EFFECT OF AMBIENT HEAT-IN-LEAK AND LONGITUDINAL WALL CONDUCTION ON A THREE-FLUID PARALLEL FLOW CRYOGENIC HEAT EXCHANGER

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ABSTRACT

A three-fluid parallel-flow cryogenic heat exchanger, involving two thermal interactions, is investigated for the effect of ambient heat-in-leak and longitudinal wall conduction, using FEM for four different flow arrangements. The three fluids are referred to as hot, cold and the intermediate fluids. Seven non-dimensional parameters inclusive of those to account for ambient heat-in-leak and longitudinal wall conduction are defined to present the results.

INTRODUCTION

Three fluid heat exchangers find wide usage in chemical processes and cryogenics [1]. Systems that deal with ammonia gas synthesis, purification and liquefaction of hydrogen, air separation systems, helium-air separation units are typical applications which make use of three-fluid heat exchangers. In addition to operating and design parameters, the thermal performance of heat exchangers, working in cryogenic temperature range, is strongly governed by losses such as longitudinal conduction through the wall, heat-in-leak from the surroundings, flow maldistribution, etc. The assumption of negligible longitudinal heat conduction along the separating walls between the streams is generally valid except when the heat exchanger effectiveness is expected to be high (above 0.9) or when the length of the flow passage is short. Aulds and Barron [3] mention that this aspect of the problem is one which deserves further investigation, since heat exchanger effectiveness of 0.95 and above are not uncommon in cryogenic systems.

A unified, flow direction independent, non-dimensional model for three-fluid heat exchangers with two thermal communications has been developed, for all possible fluid flow cases, by Sekulic and Shah [2]. Barron and Yeh [4] have obtained a numerical solution for the temperature distribution and overall heat exchanger effectiveness of a three-fluid heat exchanger considering the effect of longitudinal heat conduction between the surfaces separating the fluids. They have compared their numerical methodology with the analytical one proposed by Aulds and Barron [3] for a three fluid heat exchanger without longitudinal wall conduction and obtained favorable results. The general conclusion is that the effect of longitudinal conduction does not influence the overall effectiveness very much, but its influence is not negligible.

Analysis of three-fluid heat exchangers has been reported in the literature [5, 6, 7]. Krishna et al, [8] have studied the effect of longitudinal wall conduction for a three-fluid heat

NOMENCLATURE

T	temperature (K)
C	heat capacity rate of the fluids (W/K)
L	heat exchanger length (m)
L_e	effective length of heat exchanger as defined by L/number of elements (m)
A_1, A_3, A_4	areas as illustrated in Figure 1
U	overall heat transfer coefficient (W/m^2-K)
U_1, U_3, U_4	overall heat transfer coefficients as illustrated in Figure 1
H_2	dimensionless parameter as defined in Eq. (6)
H_3	ambient heat-in-leak parameter as defined in Eq. (6)
R_1, R_2	ratio of heat capacity rates as defined in Eq. (6)
n	Local NTU for a pair of fluids across a wall as defined by Eq. (7)
x	axial co-ordinate (m)

Greek

θ	dimensionless temperature as defined in Eq. (6)
λ	Longitudinal wall conduction factor as defined in Eq. (6)
ϵ	thermal effectiveness

Subscripts

c	cold fluid
h	hot fluid
i	intermediate fluid
in	inlet
out	outlet

exchanger, with three thermal communications. They have presented an analytical solution in which the system of governing equations is solved using the method of decoupling transformations. They have also proposed an FEM model for solving this system of governing equations using the Galerkin's method. They have obtained perfectly matching results through both the methods. In the present paper, their finite element methodology is extended to investigate the effect of ambient heat-in-leak and longitudinal wall conduction in a three-fluid heat exchanger with two thermal communications.

MODEL FORMULATION

A three-fluid heat exchanger with two communications is one in which only one fluid thermally interacts with the other two. The pipe configuration for the heat exchanger chosen for analysis is shown in Figure 1. The intermediate fluid flows in the innermost pipe. The hot fluid flows in the middle annulus while the cold fluid flows in the outermost annulus. The two thermal communications involve the hot fluid interacting with both the cold and intermediate fluids, while they themselves have no thermal contact. Depending on the flow directions, four different flow arrangements, P1 – P4 are possible. These are provided in Figure 2. The model is a general one that can be applied for all flow arrangements.

