Computation of Probabilistic Critical Centers and Reliability Indices of MSW Landfill Slopes using Spencer Method of Slices

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ABSTRACT: The shear strength properties of Municipal Solid Waste (MSW) are of special importance when evaluating the stability of landfill slopes. Geoenvironmental engineers are well aware of the existence of many sources of uncertainties associated with shear strength parameters of MSW due to various reasons. The significant uncertainties associated with the shear strength and shear stresses render deterministic modeling potentially misleading. The traditional engineering approaches like method of slices used for evaluating MSW slopes are frequently questionable as they do not adequately account for uncertainties included in analytical modeling and natural variability. In order to quantify the slope stability precisely by taking into account the variability, the Reliability Based Design Optimization (RBDO) framework is presented. The mean and standard deviations associated with unit weight, cohesion and angle of internal friction of the MSW are taken into account in the probabilistic optimization. Reliability analysis is performed using first order reliability method (FORM). A limit state function is formulated against sliding slope failure using Spencer method of slices. The influence of coefficients of variation (COV) of stability number and friction angle on critical center coordinates and reliability index is presented in the form of charts.

INTRODUCTION

It is estimated that the MSW generation in India would reach nearly 18 billion tones by the year 2020 (Jain et al., 2014). The exponential growth in the generation of MSW is critically problematic in disposing off the waste. On the other hand, most of the landfills in major metropolitan cities of India have reached their maximum limits in accommodating waste. Due the scarcity of land and capital, the heights of existing landfills have been exceeding the designed limits to accommodate more waste. This results in steep slopes and thereby increasing the potential for slope failure. Unlike homogeneous slopes, MSW landfill slopes involve a wide a range of variability and uncertainty in the shear strength parameters.

Stability analysis of slopes without considering the possible range of variability leads to deficient designs. Improperly designed slopes can pose severe physical and environmental threats (Reddy and Basha, 2014). MSW slopes may fail pre or post closure of the landfill. Improper compaction and poor interface frictional resistance lead to pre closure failures while the leachate or rainfall induced seepage forces triggers post closure failures. Recent and past landfill slope failures evidence the significance of considering the variability associated with shear strength parameters in the slope design. Koerner and Soong (2000) reported a case study on the slope failures and triggering factors of ten major lined and unlined landfills. Merry et al., (2005), Blight (2008and Reddy and Basha (2014) discussed several failure mechanisms of major engineered landfills and MSW dumps throughout the world. The stability of an MSW slope is mainly influenced by the unit weight (γ), cohesion (c), internal friction angle (ϕ) of the solid waste and interface friction angle (δ) for lined slopes. The stability analysis is to find out whether the resisting forces are substantially higher than the driving forces for a potential failure.

Use of conventional methods which do not account for the variability associated with the material properties leads to potential slope failures. As suggested by Bowels (1996), a high factor of safety needs to be used to ensure the long term stability of slopes in case of uncertainties. However, it is not logical to use the same value of the factor of safety without considering the degree of variability involved (Duncan, 2000). The shear strength parameters of MSW should be given due consideration as there is an evidence of the Cincinnati landfill slope failure in USA. Experimental evaluation and back analysis of the failed waste slopes evidenced the key role of shear strength parameters in slope failure (Eid et al., 2000). Babu et al. (2014) reported the variability associated with the geotechnical parameters of MSW and its influence on the stability of landfill slopes.

Studies Pertaining to Reliability Analysis of Soil Slopes

Hassan & Wolff (1999) proposed an algorithm to search for the minimum reliability index for soil slopes. They investigated the similarities and differences of the surface of minimum factor of safety (FS) and the surface of minimum reliability index (β). Malkawi and Abdulla (2000) used the first-order second-moment method (FOSM) and Monte Carlo simulation (MCS) method of reliability analysis of the soil slopes. Low (2003) implemented Spencer's method of slices for probabilistic approach to slope stability. Xue and Gavin (2007) reported a new approach to calculate the minimum reliability index considering the uncertainty associated with the soil properties. Bhattacharya and Dey (2010) coupled FORM with the ordinary method of slices for evaluation of factor of safety and reliability index.

