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Author(s)	Chuai, J; Li, VOK
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The Temporal Value of Information to Network Protocols – An Analytical Framework

Jie Chuai and Victor O.K. Li
Department of Electrical and Electronic Engineering,
The University of Hong Kong, Pokfulam, Hong Kong, China
Email:{jchuai, vli}@eee.hku.hk

Abstract—Network protocol performance is closely related to the available information about the network state. However, acquiring such information expends network bandwidth resource. Thus a trade-off exists between the amount of information collected about the network state, and the improved protocol performance due to this information. A framework has been developed to study the optimal trade-off between the amount of collected information and network performance. However, the effect of information delay is not considered. In this paper, we extend the framework to study the impact of information delay on the value of network state information to network protocols, and based on which optimal periodic information update policies could be obtained. The framework is illustrated by an example of multiuser scheduling, and observations about the impact of information delay on network protocols are obtained.

I. INTRODUCTION

It is well known that communication protocols require network state information. Intuitively, with more network information collected, better network performance could be achieved. However, the information collection process occupies valuable bandwidth resource, which results in less bandwidth for data transmissions. Therefore, there is a trade-off between the resource used for information collection and the performance improvement due to this knowledge. In [1], we developed a framework to analyze the relationship between the amount of collected information and network performance, based on which the optimal bandwidth resource allocation scheme could be designed. The work in [1] assumes instantaneous acquisition of network state information, i.e., there is no delay between the time of information collection and the time of information utilization. In many scenarios, however, this assumption is not true: first, it takes time for the collected information to be transported to the place where it is to be utilized; in addition, information is generally not updated instantaneously. As the network state changes over time, this delay makes the information outdated and inaccurate, rendering the information less valuable and probably resulting in degradation of network performance. Thus, it is important to understand how the time delay affects the value of collected network information and hence the optimal bandwidth resource allocation scheme. Specifically, we wish to discuss the following questions: Q1: what is the amount of required information to achieve certain network performance when the information is subject to delay? Q2: assume time-varying network states, how often should the information be updated, and how much information should be collected each time to optimize the overall bandwidth efficiency?

In this paper, we extend the general framework developed in [1] to analyze the above two questions. By applying it to a multiuser scheduling problem, we demonstrate how the delay affects the value of the collected channel state information and the optimal resource allocation schemes. The results provide some general guidelines on the design of information collection schemes in time-varying networks.

The paper is organized as follows. Section II reviews the related work. Section III introduces the framework used to solve this problem. Section IV gives an example as illustration and Section V concludes the paper with suggestions for future research.

II. LITERATURE REVIEW

The impact of outdated information on network performance has been addressed in different research areas. Work has been done to investigate the impact of delayed feedback Channel State Information (CSI) on the capacity of various wireless channels, including finite-state Markov channel [2], multiuser MIMO system [3] and finite-state Multiple Access Channels [4]. The authors in [5] showed that completely outdated information is still useful to provide multiplexing gain for a MIMO broadcast channel. There are also discussions on how delayed information affects the performance of network-wide protocols. For example, [6] studied the maximum throughput of opportunistic multiuser scheduling with randomly delayed Automatic Repeat reQuest (ARQ) feedback; [7] showed how link state information update frequency affects the bandwidth blocking probability of multi-path routing protocols; [8] analyzed the relationship between the channel/queue information delay and the throughput region of routing and scheduling in wireless network.

The above work focuses on analyzing the impact of information delay on network performance. No partial information collection is considered. Therefore, there is no discussion on how the network performance changes with the quantity of collected information when there is information delay. Furthermore, the overhead of information collection is not considered. In our work, we wish to jointly consider the impacts of partial information collection and information delay on network performance, and give the optimal resource allocation scheme for information collection and data transmission.

Finally, different from the above listed work, we will use a general approach to tackle these problems as in [1].

III. THE FRAMEWORK

We introduce the analytical framework in this section. Section III-A presents the framework to analyze the minimum required information to achieve certain network performance (the performance-rate relationship) with a given information delay d. Section III-B studies the optimal performance-rate relationship under a periodic information update scheme.

