278 circuit voltage generated; (b) Dimensional displacement; (c) Non-dimensional displacement

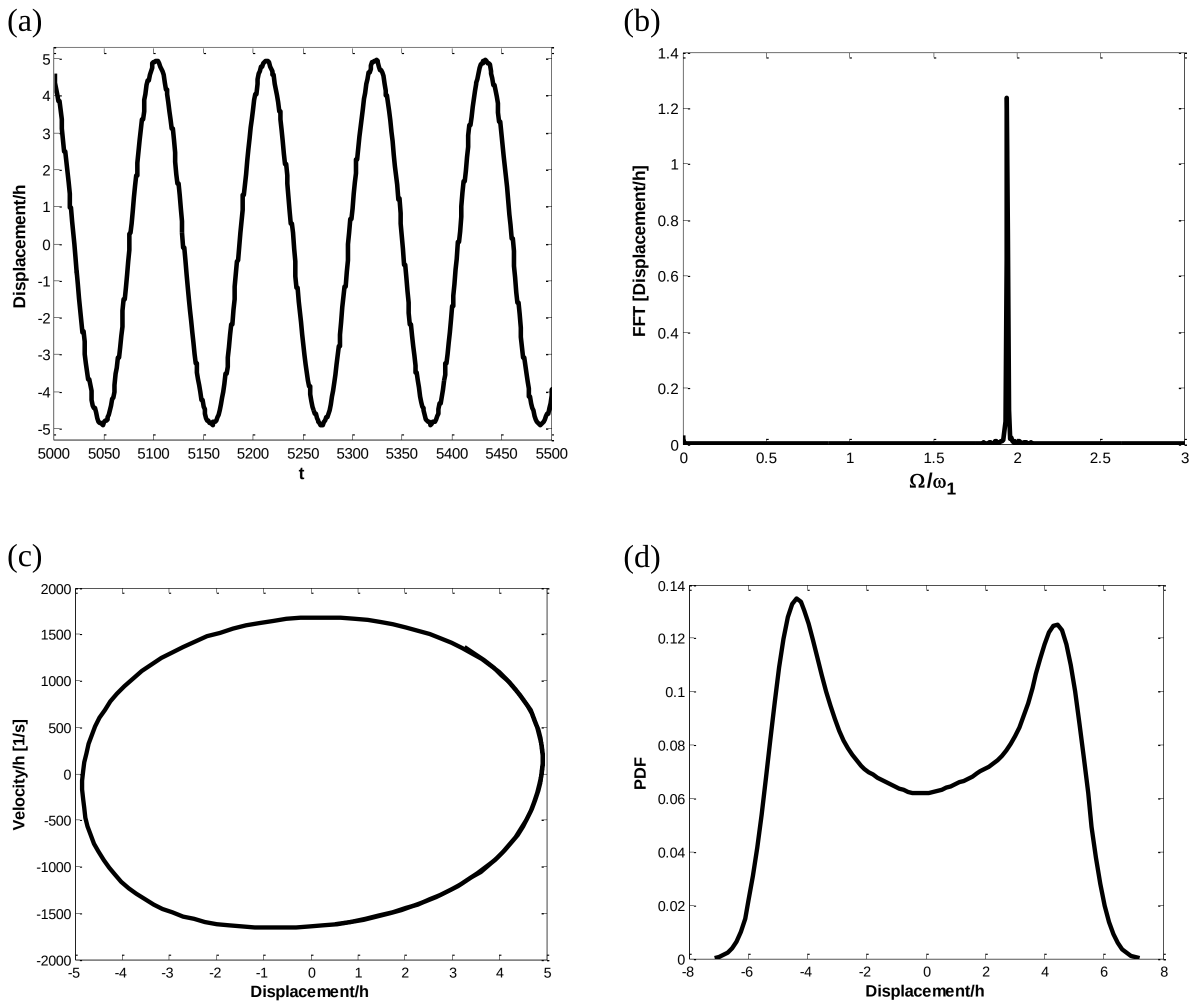


Figure 5: (a) Time trace; (b) FFT; (c) Phase plane diagram; (d) PDF at **Ω** = 54 Hz and $9ms^{-2}$ base excitation, for the system of Fig. 4, illustrating a period-2 motion

3.3 Energy harvested from a non-smooth periodic excitation

In this section, a non-smooth periodic excitation has been generated by the shaker and applied to the base of the energy harvester; the *nonlinear energy harvested* as well as the *nonlinear dynamical response* are obtained experimentally and plotted.