OBJECTIVES OF THE CURRENT STUDY

Most of the probabilistic studies reported in the literature discussed on soil slopes. Though a few studies reported regarding the probabilistic analysis of MSW slopes, there is a deficiency in understanding the mechanism behind the MSW slope failures. The present work is focused on slope reliability analysis to locate the critical slip surfaces and the corresponding reliability indices by taking into account the variability associated with the shear strength parameters. Spencer's (1967) method of slices has been chosen in this study to perform slope stability analysis. This method considers both the interslice forces, assumes a constant interslice force function and satisfies both force and moment equilibrium simultaneously and computes factor of safety.

SPENCER METHOD OF SLICES

The MSW slope geometry with method of slices is shown in Fig. 1(a). The slice considered to write force and moment equilibrium equations is shown in Fig. 1(b). Considering the force (f) and moment (m) equilibrium of the whole soil mass and solving for the factor of safety, the following two equations can be derived:

$$FS_{Spencer\_f} = \frac{\sum_{i=1}^{n} T_{i,\lim}}{\sum_{i=1}^{n} T_i} = \frac{\sum_{i=1}^{n} (cdl_i + N_i \tan\phi)\times\cos\theta_i}{\sum_{i=1}^{n} N_i \sin\theta_i} \tag{1}$$

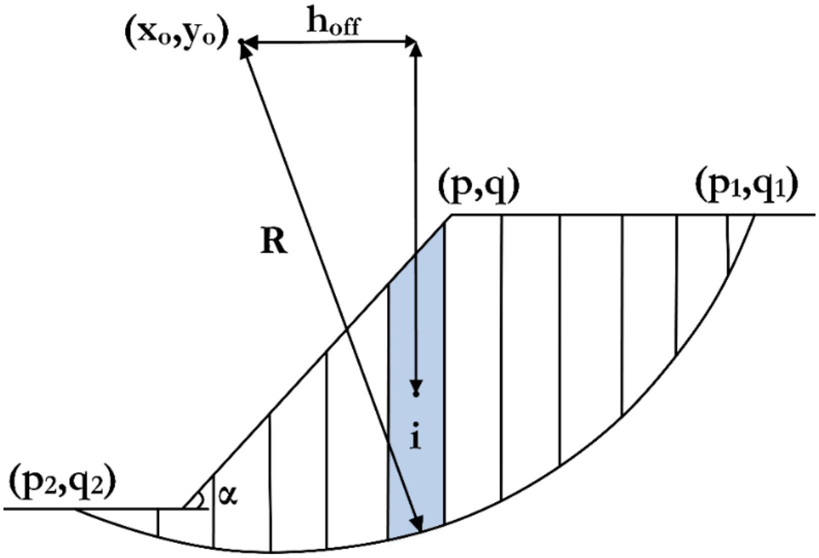

FIG. 1(a). Geometry showing the parameters used and moment arms for circular slip surface.

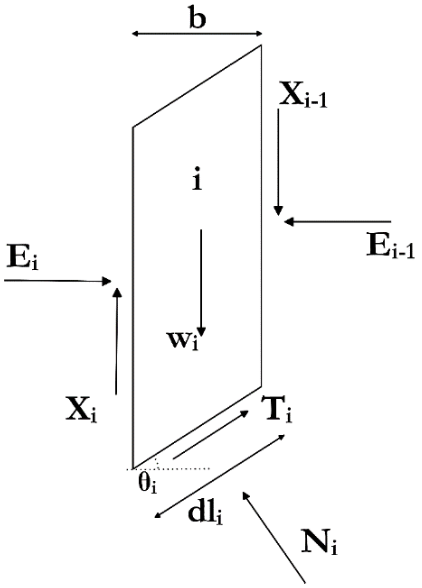

FIG. 1(b). Forces acting on ith slice.

$$FS_{Spencer\_m} = \frac{\sum_{i=1}^{n} (cdl_i + N_i \tan\phi)R}{\sum_{i=1}^{n} w_i h_{off_i}} \tag{2}$$

where , N_i = the total normal force on the base of a slice, T_i = the shear force mobilized on the base of each slice, c = cohesion, δ = internal frictional angle, E_i = the horizontal interslice normal forces, X_i = the vertical interslice shear forces, h_{off} = the horizontal distance from the centroid of each slice to the center of rotation ($= \mathrm{R}\sin\theta_i$), dl_i = length of slice along the base, θ_i = the angle between the tangent to the center of the base of each slice and the horizontal, w_i = weight of a slice, R = radius of the slip circle.

