Continuous observer design for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements

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Abstract

In this paper, continuous observer is designed for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements. The time delay may be larger or less than the sampling intervals. The sampled and delayed measurements are used to update the observer whenever they are available. Sufficient conditions are presented to ensure global exponential stability of the observation errors by constructing a Lyapunov-Krasovskii function. A numerical example is given to illustrate the effectiveness of the proposed methods.

Key words: hybrid systems, continuous observer, multi-rate, delayed measurements

1 Introduction

Recently, the problem of design global convergent observers for nonlinear systems has made great progress. For the observation of nonlinear systems, one can use extended Luenberger observers (Zeitz, 1987), normal form observers (Xia and Gao, 1988; Xia and Gao, 1989; Bestle and Zeitz, 1983; Krener and Isidori, 1983), Lyapunov based observers (Raghavan and Hedrick, 1994; Thau, 1973), high-gain observers (Gauthier, Hammouri and Othman, 1992; Gauthier and Kupka, 1994), sliding mode observers (Haskara, Özgüner and Utkin, 1998) and moving horizon/optimization based observers (Michalska and Mayne, 1995). Among these methods, high-gain observers play an important role and can be used to a large class of nonlinear systems with a triangular structure after a coordinate change. New developments of high gain observers have been carried out in various directions (Gauthier, Hammouri and Othman, 1992; Praly, 2003; Deza, 1991; Deza, Bossanne, Busvelle, Gauthier and Rakotopara, 1993; Andrieu, Praly and Astolfi, 2009). For example, the result of (Gauthier, Hammouri and Othman, 1992) is extended to a class of nonlinear systems where the nonlinear terms admit an incremental rate depending on the measured output (Praly, 2003). In (Deza, 1991), the authors considered observer design for multi-input and multi-output (MIMO) nonlinear systems. The result has been extended to a class of MIMO nonlinear systems, in which interconnection between the blocks are not allowed (Deza, Bossanne, Busvelle, Gauthier and Rakotopara, 1993). Based on the observer normal form, another extension for the multi-output systems has been studied in (Rudolph and Zeitz, 1994). However, the nonlinearity of each block does not allow the unmeasurable states of its own block. Semi-global observer has been designed for nonlinear systems with interconnections between the subsystems (Shim, Son and Seo, 2001). The estimation errors can converge to the origin in finite-time by using high gain observers in conjunction with applications of geometric homogeneity and Lyapunov theories (Shen and Xia, 2008; Shen and Huang, 2009; Li, Xia and Shen, 2013).

It should be noted that the above results on observer design are based on continuous-time analysis. However, for a networked control system, the output is only available at discrete-time instants since it is usually transmitted through a shared band-limited digital communication network. Therefore, observer design for

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continuous systems with sampled and delayed output measurements has attracted the control community wide attention. There exist three main approaches to design observer for continuous systems with sampled and delayed measurements, for example, discrete time analysis based on a discretized model (Arcak and Nešić, 2004; Barbot, Monaco and Normand-Cyrot, 1999), continuous time analysis followed by discretization (Khalil, 2004; Nešić and Teel, 2004; Nešić, Teel and Kokotović, 1999), and a mixed continuous and discrete time analysis without discretization (Deza, Busvelle, Gauthier and Rakotopora, 1992; Ahmed-Ali and Lamnabhi-Lagarrigue, 2012; Raff, Kögel and Allgöwer, 2008; Van Assche, Ahmed-Ali, Ham and Lamnabhi-Lagarrigue, 2011; Nadri, Hammouri and Grajales, 2013; Karafyllis and Kravaris, 2009; Ahmed-Ali, Van Assche, Massieu and Dorléans, 2013; Zhang, Shen and Xia, 2014). More specifically, two classes of global exponential observers have been presented for a class of continuous systems with sampled and delayed measurements in (Ahmed-Ali, Van Assche, Massieu and Dorléans, 2013). By using the same methods, exponential convergent observers were proposed for nonlinear systems with sampled and delayed measurements in (Ahmed-Ali, Karafyllis and Lamnabhi-Lagarrigue, 2013). The observers designed in (Ahmed-Ali, Van Assche, Massieu and Dorléans, 2013; Ahmed-Ali, Karafyllis and Lamnabhi-Lagarrigue, 2013) are in essence discontinuous. The authors in (Zhang, Shen and Xia, 2014) proposed a continuous observer for a class of nonlinear systems with sampled and delayed measurements based on an auxiliary integral technique. But a harsh condition is imposed on time delay, that is the maximum delay must be less than the minimum sampling interval as in (Ahmed-Ali, Van Assche, Massieu and Dorléans, 2013; Ahmed-Ali, Karafyllis and Lamnabhi-Lagarrigue, 2013).