A. Performance-rate Relationship with Fixed Delay

Assume the network state (e.g., channel states, node traffic states, etc.) at time t is a random variable X_t, and the collected information at time t is Y_t, where

$$Y_t = f(X_t) \tag{1}$$

The collected information Y_t is transmitted to a decision maker (e.g. a network controller or a single node that needs to make a decision), and a protocol decision Z_{t+d} (e.g., scheduling/routing decisions) is made based on Y_t at time $t+d$,

$$Z_{t+d} = h_d(Y_t) \tag{2}$$

The expected network performance given functions $f(\cdot)$ and $h_d(\cdot)$ is denoted by $G(f, h_d)$. Let $g(X_{t+d}, Z_{t+d})$ be the network performance (e.g., throughput, packet delivery ratio) given the true network state X_{t+d} and the protocol decision Z_{t+d} at time $t+d$. The expected network performance is then

$$G(f, h_d) = E\left[g\left(X_{t+d}, Z_{t+d}\right)\right] \tag{3}$$

With Bayes' decision rule, it is easy to see that the optimal decision function given the collected information y_t is

$$\begin{aligned} z_{t+d} &= \arg\max_z E\left[g(X_{t+d}, z)|y_t\right] \\ &= \arg\max_z \sum_{x_{t+d}} \sum_{x_t} p(x_t|y_t)p(x_{t+d}|x_t)g(x_{t+d}, z) \end{aligned} \tag{4}$$

where $p(x_{t+d}|x_t)$ is the probability that the network state transits from x_t at time t to x_{t+d} at time $t+d$.

As in [1], we use the entropy $H(Y_t)$ of random variable Y_t as a measure of the amount of collected information, and $H(Y_t)$ is defined as

$$H(Y_t) = -\sum_{y_t} p(y_t) \log_2(y_t) \tag{5}$$

where $p(y_t)$ is the probability that Y_t takes the value of y_t.

Thus, to achieve a given performance guarantee G at time $t + d$, the minimum amount of information required to be collected at time t is

$$R(G) = \min_{f, h_d} H(Y_t) \\ s.t. \quad G(f, h_d) \geq G \tag{6}$$

Equation (6) can also be transformed into an unconstrained optimization problem with $\lambda \geq 0$:

$$\min_{f, h_d} \lambda H(Y_t) - G(f, h_d) \tag{7}$$

By changing the value of λ, we can obtain different trade-offs of information collection quantity and the achieved network performance.

Problem (7) is similar to the optimal entropy-coded quantizer design problem [9], which could be solved by an iterative approach that resembles the generalized Lloyd algorithm [9].

B. Periodic Information Update

We now consider the case where network state information is updated periodically. Assume the information is updated every T time units, and the information Y_t collected at time t is used during the period $[t, t+T]$. Several protocol decisions are made at a series of distinct time instants within this duration, e.g., at time $t + d_1$, $t + d_2$, $\cdots$, $t + d_m$, where $t+d_i \in [t, t+T]$, and the decision functions are $h_{d_1}(\cdot)$, $h_{d_2}(\cdot)$, $\cdots$, $h_{d_m}(\cdot)$, respectively. If all the decisions are weighted equally, the average performance over this period is

$$G(f, \vec{h}) = \frac{1}{m} \sum_{i=1}^{m} G(f, h_{d_i}) \tag{8}$$

where $\vec{h} = (h_{d_1}, h_{d_2}, \cdots, h_{d_m})$.

In some cases different decisions may not be weighted equally. For example, the outcomes of some decisions last for a longer period, thus they have higher impacts on the overall network performance. We can assign different weights to the performance functions to reflect those unequal impacts,

$$G(f, \vec{h}) = \sum_{i=1}^{m} w_i G(f, h_{d_i}) \tag{9}$$

where w_i is the weight of the performance at time $t + d_i$ and we assume $w_1 + w_2 + ... + w_m = 1$.

We use the average amount of information collected per time unit $H(Y_t)/T$ as a measure of the overhead of information collection. The minimum information required per time unit to achieve a given average network performance is given by

$$R(G) = \min_{f, \vec{h}, T} \frac{H(Y_t)}{T} \\ s.t. \quad G(f, \vec{h}) \geq G \tag{10}$$

Similarly, the optimal relationship between the average information per time unit and the average network performance can be found by the following problem ($\lambda \geq 0$)

$$\min_{f, \vec{h}, T} \lambda \frac{H(Y_t)}{T} - G(f, \vec{h}) \tag{11}$$

IV. EXAMPLE

In this section, we use a multiuser scheduling problem to illustrate the use of our framework. We first introduce the scenario in Section IV-A, and then use the framework to analyze the problem in Section IV-B, and finally present the results in Section IV-C with discussions.