Now considering the equilibrium of an individual slice, the magnitude of the shear force mobilized at the base of a slice, T_i can be written in terms of the Mohr-Coulomb failure criterion as

$$T_i = \tau_i dl = \frac{\tau_{fi} dl_i}{FS_{Spencer}} = \frac{(c + \sigma \tan\phi) dl_i}{FS_{Spencer}} \tag{3}$$

where τ_i and τ_{fi} are the shear stress and shear stress at failure for the ith slice.

also, substituting, $\sigma = N_i / dl$ in Eq. (3)

Which gives

$$T_i = \frac{cdl_i + N_i dl_i \tan\phi}{FS_{Spencer}} \tag{4}$$

Considering the force equilibrium in the vertical direction, we get

$$w_i + X_i - X_{i-1} - N_i \cos\theta_i - T_i \sin\theta_i = 0 \tag{5}$$

Substituting the value of T_i in the above equation and solving for N_i, we get,

$$N_i = \frac{w_i + X_i - X_{i-1} - \dfrac{cdl_i \sin\theta_i}{FS_{Spencer}}}{\cos\theta_i + \dfrac{\tan\phi \sin\theta_i}{FS_{Spencer}}} \tag{6}$$

Now, consider the equilibrium of the slice in the horizontal direction, we get

$$E_{i-1} - E_i - N_i \sin\theta_i + T_i \cos\theta_i = 0 \tag{7}$$

$$E_i = E_{i-1} - N_i \sin\theta_i + T_i \cos\theta_i \tag{8}$$

The inter-slice forces within the sliding mass is defined through a function $f(x_i)$ and a scalar coefficient λ as:

$$X_i / E_i = \lambda f(x_i) \tag{9}$$

In the above equation x_i is the abscissa of the ith slice of the slope, $f(x_i) = \tan\delta$ describes the variation of the inter-slice shear (X_i) and normal (E_i) forces across the slope; the coefficient 'λ' represents the percentage of $f(x_i)$ used in the solution. Spencer (1967) assumed $f(x_i)$ is equal to 1 and then λ is equal to $\tan\delta$, where, δ = angle of the resultant interslice force with the horizontal. All the above equations are then collectively used to determine the factor of safety $FS_{Spencer}$ which satisfies both the moment and force equilibrium simultaneously. In order to solve for Spencer method, we initially set, $X_i - X_{i-1} = 0$. The equations of $FS_{f\_Spencer}$ and $FS_{m\_Spencer}$ are then calculated to obtain a first set. Also for the first slice, X_i is equal to 0. Then a trial value of 'δ' to obtain new estimates for the values of X_i and E_i. Having these values in hand $FS_{f\_Spencer}$ and $FS_{m\_Spencer}$ are recalculated to obtain the new estimates of the factors of safety. This computation is then repeated until the values of the interslice force function converge. The values of $FS_{f\_Spencer}$ and $FS_{m\_Spencer}$ computed in the above step are not necessarily equal. If $FS_{f\_Spencer} \neq FS_{m\_Spencer}$ means that the moment and force equilibrium are not satisfied simultaneously. Hence the computation must be repeated with various trial values of 'δ' until $FS_{f\_Spencer} = FS_{m\_Spencer}$. When the convergence is obtained, that value is then taken as the factor of safety $FS_{Spencer} = FS_{f\_Spencer} = FS_{m\_Spencer}$ for the slope. The performance function of MSW slope against sliding failure can be expressed as.

