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imitation regardless of the cognitive domains of the students' exercises. The students' perspectives on the instructional practice expressed in the post-lesson interviews were used as a triangulation for the results. The results showed that the students appreciated the teacher's explanation and demonstration in the teacher's exposition. Finally, the authors argue that the high percentages of imitation of teacher's methods not only are due to the students' choice, but also are influenced by the Confucian heritage cultures.

- 25 Keywords separated Students' private work Learning Cognitive domains Imitation by ' '
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Insights from students' private work in their notebooks:4how do students learn from the teacher's examples?5

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Abstract Students' seatwork plays an important part in their learning in their lessons, and 10 very often, students record their private work in the notebooks during seatwork. The students' 11 private work in their notebooks reflects students' learning and thinking, representing explicit 12learning outcomes. The students' private work in their notebooks of 14 mathematics lessons of 13an eighth-grade Hong Kong classroom was analyzed. The mathematical tasks used in the 14 lessons were categorized with the Trends in International Mathematics and Science Study 15(TIMSS) cognitive domains framework. The implementation of the tasks was recorded in 16cycles of teacher's examples (TEs) and students' exercises (SEs). By comparing the methods 17employed by the students and the teacher, the students' methods were found to be mainly 18 imitation or partial imitation regardless of the cognitive domains of the students' exercises. The 19students' perspectives on the instructional practice expressed in the post-lesson interviews 20were used as a triangulation for the results. The results showed that the students appreciated the 21teacher's explanation and demonstration in the teacher's exposition. Finally, the authors argue 22that the high percentages of imitation of teacher's methods not only are due to the students' 23choice, but also are influenced by the Confucian heritage cultures. 24

Keywords Students' private work · Learning · Cognitive domains · Imitation	
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1 Introduction

Comparative studies such as Trends in International Mathematics and Science Study (TIMSS,28Mullis, Martin, Foy & Arora, 2012) and Programme for International Assessment (PISA,29OECD, 2010) have reported that students in East Asian regions such as Hong Kong, Korea,30Singapore, and Taiwan have results outperform their counterparts in the non-Asian regions. As31a result, much interest has been made in studies about East Asian classrooms and many studies32

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of the instructional practices in East Asian regions such as Singapore, Shanghai and Hong 33 Kong (Kaur, 2009; Leung, 2005; Mok & Lopez-Real, 2006; Mok, 2009), and Korea (Park & 34Leung, 2006) have been reported. The results of these studies show not only some similarities 35consistent with the teacher-led directive style but also unfolding, at a deeper level, some 36 features conducive to learning in the cultural contexts of East Asian classrooms, hence, 37 explaining, to a certain extent, the good performance of East Asian students. In general, 38 students engage themselves in a lot of classroom activities under the teacher's instruction. This 39happens when students work individually or in a small group, and such organization of 40activities is called "seatwork" (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999, p.74). 41 Students may produce different kinds of outcomes depending on the nature of the teacher's 42assigned tasks. In the case of Hong Kong, seatwork often serves the purpose for the students to 43practice what they have just learned by doing exercises in their notebooks privately. Such 44 private work in the students' notebooks often matters to what the students have learned in that 45particular lesson and directly represents the explicit learning outcomes achieved by the 46 students in the lesson (Fried & Amit, 2003; Jablonka, 2006). However, there are very few 47 studies on the students' private work in their notebooks. The aim of this paper is to fill the gap 48 with a case study in the context of Hong Kong mathematics lessons putting the focus on 49students' private work in their notebooks, hoping to provide a gateway for understanding the 50nature of the students' learning in the classrooms, 51

Learning activities in a mathematics classroom are usually organized via mathematical 52tasks. A mathematical task may be a set of problems or a single problem for drawing students' 53attention on a particular mathematical idea (Stein, Grover, & Henningsen, 1996). These tasks 54that include teachers' examples and students' exercises in the lessons may come directly from 55the textbooks or the teacher's improvisation depending on the teacher's enactment of the 56lesson. The lessons in this study demonstrated a very typical feature in East Asian mathematics 57lessons; that is, the teacher's expository explanation through the teacher's examples formed a 58very important component of the instructional practice. How did the students learn from the 59teacher's exposition? This study attempted to investigate the relationship between the teacher's 60 examples in the teacher's exposition and the students' private work through a detailed 61 examination of the students' private work. The analysis was carried out in four aspects: (1) 62 the cognitive domains of mathematical tasks, (2) the pattern of the teacher's examples and the 63 students' exercises in the lessons, (3) the degree of imitation of the teacher's methods in the 64students' private work, and (4) the students' perspectives on the instructional practice. 65

While filling in the literature gap on students' private work in their notebooks, this paper66aims to contribute in several aspects: to show how the role of the students' notebooks may67serve as a locus wherein the public world of the classroom may be transformed into students'68own private world of engagement with mathematical materials; the potentials and pitfalls of69cognitive import in imitation; and the cultural aspect of imitation with respect to teacher's70authority and students' patterns of learning with respect to the Confucian tradition.71

2 Theoretical perspectives and terminology

2.1 Cognitive domains of mathematical tasks

The mathematical tasks are important vehicles for students to develop their mathematical 74 learning and thinking because, on the one hand, mathematical tasks and the teacher's 75