Figure 6 illustrates the non-smooth periodic excitation signal from the electrodynamic shaker where Sub-figures 6 (a) and (b) show the time trace and FFT, respectively (there is noise along with a strong periodic component); the input signal is controlled through the accelerometer attached to the shaker (see Figure 2).

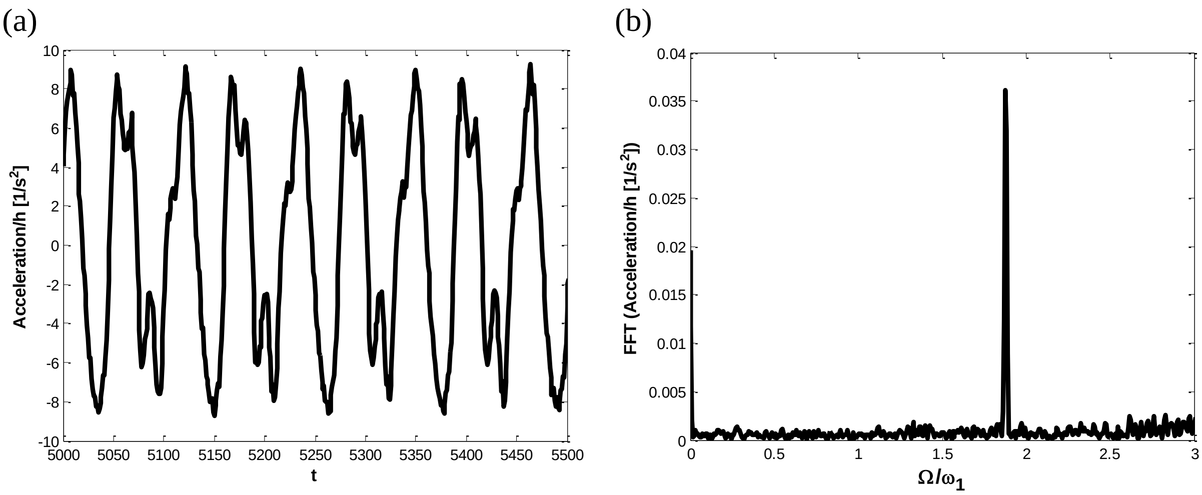
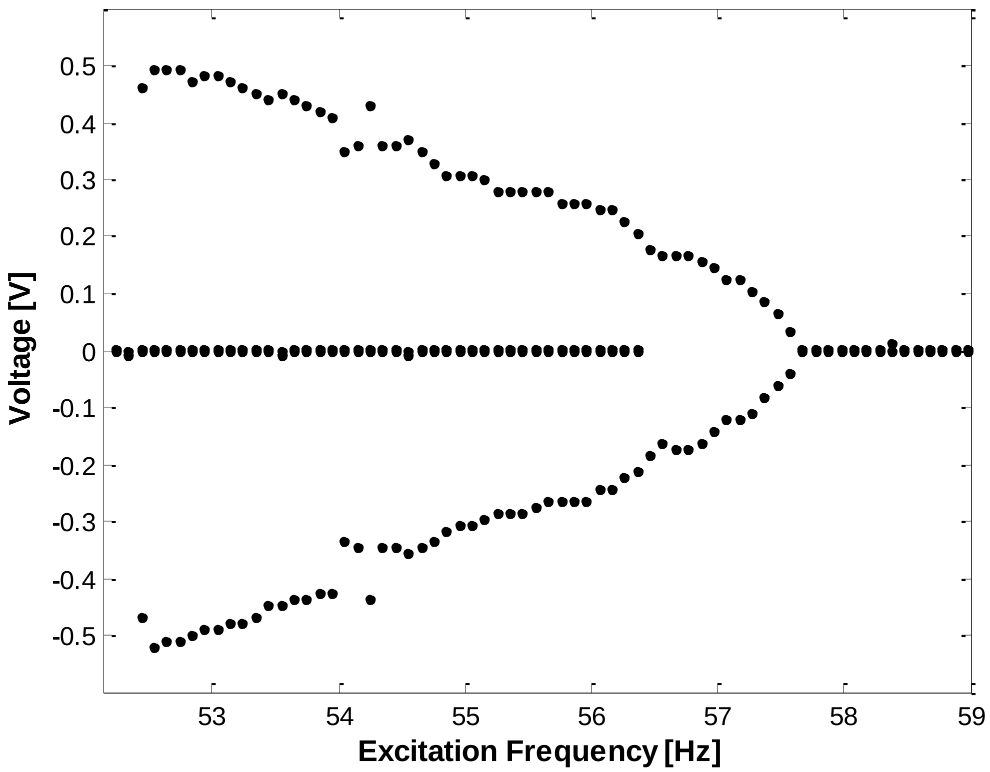