$$g\left(x\right)=FS_{Spencer}-1 \tag{10}$$

The performance function g(x) ≤ 0, indicates the slope failure and g(x) > 0 indicates the stable slope. Now the design point in the standard normal space (u_k) can be expressed as

$$u_k=-\beta_{Spencer}\frac{\sum_{i=1}^{n}\frac{\partial g}{\partial x_k}\left(\sigma_i\right)}{\sqrt{\sum_{j=1}^{n}\left\{\sum_{i=1}^{n}\frac{\partial g}{\partial x_i}\left(\sigma_i\right)\right\}^2}}=-\alpha_k\beta_{Spencer} \tag{11}$$

where k = 1,2… to n. Rearranging the above Eq. (11), we get

$$\beta_{Spencer}=\frac{-\sum_{j=1}^{n}u_k\left[\sum_{i=1}^{n}\frac{\partial g}{\partial x_k}\left(\sigma_i\right)\right]}{\sqrt{\sum_{j=1}^{n}\left\{\sum_{i=1}^{n}\frac{\partial g}{\partial x_k}\left(\sigma_i\right)\right\}^2}} \tag{12}$$

Design point (x_k) can be written as

$$x_k=\sigma_k\sum_{i=1}^{n}\left(-\alpha_i\beta_{Spencer}\right)+\mu_k \qquad \text{where, } k=1,2,\ldots\ldots,n. \tag{13}$$

The mean, standard deviation and coefficient of variation (COV) of the shear strength parameters are computed from the data which are available in the published literature. The target reliability based design optimization (TRBDO) approach illustrated by Basha and Babu (2008) is adopted in this paper for the reliability analysis. In this study, for reliability indices, the stability number c/γH is considered for calculations. When the values of mean and COV of c and γ (COV_c and COV_γ) are known, the COV value of c/γH can be calculated as follows:

$$COV = \sigma / \mu \tag{14}$$

The variance of a fraction, c/γH can be written as

$$\sigma^2\left(\frac{c}{\gamma H}\right) = \frac{\mu_{\gamma H}^2\sigma_c^2 + \mu_c^2\sigma_{\gamma H}^2}{\mu_{\gamma H}^4} = \frac{\mu_{\gamma H}^2\mu_c^2 COV_c^2 + \mu_c^2\mu_{\gamma H}^2 COV_{\gamma H}^2}{\mu_{\gamma H}^4} \tag{15}$$

$$\sigma\left(\frac{c}{\gamma H}\right) = \frac{\mu_c}{\mu_{\gamma H}}\sqrt{COV_c^2 + COV_{\gamma H}^2} \tag{16}$$

$$\sigma\left(\frac{c}{\gamma H}\right) = \mu\left(\frac{c}{\gamma H}\right)\sqrt{COV_c^2 + COV_{\gamma H}^2} \tag{17}$$

$$COV\left(\frac{c}{\gamma H}\right) = \sqrt{COV_c^2 + COV_{\gamma H}^2} \tag{18}$$

Therefore, by substituting the values of COV_c and COV_γ, COV of c/γH can be computed from Eq. (18). The range of random parameters considered in the present study are presented in Table 1.

Table 1. Range of parameters considered in the present study

Random Variable	Statistics		
	Mean	COV (%)	Distribution
Stability number, $(c/\gamma H)$	0.1	20-120	Normal
Friction angle, (ϕ)	32°	5-35	Log-Normal
Slope angle, (α)	33.69°	-	-

RESULTS AND DISCUSSION

The range of deterministic parameters and random variables considered in the present study are given in Table 1. In this study, a MSW landfill with a height (H) of 10m, slope angle (α) of 33.67° (1:1.5), unit weight (γ) = 10 kN/m^3, cohesion (c) = 10 kN/m^2 and friction angle (ϕ) = 32° are considered for the reliability analysis. Reliability analysis of MSW slopes is performed using TRBDO approach which makes use of FORM, i.e. Hasofer-Lind method. Since the surfaces of sliding for many MSW landfill slope failures have been observed to follow approximately the arc of a circle, it is assumed that the shape of slip surface is circular in the current study. Further, a limit state function is formulated for a circular sliding failure. Reliability indices against sliding failure using Spencer method of slices are computed. The effect of COV of $c/\gamma H$ and COV of ϕ on critical center coordinates (x_c/H, y_c/H), critical radius of circle (R_c/H) and critical reliability index (β_{Spencer}) is presented in Figs. 2 to 9.