interpretation of the tasks determine the students' experience in their lessons (Doyle, 1988; 76 National Council of Teacher and Mathematics, 1991). Different attempts have been made to 77 study the cognitive demand of mathematical tasks that plays a pertinent role in defining the 78premises of the students' work. Doyle (1988) discussed the cognitive demand of an academic 79task in terms of the cognitive process that varies from low level of memory such as 80 multiplication tables, to high level of decisions in problem solving or more advanced math-81 ematical work. Stein et al. (1996) defined mathematical tasks as a class activity focusing 82 students' attention on a particular mathematical idea, which could be examined in the 83 dimensions of task features and cognitive demands. Mathematical features were referred to 84 aspects of tasks for engaging student thinking, reasoning, and sense making. The cognitive 85 demand of the task-set-up phase referred to the kind of process entailed in the teacher's 86 announcement, whereas the cognitive demands at the implementation stage in the classrooms 87 referred to the actual cognitive processes in which the students engaged while carrying out the 88 tasks, that is, whether the students actually recalled facts and formulas or engaged in high-level 89 thinking and reasoning. Cognitive demand or level defined in such way referring the actual 90process of students' engagement is dynamic and difficult to measure. Nonetheless, for 91 studying the students' learning outcomes, it is important to have indicators for measuring 92the potential cognitive demand of the mathematical problems that the students engage in. By 93 classifying the assessment items, TIMSS attempts to assess students' understanding at multiple 94 levels in three cognitive domains, namely, knowing, applying, and reasoning (Mullis, Martin, 95Ruddock, O'Sullivan, & Preuschoff, 2009), hence, giving a valid inference of how students 96 may perform on specific tasks (Nixon & Barth, 2014). 97

The TIMSS categories of cognitive domains were applied in the analysis and recapitulated 98 here (Mullis et al., 2009, pp. 40-46): 99

- Knowing: covers the facts, concepts, and procedures that students need to know. The 100 subcategories are recall, recognize, compute, retrieve, measure, and classify/order. 101
- Applying: focuses on the ability of students to apply knowledge and conceptual understanding to solve problems or answer questions. The subcategories are select, represent, model, implement, and solving routine problems.
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- Reasoning: goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems. The subcategories include analyze, generalize/specialize, integrate/synthesize, justify, and solving non-routine problems
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2.2 Students' seatwork and private work in the classrooms

When classroom activities are organized in such way that students may engage 109themselves in mathematical materials in their seats either individually or in small 110 groups, such organization of activities is called seatwork (Stigler et al, 1999, p.74). 111 Seatwork often occupies a significant portion of the mathematics lessons in different 112places in the world (Stigler et al., 1999), and quite a few researchers have attempted 113to study seatwork in different cultural contexts. For example, Hino (2006) studied the 114 role of seatwork in Japanese classrooms and found that the placement of the seatwork 115prior to the presentation of the main content of the lessons provided opportunities for 116 students to share and exchange their ideas, and the main content could make a 117connection to their seatwork in the earlier part of the lesson. Serrano (2012) compared 118

the seatwork in Germany, Japan, and USA in the TIMSS videos to investigate the 119 influence of seatwork activities on students' thinking in the lessons. 120

Fried (2008) discussed public domain and private domain in mathematics classroom 121practice. The same mathematical activity such as seatwork or writing in students' notebook 122can be termed as private or public depending on the pedagogical practice. In particular, Fried 123and Amit (2003) investigated students' notebooks, one of the products of the seatwork in the 124lessons, in two Israel eighth-grade mathematics classes and found that the work in the students' 125notebooks was the rehearsals for public display as the students' work was open for inspection 126There is a certain tension between the private domain and the public domain of the treatment of 127the notebooks, but the work in the notebooks becomes a finished product by public inspection 128(Fried, 2008). In the case of Hong Kong mathematics classrooms, occasionally, the teacher 129might select the students' work to show on the board, to show the students' ideas, and to share 130alternative solutions (Jablonka, 2006). However, for most of the cases in Hong Kong, students' 131notebooks were often individual and private although there might be limited sharing between 132students when they talked to their classmates sitting next to them (Lui & Leung, 2013). 133

2.3 The framework of the study

The process of teaching and learning in mathematics classrooms is complex in the socio-135cultural context. According to Vygotsky (1978), the interpersonal (the interaction between the 136teacher and peers) process is transformed into an intrapersonal (the student) one. Within the 137zone of proximal development (ZPD), students may handle problems beyond the capability of 138their mental age when they are under guidance or in collaboration with peers. Activities in a 139lesson are arranged based on mathematical tasks that may appear in the form of a problem 140statement going through three stages: the text format of the tasks, the setting up by the 141 teachers, and the implementation by the students in the classrooms (Henningsen and Stein, 1421997). In the case of Hong Kong classrooms, the social space consists of the teacher-led whole 143class interaction and the seatwork period when the students may occasionally talk to the 144classmate sitting next to them. When students interact within the social space in the lesson, 145their learning takes place when observing and imitating of teacher's procedures. This imitation 146147is not necessary a purely mechanical process. Students imitate the teacher's procedures and later become independent through their minds. 148

In the lessons in this study, the text format of the tasks might be either worksheets designed 149by the teacher or problems adapted from the textbooks. The mathematical tasks might be used 150for teacher's expository work or assigned exercises for student seatwork, which occupied a 151significant component of the lessons. The students' work during seatwork was directly 152influenced by the design of the tasks, the teacher's exposition and demonstration, and the 153students' own implementation of the tasks. The methods demonstrated in the teacher's 154examples often acted as a model for students to imitate in their work. Thus, the methods 155employed in the students' work might infer how students learnt from the teacher's exposition. 156The students' private work is the focus in the study. The key terms are defined below. 157

A *task/mathematical task* in this paper is defined as a mathematical problem, which 158 can either be used as an example in the teacher's expository explanation or demonstration, known as *teacher's example* (TE), or an exercise assigned for students to 160 work during seatwork, known as *students' exercise* (SE). The problem statements of 161 TE and SE might appear in the text form of a mathematical problem in the textbooks, 162 a teacher example shown on the board, or a problem in the teacher-designed 163

worksheets. Consequently, a lesson can be represented as a sequence of TE episodes 164 and SE episodes, forming *TE-SE cycles*. 165