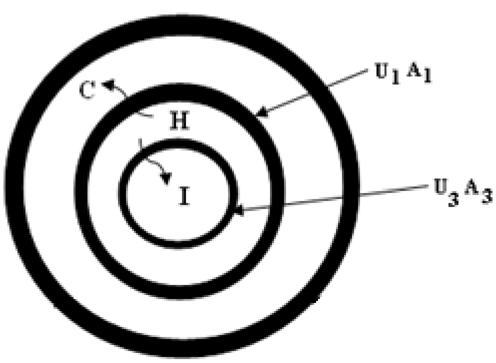
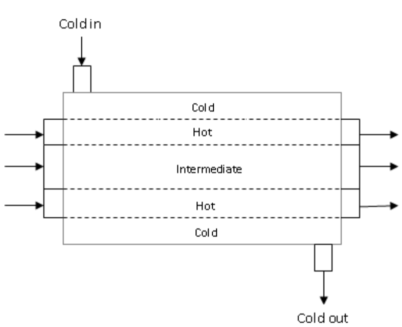
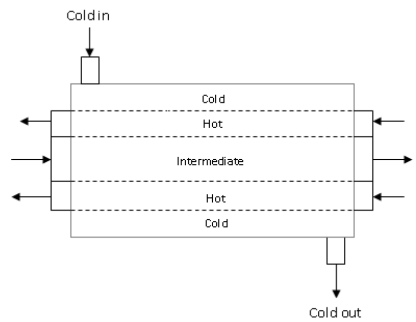


Figure 1: Tubular arrangement of the three-fluid heat exchanger with two thermal communications



P1



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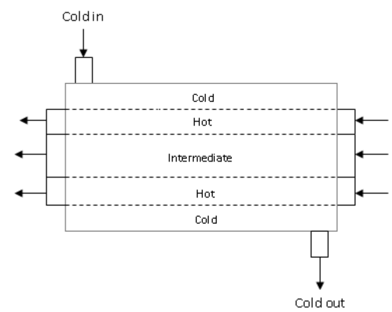

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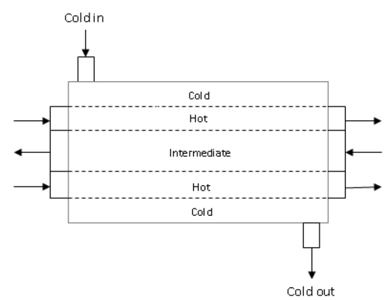

P4

Figure 2: Flow arrangements for the three-fluid heat exchanger with two thermal communications

The following assumptions have been made for the analysis: (a) The heat exchanger is in a steady state (b) All properties are constant with time and space. (c) There is no interaction with ambient and the heat exchanger is completely insulated (d) Within a stream the temperature distribution is uniform in the transverse direction and equal to the average temperature of the fluid. (e) There is no heat source or sink in the heat exchanger or in any of the fluids (f) There is no phase change in the fluid streams (g) The heat transfer area is constant along the length of the heat exchanger.

The governing equations for the hot, cold and the intermediate fluids obtained by energy balance, are as follows:-

Hot Fluid:

$$i_H \frac{d\theta_H}{dX} + n_{H-W1}(\theta_H - \theta_{W1}) + n_{H-W3}(\theta_H - \theta_{W3}) = 0 \quad (1)$$

Intermediate Fluid: $i_I \frac{d\theta_I}{dX} - n_{W3-I}(\theta_{W3} - \theta_I) = 0 \quad (2)$

Cold Fluid:

$$i_C \frac{d\theta_C}{dX} - \mathrm{n_{W1-C}}(\theta_{W1} - \theta_C) - \mathrm{NTU} \times \mathrm{H_3}(\boldsymbol{\theta_a} - \theta_C) = 0 \quad (3)$$