Effect of COV of $c/\gamma H$ on critical center (x_c/H, y_c/H) and critical radius (R_c/H)

The geometry of MSW slope along with critical slip circles is shown in Figs. 2 to 7 on normalized X- axis and Y-axis. Fig. 2 shows the effect of COV of $c/\gamma H$ on critical center (x_c/H, y_c/H) and critical radius (R_c/H) for COV of ϕ = 15%. It can be noted from Fig. 2 that the radius of the slip circles is increasing significantly from 2.19 to 3.41 as the COV of $c/\gamma H$ increases from 20% to 80%. It indicates that the slip circles are moving away from the slope when high degree of variability associated with $c/\gamma H$ ratio. The critical slip circles shown in Fig. 3 for various values of COV of $c/\gamma H$ are magnified version of Fig. 2. An observation that can be made from Fig. 2 is that the magnitude of reliability index (β_{Spencer}) reduces from 4.40 to 1.23 as the COV of $c/\gamma H$ increases from 20 to 80%. Increase in COV of $c/\gamma H$ value reduces the expected performance of slope from good to poor as per USACE (1997). It may also be noted from Figs. 3 that the failure surface passing through the toe of slope, which is known as toe failure when COV of $c/\gamma H$ = 20% and COV of ϕ = 15%. In addition, the failure surface passing above the toe of slope, which is known as slope failure when COV of $c/\gamma H$ is more than or equal to 40%. Moreover, the toe failures can be observed from Figs. 4 to 7 when COV of $c/\gamma H$ = 20 - 40% for COV of ϕ = 25% and COV of $c/\gamma H$ = 20 - 60% for COV of ϕ = 35% respectively. The base failure may not be expected as the soil beneath the MSW landfill is not softer than the slope forming MSW landfill.

Most Sensitive COV out of COV of $c/\gamma H$ and COV of ϕ

An attempt has been made to study the most sensitive COV out of COV $c/\gamma H$ and COV of ϕ. It is noted from Fig. 8 that, the magnitude of $\beta_{Spencer}$ is found to be decreased from 4.9 to 3.1 when COV of ϕ increases from 5-35%. Whereas, the reliability index, $\beta_{Spencer}$ is decreased from 5 to 1 as the COV of $c/\gamma H$ increases from 20-120%. Fig. 9 shows the effect of COV of $c/\gamma H$ and COV of ϕ on reliability index ($\beta_{Spencer}$) for $c/\gamma H$ = 0.1, ϕ = 32^{o} and typical values chosen in the above sections. It is noted from Fig. 9 that the magnitude of $\beta_{Spencer}$ decreases significantly with the increase in COV of $c/\gamma H$ from 20 to 120%.

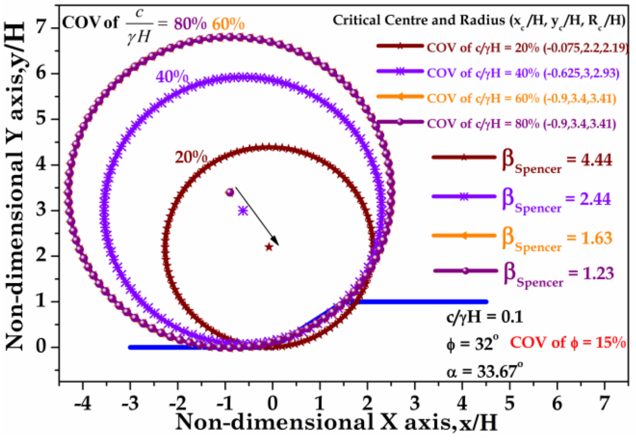

FIG. 2. Effect of COV of $c/\gamma H$ on $\beta_{Spencer}$, x_c/H, y_c/H and R_c/H for COV of ϕ = 15%