In this paper, we address continuous observer design for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements. This paper is organized as follows. In Section 2, continuous observers are presented for a class of multi-output nonlinear systems with multi-rate sampled and time delayed measurements. In Section 3, an example is used to illustrate the validity of the proposed design methods. Finally, Section 4 concludes the paper.

2 Main results

In this section, we consider the following multi-output nonlinear systems

$$\begin{cases} \dot{x}(t) = Ax(t) + B(x(t), u(t)), \\ y(t) = Cx(t) = [C_1 x^1(t), \ \cdots \ C_m x^m(t)]^\top, \end{cases} \tag{1}$$

where the state $x(t) \in \mathbb{R}^n$, the input $u(t) \in \mathbb{R}^p$, the output $y(t) \in \mathbb{R}^m$, $x(t) = [x^1(t)^\top, \cdots, x^m(t)^\top]^\top$, $x^i(t) \in \mathbb{R}^{\lambda_i}$ $(1 \le i \le m)$ is the ith partition of the state $x(t)$; $A = \mathrm{diag}\{A_1, \cdots, A_m\}$, A_i is $\lambda_i \times \lambda_i$ matrix of Brunovsky form, that is $A_i = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}$, $C = \mathrm{diag}\{C_1, \cdots, C_m\}$, $C_i = [1, 0, \cdots, 0]_{1\times\lambda_i}$, and $B(x(t), u(t)) = [b^1(x(t), u(t))^\top, \cdots, b^m(x(t), u(t))^\top]^\top$ in which the jth element of $b^i(\cdot)$, $b^i_j(\cdot)$ has the following structural dependence on the states:

$$b^i_j(t) = b^i_j(x^1(t), \cdots, x^{i-1}(t); x^i_1(t), \cdots, x^i_j(t); u(t)),$$

for all $1 \le i \le m$ and $1 \le j \le \lambda_i$. Thus, b^i_j is independent of the lower states $(x^i_{j+1}, \cdots, x^i_{\lambda_i})$ of the ith block and the states of the lower blocks $(x^{i+1}, \cdots, x^m)$. The ith block of the above system can be expressed as follows

$$\begin{cases} \dot{x}^i_1(t) = x^i_2(t) + b^i_1(x(t)^{[1,i-1]}; x^i_1(t); u(t)), \\ \quad \vdots \\ \dot{x}^i_{\lambda_i-1}(t) = x^i_{\lambda_i}(t) \\ \qquad\qquad + b^i_{\lambda_i-1}(x(t)^{[1,i-1]}; x(t)^i_{[1,\lambda_i-1]}; u(t)), \\ \dot{x}^i_{\lambda_i}(t) = b^i_{\lambda_i}(x(t)^{[1,i-1]}; x(t)^i_{[1,\lambda_i]}; u(t)), \end{cases} \tag{2}$$

where $x^i_j(t)$ is the jth element of the ith block $x^i(t)$. The abbreviation $x(t)^{[1,k]} := [x^1(t)^\top, \cdots, x^k(t)^\top]^\top$ and $x(t)^i_{[1,j]} := [x^i_1(t), \cdots, x^i_j(t)]^\top$ can be used to simplify the notation. We assume that there are m sensors in m channels to sample the output y at sampling instants t^i_k, and $t^i_k < t^i_{k+1}$ $(i = 1, \cdots, m$ and $k = 0, 1, 2, \cdots, \infty)$, where $\{t^i_k\}$ $(i = 1, \cdots, m)$ are strictly increasing sequences and satisfy that $\lim_{k\to\infty} t^i_k = \infty$. The sampled measures are available at instants $t^i_k + \tau^i_k$ $(i = 1, \cdots, m)$, where $\tau^i_k > 0$ $(i = 1, \cdots, m)$ denote the transmission delay, which are unknown but have an upper bound $\bar{\tau}_i$. The nonlinear terms $b^i_j(\cdot)$ are assumed to satisfy the following global Lipschitz conditions with Lipschitz constant $l_1 > 0$,

$$\begin{aligned} &\big|b^i_j(x^1, \cdots, x^{i-1}; x^i_1, \cdots, x^i_j; u) \\ &\quad -b^i_j(\hat{x}^1, \cdots, \hat{x}^{i-1}; \hat{x}^i_1, \cdots, \hat{x}^i_j; u)\big| \\ &\le l_1\big(|x^1_1 - \hat{x}^1_1| + |x^1_2 - \hat{x}^1_2| + \cdots |x^i_j - \hat{x}^i_j|\big), \\ &\qquad 1 \le i \le m, 1 \le j \le \lambda_i. \end{aligned} \tag{3}$$