Student's private work (SW) refers to the records of the students' work in their notebook 166during seatwork. A preliminary analysis showed that the students' private work contained 167some direct copies of the TEs shown on the board and the students' private work when they 168engaged in the exercises on their own. The students' private work also contained some 169incomplete items (including the unattempted items) and some complete items. The reason 170for incomplete items might be due to insufficient time to complete the assigned exercises 171during the lessons. The complete items of SW were further analyzed. To make a differentia-172tion, SE referred to the task problem statement of the SE, whereas SW referred to the students' 173private work when they completed the exercises in their notebooks during seatwork. SW 174includes the students' own answers worked out by themselves for the teacher's assigned 175student exercises (SE) or the students working on extra exercises not assigned by the teacher. 176

The *cognitive domains* of the mathematical tasks, including both TE and SE, were analyzed 177 according to the TIMSS cognitive domain categories. Some examples are shown in Table 1. 178

The *degree of imitation* refers to the degree of similarity when the method employed in the 179 complete items of students' private work (SW) was contrasted with the method employed in 180 the TE. The degree of imitation of the SW thus gives an indicator on how the students learn 181 from the teacher's expository demonstration. 182

3 Source of data: the LPS

The data consisted of 14 consecutive lessons of a Hong Kong school (HK3) taken from the 184Learner's Perspective Study (LPS) which was an international research collaboration to 185examine the patterns of participation in competently taught eighth-grade mathematics class-186rooms (Clarke, Keitel, & Shimizu, 2006). The 14 lessons covered two topics: slopes of lines 187 and a system of simultaneous linear equations in two unknowns (Table 2). The class size was 18840 and the mean International Benchmark Test (IBT)¹ scores of the class were 38.4 over 50 189(77 %). The teacher had 12 years of secondary mathematics teaching experience and was 190identified as a competent teacher locally by the researchers and the school principal. 191

The data collection procedures followed the LPS design which aimed to collect a rich data 192set for allowing the researchers to reconstruct the lesson scenario from different perspectives 193including the learners' perspectives to make possible analysis under different themes and 194frameworks (Clarke et al., 2006). An integrated system of three cameras was used to collect 195data in which one was for the whole class, one was for the teacher, and one was for a group of 196two focus students. A total of 14 consecutive lessons of the same class were recorded. Two 197different students were chosen to be the focus for each lesson, and they were invited to take a 198post-lesson interview. All the lesson materials including the focus students were collected at 199the end of the lesson. The video-stimulated recall interview technique was used, and the 200students were asked to stop the video at episodes that they saw as important and explained why 201they saw the importance. The data used in this study consisted of the videos and transcripts, 202focus students' notebooks and worksheets, and interview transcripts. 203

¹ The International Benchmark Test for Mathematics (IBT) is norm-referenced and evaluates student achievement on mathematical content for eighth grade. Items are taken from the TIMSS Student Achievement Study (population 2).

pring	Table 1 The examp	Table 1 The examples of cognitive domains of mathematical tasks in the lessons	essons	
ti ti	Cognitive domains	Cognitive domains Teacher's examples (TE)	Students' exercises (SE)	Cognitive demands
t1.3	Knowing	5		
t1.4	Compute	Solving the simultaneous equations by the method	Use the method of elimination to solve the $\int_{-\infty}^{+\infty}$	Carry out algebraic procedures in solving
		of substitution $\begin{cases} 2x-5y=3\\ 3x+4y=13 \end{cases}$	simultaneous equations $\begin{cases} 2x-5y = 3\\ 3x+5y = 20 \end{cases}$	the equations
t1.5	Recognize		Compare the slopes of L1 and L2. What do you find?	Recognize the values of slopes
t1.6	Retrieve		Using the graph drawn in (a) to answer the following questions	Retrieve the information from the graph
t1.7	Applying			
t1.8	Select	A (6, 4) and B (2, -3) are two points on straight line L ₁ . Find the slope of L ₁ .	The vertices of a quadrilateral PQRS are P (2, 4), Q (3, 5), R (-3 , 1), and S (-5 , -1). Find the slope of each side of the quadrilateral PQRS.	Find the slope by using the formula of slope
t1.9	Model	The selling price of three tables and four chairs is \$6400. The selling price of four tables and one chair is \$5500. What are the selling prices of each table and each chair?	The cost of four apples and two oranges is \$14. The cost of six apples and five oranges is \$27. Find the cost of each fruit.	Generate the equations for solving the problem
t1.10	Implement	Draw the graph of the linear equation in two unknowns y = 2x + 3 where x takes values from -2 to 2.	Draw the graph of the linear equation $y = 2^{-\frac{1}{2}}x$ from $x = -2$ to $x = 6$.	Implement a set of mathematical instruction in drawing the graph
t1.11	Reasoning			
t1.12	Analyze	1	(i) What is the size of $\angle CAB$ and $\angle ACB$? (ii) What is the size of $\angle AEF$ and $\angle EAF$? (iii)What can you say about $\triangle ABC$ and $\triangle AEF$?	(iii) Determine the relationships between two triangles according to the values of the angles from (i) and (ii).
t1.13	Generalize	1	From these results (in a), what conclusion can you make?	Generalize the results in (a) in a more general statement.
t1.14	Justify	Given that the vertices of $\triangle ABC$ are A (-3, 2), B (-1, -2), and C (1, 4), prove that $\triangle ABC$ is a right-angled triangle.	Given that the vertices of \triangle ABC are A (-1, 4), B (9, -11), and C (-16, -6), prove that \triangle ABC is a right-angled triangle.	Find the slope of the three lines, and connect the results to the properties of right-angled triangle for justification

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Topics	Lessons
Slopes of lines	L01 to L04
A system of simultaneous linear equations in two unknowns (i) The graphical method	L05 to L07
(ii) The method of substitution	L08 to L09
(iii) The method of elimination	L10 to L11
(iii) The word problems	L11 to L14

4 Methods of analysis

4.1 The cognitive domains of mathematical tasks

A total of 116 mathematical tasks, which might be used as either a TE or a SE, were 206implemented in the 14 lessons. The cognitive domains of the mathematical tasks were 207classified into knowing, applying, or reasoning with their corresponding subcategories 208(Table 1). 209

