An analytical investigation of the physical dimensions of natural convection flow on a vertical heated Plate

Ali S. Alzwayi and Manosh C. Paul

Abstract— **Analytical results of the boundary layer of natural convection air flow on a vertical heated plate are presented for both isothermal and constant heat flux plates. We investigate the effects of the heat flux and the temperature of the plate on the development of the physical boundary layer along the plate. The ranges of temperature and heat flux examined are 10°C ≤ T ≤ 100°C and 50 ≤ q_P ≤ 600 (w/m^2) respectively. The results show that the variation in the plate temperature has a significant effect on the transition of air on the isothermal plate, and the difference between the temperature of air and plate has more effect on the transition especially when T_P ≤ 60°C. In addition, the temperature of the flow on the constant heat flux plate shows a little effect on the transition, where the transition mostly depends on the heat flux. At q_P = 220 (w/m^2) and T_a=29°C with declining the temperature of air by 69%, the critical height declines by 3.8%. The analytical results are compared with different previous experimental studies for different operation conditions, where a satisfactory agreement is achieved.**

Keywords — **Boundary layer, Critical height, Free convection, Heat flux, Isothermal plate, Turbulent flow, Vertical plate.**

I. INTRODUCTION

Boundary layer of natural convection flow has extensively been studied for long years. Many experimental and theoretical investigations have been carried out to study the behaviour of the development of boundary layer flow on various geometries, for instance, cylinder, sphere and flat plate with different boundary conditions. In fact, the natural convection on a vertical plate has received more attention because the phenomenon of free convection is employed in many engineering applications, for example, cooling industrial equipments or circuit boards in electric models. Moreover, the major applications of free convection link with solar energy systems, for example, natural ventilation uses natural convection to supply different building with fresh air.

Simon [1] performed numerical simulations to study laminar natural convection flow on an isothermal vertical plate for different Prandtl numbers, 0.01, 0.72, 0.733, 1.0, 2.0, 10.0 and 1000. They investigated extensively the effects of the Prandtl number on the rate of heat transfer and reported the results of the velocity, temperature and boundary layer thickness of flow, which become a reference data to many recent studies of laminar convection on parallel heated plates.

Boundary layer of turbulent free convection of air on a vertical heated plate was investigated early by Eckert and Jackson [2] with Grashof number ranging from 10^{10} to 10^{12}. They determined shear stress, its effect on the rate of heat transfer and the thickness of the boundary layer, and compared these to the experimental results of Griffiths and Davis [3]. The quantities characterizing the turbulent free convection on a uniform heat flux were measured by Charles and Vedat [4] on the heated plate which was made of aluminium with dimensions 0.5 in, 24 in and 146 in. Two parallel glasses were used to reduce the loss of heat convection of air from two sides of the plate, and the result was presented for the Rayleigh number over 10^{12}. In addition, smoke was employed to determine the thickness of boundary layer and the location where the flow became turbulent. The thermal boundary layer thickness was then presented as a function of the dimensional temperature of the flow, and the results show that the transition started at $Ra = 4\times10^8$.

Variation of the thermal boundary layer thickness with Prandtl number of laminar free convection on a vertical uniform heat flux plate was measured by Carey and Mollendorf [5]. The investigation was on flows of air, water, ethylene glycol, tri ethylene glycol and SF-97 silicone fluid. Good experimental data was reported for air and water. However, the measured results for SF-97 silicone, which has the highest Prandtl number of 8940, were higher by 20% compared to theoretical result. Some numerical results of the two dimensional flow were reported recently by Wenxin et al [6]. Applying direct numerical simulation (DNS) he calculated Nussle number and the thickness of boundary layer of turbulent free convection on a heated plate. The computations were carried out for $Pr \leq 1$.

Majority of the experimental and numerical works presented the physical properties of the boundary layer as a dimensional quantity in local velocity and temperature. However, these quantities should be known at different location and different thermal condition for the flow to predict the thickness of boundary layer. In the present paper, the critical distance and the thickness of boundary layer will be investigated at different positions of wall and flow temperature.

Manuscript received March 9, 2010. This work was supported by Libyan General People's Committee of Higher Education.

A. S. Alzwayi is with the Department of Mechanical Engineering, University of Glasgow, Glasgow, G12 8QQ, UK
(e-mail: a.alzwayi@mech.gla.ac.uk).