Now, the explicit form of the ith block of the observer is given as follows:

$$\dot{\hat{x}}^i_1(t) = \hat{x}^i_2(t) + L_i a^i_1 e^i_1(t^i_k)$$

$$
\begin{aligned}
&\quad +b_1^i(\hat{x}(t)^{[1,i-1]};\hat{x}_1^i(t);u(t)),\\
&\vdots\\
&\dot{\hat{x}}_{\lambda_i-1}^i(t)=\hat{x}_{\lambda_i}^i(t)+L_i^{\lambda_i-1}a_{\lambda_i-1}^i e_1^i(t_k^i)\\
&\qquad +b_{\lambda_i-1}^i(\hat{x}(t)^{[1,i-1]};\hat{x}(t)_{[1,\lambda_i-1]}^i;u(t)),\\
&\dot{\hat{x}}_{\lambda_i}^i(t)=L_i^{\lambda_i}a_{\lambda_i}^i e_1^i(t_k^i)\\
&\qquad +b_{\lambda_i}^i(\hat{x}(t)^{[1,i-1]};\hat{x}(t)_{[1,\lambda_i]}^i;u(t)),\\
&\hat{x}_j^i(t_{k+1}^i+\tau_{k+1}^i)=\lim_{t\to t_{k+1}^i+\tau_{k+1}^i{}^-}\hat{x}_j^i(t),\\
&j=1,2,\cdots,\lambda_i,t\in[t_k^i+\tau_k^i,t_{k+1}^i+\tau_{k+1}^i),k\geq 0,
\end{aligned}
\tag{4}
$$

where $\hat{x}_j^i(t)=\hat{x}_{j_0}^i$ for $t\in[t_0,t_0+\tau_0^i]$ $(t_0=t_0^i)$, $i=1,\cdots,m$ and $j=1,\cdots,\lambda_i$, $e_1^i(t_k^i)=x_1^i(t_k^i)-\hat{x}_1^i(t_k^i)$, $L_i\geq 1$ and a_j^i $(1\leq i\leq m,1\leq j\leq\lambda_i)$ are positive real numbers, and will be given later. The definition of global exponential stable observer for the system (2) is given as follows.

Definition 1 *We say that the system (4) is a global exponential stable observer for the system (2), if there exist a non-decreasing function $N:\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}^+$ and a positive constant κ such that $\|\hat{x}(t)-x(t)\|\leq\exp(-\kappa(t-t_0))N(\|x_0\|,\|\hat{x}_0\|)$ for any $x_0\in\mathbb{R}^n$, $\hat{x}_0\in\mathbb{R}^n$.*

Remark 1 *The outputs y_i $(i=1,\cdots,m)$ are transmitted through m channels, respectively. We can use m sensors to detect them. Therefore, although τ_k^i are unknown, we can obtain the instant that the sampled data at instants t_k^i is available. In other word, $e_1^i(t_k^i)$ is updated automatically whenever the sampled and delayed measurement $y_i(t_k^i)$ arrives.*

From (2) and (4), the dynamics of the state error can be obtained

$$
\begin{cases}
\dot{e}_1^i(t)=e_2^i(t)-L_ia_1^ie_1^i(t_k^i)+\tilde{b}_1^i,\\
\qquad\vdots\\
\dot{e}_{\lambda_i-1}^i(t)=e_{\lambda_i}^i(t)-L_i^{\lambda_i-1}a_{\lambda_i-1}^ie_1^i(t_k^i)+\tilde{b}_{\lambda_i-1}^i,\\
\dot{e}_{\lambda_i}^i(t)=-L_i^{\lambda_i}a_{\lambda_i}^ie_1^i(t_k^i)+\tilde{b}_{\lambda_i}^i,\\
e_j^i(t_{k+1}^i+\tau_{k+1}^i)=\lim_{t\to t_{k+1}^i+\tau_{k+1}^i{}^-}e_j^i(t),\\
\ j=1,\cdots,\lambda_i,\ t\in[t_k^i+\tau_k^i,t_{k+1}^i+\tau_{k+1}^i),\ k\geq 0,
\end{cases}
\tag{5}
$$