Figure 6: (a) Time trace; (b) FFT of input signal from electrodynamic shaker

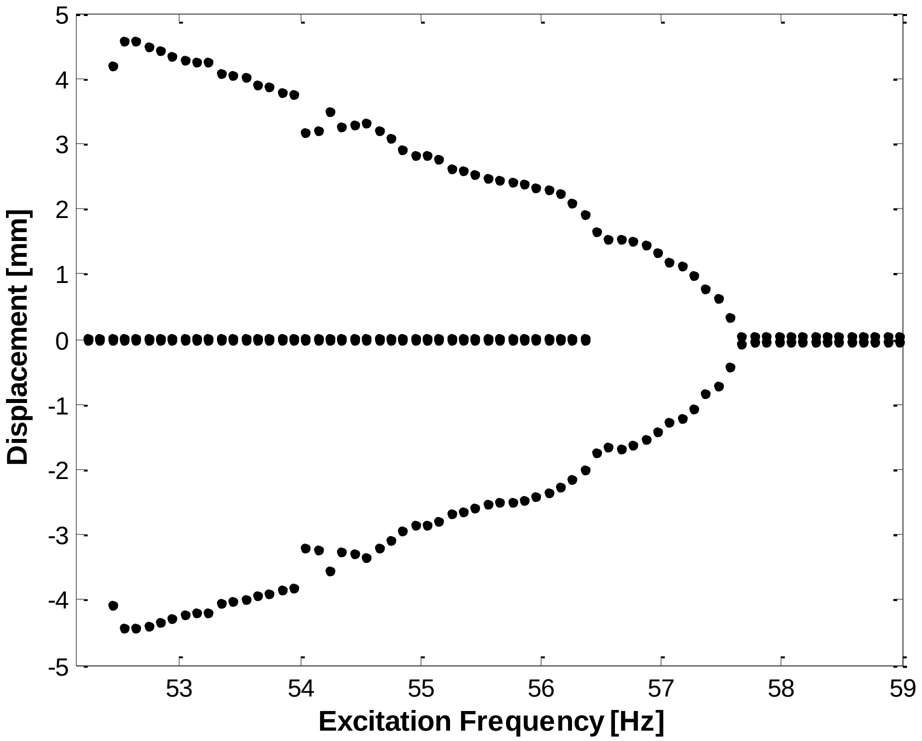
Figure 7 shows the nonlinear voltage harvested (a), the dimensional nonlinear response (b) and the dimensionless nonlinear response (c); the energy harvester still displays a strong softening-type nonlinearity with discontinuous points occurring in the forward and reverse frequency sweeps at Ω = 56.36 and 52.43 Hz, respectively. Comparing Figs. 7 and 4 (with non-smooth and smooth periodic excitations, respectively), the energy harvester performs better for non-smooth periodic excitations than with purely harmonic sinusoids which are more realistic in practical implementation; moreover, the effective operating bandwidth is increased over 20% and a maximum open-circuit voltage of 0.5V is harvested at 52.53 Hz. There are also traces of weak internal resonances present, for instance, in the vicinity of Ω = 53.94 and 54.55 Hz. The experimentally obtained time trace, FFT, phase-plane diagram and PDF are plotted in Figs.8(a)–(d) for Ω = 52.52 Hz; the energy harvester displays a period-2 motion which is slightly non-symmetric due to the initial geometric imperfections.