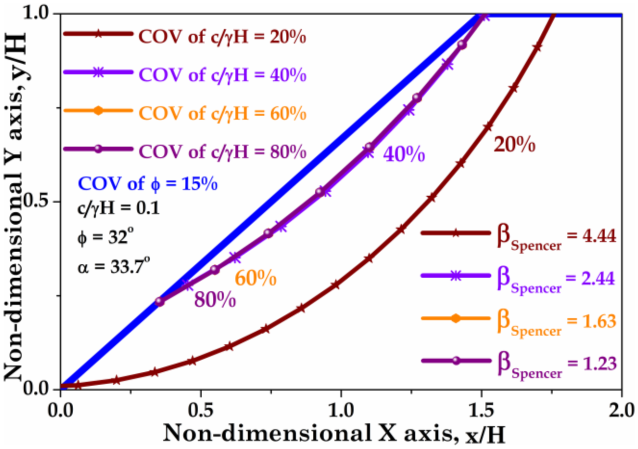

FIG. 3. Effect of COV of $c/\gamma H$ on slip circles for COV of ϕ = 15%(magnified view of Fig. 2)

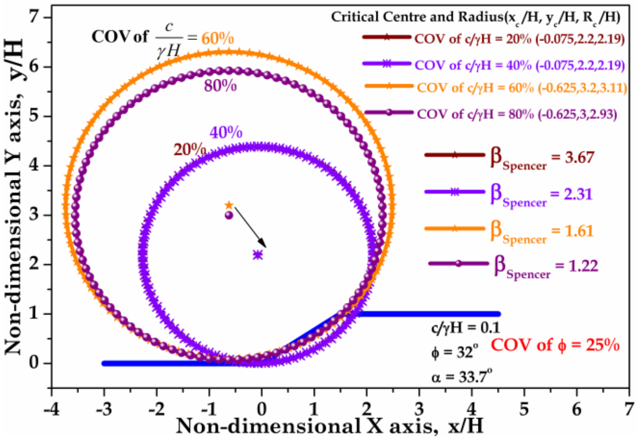

FIG. 4. Effect of COV of $c/\gamma H$ on $\beta_{Spencer}$, x_c/H, y_c/H and R_c/H for COV of ϕ = 25%

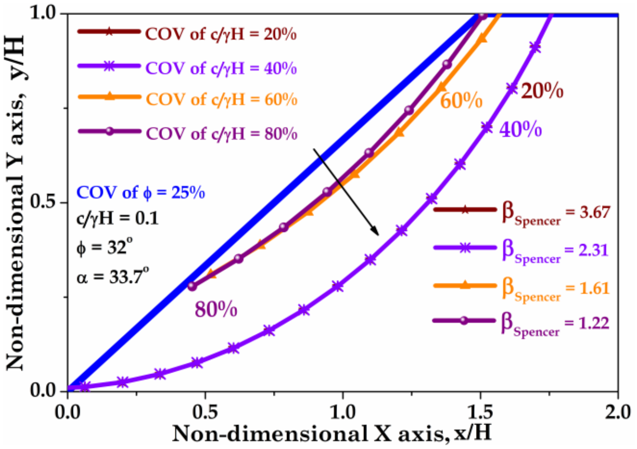

FIG. 5. Effect of COV of $c/\gamma H$ on slip circles for COV of ϕ = 25%(magnified view of Fig. 4)

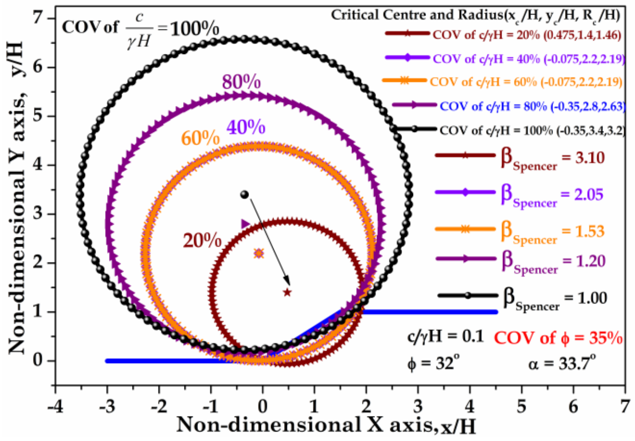

FIG. 6. Effect of COV of $c/\gamma H$ on $\beta_{Spencer}$, x_c/H, y_c/H and R_c/H for COV of ϕ = 35%