or,

$$
\begin{aligned}
&\dot{e}_1^i(t)=e_2^i(t)-L_ia_1^ie_1^i(t)+L_ia_1^i\textstyle\int_{t_k^i}^t\dot{e}_1^i(s)ds+\tilde{b}_1^i,\\
&\quad\vdots\\
&\dot{e}_{\lambda_i-1}^i(t)=e_{\lambda_i}^i(t)-L_i^{\lambda_i-1}a_{\lambda_i-1}^ie_1^i(t)\\
&\qquad +L_i^{\lambda_i-1}a_{\lambda_i-1}^i\textstyle\int_{t_k^i}^t\dot{e}_1^i(s)ds+\tilde{b}_{\lambda_i-1}^i,\\
&\dot{e}_{\lambda_i}^i(t)=-L_i^{\lambda_i}a_{\lambda_i}^ie_1^i(t)+L_i^{\lambda_i}a_{\lambda_i}^i\textstyle\int_{t_k^i}^t\dot{e}_1^i(s)ds+\tilde{b}_{\lambda_i}^i,\\
&e_j^i(t_{k+1}^i+\tau_{k+1}^i)=\lim_{t\to t_{k+1}^i+\tau_{k+1}^i{}^-}e_j^i(t),\\
&j=1,\cdots,\lambda_i,\ t\in[t_k^i+\tau_k^i,t_{k+1}^i+\tau_{k+1}^i),\ k\geq 0,
\end{aligned}
\tag{6}
$$

where $e=[e^1(t)^\top,\cdots,e^m(t)^\top]^\top$, $e^i(t)=[e_1^i(t),\cdots,e_{\lambda_i}^i(t)]^\top$, $e_j^i(t)=x_j^i(t)-\hat{x}_j^i(t)$, $\tilde{b}_j^i=b_j^i(x(t)^{[1,i-1]};x(t)_{[1,j]}^i;u(t))-b_j^i(\hat{x}(t)^{[1,i-1]};\hat{x}(t)_{[1,j]}^i;u(t))$, $(1\leq i\leq m,1\leq j\leq\lambda_i)$.

Next, we represent t_k^i in (6) as

$$
t_k^i=t-\eta_i(t),\ \eta_i(t)=t-t_k^i. \tag{7}
$$

Then,

$$
0<\eta_i(t)=t-t_k^i\leq t_{k+1}^i+\tau_{k+1}^i-t_k^i<h_i,
$$

where $h_i>0$. Our aim is to find the bounds of h_i such that the error system (5) is globally exponentially stable.

Remark 2 *Note that $\lim_{t\to t_{k+1}^i+\tau_{k+1}^i}e_j^i(t)=e_j^i(t_{k+1}^i+\tau_{k+1}^i)$, then $e^i(t)$ is continuous on $[t_k^i+\tau_k^i,t_{k+1}^i+\tau_{k+1}^i]$. On the other hand, the evolution process $e_1^i(t_k^i)=x_1^i(t_k^i)-\hat{x}_1^i(t_k^i)$ is updated at instants $t_k^i+\tau_k^i$, whereas the sampled measurement $y_i(t)$ is sampled at instants t_k^i. Therefore, the system (6) is continuous, delayed and hybrid in nature. Similar systems have been investigated in (Karafyllis, 2007a; Karafyllis, 2007b; Karafyllis and Jiang, 2007; Ahmed-Ali, Van Assche, Massieu and Dorléans, 2013).*

Consider the following change of coordinates $\varepsilon_j^i=\frac{e_j^i}{L_i^{\lambda_j^i-1}}$, $1\leq i\leq m,1\leq j\leq\lambda_i$, where $\lambda_j^i=\Sigma_{k=1}^{i-1}\lambda_k+j$, $(1\leq i\leq m,1\leq j\leq\lambda_i)$. Then,