Wall-1:

$$\frac{\lambda_1}{R_2}\frac{d^2\theta_{W1}}{dX^2} + n_{H-W1}(\theta_H - \theta_{W1}) - \frac{\mathrm{n_{W1-C}}}{R_2}(\theta_{W1} - \theta_C) = 0 \quad (4)$$

Wall-3:

$$\frac{\lambda_3}{R_2}\frac{d^2\theta_{W3}}{dX^2} + n_{H-W3}(\theta_H - \theta_{W3}) - \frac{n_{W3-I}}{R_1}(\theta_{W3} - \theta_I) = 0 \quad (5)$$

In the above expressions θ_h, θ_i and θ_c represent the dimensionless temperatures of the hot, intermediate and the cold fluids respectively. Directional constants - i_h, i_i & i_c - are introduced to the governing equations to make them applicable for all four flow arrangements. Their values are +1 for the positive x direction and -1 for negative x direction. For the flow arrangement of P2 analyzed in this paper, $i_h = -1$, $i_c = +1$ and $i_i = +1$. The different non-dimensional terms used in the analysis are defined as mentioned below:-

$$\theta = \frac{T - T_{c,in}}{T_{h,in} - T_{c,in}}, \quad X = \frac{x}{L}, \quad R_1 = \frac{C_h}{C_i}, \quad R_2 = \frac{C_h}{C_c},$$

$$\lambda = \frac{kA_c}{C_c L_e}, \quad \mathrm{NTU} = \frac{U_1 A_1}{C_c}, \; H_3 = \frac{U_4 A_4}{U_1 A_1} \quad (6)$$

$$n_{H-W1} = \left(\frac{hA_1}{C_h}\right)_{H-W1}, \quad n_{W1-C} = \left(\frac{hA_1}{C_c}\right)_{W1-C}$$

$$n_{H-W3} = \left(\frac{hA_3}{C_h}\right)_{H-W3}, \quad n_{W3-I} = \left(\frac{hA_3}{C_i}\right)_{W3-I} \quad (7)$$

It is assumed that the local Ntu of each pair of fluids across a given wall is same [9], i.e.,

$n_{H-W1} = n_{W1-C} = n_1$

$n_{H-W3} = n_{W3-I} = n_3$

and $n_4 = \mathrm{NTU} \times H_3$ (8)

The local Ntu of each fluid is related to the overall NTU as given below:-

$$\mathrm{NTU} = n_1\left[\frac{R_2}{1+R_2}\right] = n_3\left[\frac{R_2}{H_2(1+R_1)}\right] \quad (9)$$

Assuming that the walls are insulated at the two ends, the boundary conditions for the walls are expressed as follows:

$\frac{d\theta_{wi}}{dX} = 0$ at $X = 0$ and $X = 1$, where $i = 1$ and 3 for the two walls (10)

FEM MODELLING

The heat exchanger is discretised into a number of elements. A linear variation is assumed for the hot, intermediate and the cold fluids in a single element. The fluid temperature at any point, for the flow arrangement P2, is given by the following equations:

$$\left.\begin{aligned} \theta_h &= N_1\theta_{h,out} + N_2\theta_{h,in} \\ \theta_i &= N_1\theta_{i,in} + N_2\theta_{i,out} \\ \theta_c &= N_1\theta_{c,in} + N_2\theta_{c,out} \end{aligned}\right\} \quad (11)$$

Assuming a linear variation for the temperatures of the walls, the temperature for the walls at any point is given by:

$$\left.\begin{aligned} \theta_{W1} &= N_1\theta_{W1,in} + N_2\theta_{W1,out} \\ \theta_{W2} &= N_1\theta_{W2,in} + N_2\theta_{W2,out} \\ \theta_{W3} &= N_1\theta_{W3,in} + N_2\theta_{W3,out} \end{aligned}\right\} \quad (12)$$

where N_1 and N_2 are the shape functions and given by

$N_1 = 1 - X$ and $N_2 = X$. (13)

Using the Galerkin's method of minimizing the weighted residual (Lewis et al, [10]), the governing equations are reduced to a set of algebraic equations. The discretised governing equations are written in matrix form for each element as:

$[K]\{\theta\} = \{f\}$ (14)

where [K] is the local stiffness matrix for each element, $\{\theta\}$ is the non-dimensional temperature vector and {f} the loading terms. Assembling the local stiffness matrix for all the elements in the solution domain leads to the formation of the global stiffness matrix. Following this, the boundary conditions are enforced on the global stiffness matrix and the loading vector and the equations are solved by MATLAB to obtain the dimensionless temperatures along the heat exchanger.