4.2 TE-SE cycles of the lessons

The mathematical tasks implemented in the lessons were identified as either TEs or SEs 211according to the lesson videos. The teacher usually demonstrated principles or procedures in 212

Lessons		Patter	ns of T	E – SE cycle	es		Number of TE – SE cycles in the lessons
L01			TE (2)	SE (5)			1
L02			SE	. (9)			0
L03			SE	(11)			0
L04	SE (2)	TE (1) S	E (7)	TE (1) SE	(2)	TE (1) SE (5)	3
L05	TE (4)	SE (2)		Т	E (1) SE (1)	2
L06			TE (1)	SE (1)			1
L07			SE	(2)			0
L08) SE (2)	,	TE (1) SE (2)	3
L09				SE (6)			1
L10				TE (1) SE	(5)	TE (1) SE (3)	4
L11				Т	TE (2) SE (1)		2
L12	TE (1)	SE (1)			TI	E(1)	1
L13	TE (1)	SE (2)			TI	E(1)	1
L14	TE (1)	SE (2)			TE	E (2)	1

Legend:

represents the border between each TE -SE cycle

TE: Teacher's example (number of task); SE: Students' exercise (number of task) Note: The lengths of segments do not reflect the duration of the lessons.

Fig. 1 The structural patterns of teacher's examples and students' exercises in the lessons

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solving the TEs and then assigned exercises for student to practice forming a TE-SE cycle; 213 hence, a lesson could be seen as a chain of TE-SE cycles (Fig. 1). 214

4.3 The degree of imitation when contrasting SW with the teacher's method in TE 215

After the classification of cognitive domains of mathematical tasks and the pattern of the TE-216SE cycles, the students' private work in the students' notebooks done by the focus students 217was analyzed. A total of 252 items of SW in the students' notebooks were collected from 27 218students in 14 lessons. Altogether, there were 136 complete exercise items (labeled as SW), 219which were further analyzed by comparing the methods employed by the students with the 220methods demonstrated in the TEs. For instance, Janice and Gary worked on the same SE and 221produced their own SW; therefore, the counting of SE was 1 and the counting of SW was 2 in 222this case (Fig. 2). 223

Very often, the teacher's demonstration of principles and procedures for solving a particular 224 task in TE was prior to the SEs. Therefore, there was often a high degree of similarity between 225 the TE and the SEs in a TE-SE cycle. Two examples of TE-SE cycles are given in Table 5. The 226 methods employed in the students' private work (SW) were compared with the method in the 227 TEs, the degree of imitation was categorized based on how closely the students imitated the 228 teacher's methods, and the categories were as follows: imitation, partial imitation, and 229 students' own method (Table 3).

4.3.1 Examples of imitation and partial imitation

The students' private work (SW) by Gary and Janice (Fig. 2) is used here to illustrate the 232 differentiation between imitation and partial imitation in the coding. The lesson (L06) was 233about graphical method for solving a pair of simultaneous linear equations. The teacher's 234method was to use three points with the values of x coordinates 1, 3, and 5 in two tables, 235respectively, to draw the two lines. Gary copied the TE in solving the equations (4x-5y=2,2367x-10y=2); he imitated completely the teacher's method by using the same values of x 237coordinates (1, 3, 5, respectively) for plotting the two lines. His private work was coded as 238"imitation." In contrast, Janice also imitated the teacher's method of using three points, but she 239chose different values of x (1, 3, 5 for one equation and 1, 2, 3 for another equation). Janice's 240private work was classified as "partial imitation." 241

4.4 Students' perspectives on the instructional practice

Twenty-six student interview transcripts were analyzed to give the students' perspectives of the 243 instructional practice. The stimulated-video-recall method was used in the post-lesson 244

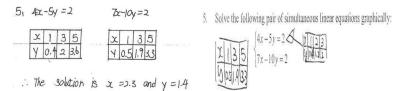


Fig. 2 Gary's private work (*left*) was classified as imitation, and Janice's private work (*right*) was classified as partial imitation

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Degrees of imitation	Descriptions
Imitation	Students reproduce the methods used in the teacher's example exactly in solving the task.
Partial imitation	Students imitate the teacher's methods incompletely, such as skip/miss some steps or use other values that were not same as in the teacher's examples.
Student's own method	Students use a method different from the teacher's examples or no corresponding teacher's example for imitation.

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interviews. The students were invited to stop the lesson videos at moments where they saw as 245 important and give their comments. In general, the instructional practice could be categorized 246 into exposition, seatwork, and review, which could be further break down into subcategories 247 (for details, Mok, Kaur, Zhu, & Yau, 2013). The lesson video segments and the students' 248 attached importance for the video segment were coded by three sets of codes: (1) TE/SE, (2) 249 exposition/seatwork/review, and (3) subcategories under exposition, seatwork, and review that are as follows: 251

- Exposition: teacher's explanation (EC), teacher's demonstration (D), new knowledge (NK), giving instruction (GI), and uses real-life examples during instruction (RE).
- Seatwork: students working individually/copying notes (IW), students working in groups/ group discussion (GW), and material used as part of instruction (M).
- Review: reviews prior knowledge (PK), uses student's presentation or work to give 257 feedback for in class work or homework (SP), gives feedback to individuals during lesson 258 (IF), and gives feedback through grading of written assignments (GA). 259

4.5 Reliability and validity

Two researchers carried out the coding independently on the cognitive domains of 261 mathematical tasks, the classification of TEs and SEs, the degree of imitation of 262 teacher's methods in students' private work, and the exposition codes. The percentages of agreement were over 84 %. 264

5 Results: how did the students learn from the teacher's exposition? 265

5.1 The cognitive domains of mathematical tasks

The distribution of the cognitive domains of the tasks in the 14 lessons is shown in 267 Table 4. The ratio of SEs to TEs was about 2.6 (84:32). The distributions of the 268 cognitive domains of the TEs were knowing (47 %), applying (50 %), and reasoning 269 (3 %), whereas those of SEs were knowing (33 %), applying (45 %), and reasoning 270 (21 %). Therefore, the students had more practice on the knowing and applying tasks 271 in comparison with the reasoning tasks. The proportion of reasoning tasks for SE was greater than that for TE. 273