$$
\begin{cases}
\dot{\varepsilon}_1^i(t)=L_i\varepsilon_2^i(t)-L_ia_1^i\varepsilon_1^i(t)\\
\qquad +L_ia_1^i\int_{t-\eta_i(t)}^t\dot{\varepsilon}_1^i(s)ds+\frac{\tilde{b}_1^i}{L_i^{\lambda_1^i-1}},\\
\quad\vdots\\
\dot{\varepsilon}_{\lambda_i-1}^i(t)=L_i\varepsilon_{\lambda_i}^i(t)-L_ia_{\lambda_i-1}^i\varepsilon_1^i(t)\\
\qquad +L_ia_{\lambda_i-1}^i\int_{t-\eta_i(t)}^t\dot{\varepsilon}_1^i(s)ds+\frac{\tilde{b}_{\lambda_i-1}^i}{L_i^{\lambda_{\lambda_i-1}^i-1}},\\
\dot{\varepsilon}_{\lambda_i}^i(t)=-L_ia_{\lambda_i}^i\varepsilon_1^i(t)+L_ia_{\lambda_i}^i\int_{t-\eta_i(t)}^t\dot{\varepsilon}_1^i(s)ds\\
\qquad +\frac{\tilde{b}_{\lambda_i}^i}{L_i^{\lambda_{\lambda_i}^i-1}},\quad i=1,\cdots,m.
\end{cases}
\tag{8}
$$

Now, we give the following result for the system (2).

Theorem 1 *Consider the system (2) with the condition (3). If L_i satisfy $L_i>\max\{1,l_1,8\lambda_{\lambda_i}^il_1\bar{p}_2^i,L_{i-1}\}$, and $a_j^i>0$ $(1\leq i\leq m,1\leq j\leq\lambda_i)$ are given such that* *there exists a symmetric positive definite matrix P such that*

$$\bar{A}^\top P + P\bar{A} \ \leq \ -I, \tag{9}$$

and

$$\begin{aligned} h_i < \min\Big\{ & \tfrac{1}{4L_i(3+\lambda_1^i)^2(1+{a_1^i}^2)}, \tfrac{1}{16L_i\lambda_i\bar{\lambda}_i^2\bar{a}_i}, \\ & \tfrac{1}{2L_i\sqrt{(3+\lambda_1^i)a_1^i}} \Big\}, i = 1, \cdots, m, \end{aligned} \tag{10}$$

then, the system (4) is a global exponential stable observer for the system (2), where $L_0 \geq 1$, $\bar{A} = diag\{\bar{A}_1, \cdots, \bar{A}_m\}$, $P = diag\{P_1, \cdots, P_m\}$, $\underline{\lambda}_i = \lambda_{\min}(P_i)$, $\bar{\lambda}_i = \lambda_{\max}(P_i)$, $\bar{\lambda} = \max_{\{1\leq i\leq m\}}\{\bar{\lambda}_i\}$, $\bar{a}_i = \max_{\{1\leq j\leq\lambda_i\}}\{(a_j^i)^2\}$, $P_{j,r}^i$ *is the element of* P_i *at the* j*th line and* r*th column,* $\bar{p}_2^i = \max_{\{1\leq j\leq\lambda_i, 1\leq r\leq\lambda_i\}}\{|P_{j,r}^i|\}$, $(1 \leq i \leq m)$, *and* $\bar{A}_i = \begin{bmatrix} -a_1^i & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ -a_{\lambda_i-1}^i & 0 & \cdots & 1 \\ -a_{\lambda_i}^i & 0 & \cdots & 0 \end{bmatrix}$.

Proof: Consider the positive definite function

$$V_1(t) = \varepsilon(t)^\top\varepsilon(t) = \sum_{i=1}^m \varepsilon^i(t)^\top P_i\varepsilon^i(t),$$

where $\varepsilon(t) = [\varepsilon^1(t)^\top, \cdots, \varepsilon^m(t)^\top]^\top$, $\varepsilon^i(t) = [\varepsilon_1^i(t), \cdots, \varepsilon_{\lambda_i}^i(t)]^\top$, $(1 \leq i \leq m)$. Then, the derivative of $V_1(t)$ along the system (8) is given by

