OPTIMAL PUBLIC INVESTMENT, GROWTH, AND CONSUMPTION: EVIDENCE FROM AFRICAN COUNTRIES*

AUGUSTIN KWASI FOSU[a], **YOSEPH YILMA GETACHEW**[b] and **THOMAS H. W. ZIESEMER**[c]

[a]Institute of Statistical, Social and Economic Research (ISSER), University of Ghana, Legon, Ghana. tel.: +233-032 512502, fax: +233-057-7699902, e-mail: afosu@isser.edu.gh

[b]Department of Economics, University of Pretoria, Pretoria 0002, South Africa; tel.: +27 124204509, fax: +27 123625168, e-mail: yoseph.getachew@up.ac.za

[c]Department of Economics and UNU-MERIT, Maastricht University, Maastricht, Netherlands: tel.: +31 43 3883872; fax: +31 43 3884499, e-mail: t.ziesemer@maastrichtuniversity.nl

*We benefitted from comments on an earlier version of the paper from the participants of the CSAE Conference 2012: Economic Development in Africa, Oxford, UK. Thanks are specially due to Erik de Regt and anonymous referees. The usual disclaimer applies.

AFRICA'S OPTIMAL PUBLIC INVESTMENT & GROWTH

- **THOMAS H. W. ZIESEMER**
  - Department of Economics and UNU-MERIT, Maastricht University, Maastricht, Netherlands: tel: 31433883872, fax: 31433884150, e-mail: t.ziesemer@maastrichtuniversity.nl

This paper develops a model positing a nonlinear relationship between public investment and growth. The model is then applied to a panel of African countries using nonlinear estimating procedures. The growth-maximizing level of public investment is estimated at about 10% of GDP based on System GMM estimation. The paper further runs simulations, obtaining the constant optimal public investment share that maximizes the sum of discounted consumption as between 8.1% and 9.6% of GDP. Compared with the observed end-of-panel mean value of no more than 7.26 percent, these estimates suggest that there has been significant public under-investment in Africa.

Keywords: Public investment; Economic Growth; Nonlinearity

1. Introduction

The gap separating the world's rich and poor countries remains startling. In 2007, per-capita income in the United States was at least thirty times higher than in eighteen Sub-Saharan African (SSA) countries.[1] Compared to Ethiopia and Tanzania, for instance, the United States has a per-capita income that is more than thirty-eight and forty-six times larger, respectively, when measured in terms of purchasing power parity. Put differently, a typical individual in Tanzania has to work more than a month and a half to earn what his counterpart in the United States earns in a day. Differences in economic growth rates compounded over long periods of time account for these differences. Fortunately, endogenous growth theory suggests that there is something we can do about it.

One of the most important contributions of the "new" growth theory is the insight into the role of fiscal policy in long-run growth. In his seminal contribution, Barro (1990) argues that when the private rate of return of capital is lower than its social rate, optimal allocation calls for further capital accumulation. In this case, public investment becomes important for long-run growth. A vast theoretical literature on endogenous growth underscores the importance of fiscal policy, in the form of public capital flow and stock, for economic growth (e.g., Ziesemer, 1990, Futagami et al., 1993, Glomm and Ravikumar, 1994, 1997, Turnovsky, 1997, 2000, Agenor et al., 2008, Dioikitopoulos and Kalyvitis, 2010).

Existing empirical evidence is mixed, however, due to mainly methodological and model specification issues as well as due to differences in samples. Recent estimates of the elasticity of output with respect to public capital range from zero to a value that is higher than the output elasticity of private capital, for instance.[2] Fedderke and Bogetic (2009) presented five reasons for the contradictory empirical findings: the presence of nonlinearity; crowding out effect; endogeneity; an indirect or complementarity effect (rather than a direct productivity effect); or problems of aggregation. We address in this paper the first four of these reasons while providing a more comprehensive analysis of optimal public investment, with a focus on SSA countries.