A. Scenario

Suppose a controller has data to transmit to N users. Assume time is slotted and the length of each slot is L bit time ($L \geq N$), where 1 bit time is the time taken to transmit 1 data bit. The channel between the controller and an arbitrary user is modeled as a stationary two-state Markov chain, i.e., the channel state at any time is one of the two states $\{Good, Bad\}$. This model is suitable to represent a channel with burst noise. The channel state transits at the beginning of each slot. We assume the transition matrix of the two-state Markov chain is symmetric, i.e., $Pr(Good|Good) = Pr(Bad|Bad) = p$ and $Pr(Bad|Good) = Pr(Good|Bad) = 1-p$. Let $p \geq 0.5$, that is, the states of the channel are positively correlated over time. It is easy to see that the stationary probability distribution of the channel state is $Pr(Good) = Pr(Bad) = 0.5$. We assume that the channels between the controller and the users are independent Markov chains with identical statistics.

We consider the case where the controller can transmit data to only one user each time. Suppose at the beginning of Slot t_0 state transitions occur. Then the controller collects channel state information from the users (e.g. by probing the channels) and chooses one of them to transmit data based on the collected channel state information at t_0. We use throughput to measure the network performance[1]. Assume the data throughput is 1 data bit per bit time if the channel is Good, and is 0 data bit per bit time otherwise. The data transmission session ends at the beginning of Slot $t_0 + T$, when the channel state information is updated again and a new user is selected for data transmission based on the updated information at time $t_0 + T$.

B. Analysis

Denote the true network state at the beginning of Slot t_0 by $\vec{X} = (X_1, X_2, \cdots, X_N)$, where X_i is the channel state of User i in Slot t_0, and $X_i = 0$ and 1 represent the events that the channel is in *Bad* and *Good* states, respectively. Channel state information is collected from the users independently. The collected information from User i at time t_0 is denoted by Y_i, and $Y_i = f_i(X_i)$. Since X_i only has two states, there are only two options of $f_i(\cdot)$: (1) $Y_i = X_i$ (complete information is collected from User i); (2) $Y_i = \_\_$ (no information is collected from User i). We use $\vec{Y} = (Y_1, Y_2, \cdots, Y_N)$ to represent all the collected information.

Since the random variables Y_i are mutually independent, the total quantity of collected information is $H(\vec{Y}) = \sum_i H(Y_i)$. When $Y_i = X_i$, $H(Y_i) = H(X_i) = 1$ bit since $Pr(X_i = 1) = Pr(X_i = 0) = 0.5$; and $H(Y_i) = 0$ bit if $Y_i = \_\_$. Therefore, when complete channel state information of K users are collected, the total amount of information updated at Slot t_0 is K bits, and the average amount of collected information per time slot is

$$R_{ave} = \frac{K}{T} \tag{12}$$

[1]We use *throughput* in this paper to measure the performance of the duration for data transmission, i.e., the overhead of information collection is not considered.

The user selection scheme with K bits channel state information collected is as follows:
1) If at least one of the K channels is in Good state, select one Good channel to transmit data;
2) If none of the K channels are Good, select a user randomly from the remaining $N - K$ users (if $N > K$) or from the N users (if $N = K$) to transmit data.

The above user selection scheme maximizes the expected throughput of Slot $t_0 + d$ (for any finite $d \geq 0$) given information $\vec{Y}$. Since we assume positively correlated Markov channels, a channel known to be Good is more likely to be Good after d slots than an unknown channel; similarly, a channel with unknown state is more likely to be Good after d slots than a channel known to be Bad. The above user selection scheme always chooses a user that has the highest conditional probability to be Good given $\vec{Y}$, therefore it is optimal.

Performance at Slot $t_0 + d$: First we discuss the temporal change of the value of information, i.e., given the amount of collected information, how the system performance changes as time passes.

Assume channel state information is collected from K users at Slot t_0. The throughput at Slot $t_0 + d$ is

$$G_d = \alpha \pi_{g \to g}^d + (1-\alpha)\pi_g^d \tag{13}$$

where $\alpha = 1 - 0.5^K$ is the probability that at least one of the K channels is in Good state at Slot t_0; $\pi_{g \to g}^d$ is the conditional probability that a channel is in Good state at Slot $t_0 + d$ given that it is Good at Slot t_0; and π_g^d is the probability that the selected channel is Good at Slot $t_0 + d$ when none of the K channels are Good at Slot t_0.