$$\begin{aligned} \tfrac{d}{dt}V_1(t)|_{(8)} &= \textstyle\sum_{i=1}^m[\dot{\varepsilon}^i(t)^\top P_i\varepsilon^i(t) + \varepsilon^i(t)^\top P_i\dot{\varepsilon}^i(t)] \\ &= \textstyle\sum_{i=1}^m L_i\varepsilon^i(t)^\top(\bar{A}_i^\top P_i + P_i\bar{A}_i)\varepsilon^i(t) \\ &\quad +2\textstyle\sum_{i=1}^m L_i(a_1^i, a_2^i, \cdots, a_{\lambda_i}^i)(\int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)ds)P_i\varepsilon^i(t) \\ &\quad +2\textstyle\sum_{i=1}^m\sum_{r=1}^{\lambda_i}\sum_{j=1}^{\lambda_i} \frac{\bar{b}_j^i}{L_i^{\lambda_j^i-1}}\varepsilon_j^i(t)P_{j,r}^i \\ &\leq -\textstyle\sum_{i=1}^m L_i\varepsilon^i(t)^\top\varepsilon^i(t) + \frac{1}{4}\sum_{i=1}^m L_i\varepsilon^i(t)^\top\varepsilon^i(t) \\ &\quad +4\textstyle\sum_{i=1}^m L_i(a_1^i, a_2^i, \cdots, a_{\lambda_i}^i) \\ &\quad \times\textstyle\int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)dsP_iP_i(a_1^i, a_2^i, \cdots, a_{\lambda_i}^i)^\top\int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)ds \\ &\quad +2l_1\textstyle\sum_{i=1}^m\sum_{r=1}^{\lambda_i}\sum_{j=1}^{\lambda_i}|\varepsilon_j^i(t)P_{j,r}^i| \\ &\quad \times(\textstyle\sum_{k=1}^j\sum_{a=1}^{\lambda_b}\sum_{b=1}^{i-1}|\varepsilon_a^b| + |\varepsilon_k^i|) \\ &\leq -\textstyle\sum_{i=1}^m(\frac{3}{4}L_i - 2\lambda_{\lambda_i}^i l_1\bar{p}_2^i)\varepsilon^i(t)^\top\varepsilon^i(t) \\ &\quad +4\textstyle\sum_{i=1}^m L_i\lambda_i\bar{\lambda}_i^2\bar{a}_i(\int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)ds)^2. \end{aligned}$$

Note that $L_i > \{8\lambda_{\lambda_i}^i l_1\bar{p}_2^i\}$. Then, we have

$$\begin{aligned} &\tfrac{d}{dt}V_1(t)|_{(8)} \leq -\tfrac{1}{2}\textstyle\sum_{i=1}^m L_i\varepsilon^i(t)^\top\varepsilon^i(t) \\ &+4\textstyle\sum_{i=1}^m L_i\lambda_i\bar{\lambda}_i^2\bar{a}_i(\int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)ds)^2. \end{aligned} \tag{11}$$

By Lemma 1 in (Gu, 2000), we have

$$| \int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)ds |^2 \leq h_i \int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds. \tag{12}$$

It follows from (11) and (12) that

$$\begin{aligned} &\tfrac{d}{dt}V_1(t)|_{(8)} \leq -\tfrac{1}{2}\textstyle\sum_{i=1}^m L_i\varepsilon^i(t)^\top\varepsilon^i(t) \\ &+4\textstyle\sum_{i=1}^m L_i\lambda_i\bar{\lambda}_i^2\bar{a}_ih_i\int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds. \end{aligned} \tag{13}$$

Consider the following auxiliary integral function

$$V_2(t) = \sum_{i=1}^m \int_{t-h_i}^t \int_\rho^t \dot{\varepsilon}_1^i(s)^2dsd\rho, t \geq t_0 + \bar{h},$$

where $\bar{h} = \max_{1\leq i\leq m}\{h_i\}$. We have,

$$\begin{aligned} \tfrac{dV_2(t)}{dt} &= \textstyle\sum_{i=1}^m h_i\dot{\varepsilon}_1^i(t)^2 - \sum_{i=1}^m\int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds \\ &\leq \textstyle\sum_{i=1}^m L_i^2h_i(3+\lambda_1^i)[\varepsilon_2^i(t)^2 + {a_1^i}^2\varepsilon_1^i(t)^2 \\ &\qquad +h_i{a_1^i}^2\textstyle\int_{t-\eta_i(t)}^t \dot{\varepsilon}_1^i(s)^2ds \\ &\quad +\lambda_1^i\frac{\varepsilon_1^1(s)^2 + L_1^2\varepsilon_2^1(s)^2 + \cdots + L_{i-1}^{2\lambda_1^i-6}\varepsilon_{\lambda_{i-1}}^{i-1}(s)^2}{L_i^{2(\lambda_1^i-2)}}] \\ &\qquad -\textstyle\sum_{i=1}^m\int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds \\ &\leq \textstyle\sum_{i=1}^m L_i^2(3+\lambda_1^i)^2(1+{a_1^i}^2)h_i\varepsilon^i(t)^T\varepsilon^i(t) \\ &\quad +\textstyle\sum_{i=1}^m L_i^2(3+\lambda_1^i){a_1^i}^2h_i^2\int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds \\ &\quad -\textstyle\sum_{i=1}^m\int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds, \ t \geq t_0 + \bar{h}, \end{aligned}$$

and

$$V_2(t) \leq \sum_{i=1}^m h_i \int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds. \tag{14}$$