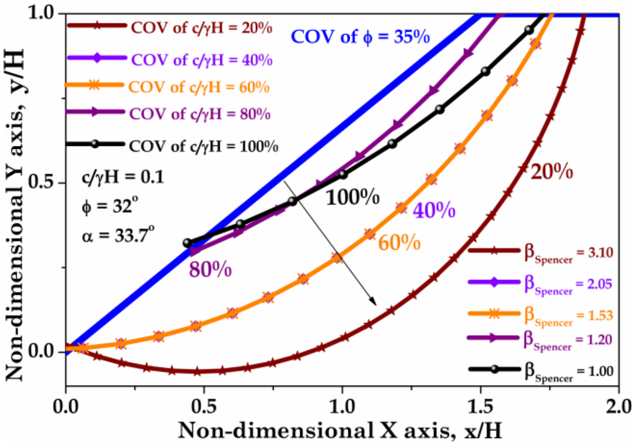

FIG. 7. Effect of COV of $c/\gamma H$ on slip circles for COV of ϕ = 35%(magnified view of Fig. 6)

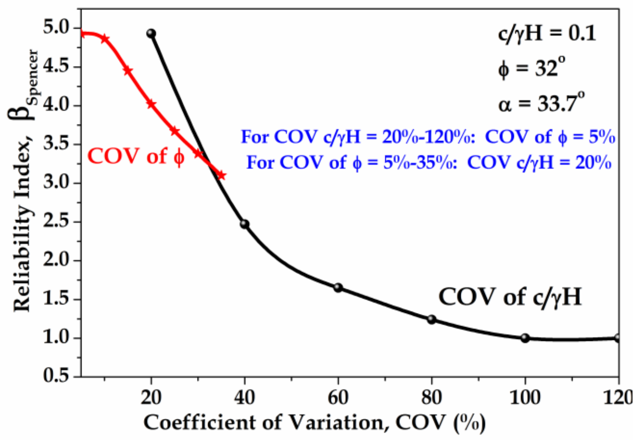

FIG. 8. The most sensitive COV out of COV of $c/\gamma H$ and COV of ϕ on $\beta_{Spencer}$ for $c/\gamma H$ = 0.1, ϕ = 32° and α = 33.7°

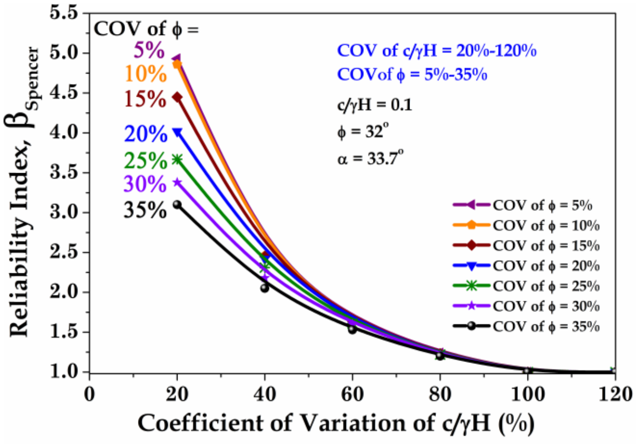

FIG. 9. Effect of COV of $c/\gamma H$ and ϕ on $\beta_{Spencer}$ for $c/\gamma H$ = 0.1, ϕ = 32° and α = 33.7°

In addition, COV of ϕ which ranges from 0-35% has relatively less influence on $\beta_{Spencer}$ than COV of $c/\gamma H$. Therefore, COV of $c/\gamma H$ and COV of ϕ both should be given due consideration as they are significant variables.