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t4.1	Table 4 The cognitive domains of mathematical tasks	s								
t4.2		Teacher's examples	mples		Students' exercises	ercises		Total		
t4.3 t4.4	Topics	Knowing CO/RC/RE	Applying SE/IM/MO	Reasoning JU/GE/AN	Knowing CO/RC/RE	Applying SE/IM/MO	Reasoning JU/GE/AN	Knowing CO/RC/RE	Applying SE/IM/MO	Reasoning JU/GE/AN
t4.5	Slope of the lines	0/0/0	4/0/0	1/0/0	1/1/0	21/0/0	12/2/4	1/1/0	25/0/0	13/2/4
t4.6	The graphical method	4/0/0	0/2/0	0/0/0	1/0/1	0/4/0	0/0/0	5/0/1	0/9/0	0/0/0
t4.7	The methods of substitution and elimination	11/0/0	0/1/0	0/0/0	24/0/0	6/1/0	0/0/0	35/0/0	6/2/0	0/0/0
t4.8	Word problems	0/0/0	6/0/0	0/0/0	0/0/0	9/0/0	0/0/0	0/0/0	0/0/15	0/0/0
t4.9	Total number of tasks in each subcategory	15/0/0	4/3/9	1/0/0	26/1/1	27/5/6	12/2/4	41/1/1	31/8/15	13/2/4
t4.10	Total number of tasks in each cognitive domain (%)	15 (47 %)	16 (50 %)	1 (3 %)	28 (33 %)	38 (45 %)	18 (21 %)	43 (37 %)	54 (47 %)	19 (16 %)
t4.11	Total number of mathematical tasks (%)	32 (28 %)			84 (72 %)			116 (100 %)		
	~				,			, ,		
	CO knowing —compute, RC knowing—recognize, RE knowing—retrieve, SE applying—select, IM applying—implement, MO applying—Model, JU reasoning—Justify, GE	RE knowing-	-retrieve, SE	applying-sel	ect, IM apply	ing-implemen	nt, MO apply:	ing-Model, J	IU reasoning-	Justify, GE
	reasoning—generalize, AN reasoning—analyze									
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5.2 The TE-SE cycles in the lessons

The pattern of TE-SE cycles in each lesson is shown in Fig. 1. There were 11 lessons 275containing TE-SE cycles with different length of SE items, showing a variation in the emphasis 276of SEs in these cycles. Three lessons L02, L03, and L07 did not have TEs. When we examined 277the TEs and the SEs, the tasks used for TE and SE were very similar in each cycle. Using 27804 Lesson L08 as an example, the first cycle was TE(2)-SE(2). The teacher first introduced the 279lesson with one TE on the board; then, he used the first item (question 1 (a)) of his self-280designed worksheet as the second TE. The worksheet consisted of 18 items that were grouped 281into six questions. All the items were very similar with minor changes, and the teacher gave 282emphasis in different part of the computation procedures in his explanation for different 283examples. The students were expected to use the teacher's methods in TE to complete the 284assigned SE (Table 5). 285

5.3 The students' private work in their notebooks

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The students' private work (SW) in their notebooks was analyzed. One hundred thirty-six 287 items of students' complete private work were coded for the degree of imitation. Among the 288 items, 116 items belonged to teacher-assigned exercises and 20 items belonged to items that 289 were not assigned by the teacher but completed on the students' self-initiative because they 290 completed the assigned work early. The distribution of the different degrees of imitation in the 291 students' private work is given in Table 6. Imitating from the teacher's method in the 292

Mathematical tasks	Classification
The first TE-SE cycle, TE(2)-SE(2)	
$\begin{cases} y = x + 1 \\ 2x - y - 5 = 0 \end{cases}$	Teacher's example (written on the board)
1(a) Solving the simultaneous equations by the method of substitution $\begin{cases} y = x + 1 \\ 3x + 4y = 11 \end{cases}$	Teacher's example (item on the worksheet)
1(b) Solving the simultaneous equations by the method of substitution $\begin{cases} y = 3x + 1 \\ y = x + 7 \end{cases}$	Students' exercise
1(c) Solving the simultaneous equations by the method of substitution $\begin{cases} x = 4y + 7 \\ x + 4y - 7 = 0 \end{cases}$	Students' exercise
The second TE-SE cycle, TE(1)-SE(2)	
2 (a) Solving the simultaneous equations by the method of substitution $ \begin{cases} 2x + 3y = 5 \\ x - y = 5 \end{cases} $	Teacher's example (item on the worksheet)
2 (b) Solving the simultaneous equations by the method of substitution $ \begin{cases} 2x + y = 9 \\ x - y = 3 \end{cases} $	Students' exercise
2 (c) Solving the simultaneous equations by the method of substitution $\begin{cases} 5x + 7y = 18 \\ x + y = 6 \end{cases}$	Students' exercise

Table 5 The teacher's examples and the students' exercises implemented in L08 (the first seven tasks)

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demonstration was the major feature in the students' private work. There were 60 items of 293 imitation and 73 items of partial imitation, making up a total of 133 out of 136 items of SW 294 regardless of the cognitive domain of the tasks. One possible reason for large number of 295 imitation might be due to the TE-SE pattern in which the TEs were always arranged before the 296 SEs and the TE and SE tasks for each cycle were similar in nature. Furthermore, the teacher 297 demonstrated detailed procedures or instructions, giving a model for students to imitate. These 298 features help the students to recognize and imitate the teacher's methods easily. 299