Construct the following Lyapunov-Krasovskii function

$$V(t) = V_1(t) + V_2(t), t \geq t_0 + \bar{h}. \tag{15}$$

From (13) and (14), we have

$$\begin{aligned} \tfrac{dV(t)}{dt}|_{(8)} &\leq -\sum_{i=1}^m(\tfrac{1}{2} - (3+\lambda_1^i)^2(1+{a_1^i}^2)h_iL_i) \\ &\quad \times L_i\varepsilon^i(t)^\top\varepsilon^i(t) + \textstyle\sum_{i=1}^m(4L_i\lambda_i\bar{\lambda}_i^2\bar{a}_ih_i \\ &\quad +(3+\lambda_1^i){a_1^i}^2h_i^2L_i^2 - 1)\textstyle\int_{t-h_i}^t \dot{\varepsilon}_1^i(s)^2ds, \ t \geq t_0 + \bar{h}. \end{aligned}$$

From (10) it follows that

$$\frac{d}{dt}V(t)|_{(8)} \leq -\frac{\underline{L}}{4\bar{\lambda}}V(t), t \geq t_0 + \bar{h},$$

where $\underline{L} = \min_{\{1\leq i\leq m\}}\{L_i\}$. Then, $V(t) \leq \exp(-\frac{\underline{L}}{4\lambda}(t-t_0-\bar{h}))V(t_0+\bar{h})$, $t \geq t_0+\bar{h}$. Since the nonlinear terms in the system (2) and (4) satisfy the global Lipschitz conditions (3), then, the solutions of (2) and (4) exist and are continuous on $[t_0, t_0+\bar{h}]$. Therefore, there exists a non-decreasing function $N : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ such that $\|\hat{x}(t) - x(t)\| \leq \exp(-\frac{\underline{L}}{4\lambda}(t-t_0-\bar{h}))N(\|x_0\|, \|\hat{x}_0\|)$ for any $x_0 \in \mathbb{R}^n$, $\hat{x}_0 \in \mathbb{R}^n$. Thus, the system (4) is a global exponential stable observer for the system (2).

3 Numerical simulation

In this section, we use an example to show the effectiveness of our high gain observer design for nonlinear systems with sampled and time delay measurements. Consider the following multi-output nonlinear system (Shim, Son and Seo, 2001):

$$\begin{cases} \dot{x}_1(t) = x_2(t) + 0.01u(t), \\ \dot{x}_2(t) = -x_1(t) + 0.1(1 - x_1^2(t))x_2(t) + 0.1x_2(t)u(t), \\ \dot{x}_3(t) = x_4(t) + 0.01x_2(t)x_3(t)\exp(u(t)), \\ \dot{x}_4(t) = -x_3(t) + 0.1(1 - x_3^2(t))x_4(t) \\ \qquad +\frac{1}{1+(x_2(t)x_4(t))^2}u(t), \\ y_1(t) = x_1(t), \\ y_2(t) = x_3(t), \end{cases}$$

where $x(t) = (x_1(t), x_2(t), x_3(t), x_4(t))^T$, which is in the form (3) with $m = 2$ and $x^1(t) = (x_1(t), x_2(t))^T$, and $x^2(t) = (x_3(t), x_4(t))^T$. By (4), the observer is given by