(a)



Voltage [V]
Excitation Frequency [Hz]

(b)



Displacement [mm]
Excitation Frequency [Hz]

(c)

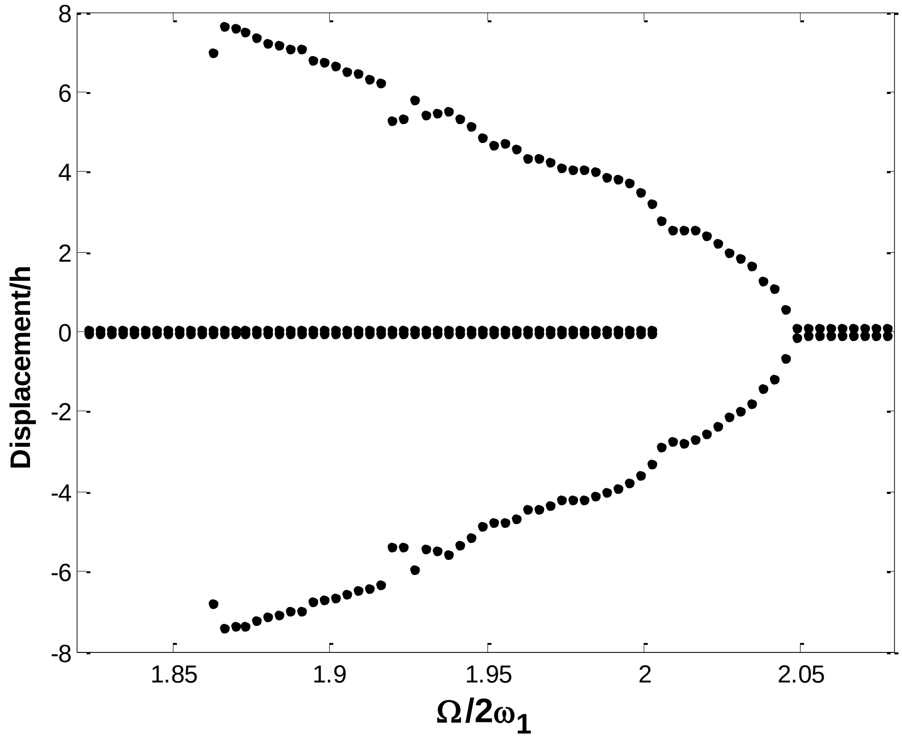


Figure 7: Frequency-response curves for the energy harvester in the vicinity of the primary resonance (a) Open-circuit voltage generated; (b) Dimensional displacement; (c) Non-dimensional displacement