For example, the teacher used two lessons (L08 and L09) for teaching the method of 300 substitution and he based his lessons on a self-designed worksheet. The worksheet consisted of 301 six questions of different variations of the coefficients and forms of the equations. Each 302 question consisted of three similar items, making up a total of 18 items of very similar format. 303 Each item was a pair of simultaneous equations that might either be a TE or SE. The lesson 304 pattern of L08 consisted three TE-SE cycles (TE(2)-SE(2), TE(1)-SE(2), and TE(1)-SE(2)) 305 where each cycle had two assigned items for SEs. L09 was the second lesson for the topic 306 aiming to give more practice on the method with only one TE-SE cycle, TE(1)-SE(6). That is, 30705 in L09, the teacher used one item in the worksheet as TE and assigned six items as SEs. 308 Joanne's notebook was collected by the end of L09, therefore, contained her private work for 309 both L08 and L09. When we examined Joanne's notebook in L09 in details, she did the 310assigned SE selectively. In L08, she did only one SE (producing one SW) in each TE-SE, and 311 in L09, she produced three SWs out of six SEs in her notebook. In the post-lesson interview, 312she explained that she discerned between similar methods and seemed to be reluctant to do 313 items with repetitive calculation methods. She said, "The same calculation method, but not the 314same numbers, just for familiarizing, see whether you understand it or not." Her private work 315in L09 was coded as imitation (1), partial imitation (1), and students' own method (1). By 316 partial imitation, there were some skipping steps in the students' private work, but these 317 skipping steps did not hinder the students to get the correct answers while repeating the 318 teacher's method. These skipping steps such as missing labels of some equations during 319substitution sometimes might cause some ambiguity in the presentation of answers. 320

Cognitive domair	s of the students' exercises	Degrees of	imitation in stude	ents' private work
	Subcategories	Imitation	Partial imitation	Student's own method
Knowing	Compute	4	32	1
	Recognize	2	0	0
	Retrieve	2	0	0
Applying	Select	32	30	1
	Implement	4	5	0
	Model	6	4	0
Reasoning	Justify	6	2	0
	Generalize	0	0	0
	Analyze	4	0	1
Total number of	SW in different degree of imitation (%)	60 (44 %)	73 (54 %)	3 (2 %)
Total number of	completed students' private work	136		

t6.1 **Table 6** The relationship between the cognitive domains of the students' exercises and the degrees of imitation of the students' private work

The analysis showed that 20 out of 136 items of students' private work were items not 321 assigned by the teacher. Six students in three different lessons worked on extra tasks after they 322 had completed the teacher-assigned exercises. The extra tasks were similar to those assigned 323 SE in nature and belonged to the same topic. For instance, the 14 tasks in worksheets used in 324Lesson L02 were about the slopes of parallel lines. The first eight assigned SEs were to prove a 325pair of parallel lines or four points forming a trapezium. Shown in Helen's private work, the six 326 extra tasks demanded the students to solve similar problems (Fig. 3). In the post-lesson 327 interview, Helen explained why she carried out the six extra tasks after completing the eight 328 assigned SEs. She said, "I worked on the later questions in this worksheet because I know how 329to do it." When the interviewer asked whether she could do all these. Helen said, "Yes, I can." 330 Upon further probing, Helen added, "If I can't, I will ask my neighbor, because she is strong in 331 calculation. So, I ask her most of the time." Helen's case, unfolded how the student might work 332 through the ZPD. At the beginning, her work was mostly imitating the TEs under the teacher's 333 guidance, supplemented with interaction with a more capable peer. Achieving the skills, the 334 student developed her confidence and motivation to do additional exercises on her own. Such 335 phenomenon might happen for students of different degree of fondness for mathematics. 336

5.4 What were the students' perspectives on the instructional practice?

Table 7 summarizes the number of video segments at which the students stopped the video to338say the instructional practice at that moment was important. Forty-two percent of the video339segments were TE, and 55 % of the video segments were SE; therefore, both TE and SE were340important while SE was slightly more important than TE. The teacher's exposition was the341most important when comparing with seatwork and review. In the further breakdown of the342subcategories for exposition, the teacher's demonstration of procedures (D) in TE and SE (16343segments in TE and 15 segments in SE) was important. The fact that the teacher demonstrated344

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Examples
                                  In each of the following, which pairs of the lines is/are parallel?
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Straight Line Points on the line
                                                                                                                                                                                                                                                                                                                                                                                      (1, 3), (-2, 6)
                                                                                                                      L_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \frac{1 \cdot (\cdot)_{2}}{2 \cdot 2} = \frac{2}{\cdot 2}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Lisle
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           11. A(-2, 3), B(3, 5), C(-3, 1) and D(4, k) are 4 points. If AB / CD, find the value of k.

 \sum_{k=1}^{k-1} \sum_{j=1}^{k-1} \sum_{j=1}
                                                                                                                                                                                                                                                                                                                                                                                           (4, 1), (-2, 3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                M2. + M/s
                                                                                                                                                                                                                                                                                                                                                                                      (3, -1), (1, 3)
                                                                                                                                                                                                                                                                                                                                                                                      (5, 7), (-4, 6)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              7-6 - 7
                                                                                                                                                                                                                                                                                                                                                                                           (2, 4), (5, 1)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           13. Prove that the 4 points A(-3, 0), B(-1, -1), C(3, 1) and D(1, 2) form a parallelogram \mathbb{M}_{b, 5}, \frac{p+p_1}{p+p_2}, \frac{1}{2k}, 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               AB: 0-(-1) + 1/2 + -1/2
                                                 The slope of the straight line L_1 is -2. The straight line L_2 passes through the points C(-8, 9) and
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                14. Prove that the 4 points A(-8, 8), B(-13, -2), C(-8, -17) and D(-3, -7) form a parallelogram
                                                                                                                                                                                                                                                                                                                                                                                                                    \frac{1}{2} = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \frac{1}
                                                      D(-1, -5). Prove that L1 // L2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 MAD: 8-(-71)
-8-(-3)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               AB + 1-1-2)
-1-(-R)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   MBC = -11.(-2)
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    3. The slope of the straight line L_1 is 1. The straight line L_2 passes through the points A(5, 4) and
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     = -12
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                                                      B(-2, -3). Prove that L1 // L2.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             MR (-1-3)
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       524
4. The slope of the straight line L_1 is -\frac{3}{2}. The straight line L_2 assess through the points A(2, 3) and M_1 = \frac{3}{2}.
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       5 -3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 M_{2} = \frac{3 - 12}{2 - 1 - 12} = -\frac{q}{6} = -\frac{3}{2}
                                                 B(-4, 12). Prove that L1 // L2.