The issue of the optimal level of public investment is under-researched for SSA, as much of the discussion in the literature has been on attracting private investment to this region. However, Foster and Briceno-Garmendia (2010) argue that countries in SSA lag behind their developing countries' peers in any measure of infrastructure. According to these authors, there are in particular significant differences among SSA and other low- and middle-income countries in terms of paved roads, telephone mainlines, and power generation, with SSA possessing less than four, seven and eight times the respective infrastructure units than their counterparts. The cost of infrastructure service in SSA is, furthermore, twice more expensive than elsewhere. In contrast, Devarajan et al. (2001, 2003) argue that most African countries have already public over-investment, probably the result of creating rent-seeking opportunities. This ambiguity is likely explained by the implied low 'quality' of public investment due to inefficient public allocation. However, as African governments seem to have improved governance in the more recent period, it is expected that higher quality would now accompany a given quantity of public investment.

Moreover, although the literature on the impact of public capital on economic growth has grown voluminous in the past few decades, only very few studies have addressed Africa (Ayogu, 2007). In particular, the issue of the growth-maximizing levels of public capital for African economies is yet to be addressed, as existing studies tend to employ linear models.[3] Nonlinear models have been applied to data from other parts of the world, however.[4]

The relevant question for policy is not only whether public capital is productive, that is, whether or not a unit increment on public capital stock increases output or growth, but also whether public capital is overall growth-enhancing given that it diverts resources from other activities (Romp and de Haan, 2007, Canning and Pedroni, 2008). The reason is that public capital can have a negative as well as a positive effect on the economy. Even though an adequate and efficient supply of public capital promotes output and growth, the burden resulting from financing it may have an adverse effect as well, such as the crowding-out of private capital. A highly enhanced transportation system, for instance, could improve the efficiency of trucks, but overly burdensome taxes to finance it could deter the accumulation of these trucks (Aschauer, 1998). Should the private sector not receive a net advantage from the infrastructure development, there would be no increase in output. It is this phenomenon that mainly gives rise to the nonlinearity between public capital and growth.

This paper first develops a simple endogenous growth model in an overlapping-generation framework. It then estimates the implied nonlinear relationship between public investment and economic growth, resulting from a positive infrastructure effect but a potentially negative taxation effect.[5] The growth-maximizing level of public investment is determined by applying nonlinear estimation techniques to dynamic panel data from SSA countries. Estimation of dynamic panel models with fixed effects gives consistent estimates, with only a weak bias when there is a sufficiently long time period. Given the relatively small sample in time dimension, we estimate the growth model using non-linear System GMM. In contrast, earlier studies that estimate the elasticity of output of public capital in nonlinear models usually apply simple calibration (e.g., Aschauer, 2000a, Miller and Tsoukis, 2001) or nonlinear least squares methods (e.g., Kamps, 2005), or simply use cross-country analysis, which runs the risk of taking into account only the short-term effects (see Glomm and Ravikumar 1997).

Limiting the growth impact of public investment to its direct effects, however, may provide a poor indicator of its importance in the economy. This is because public investment is likely to affect other important variables such as private investment. Moreover, policy-induced changes of growth may in turn influence population growth, for instance, with further implications for growth. The current paper, therefore, attempts to capture these indirect effects through formulating and estimating a system of difference equations that account for the mutual interaction among output growth, public and private investments and population growth.

In addition to estimating the growth equation, we regress public investment on private investment and conversely, in order to account for possible crowding-in (complementarity) and crowding-out effects. We also treat population growth endogenously. The resulting equations are estimated separately and also together as a system of simultaneous equations in order to account for possible correlation across equations. Finally, we run simulations in order to further examine the issue of policy optimality using coefficient estimates from both the separate- and simultaneous-equations models.

Among our findings is that public investment has a positive effect on growth. Perhaps more interestingly, the growth maximizing public investment/GDP ratio is estimated to be above 10.0 percent, which is larger than the mean observed value of no more than 7.26 percent at the end of the sample period.[6] Furthermore, from the policy simulation experiment, the sum of the discounted future consumption gain is maximized when there is a public investment share in GDP of between 8.1 and 9.6 percent.

We organize the rest of the paper as follows. In Section 2, we provide the theoretical model. Sections 3 and 4 present the empirical estimation and the simulations, respectively. Section 5 contains the conclusion.