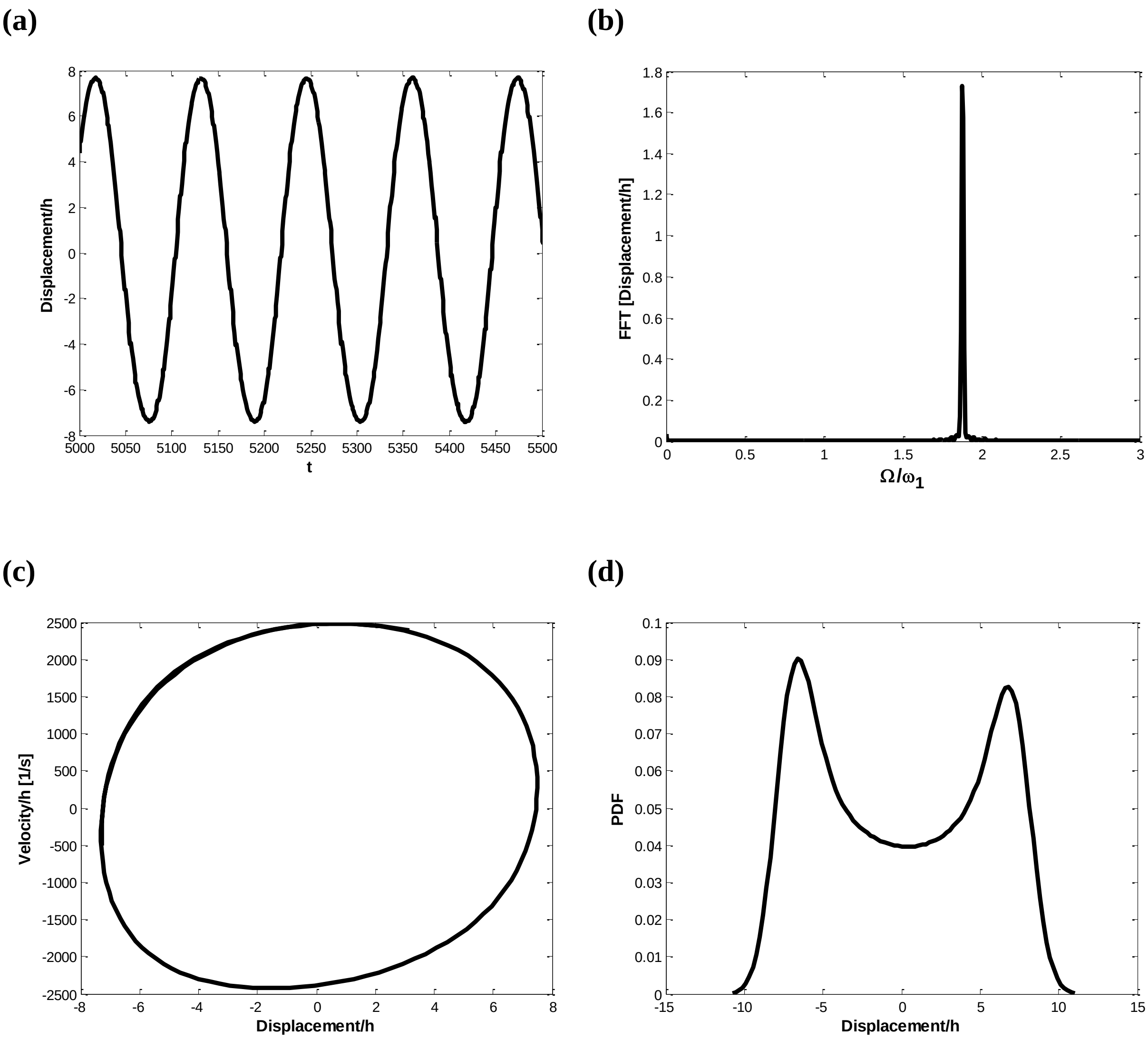


Figure 8: (a) Time trace; (b) FFT; (c) Phase plane diagram; (d) PDF at **Ω** = 52.53 Hz, for the system of Fig. 7, illustrating a period-2 motion

Table 1: Comparison of the excitation types

Excitation	**Bandwidth [Hz]**	**Max. Open Circuit Voltage [V]**	**Max. Displacement/*h***
Harmonic	3.02	0.34	4.96
Non-smooth periodic	5.24	0.52	7.624

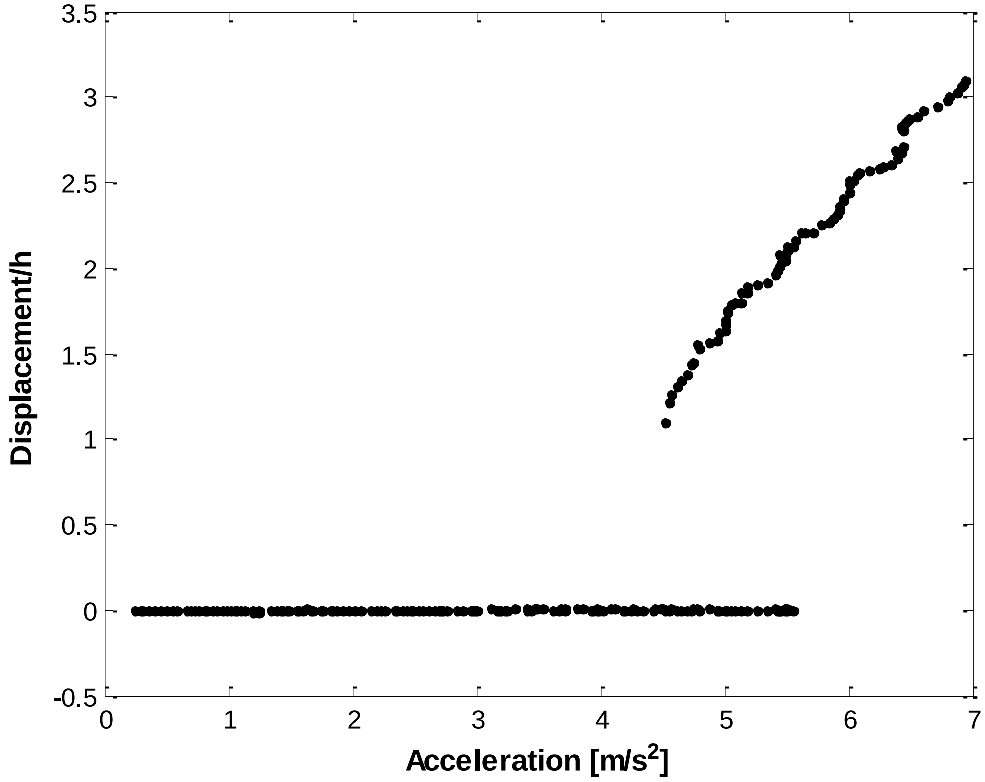
A comparison of the two different excitation types, i.e. harmonic and non-smooth periodic is shown in Table 1. For the comparisons, the harmonic excitation was selected at 9 m/s^2 from Fig. 4; while for the non-smooth periodic excitation case, the system shown in Fig.