M. C. Paul is with the Department of Mechanical Engineering, University of Glasgow, Glasgow, G12 8QQ, UK (corresponding author. Tel: +44 (0)141 330 8466, Fax: +44 (0)141 330 4343; e-mail: m.paul@mech.gla.ac.uk).

II. MATHEMATICAL FORMULATIONS OF BOUNDARY LAYER THICKNESS

A. Laminar natural convection on an isothermal heated vertical plate

The growth of boundary layer in natural convection flow depends on many parameters such as, T_a, T_P, Pr and L. These reflect in the calculation of Gr and Ra. Moreover, all these parameters and the roughness of surface plate have a great effect on transition of flow from laminar to turbulent. For free convection flow on a vertical plate, the integral momentum equation becomes

$$\frac{d}{dx}\left(\int_0^\delta \rho u^2 dy\right) = -\mu \left.\frac{du}{dy}\right|_{y=0} + \int_0^\delta \rho g \beta (T - T_a)\,dy\,, \tag{1}$$

where x-coordinate is chosen along the plate and distance y is normal to the plate. Boundary conditions for the velocity profile and temperature distribution are presented in Holman [7], and the data suggest that the flow on a vertical isothermal plate becomes turbulent at $Ra \geq 10^9$, Incropera et al [8] and Luice [9]. Approximately at this critical height ($L=Lc$) of the plate, the thickness of the boundary layer ($\delta=\delta c$) in laminar flow is largest. Both these parameters are very important to study free convection, and equation (1) can be solved to calculate the thickness of the boundary layer of the laminar free convection on an isothermal heated plate as,

$$\frac{\delta}{L} = 3.93\,\mathrm{Pr}^{-1/2}(0.952 + \mathrm{Pr})^{1/4}\left(Ra/\mathrm{Pr}\right)^{-1/4}\,, \tag{2}$$

where the Rayleigh number of the flow is defined as

$$Ra = Gr\,\mathrm{Pr} = \frac{g\beta (T_P - T_a) L^3\,\mathrm{Pr}}{\nu^2}\,. \tag{3}$$

To calculate the critical length, Lc, equation (4) is used where $Ra = 10^9$,

$$L_C = \sqrt[3]{\frac{Ra\,\nu^2}{g\beta(T_P - T_a)\mathrm{Pr}}}\,. \tag{4}$$

B. Turbulent flow regimes on an isothermal plate

The mechanism of the development of boundary layer in turbulent flow is very complex to understand compared to a laminar flow, so it has received more theoretical investigations and experimental tests. The rate of heat transfer in turbulent flow is larger than laminar, which is more important in engineering application. Moreover, shear stress has a large impact on local velocity and temperature in the region of turbulent flow. Majority of equations to determine the thickness of boundary layer contain shear stress, and fluctuating components which are unavailable in some calculations. The thickness of the boundary layer of turbulent free convection flow on a vertical plate is presented here as a function of the mean velocity and temperature of the plate and flow by neglecting the fluctuating components. According to Kays and Crawford [10], the equation of shear stress is defined as

$$\tau_o = 0.0225 \rho U^2 \left(\frac{\nu}{U\delta}\right)^{1/4} \tag{5}$$

where U is the mean velocity in the flow, which is caused by the buoyancy and obtained from

$$U = \frac{\nu\sqrt{Gr_L}}{L}\,. \tag{6}$$

The rate of heat convection, defined as Stanton number, can be determined as a function of the shear stress and Prandtl number as

$$St = \frac{q_P}{\rho C_P U (T_P - T_a)} = \frac{\tau_o}{\rho U^2}\mathrm{Pr}^{-2/3}\,. \tag{7}$$

The thickness of the boundary layer of turbulent flow is then approximately determined from

$$\delta = \left(0.0225\left(\frac{\nu}{U}\right)^{1/4}\mathrm{Pr}^{-2/3}\,St^{-1}\right)^4\,. \tag{8}$$

However, both the heat convection, q_P, on the plate and the plate temperature, T_P, should be known. To determine the thermo physical properties (ν, Pr, k, C_p and ρ) of the flow, reference temperature, T_f, is first defined as

$$T_f = T_P + 0.38(T_a - T_P)\,. \tag{9}$$

The volumetric thermal expansion, β, is determined from

$$\beta = -\frac{1}{\rho}\left.\frac{\partial\rho}{\partial T}\right|_{P=const}\,. \tag{10}$$