EFFECTIVENESS EXPRESSIONS

Cooling of the hot fluid has been identified as the objective of the three-fluid heat exchanger adopted for analysis. Cooling effectiveness of the hot fluid for any three fluid heat exchanger can be defined based on its temperature effectiveness or thermal effectiveness [6, 7]. In this paper, the performance of the heat exchanger has been presented only in terms of the thermal effectiveness of the hot fluid, ϵ_h, which is given by the equation:

$$\epsilon_h = \frac{\dot{Q}_{h,actual}}{\dot{Q}_{h,max}} \quad (15)$$

The expressions for $\dot{Q}_{h,actual}$ and $\dot{Q}_{h,max}$, for various combinations of thermal capacities of the three fluids, have been presented earlier [6, 8]. Using these, expressions are presented for ε_h, in terms of non-dimensional parameters, hereunder.

When the thermal capacity of the hot fluid is greater than the thermal capacities of the other two fluids and when the hot fluid is flowing counter to the other two streams,

$$\epsilon_h = \frac{(1-\theta_{h,out})}{\left[\frac{1}{R_2} + \frac{(1-\theta_{i,in})}{R_1}\right]} \quad (16)$$

For all other possible combinations of thermal capacities of the three fluids,

$$\epsilon_h = (1 - \theta_{h,out}) \quad (17)$$

RESULTS AND DISCUSSION

The model proposed is a general model and can be applied for all three-fluid, single pass, parallel flow heat exchangers considering two thermal communications and all flow arrangements. The set of non-dimensional governing equations (1)-(5) are solved by FEM using the Galerkin's method. An exponentially distributed grid is used for the FEM analysis. Validation is carried out by comparing the present values with those obtained by Sekulic [1] and shown in Figure 3. The excellent match between the two validates the solution methodologies used. The effect of ambient heat-in-leak and longitudinal wall conduction is studied for the four flow configurations shown in Figure 2. Cooling of the hot fluid is chosen as the objective of the heat exchanger and the arrangement that best satisfies this objective is chosen for analysis. For the best flow arrangement, the effect of ambient heat-in-leak, longitudinal wall conduction and that of varying design parameters are studied for their effect on the temperature profile of the hot fluid and its effectiveness. The value of $\theta_{i,in}$ is taken to be 0.5 for the temperature profiles and 0.3 for the effectiveness evaluations. However, this could be any value between 0 – 1. The non-dimensional ambient temperature is taken equal to 1, while the value of ambient heat-in-leak parameter H_3 is taken to be 0.1 and that of the longitudinal wall conduction parameter λ is taken as 0.08 for all the walls. These are indicative values, used in earlier studies on cryogenic heat exchangers [8].

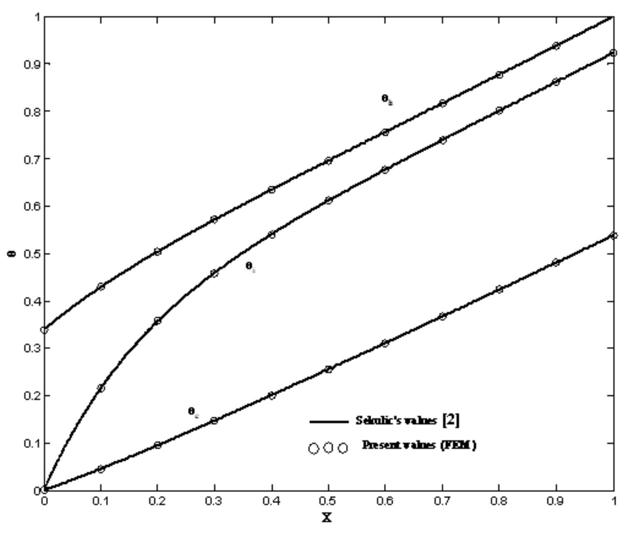

Figure 3: Temperature profiles for a three-fluid heat exchanger with two thermal communications with no heat-in-leak and longitudinal wall conduction. Comparison of present values (FEM) with Sekulic's values [2]. Values of non-dimensional parameters: $H_2 = 2$, $H_3 = 0$, $\lambda = 0$, NTU = 1.25, $R_1 = 4$, $R_2 = 1.25$, $\theta_{i,in} = 0$.