7 has been used because it has a maximum base excitation of 9 m/s^2 (which is the same as the harmonic case). It was observed that under a non-smooth periodic excitation the fabricated energy harvester has a larger bandwidth of 2.22 Hz compared to the harmonic excitation case; this result was due to the non-smooth excitation causing more instability of the core element which in turn activates motion attractors which are more beneficial for harvesting energy.

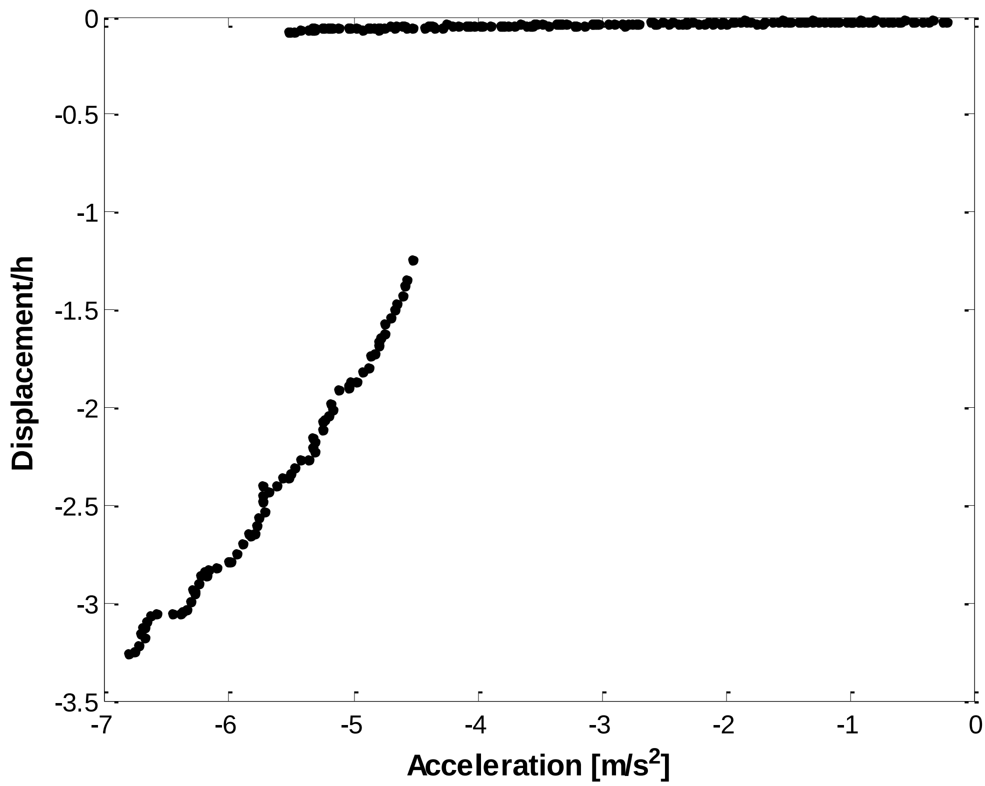
3.4 Acceleration-response curves

To further demonstrate the nonlinear parametric behaviour, experiments were also conducted for the acceleration-response curves at a fixed frequency of 56 Hz; this was done as a design requirement to verify the minimum parametric threshold amplitude required to activate a principal parametric resonance as shown in Figure 9. It was observed there are two distinct discontinuous bifurcations at Y = 4.525 m/s^2 and at Y = 5.56 m/s^2; moreover, this behaviour was due to the geometric extensibility at the centreline of the core element, the geometric imperfection and the parametric excitation which have all been used in the design of the fabricated energy harvester to further enhance the operating bandwidth of the device.