The equations, which are used to determine the thermal properties, can be written as

$$\phi = \sum_{i=1}^{i=n} a_i T_f^{\,i} + b \qquad i \neq 0 \tag{11}$$

where Φ indicates the thermal property of the flow, and a and b are unknown parameters. The results of the thermal properties of air are presented in equation (12), Kays and Crawford [10]

Properties of air flow, $273 \le T_f\,(\mathrm{K}) \le 373$

$$\begin{bmatrix} \nu = \left(43.78894 - 0.4261292 \times T_f + 1.5941113 \times 10^{-3} T_f^2 - 1.618569 \times 10^{-6} T_f^3\right) \times 10^{-6} \\ \Pr = 1.757623 - 9.3943793 \times 10^{-3} T_f + 2.8247901 \times 10^{-5} T_f^2 - 2.8561855 \times 10^{-8} T_f^3 \\ \rho = 0.9380475 + 1.3550166 \times 10^{-2} T_f - 6.9079673 \times 10^{-5} T_f^2 + 8.8553321 \times 10^{-8} T_f^3 \\ C_P = \left(1.290128 - 2.8180762 \times 10^{-3} T_f + 9.0782796 \times 10^{-6} T_f^2 - 9.5160644 \times 10^{-9} T_f^3\right) \times 10^3 \\ k = 58.8769 - 0.4907669 \times T_f + 1.9291618 \times 10^{-3} T_f^2 - 2.1899 \times 10^{-6} T_f^3 \end{bmatrix} \quad (12)$$

C. Critical distance on a flux plate

The transition of free convection flow on a vertical flux plate, where the temperature of surface will be a function of the height of the plate, has different characteristics than an isothermal plate. Results in Holman [7] show that the transition on the uniform heat surface flux occurs late compared to the isothermal plate. The transition started at $3\times10^{12} \le Ra^* \le 10^{13}$ and the flow became fully developed turbulent when $2\times10^{13} \le Ra^* \le 10^{14}$, where the modified Rayleigh number is defined as $Ra^* = Gr^*Pr$ with the flux Grashof number, $Gr^* = GrNu$. The critical height in this case can be determined from

$$L_C = \left(\frac{Gr^* k \nu^2}{g\beta\, q_P}\right)^{1/4} . \quad (13)$$

III. RESULTS AND DISCUSSION

Results of the analytical investigation of both laminar and turbulent flow of air under the isothermal condition of the heated plate are reported first in Section A, followed by the results of the effects of the plate heat flux on the development the boundary layer of flow in Section B. Some comparisons between our analytical results with experimental studies are presented in Section C. Finally, conclusion on our findings is made in Section IV.

A. Isothermal plate

Laminar air flow

Figs.1 and 2 respectively show the distribution of the Rayleigh number and the thickness of the boundary layer at different location for three different plate temperatures, 40°C, 70°C and 90°C. The ambient temperature of the air flow is kept at 25°C and the results are calculated at every 0.05m interval from the leading edge of the plate. From Fig.1 it is clear that, with an increase in the temperature of plate the-transition from laminar to turbulent occurs early. For instance, at T_P= 40°C the flow becomes turbulent at 0.77m compared to T_P = 90°C where the transition approximately begins at 0.60m

Influence of the temperature between the wall and air on the critical distance and the growth of the boundary layer is presented in Figs. 3 and 4 respectively. When T_P =30°C and T_a =27°C, the critical height is approximately 1.56 m (Fig. 3) and the thickness at this location is 4.20E-2 m (Fig. 4). However, at T_P =30°C and T_a =15°C, both the critical height and the thickness of the boundary layer drops to 0.88m and 2.35E-2m respectively. Furthermore, fixing the air temperature at T_a =24°C, if we increase the plate temperature (T_P) from 35°C to 60°C, which means increasing the plate temperature by 70%, both the critical height and the thickness of the boundary layer drop approximately by 30% and 36% respectively.

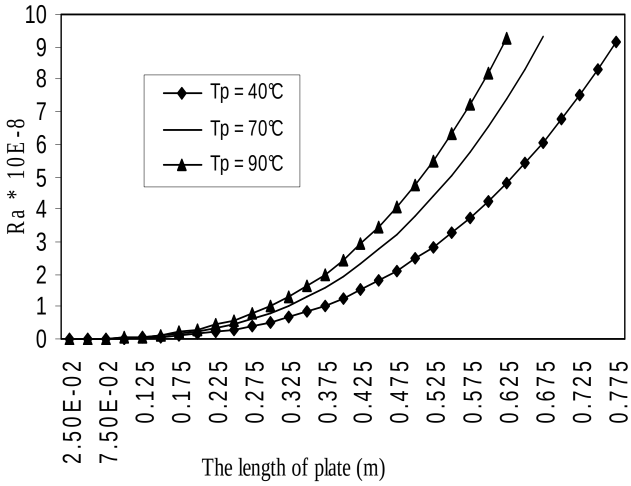

Fig 1: Rayleigh number at different location on the vertical plate at T_a =25°C.