We first consider the case $N > K$. We have $\pi_g^d = 0.5$. Based on the transition matrix of the Markov chain, we can calculate the value of $\pi_{g \to g}^d$ as follows (for $d \geq 1$):

$$\pi_{g \to g}^d = (1 - \pi_{g \to g}^{d-1})(1-p) + \pi_{g \to g}^{d-1}p$$
$$= 1 - p + (2p-1)\pi_{g \to g}^{d-1} \tag{14}$$

By using the above recurrence relation, we can get

$$\pi_{g \to g}^d = (1-p)\sum_{i=0}^{d-1}(2p-1)^i + (2p-1)^d \pi_{g \to g}^0$$
$$= 0.5 + 0.5(2p-1)^d \tag{15}$$

where we use the formula for the sum of geometric series and the fact that $\pi_{g \to g}^0 = 1$.

Therefore, we have the value of G_d as

$$G_d = 0.5 + 0.5(1 - 0.5^K)(2p-1)^d \tag{16}$$

When $N = K$, $\pi_g^d = \pi_{b \to g}^d$ where $\pi_{b \to g}^d$ is the conditional probability that a channel is in Good state at Slot $t_0 + d$ given that it is Bad at Slot t_0. From Equation (15) and the symmetry of the Markov chain, we have

$$\pi_{b \to g}^d = 0.5 - 0.5(2p-1)^d \tag{17}$$

and we can obtain the value of G_d when $N = K$
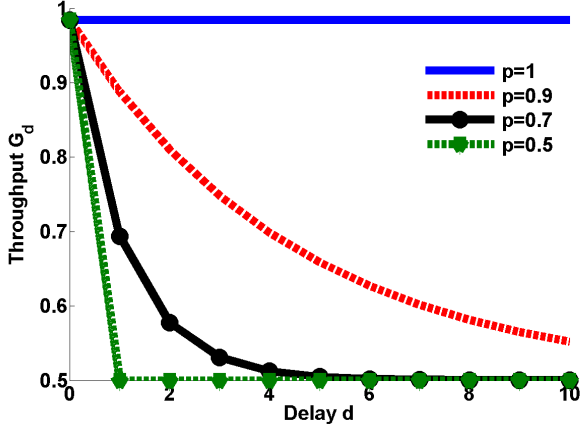
$$G_d = 0.5 + 0.5(1 - 0.5^{K-1})(2p-1)^d \tag{18}$$

Fig. 1: The network throughput G_d as a function of d and p with $K = 5$ and $N = 10$

Fig. 2: The Relationship between R_{ave} and G_{ave} ($L = 15$, $N = 10$, $p = 0.85$)

Note that Equations (16) and (18) apply to all $p \in [0.5, 1]$ and $d \geq 0$.

Average Throughput of Data Transmission: We next calculate the average throughput of the data transmission duration. Since the information collection process costs K bit time, the data transmission duration of Slot t_0 is $L - K$ bit time; all the remaining $L(T-1)$ bit time within this period of T slots is used for data transmission. The average throughput of the data transmission duration is

$$G_{ave} = \frac{L \sum_{d=0}^{T-1} G_d - K G_0}{TL - K} \qquad (19)$$

which is a weighted average of the values of G_d with different delay d.

The optimal trade-off between R_{ave} and G_{ave} can be obtained by varying the value of λ ($\lambda \geq 0$) in the following problem

$$\min_{K,T} \lambda R_{ave} - G_{ave} \qquad (20)$$

Net Data Rate: The net data rate, denoted by R_e, is calculated as

$$R_e = \frac{L \sum_{d=0}^{T-1} G_d - K G_0}{TL} \qquad (21)$$

We can obtain the optimal resource allocation scheme, i.e., the optimal values of K and T, by maximizing R_e in the above equation.

C. Results

1) Temporal Decline of the Value of Information: We first plot the temporal decline of the network throughput G_d as a function of d and the state transition probability p in Figure 1.

As expected, the throughput declines as the delay increases when $p < 1$. The decline rate is larger with a smaller value of p since the channel state changes more rapidly in this case. When $p = 0.5$, the channel states of consecutive slots are independent, and throughput G_d drops to 0.5 for $d \geq 1$
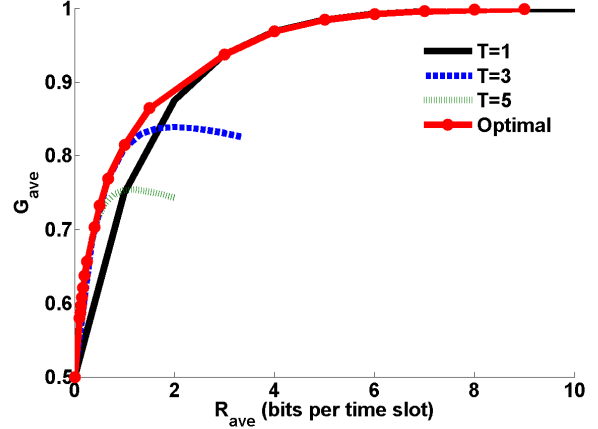
indicating that the collected information becomes completely useless when it is independent from the channel states[2].