    Given 4 points A(2, 4), B(3, 5), C(-3, 1) and D(-5, -1). Prove that AB // CD.

<sup>W</sup>R<sup>B</sup> = \frac{1}{3+2} = \frac{1}{3+2} = \frac{1}{1+1} = \frac{1}{3} (C) \frac{L_{1}(1)}{2(2+3)} = \frac{3}{2} = 1
6. Given 4 points P(7, -1), Q(3, -3), R(-4, 2) and S(2, 5). Prove that PQ/I/RS
                                            \begin{array}{c} \text{Prove that } A(0,0), B(2,1), C(1,3), D(2,3) \\ \text{Prove that } A(0,0), C(1,3), C(1,3), D(2,3) \\ \text{Prove that } A(0,0), C(1,3), C(1,3), D(2,3) \\ \text{Prove that } A(0,0), C(1,3), 
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              *RS:蒜·芫·春生
                                  (a) The vertices of a quadrilateral are P(2, 4), Q(3, 5), R(-3, 1) and S(-5, -1). Find the slope of each side of the quadrilateral PQRS. <sup>N</sup>Fa : <sup>1</sup>/<sub>2</sub> = <sup>1</sup>/<sub>1</sub> = <sup>1</sup>/<sub>1</sub> = <sup>1</sup>/<sub>1</sub> = <sup>1</sup>/<sub>2</sub> = <sup>1</sup>/<sub>2</sub>
                                                                                                       side of the quadrilateral PQRS. {}^{\mu}Pa = \frac{1}{2} + \frac{1}{2} +
                                                      (b) What type of quadrilateral is this?
    9. A(2, -2), B(3, 2), C(-3, -3) and D(h, 1) are 4 points. If AB // CD, find the value of h.
                                                                     \frac{h_{0}}{h_{0}} = \frac{2\cdot (1)}{3\cdot 2} \qquad h = \frac{h_{1}(\cdot 2)}{1\cdot (\cdot 2)} \qquad h = 13 \qquad (1 \text{ the value of } 0 - 13).
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Fig. 3 Helen's private work. Tasks 1 to 8 were the assigned SE items, and tasks 9 to 14 were the extra items that Helen worked by her own

detailed procedures in TE and gave detailed instructions prior to students working on SEs had 345 a strong impact on how the students learned. Referring to what the students said in the post-346 lesson interviews, the students appreciated and learned from the teacher's explanation and 347 demonstration. For example, Iris in L04 said, "Before here I didn't quite understand, after 348 listening the teacher's explanation, I started to understand a little bit." Joanne in L09 thought 349"The teacher was doing the example. I don't know how to do it without examples. Example is 350for you to see how to do it." Students believed doing the SEs independently (IW) was 351important for their learning, for example, "Do it yourself, don't know if you don't do it." 352(Janice in L06) and "Because you have to work. You have to work it out for sure after the 353teacher has taught you things." (Gordon in L07). These results showing the strong students' 354appreciation for teacher's demonstration and explanation and working on SEs were consistent 355with the results for other East Asian classrooms reported in the work of Mok and others (Mok 356 et al., 2013) 357

6 Discussion and conclusions

In our study, the students' private work of an eighth-grade mathematics classroom in Hong 359Kong was analyzed. The cognitive domains of TEs and SEs were mainly belonged to the 360 knowing and applying, whereas relatively fewer tasks belonged to the domain of reasoning. In 361the 14 consecutive lessons, the mathematical tasks were arranged as TE-SE cycles of TEs and 362 SEs. Regardless of the cognitive domains of SEs, the methods employed by the students in 363 their private work were mainly the imitation of teacher's methods. This imitation was not only 364simply determined by the students' choice in learning mathematics, but also influenced by the 365 TE-SE arrangement and the similarities of tasks in each TE-SE cycle. In the students' 366 perspectives, the teacher's demonstration was the most important. 367

The finding of high proportion of imitation of teacher's methods was not a surprise because 368Q6 education in Hong Kong and other East Asian regions is often reported to be much influenced 369 by the Confucian philosophy (e.g., Watkins & Biggs, 2001), emphasizing that the teachers are 370 the role models of subject matter (Leung, 2001). Very often, teachers play a significant guiding 371role in the mathematics classrooms (e.g., Mok, 2009; Leung & Park, 2002). The teacher 372 facilitated the role of learning by demonstrating the TEs or giving hints before the SEs. 373 Moreover, with the image of scholar-teacher deeply rooted in Confucian culture, it is very 374likely that the students believed that the methods used in solving the TEs were the best. In East 375376 Asian classrooms, the emphasis on practice is an important feature in the pedagogical philosophy. The traditional Chinese beliefs of "practice makes perfect" (Li, 2006) and 37707 memorization which could come before understanding (Cai & Wang, 2010) may explain for 378 the high percentage of SEs (72 % of total tasks) in the lessons. However, practice is not 379 equivalent to repetition by rote. The variations embedded in the TEs and SEs, in fact, help 380students to experience the object of learning in a deep sense leading to an understanding of the 381 mathematical concepts and procedures from multiple perspectives (e.g., Gu, Huang & Marton, 382 2004; Huang & Leung, 2004; Wong, 2006). Huang and Leung (2004) studied the mathemat-383 ical tasks in Shanghai and Hong Kong classrooms and found that the tasks might serve the 384purpose of consolidation and help developing proficiency and understanding of the topic. 385