$$\begin{cases} \dot{\hat{x}}_1(t) = \hat{x}_2(t) + 0.01u(t) + 3L_1(y_1(t_k^1) - \hat{x}_1(t_k^1)), \\ \dot{\hat{x}}_2(t) = -\hat{x}_1(t) + 0.1(1 - \hat{x}_1^2(t))\hat{x}_2(t) + 0.1\hat{x}_2(t)u(t) \\ \qquad +2L_1^2(y_1(t_k^1) - \hat{x}_1(t_k^1)), \\ \quad t \in [t_k^1 + \tau_k^1,\ t_{k+1}^1 + \tau_{k+1}^1), k \geq 0, \\ \hat{x}_i(t_{k+1}^1 + \tau_{k+1}^1) = \lim_{t\to t_{k+1}^1+\tau_{k+1}^1{}^-} \hat{x}_i(t),\ i = 1, 2, \\ \dot{\hat{x}}_3(t) = \hat{x}_4(t) + 0.01\hat{x}_2(t)\hat{x}_3(t)\exp(u(t)) \\ \qquad +2L_2(y_2(t_k^2) - \hat{x}_3(t_k^2)), \\ \dot{\hat{x}}_4(t) = -\hat{x}_3(t) + 0.1(1 - \hat{x}_3^2(t))\hat{x}_4(t) \\ \qquad +\frac{1}{1+(\hat{x}_2(t)\hat{x}_4(t))^2}u(t) + L_2^2(y_2(t_k^2) - \hat{x}_3(t_k^2)), \\ \quad t \in [t_k^2 + \tau_k^2,\ t_{k+1}^2 + \tau_{k+1}^2), k \geq 0, \\ \hat{x}_i(t_{k+1}^2 + \tau_{k+1}^2) = \lim_{t\to t_{k+1}^2+\tau_{k+1}^2{}^-} \hat{x}_i(t),\ i = 3, 4, \end{cases}$$

where $t_k^1 = kT_1 - (1.1 \cdot rand)T_1$ and $t_k^2 = kT_2 - (1.5 \cdot rand)T_2$, $rand$ is a random number in the interval $[0, 1]$, τ_k^1 and τ_k^2 denote the transmission delays, T_1 and T_2 are two positive real constants and will be given later. By simple computation, $P = \mathrm{diag}\{P_1, P_2\}$, where $P_1 = \begin{bmatrix} 0.8917 & -0.5695 \\ -0.5695 & 1.1735 \end{bmatrix}$, $P_2 = \begin{bmatrix} 0.5062 & -0.5052 \\ -0.5052 & 1.5124 \end{bmatrix}$. Then $\lambda_{\max}(P) = 1.7223$, $\lambda_{\min}(P) = 0.2963$. The other parameters are given as: $l = 1.6$, $L_1 = 40$, $L_2 = 90$. τ_k^1 and τ_k^2 are simulated by random numbers in the interval $[0, 1.5T_1]$ and $[0, 1.8T_2]$. From the condition (10), we have $h_1 = 3.3 \times 10^{-5}$s and $h_2 = 1.5 \times 10^{-5}$s. Let $T_1 = 1.0 \times 10^{-5}$s and $T_2 = 0.5 \times 10^{-5}$s. Fig. 1 shows the simulation results with the initial condition of observer $\hat{x}(0) = [-10, -10, -10, -10]$. The observer can also work for $T_1 = T_2 = 0.01s$, the simulation results are shown in Fig. 2.

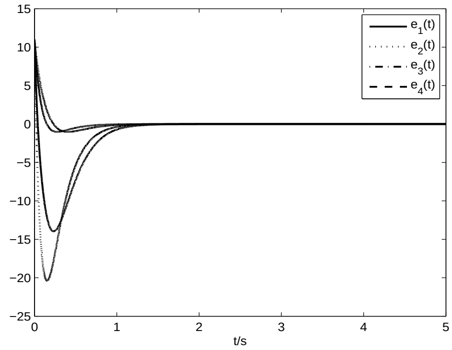

Fig. 2 Trajectories of the error states $e_i(t)(1 \leq i \leq 4)$ with $\hat{x}(0)$.

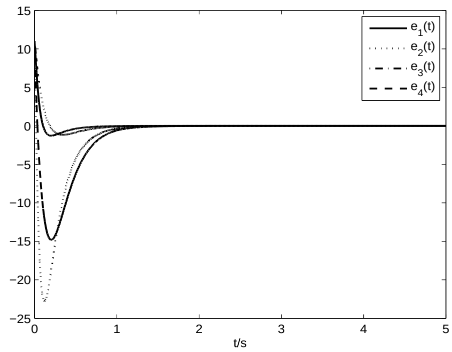

Fig. 3 Trajectories of the error states $e_i(t)(1 \leq i \leq 4)$ with $\hat{x}(0)$.

4 Conclusion

In this paper, continuous observers were designed for a class of multi-output nonlinear systems with multi-rate sampled and delayed output measurements. The time delay might be larger or less than the sampling intervals. Sufficient conditions were presented to ensure global exponential stability of the observation errors by constructing a Lyapunov-Krasovskii function.

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