(a)



(b)



352 Figure 9: Acceleration-response curves for the core element at 56 Hz (a) Positive displacement (i.e. when moving away from
353 the laser displacement sensor); (b) Negative displacement (i.e. when moving towards the laser displacement sensor)

4. Theoretical verification

In this section, theoretical verifications of the experimentally obtained data has been conducted. COMSOL Multiphysics 5.1, using the solid mechanics module, has been used for the theoretical verifications; a geometry based on the schematic shown in sub-Figure 1 (b) has been developed—moreover, the geometry has been meshed using 375907 elements. The theoretically calculated primary resonance of the energy harvester was 29.687 Hz while the experimentally obtained natural frequency was 28.27 Hz; furthermore, the theoretical parametric resonance was calculated to be 59.374 Hz while the experimentally obtained parametric was in the vicinity of 57.5 Hz—these results are within very good agreement of each other. From theoretical simulations the maximum parametric motion amplitude occurred at the centre of the beam as shown in Figure 10.

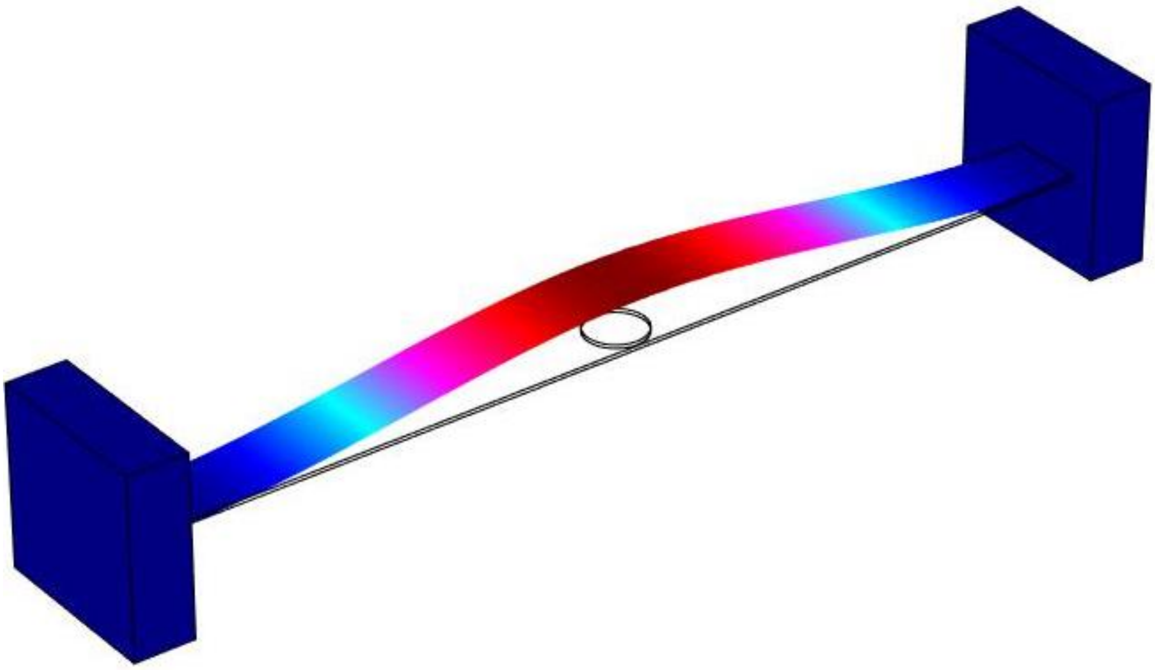


Figure 10: Theoretical mode shape of the fundamental and parametric resonance

Theoretical simulations for the frequency-response curves have also been done using COMSOL Multiphysics to verify the experimentally obtained results at the fundamental resonance as shown in Figure 11; results are within good agreement close to resonance behaviour and post resonance, however, prior to $\Omega/\omega_1 \approx 0.98$ a discrepancy between the two results was obtained—this was due to the axial excitation resulting in a zero-response experimentally, however, results are within good agreement.