CONCLUSIONS

This paper outlines a procedure of probabilistic analysis of MSW landfill slope stability using first order reliability method. The results are computed using well recognized method of slope stability i.e. Spencer's method. The proposed procedure illustrates the determination of the probabilistic critical surface where the maximum probability of failure is calculated, compared to the use of the lowest value of critical factor of safety. The finding of this research warrant the following conclusions:

1. The study indicates that the slip circles are moving away from the slope when high degree of variability associated with $c/\gamma H$ of MSW.
2. The study reveals that the magnitude of reliability index ($\beta_{Spencer}$) reduces considerably as the COV of $c/\gamma H$ increases from 20% to 80% and COV of ϕ increases from 5 to 35%.
3. An important observation that can be made from the present study is that the increase in the COV of $c/\gamma H$ value reduces the expected performance of MSW slope from good to poor as per USACE (1997).
4. The results presented in the charts also reveals that the COV of ϕ associated with MSW (ranges from 0 to 35%) has relatively less influence on $\beta_{Spencer}$ than COV of $c/\gamma H$ associated with MSW (ranges from 20 to 120%).

REFERENCES

Babu, G.L.S., Reddy, K.R. and Srivastava, A. (2014). "Influence of spatially variable geotechnical properties of MSW on stability of landfill slopes." *J. Hazard. Toxic Radioactive Waste*, Vol. 18: 27-37.

Bhattacharya, G. and Dey, S. (2010). "Reliability evaluation of earth slopes by first order reliability method." Proc. of Indian Geotechnical Conference, IGC-2010, Dec. 16-18, Bombay: 1069-1072.

Blight, G. (2008). "Slope failures in municipal solid waste dumps and landfills: a review." *Waste Management & Research,* Vol. 26: 448–46.

Duncan, J.M. (2000). "Factors of safety and reliability in geotechnical engineering." *J. Geotechnical & Geoenv. Engrg.*, Vol. 4: 307–316.

Eid, H.T, Stark, T.D., Evans, W.D. and Sherry, P.E. (2000). "Municipal solid waste slope failure. I: waste and foundation soil properties." *J. Geotechnical & Geoenv. Engrg.*, Vol. 126: 397-407.

Hassan, F., Ahmed M., and Wolff, T.F. (1999). "Search algorithm for minimum reliability index of earth slopes." *J. Geotechnical & Geoenv. Engrg.*, Vol. 125(4): 301-308.

Koerner, R.M and Soong, T (1999). "Stability assessment of ten large landfill failures." *Advances in Transportation and Geoenvironmental Systems Using Geosynthetics (GSP 103):* ASCE, Reston/VA: 1-38.

Low, B.K. (2003). "Practical probabilistic slope stability analysis." *Proc. soil and rock America*, Vol. 2: 2777-2784.

Malkawi, A.I., Husein, W., Hassan, F. and Sarma, K.S. (2001). "An efficient search method for finding the critical circular slip surface using the Monte Carlo technique." *Canadian Geotechnical Journal*, Vol. 38(5): 1081-1089.

Merry, S.M., Kavazanjian, E. and Fritz, W.U. (2005). "Reconnaissance of the July 10 2000, Payatas landfill failure." *J of Performance of Constructed Facilities*, Vol. 19: 100–107.

Preeti, J., Handa, K. and Paul, A. (2014). "Studies on waste-to-energy technologies in india & a detailed study of waste-to-energy plants in Delhi." *Int. J . Adv. Res*, Vol. 2: 109-116.

Reddy K.R. and Basha, B.M. (2014). "Slope stability of waste dumps and landfills: state-of-the art and future challenges." *Proc. of Indian Geotechnical Conference IGC-2014,* Dec. 18-20, Kakinada, India: 2311 - 2337.

US Army Corps of Engineers (1997). *Engineering and Design: Introduction to probability and reliability methods for use in geotechnical engineering*. Department of the Army, U.S. Army Corps of Engineers, Washington, DC 20314-1000.