Another factor shaping the students' learning in mathematics is the cognitive domains of 386 tasks. Examining the cognitive domains of tasks in the SEs across 14 consecutive lessons 387 showed that the majority was knowing and applying tasks, with relatively lower percentage of 388

				, MACL							
t7.1	Table 7 The students' perspectives of the instructional practice in post-lesson student interviews	' perspectives of the	instructional ₁	practice in post-le	sson student intervi	iews					
t7.2		Teacher's examples	S		Students' exercises	s		Off tasks			Total
t7.3 t7.4		Exposition Seatwork Review EC/D/NK/GI/RE IW/GW/M PK/SP/IF/GA	Seatwork IW/GW/M	Review PK/SP/IF/GA	Exposition EC/D/NK/GI/RE	Seatwork IW/GW/M	Review PK/SP/IF/GA	Exposition Seatwork Review Exposition Seatwork Review EC/D/NK/GI/RE IW/GW/M PK/SP/IF/GA EC/D/NK/GI/RE IW/GW/M PK/SP/IF/GA	Seatwork IW/GW/M	Review PK/SP/IF/GA	
t7.5	No. of segments	4/16/7/0/0	1/0/1	0/1/2/0	2/15/3/0/0	9/4/0	1/5/3/0	0/0/0/2/0	0/0/0	0/0/0/0	
t7.6	Total no. of segments 32/76 (42 %)	32/76 (42 %)			42/76 (55 %)			2 (3 %)			76
	<i>EC</i> explains/explains clearly, <i>D</i> demonstrates a procedure: "teaches the method" or shows using manipulative a concept/relationship, <i>NK</i> introduces new knowledge, <i>GI</i> gives instructions (assigning homework/how work should be done/when work should be handed in for grading, etc.), <i>RE</i> uses real-life examples during instruction, <i>IW</i> students working individually/conving notes <i>GW</i> students working in granus/oronal discussion <i>M</i> material used as not of instructions (worksheet or any other mint resource) <i>PK</i> eviews mint	clearly, D demonstra homework/how wor otes GW students y	ates a procedu k should be c vorking in on	ure: ''teaches the lone/when work	method" or shows should be handed in scion M material	s using manip n for grading, used as part of	ulative a concep etc.), <i>RE</i> uses re of instruction (w	trelationship, NK i sal-life examples du orksheet or any oth	ntroduces new ring instruction	knowledge, <i>Gl</i> 1, <i>IW</i> students w	gives orking

knowledge, SP uses student's presentation or work to give feedback for in class work or homework, IF gives feedback to individuals during lesson, GA gives feedback through grading of written assignments

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reasoning tasks. One of the possible reasons for the phenomenon may be due to the nature of 389 the topics. Another possible reason may be the teacher's expectation of students' ability and 390pedagogical style. Although the opportunity for practice in reasoning was relatively fewer, the 391proportion of reasoning items in SE was greater than that in TE and also sufficient practice on 392all three domains was guaranteed by the high amount of exercises. The findings were 393 consistent with the TIMSS 2011 results in which Hong Kong students (eighth grade) got the 394mathematics high average scores of 591, 587, and 580 for in knowing, applying, and reasoning 395 domains, respectively (Mullis et al., 2012, p. 150). 396

Seventy-one percentage of students' private work (136 out of 190 items of students' private 397 work, excluding the copies of TEs) were completely recorded, showing students' motivation of 398 engaging themselves in the tasks. This might be due to the belief in effort and illustrated the 399 Chinese dictum "diligence could remedy mediocrity." A high expectation of parents and the 400 competitiveness of examination cultures strongly influence students' belief in working hard as the 401 route of success. Students' conceptions of mathematics and mathematics learning are obviously 402 shaped by their experience of learning (Bishop, 1991). In the student interviews, students showed 403appreciation for how they learned from the teacher's exposition. They believed that the imitation 404 of the teacher's methods with correct answers in their private work was the key of success in 405learning mathematics. So, they focused on the methods or procedures in solving the tasks. 406 Although students in East Asian classrooms might have interpreted as passive at the surface, 407 they might have been active in their minds (Biggs, 1998). In our study, six students showed their 408motivation to work on extra tasks; their private work and the students' post-lesson interviews 409showed how the teacher's demonstration in the public domain of the lesson might possibly be 410 internalized in the students' learning outcomes. While some celebrated the mastering of skills, in 411 some cases, the partial imitation instances indicated the pitfall. The missing steps might not hinder 412the students from getting the answers, but the students might lose the chance in developing the 413mathematical connections. Putting an overemphasis on the teacher's methods as the role models, 414 the motivation for exploring new methods might be lost. To conclude, imitation that might be seen 415often in East Asian classrooms does not necessarily imply mechanical learning. Suitable use of 416 imitative work, the students might possibly extend their mental capacity under the teacher's 417 guidance and peer influence in the zone of proximal development (Vygotsky, 1978), developing a 418confidence and motivation in the work and possibly a "deep" approach that brings about 419understanding beyond memorization (Biggs, 1998). 420

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AUTHOR QUERIES

AUTHOR PLEASE ANSWER ALL QUERIES.

- Q1. Author names and affiliation are taken from the manuscript draft. Please check if captured correctly.
- Q2. The sentence "Given that the vertices of \triangle ABC are A (-3, 2)..." has been edited for clarity. Please check that the intended meaning was retained.
- Q3. The sentence "Given that the vertices of $\triangle ABC$ are A (-1, 4), B (9, -11)..." has been edited for clarity. Please check that the intended meaning was retained.
- Q4. The sentence "Using Lesson L08 as an example..." has been edited for clarity. Please check that the intended meaning was retained.
- Q5. The sentence "That is, in L09, the teacher used…" has been edited for clarity. Please check that the intended meaning was retained.
- Q6. The sentence "The finding of high proportion..." has been edited for clarity. Please check that the intended meaning was retained.
- Q7. The sentence "The traditional Chinese beliefs of "practice makes perfect"..." has been edited for clarity. Please check that the intended meaning was retained.