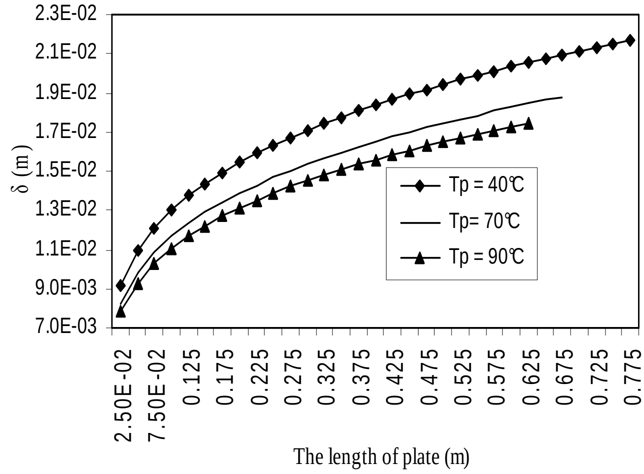

Fig 2: Thickness of the boundary layer along the vertical plate at T_a =25°C.

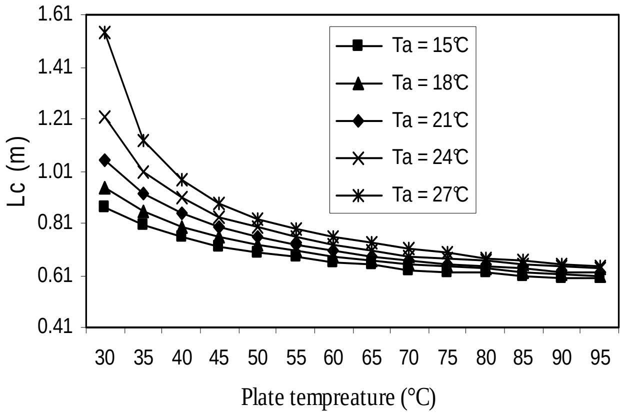

Fig 3: Critical height of the vertical plate for different plate temperatures.

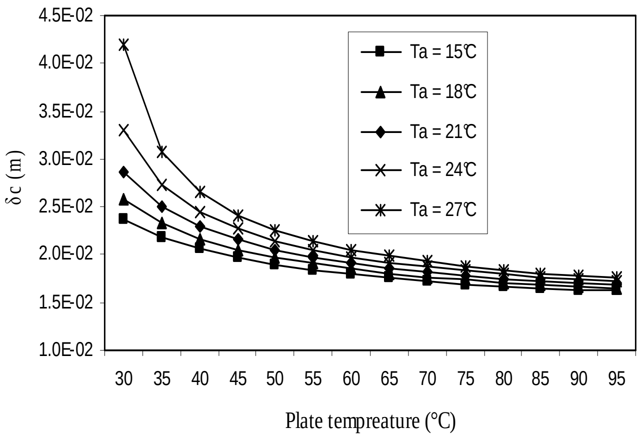

Fig 4: Thickness of the boundary layer at the critical height for different plate temperatures.

Full development of the air boundary layer

Equation (8) is used to determine the thickness of the boundary layer in turbulent flow. The measurements of T_P and q_P recorded by Turgut and Simon [11] have been used as data in this investigation. The results in Fig. 5 show that the thickness of the boundary layer increases sharply in the turbulent flow compared to that of the laminar flow. At T_a =21°C and L= 2.8 m the thickness of boundary layer becomes approximately 0.22 m. However, with an increase in the air temperature to 24°C, the thickness drops by 26%.

B. Heat flux surface

Fig. 6 shows the effects of the variation in heat flux on the critical height of the vertical plate, the results presented for air temperature at 9°C, 19°C and 29°C. The most important point from the figure is that, the temperature of the flow has a little effect on the transition from laminar to turbulent flow. For example, at q_p = 220 (w/m^2) with dropping the air temperature by 69% (29 - 9°C) the critical height declines by 3.8% (1.84 – 1.77m). However, at T_a = 19°C with declining the heat flux by 74% (553 – 145 w/m^2), the critical height increases by 18% (1.625 -1.93 m). Therefore, the critical length of the plate in the heat flux condition is mainly affected by the heat flux parameter than it does by the flow temperature.