When $p = 1$, the channel stays in the same state forever, and the throughput remains constant.

2) The Relationship between R_{ave} and G_{ave}: Figure 2 illustrates the relationship between R_{ave} and G_{ave}. The red solid line with dotted markers represents the optimal average performance G_{ave} that could be achieved given information R_{ave} by considering all possible values of T and K. This line is obtained by finding the upper convex hull of all the G_{ave}-R_{ave} curves with different values of T. Figure 2 also plots the G_{ave}-R_{ave} curves with $T = 1, 3$ and 5.

In general, the optimal average throughput G_{ave} increases as more information is collected. However, when the value of T is fixed (e.g., $T = 3$), the average throughput can decrease if too much information is collected. This is counter to our intuition that more information can result in better network performance, which is shown to be true when no information delay is considered [1]. The drop of network performance here is not due to the resource occupied by information collection, since G_{ave} represents the throughput of the data transmission duration itself without considering the overhead. The value of G_{ave} drops here because the information collection process postpones the time of information utilization, which increases the information delay, thus reducing the information's value to network protocols.

3) The Optimal Resource Allocation Scheme: We discuss the optimal resource allocation schemes in this part. In Tables I to III, we give the optimal values of T and K under different network conditions.

In Table I, with N and L fixed and the transition probability p increasing, both of T^* and K^* increases. This means, when

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[2]In [5], the authors show that completely stale information can be useful. Their model is different from ours, since the network performance in their case is not only a function of the current state and the protocol decision (as in our model), but also depends on the past channel states.

TABLE I: Optimal Resource Allocation ($N = 10$, $L = 15$)

	T^*	K^*
$p = 0.75$	1	2
$p = 0.85$	2	3
$p = 0.95$	4	4

TABLE II: Optimal Resource Allocation ($N = 5$, $p = 0.8$)

	T^*	K^*
$L = 5$	3	2
$L = 12$	2	3
$L = 18$	1	3
$L = 50$	1	4

TABLE III: Optimal Resource Allocation ($L = 20$, $p = 0.85$)

	T^*	K^*
$N = 3$	1	2
$N = 5$	2	3
$N = 20$	2	3

the network states change more rapidly (smaller p), the optimal information collection scheme is to update less information each time since the information will become useless quickly, and update more frequently to keep track of the network states. When the network conditions become more stable, the benefit of collecting more information can last for a longer period, thus there is no need to update information frequently.

Table II shows that more information should be collected (i.e., increase the update frequency and the amount of information updated each time) when the duration of the slot becomes longer. This is because the relative overhead of collecting information decreases as L grows, and the benefit of collecting information (i.e., increase data throughput) is more significant.

In Table III, we keep the values of L and p fixed and increase the network size. The results show that more information is collected each time as the network size grows. This is because, when the network is small, there are only limited number of users available for the controller to collect information to locate a good channel; as the network size increases, more channels exist in the network, which allows the controller to search more users and hence find a good channel with higher probability. This increase of K^*, however, saturates quickly as N grows, reflecting that the network size no longer constrains the controller's search scope. As shown in Table III, when collecting more information each time, the controller may decrease the update frequency, as long as the increased throughput can compensate the loss due to non-instantaneous update.

V. CONCLUSIONS

In this paper, we discussed the impact of information delay on the relationship between the amount of collected state information and network performance, and hence its effect on the optimal resource allocation scheme. We extended the framework in [1] to calculate the optimal performance-rate relationship when there is a fixed information delay. By assuming a periodic information update policy, the framework can give the optimal trade-off between the average information collected per time unit and the average network performance over the update period, based on which the optimal resource allocation scheme (i.e., the optimal information update frequency and the amount of information updated each time) could be derived.

The framework is illustrated with an example of multiuser scheduling, and some observations about the impact of information delay on network protocols are obtained. First, when there is information delay, more information collection may not result in higher network performance given a fixed information update period. Second, for a network with rapidly changing states, information should be updated frequently while only a small amount of information should be collected each time. Finally, as the slot length increases, more information should be collected.

In this work, we assume the information updates at different times are independent; that is, the controller does not have memory of the collected information in the past. However, since the channel states are temporally correlated, it would be more efficient if the controller has memory and decides the information update plan based on the past collected information. We would like to account for memory effects in the framework in our future work.

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