HOT FLUID EXIT TEMPERATURE

For the values of non-dimensional parameters chosen, it is observed that from Table 1, that the effect of heat-in-leak and longitudinal wall conduction, is to enhance $\theta_{h,out}$ in all the four arrangements. The enhancement of $\theta_{h,out}$ is highest for the P2 arrangement (12.10 %). In spite of this, the minimum values of $\theta_{h,out}$ are found for P2, suggesting that for the values of non-dimensional parameters chosen, this is the best arrangement for the purpose of cooling of the hot fluid.

Table 1: Variation of $\theta_{h,out}$ due to longitudinal wall conduction. Values of non-dimensional parameters: H_2=2, R_1=2, R_2=1.25, NTU =1, $\theta_{i,in}$ = 0.3, θ_a=1

Flow Arrangement	$\theta_{h,out}$		% increase in $\theta_{h,out}$
	λ=0 & H_3=0	**λ=0.08 & H_3=0.1**	
P1	0.5448	0.5607	2.92
P2	**0.3900**	**0.4372**	**12.10**
P3	0.5029	0.5275	4.89
P4	0.4542	0.4861	7.02

EFFECT OF DESIGN PARAMETERS & LONGITUDINAL WALL CONDUCTION ON THE HOT FLUID FOR P2 ARRANGEMENT

From Table 1, it is noticed that the P2 arrangement is the most effective for the objective of cooling of the hot fluid. The effect of the design parameters (H_2, NTU) and longitudinal wall conduction parameter (λ) are discussed for the P2 arrangement. Each fluid is affected by the longitudinal conduction in both the walls it is in contact with, besides various other factors. Longitudinal wall conduction is found to be pronounced at the heat exchanger ends. The hot fluid experiences enhanced heat transfer at its entrance and a reduced heat transfer near its exit, due to the effect of wall conduction.

Effect of H_2

The effect of H_2 and degradation parameters ambient heat-in-leak & longitudinal wall conduction is shown in Figures 4 - 5. An increase in H_2 increases the thermal interaction between the hot and the intermediate fluids relative to that between the hot and the cold fluids. Thus an increase in H_2 results in the reducing temperature difference between the hot and the intermediate fluids leading to a reduction in the hot fluid exit temperature and increased hot fluid effectiveness as seen in Figures 4 - 5. The effect of ambient heat-in-leak and longitudinal wall conduction is to increase the exit temperature of the hot fluid leading to reduced values of hot fluid effectiveness. It is observed that for H_2 = 20, hot fluid exit temperature increases from 0.41 to 0.47, an increase of

14.6 %, and effectiveness reduces from 0.55 to 0.50, a reduction of 9.1%.

Effect of NTU

The effect of NTU and degradation parameters ambient heat-in-leak & longitudinal wall conduction is shown in Figures 6 - 7. From its definition it is clear that a change in NTU will affect the overall thermal resistance between the hot and cold fluids relative to the thermal capacity of the cold fluid. If the value of H_2 is fixed, an increase in NTU will not result in any change in the thermal resistance between the hot and the cold fluids. This will only result in a decrease in the thermal capacity of the cold fluid. If the cold-fluid-specific-heat is assumed constant then an increase in NTU manifests as a decrease in the mass flow rate of the cold fluid. Since R_1 and R_2 are also constant, this results in a decrease in the flow rates of the hot and the intermediate fluids as well. The effect of all this is steeper thermal gradients for all three fluids and increased differences between the inlet and exit temperatures. As NTU is increased from 0.1 to 1, gradual changes occur in the hot fluid temperature distribution. As NTU is increased from 1 – 10, the decrease in the mass flow rates of all the three streams results in a sharp fall in the hot fluid temperature at the inlet and subsequent increase towards the exit, due to the intermediate fluid boundary condition, as seen in Figure 6. An increased difference between the hot fluid inlet and exit temperatures is also observed. As seen in Figure 7, an increase in NTU results in an increase in the effectiveness. However the rate of increase is higher at lower values of NTU and this rate gradually reduces, till, after a certain threshold NTU value, an increase in NTU does not significantly alter the effectiveness. This threshold value of NTU is crucial in the design of the heat exchanger, and is around 6, for the values of non-dimensional parameters chosen, as observed from Figure 7.