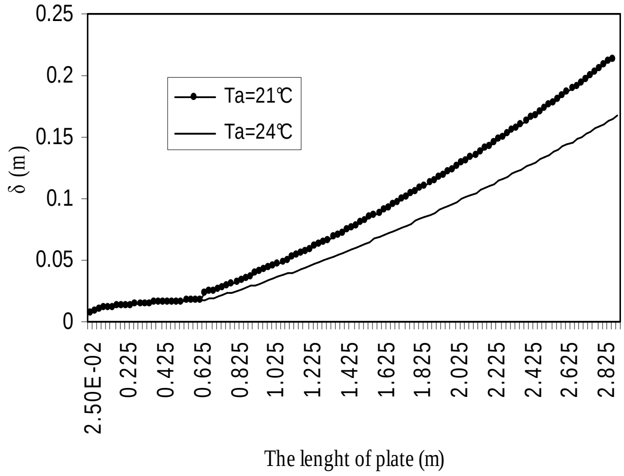

Fig 5: Boundary layer thickness of the full developed air flow at T_P =90°C.

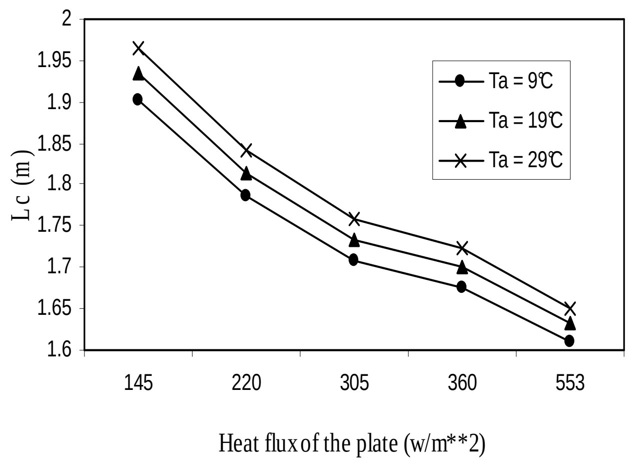

Fig 6: Critical height of the plate at different heat flux

C. Comparison with experimental studies

Some experimental results have been presented in Table 1 obtained for the air flow, but they depend on the facilities and the quantities of flow such as temperature, velocity and heat convection. Comparisons between the theoretical and experimental results show that the agreement between the results of the critical height and the thickness of the boundary layer in most cases is good. However, there are small discrepancies seen between the two results. For example, the critical height of flow on the isothermal plate presented by Cheesewrigh [12] was 1.25 m at T_P =85°C, but from our theoretical analysis the transition regime is found at 0.79 m, while in the same case some agreement is seen between the analytical and the experimental results of the thickness of the boundary layer at L=2.75m, they are 0.07 m and 0.082 m respectively.

Table 1: Comparison between experimental and present analyses

Reference	Heated plate q_P (w/m^2), T_P (°C)	Plate height × width (m×m)	Rayleigh number (Turbulent)	Lc_{exp} (m)	Lc_{cal} (m)	δ_{exp} (m)	δ_{cal} (m)
Charles and Vedat [4]	$T_P = 62$	3.7×0.60	$Ra = 4 \times 10^8$	1.01	0.79	$\delta_c = 0.0135$	$\delta_c = 0.018$
Cheesewright [12]	$T_P = 85$	2.75 × 0.6	$Ra = 5 \times 10^{10}$	1.25	0.77	$\delta_{2.75} = 0.07$	$\delta_{2.75} = 0.082$
Tsuji and Nagano [13]	$T_p = 60$	4×1	$Ra = 3 \times 10^9$	N/A	0.79	$\delta_c = 0.022$	$\delta_c = 0.018$
Yasuo et al [14]	$50 \le q_P \le 202$	4.98×0.95	N/A	1.5-2.3	1.85 - 2.7	N/A	
Chen et al [15]	$200 \le q_P \le 600$	1.5×0.62	$2 \times 10^{13} \le Ra^* \le 10^{14}$	≈2	1.67-1.85	N/A	

IV. Conclusion

Development of the shape of the boundary layer of natural convection flow on a vertical heated plate has been investigated theoretically, the surface is considered smooth and the air is used as a test flow. Various temperatures of the plate and flow have been employed and some experimental results of the plate temperature and heat convection have been used as data to some calculations.