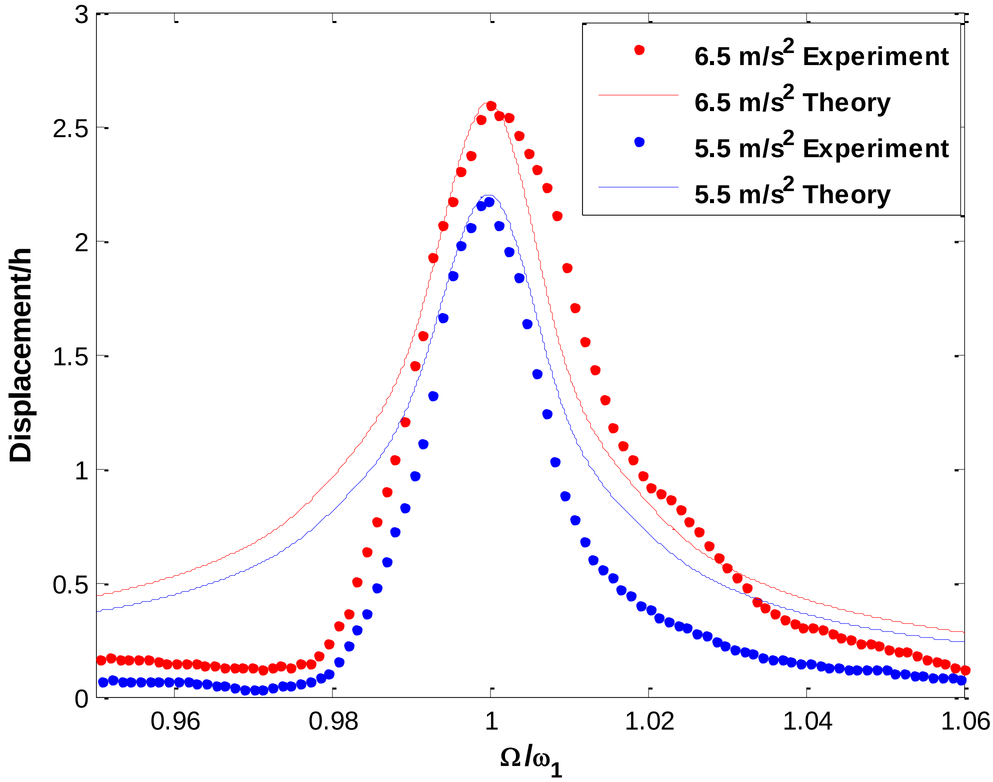


Figure 11: Comparison between theoretical and experimental results obtained at the fundamental resonance

5. Conclusions

An energy harvester has been designed, fabricated and tested, operating based on the nonlinear dynamical response of a parametrically excited clamped-clamped system using a beam with a point-mass (magnets) as the core element; as the core element resonates, the magnets pass through a coil to generate a backward electromotive force for energy harvesting. An electrodynamic shaker was used to excite the energy harvester and experiments have been conducted when the core element was excited (i) harmonically near the primary resonance; (ii) harmonically in the vicinity of the principal parametric resonance; (iii) in the vicinity of the principal parametric resonance with a non-smooth periodic excitation. Experiments (i-iii) were used to evaluate the *increased operating bandwidth* of the *nonlinear energy harvester*.

The magnets attached to the core element showed non-symmetric motion (hence energy harvested) due to a small initial geometric imperfection; the parametric displacement was larger in one direction about the initial equilibrium position. It was observed that when the energy harvester was excited near the primary resonance, linear energy was harvested. When the core element was excited in the vicinity of the principal parametric resonance, the dynamics substantially changed; a strong softening-type nonlinearity was observed near $\Omega/2\omega_1$ which was due to the effects of parametric excitations, nonlinearity and initial geometric imperfection about the equilibrium position. In the vicinity of the principal parametric resonance, the nonlinear energy harvested possesses two discontinuities (i.e. saddle-node and period-doubling bifurcations); this nonlinear behaviour is advantageous in broadening the frequency range at which the energy is harvested—the motion amplitude was also larger than its linear counterpart at the primary resonance. With a non-smooth periodic excitation in the vicinity of the principal parametric resonance, the system performed best, displaying the largest motion amplitude and the energy harvested as well as *effective operating bandwidth*; the energy harvester displayed a period-2 motion. For the theoretical investigation, the calculated fundamental and parametric resonances are within very good agreement of each other; moreover, the theoretical frequency-response curves are also within good agreement of the experimental results.

It was observed that the system harvests energy at both the primary and principal parametric resonances (displaying strong softening-type nonlinearity at the principal parametric resonance); the *parametric* design has substantial *qualitative* and *quantitative* effects on the *nonlinear dynamical response,* hence *increasing* the amount of *energy harvested* and *effective operating bandwidth* of the device.

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