The effect of ambient heat-in-leak and longitudinal wall conduction is generally pronounced at the ends, while at higher values of NTU, it is prominent even at the mid-span. It is also observed that the effectiveness in the presence of the degradation factors increases to a value of 0.6056 for an NTU value of 0.6 and subsequently reduces marginally to a value of 0.5951 for an NTU value of 10. For the NTU value of 10, the effect of ambient heat-in-leak and longitudinal wall conduction enhances the hot-fluid-exit-temperature from 0.22 to 0.37 (68.18 %) while reducing its effectiveness from 0.74 to 0.6 (18.9 %).

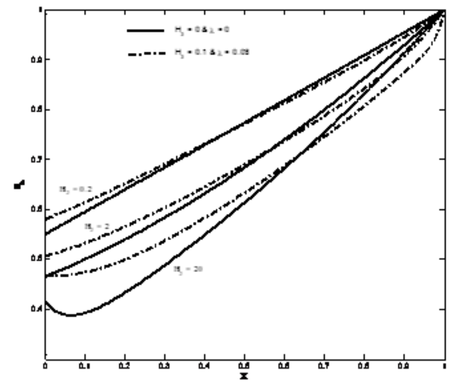

Figure 4: Effect of H_2 on the hot fluid temperature profile - P2 arrangement. Values of other non-dimensional parameters: NTU=1, R_1=2, R_2=1.25, $\theta_{i\,in}$= 0.5, θ_a=1, H_3 = 0.1, λ=0.08

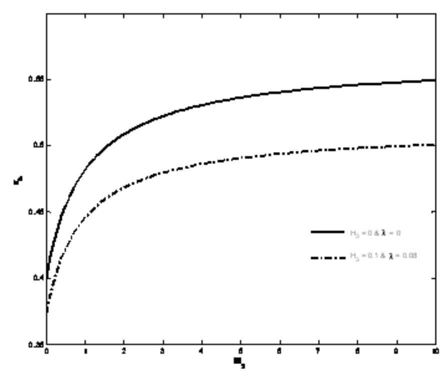

Figure 5: Effect of H_2 on the hot fluid effectiveness – P2 arrangement. Values of other non-dimensional parameters: NTU=1, R_1=2, R_2=1.25, $\theta_{i\,in}$= 0.3, θ_a=1, H_3 = 0.1, λ=0.08

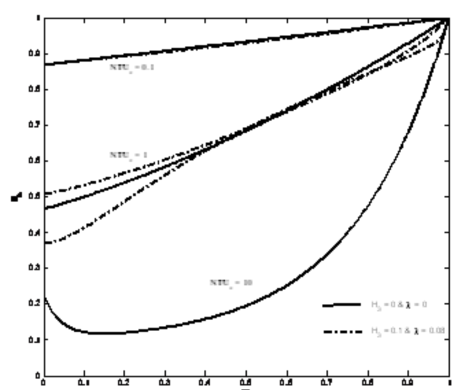

Figure 6: Effect of NTU on the hot fluid temperature profile – P2 arrangement. Values of other non-dimensional parameters: H_2=2, R_1=2, R_2=1.25, $\theta_{i\,in}$= 0.5, θ_a=1, H_3 = 0.1, λ=0.08