Overall, it is found that in the isothermal case the temperature of the flow and the plate has a major effect on the transition of the flow. However, the temperature of the flow has little effect on the transition when $T_P \geq 60$°C, where the transition approximately depends on the tempreature of the plate. Moreover, the difference of temperature between the plate and the flow has main effect on the transition of the flow, the transition occurs early when the temperature between the plate and flow is increased.

Under the conditions of the investigation on the heat flux surface, the temperature of the flow has a little effect on the transition, where the transition of the flow is mainly affected by the heat flux. From the comparison between the results of our calculation and experimental it is reported that the theoretical results predict satisfactorily the behaviour of the development of the boundary layer on the vertical plate.

Nomenclature

a, b unknown parameters (11)
C_P specific heat (J/kg.K)
g gravitational acceleration (m/s^2)
Gr Grashof number
k thermal conductivity (W/m K)
L length of the plate (m)
Nu Nusselt number
Pr Prandtl number
q_P heat convection surface (W/m^2)
Ra Rayleigh number
Ra^* modified Rayleigh number
T temperature (°C)
U mean velocity (m/s)
x, y Cartesian coordinantes (m)

Greek symbols

β thermal expansion coefficient (1/K)
ρ density (kg/m^3)
ν kinematic viscosity (m^2/s)
δ thickness of boundary layer (m)

Subscripts

a air
f reference temperature
P plate
exp experiment
cal calculation

References

[1] O. Simon, An analysis of laminar free convection flow and heat transfer about a flat plate parallel to the direction of the generation body force, NACA (1952) TN 2635.
[2] E. R. G. Eckert, and T.W. Jackson, Analysis of turbulent free convection boundary layer on vertical plate, NACA (1950) TN 2207.
[3] E. Griffiths, and A. H Davis, The transmission of heat by radiation and convection, Food investigation board, British Dept Sci and Ind, Special report (1922) No. 9.
[4] Y. W. Charles, and S. A. Vedat, An experimental investigation of turbulent natural convection in air at low pressure along a vertical heated flat plate, Int. J. Heat Mass Transfer, 11 (1968) 397-406.
[5] V. P. Carey, and J. C. Mollendorf, Measured variation of thermal boundary layer thickness with prandtl number for laminar natural convection from a vertical uniform heat flux surface, Int. J. Heat Mass Transfer, 21 (1978) 481-488.
[6] L. Wenxian, S. W. Armfield, and J. C. Patterson, Unsteady natural convection boundary layer flow of a linearly stratified fluid with Pr < 1 on an evenly heated semi-infinite vertical plate, Int. J. Heat and Mass Transfer, 51 (2008) 327-343
[7] J. P. Holman, Heat transfer, 9th Ed, McGraw Hill Companies, 2002.
[8] F. P. Incropera, D. P. Dewitt, T. L. Bergman, and A. S. Lavine, Fundamental of Heat and Mass Transfer, - 5th Ed, John Wiley & sons, 2001.
[9] C. B. Louis, Convection heat transfer, 2nd Ed, John Wiley & sons, 1993.
[10] W. M. Kays, and M. E. Crawford, Convection heat and mass transfer, 2nd Ed, the McGraw Hill Companies, 1980.
[11] Y. Turgut, and M. F. Simon, Turbulent natural convection in a vertical parallel-plate channel with asymmetric heating, Int. J. Heat Transfer 50, (2007) 2612-2623.
[12] R. Cheesewright, Turbulent natural convection from a vertical plane surface, National Science Foundation (1967) Crant G- 22529
[13] T. Tsuji, and Y. Nagano, Velocity and temperature measurements in a natural convection boundary layer along a vertical flat plate, Experimental Thermal and Fluid Science, 2 (1989) 208-215
[14] K. Yasuo, M. Masahide, K. Jyunji, and K. Shintaro, Turbulent free convection heat transfer from vertical parallel plates, JSME International Journal, 34 (1991) No. 4, 496-501
[15] Z.D. Chaen, P. Bandopadhayay, J. Halldorsson, C. Byrjalsen, P. Heiselberg, and Y. Li, An experimental investigation of a solar chimney model with uniform wall heat flux, Building and Environment, 38 (2003) 893-906.