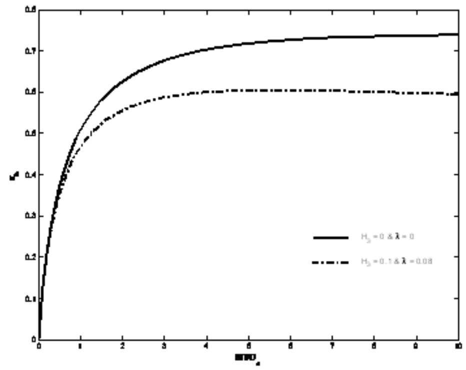

Figure 7: Effect of NTU on the hot fluid effectiveness – P2 arrangement. Values of other non-dimensional parameters: H_2=2, R_1=2, R_2=1.25, $\theta_{i\ in}$= 0.3, θ_a=1, H_3 = 0.1, λ=0.08

CONCLUSIONS

The effect of degradation parameters ambient heat-in-leak & longitudinal wall conduction in a three-fluid heat exchanger, for a cryogenic application, involving two thermal interactions, has been investigated using FEM. Cooling of the hot fluid has been identified as the objective of the three fluid heat exchanger.

For the values of the non-dimensional parameters chosen, the flow arrangement involving the hot fluid flowing counter to the other two fluids, is found to have the lowest hot fluid exit temperatures, indicating that this is the best arrangement for the cooling of the hot fluid.

An increase in the resistance ratio H_2 leads to reduced values of hot fluid exit temperature and increased hot fluid effectiveness. High values of the 'number of transfer units' – NTU – leads to steep gradients for the hot fluid at its entrance. An increase in NTU results in an increase in hot fluid effectiveness. The rate of increase is higher at lower values of NTU and the effectiveness increases only till a threshold NTU value.

The effect of ambient heat-in-leak and longitudinal wall conduction is found to be pronounced at the heat exchanger ends and results in altering of the temperature profiles at the ends. The effect of ambient heat-in-leak and longitudinal wall conduction is significant at higher values of H_2 and NTU and leads to enhanced values of hot-fluid-exit-temperature, reducing its effectiveness.

REFERENCES

[1] Dusan P. Sekulic, 'A compact solution of the parallel flow three-fluid heat exchanger problem', Int. J Heat Mass Transfer, Vol 37, (1994), No. 14, pp 2183-2187

[2] D.P. Sekulic, R.K. Shah, 'Thermal design theory of three-fluid heat exchangers', Advances in Heat Transfer 26 (1995) 219–329.

[3] D.D. Aulds, R.F. Barron, 'Three-fluid heat exchanger effectiveness', International Journal of Heat and Mass Transfer 10 (1967) 1457–1462.

[4] Barron, R. F., and Yeh, S. L., 'Longitudinal conduction in a three-fluid heat exchanger'. Am. Soc. Mech. Eng. [Pup.] (1976), 76-WA/ HT-9.

[5] D. Shrivastava, T.A. Ameel, 'Three-fluid heat exchangers with three thermal communications. Part A: General Mathematical Model', International Journal of Heat and Mass Transfer 47 (2004) 3855 - 3865

[6] D. Shrivastava, T.A. Ameel, 'Three-fluid heat exchangers with three thermal communications. Part B: effectiveness evaluation', International Journal of Heat and Mass Transfer 47 (2004) 3867 - 3875

[7] V.Krishna, Pradeep G. Hegde, N. Subramanian, K.N. Seetharamu, 'Effect of ambient heat-in-leak on the performance of a three fluid heat exchanger, for cryogenic applications, using finite element method', Int. J. Heat Mass Transfer, 55 (2012) 5459–5470,

[8] V.Krishna, Spoorthi S, Pradeep G. Hegde, K.N. Seetharamu, 'Effect of longitudinal wall conduction on the performance of a three-fluid cryogenic heat exchanger with three thermal communications', International Journal of Heat and Mass Transfer, 62, (2013) 567–577

[9] P.G. Kroger, 'Performance deterioration in high effectiveness heat exchangers due to axial conduction effects', Adv. Cryog. Eng. 12 (1967) 363–372.

[10] Roland W. Lewis, Perumal Nithiarasu, Kankanhalli N. Seetharamu, 'Fundamentals of Finite Element Method for Heat and Fluid Flow', John Wiley and Sons, 2004.