2. Theoretical model

In neoclassical growth models, exogenous technical progress is the source of long-run growth, leaving no room for policy decisions to have long-term effects on economic growth. Therefore, a shock to the public policy variable will have a transitory effect on the economy, affecting only the level of (long-run) output. By contrast, in endogenous growth models, policies may have a lasting impact on growth rates. Hence, in these models, a shock to public capital may influence both the long-run growth rate and the output level.

In this section, we develop a simple endogenous growth model in an overlapping-generations framework where agents live two periods. The model captures the nonlinear relationship between public capital and growth that will form the basis for the empirical analysis in a later section of the paper.[7]

The model allows for the capital stock to be long-lasting. In contrast to standard models (see, for e.g., Barro, 1990, Futagami et. al., 1993, Glomm and Ravikumar, 1994, 1997, Turnovsky, 2000, 2004, Hashimzade and Myles, 2010), aggregate capital may depreciate nonlinearly. Capital is assumed to be heterogeneous, so that current investment may not add to the existing stock on a one-to-one basis.[8] The model also allows adjustment cost of capital in the spirit of Lucas and Prescott (1971), Basu (1987) and Basu et al. (2012).[9] It explicitly captures the nonlinear relationship between both the flow and the stock of public capital and economic growth, and their respective growth maximizing levels are derived.

2.1. The model

Consumers

We use an overlapping-generations model with logarithmic preferences and technologies of a representative agent, as in Glomm and Ravikumar (1997). When young, that is, during the first period of life, the individual is endowed with a unit of labor, which she supplies to the representative firm inelastically. Her income is equal to the wage income (w_t). The government taxes this income at a fixed flat rate tax (ψ), in order to finance public investment. The individual allocates after-tax income between current consumption (c_t) and saving (s_t^k). When old, she consumes (c_{t+1}) what she has saved in the previous period plus the after-tax return from saving.[10]

$$u\left(c_t, c_{t+1}\right) = \ln c_t + \beta \ln c_{t+1} \tag{1}$$

$$c_t + s_t^k = (1-\psi)w_t \tag{2}$$

$$c_{t+1} = \left(1 + r_t\left(1-\psi\right)\right) s_t^k \tag{3}$$

where r_t is the interest rate, net of the depreciation and the adjustment costs of capital. Private capital is accumulated according to the following equation, following Lucas and Prescott (1981), Basu (1987) and Basu et al. (2012),

$$k_{t+1} = (k_t)^{1-\kappa} \left(k_t (1-\delta) + s_t^k \right)^{\kappa} \tag{4}$$

where δ, κ and k_t represent the depreciation cost, the adjustment cost and the private capital stock, respectively. Therefore, the model explicitly allows installation cost for new investment and depreciation cost. When $\kappa = 0$, adjustment cost is too high to change both private and public capital. But when $\kappa = 1$, adjustment cost is zero, and capital stocks are accumulated according to the perpetual inventory method (e.g., $k_{t+1} = k_t (1-\delta) + s_t^k$). When $\kappa \in (0, 1)$, adjustment cost is different from zero. Current investment adds to the stock of capital after adjustment made for installation costs.

Government

The government budget is assumed to be always balanced and given by,

$$s_t^g = y_t \psi \tag{5}$$

where s_t^g and y_t, are public investment and aggregate income, respectively.

The public capital accumulation equation is given by,

$$G_{t+1} = (G_t)^{1-\kappa} (G_t (1-\delta) + s_t^g)^{\kappa} \tag{6}$$

Similar to (4), δ and κ are the depreciation rate and the adjustment cost associated to the public capital stock (G_t).[11]

Firms

The production function of the representative firm has the Cobb-Douglas form:

$$y_t = A\left(G_t\right)^{\alpha}\left(k_t\right)^{1-\alpha} \tag{7}$$

where y_t denotes output. As a simplifying assumption, labor is standardized to be unity ($l_t = 1$).

The firm maximizes profit within a competitive economy setting, taking prices and public capital as given,

$$\max_{k_t}\left\{A\left(G_t\right)^{\alpha}\left(k_t\right)^{1-\alpha} - w_t - R_t k_t\right\} \tag{8}$$

where R_t denotes the cost of capital, including a rental price for a unit of capital paid to households (r_t) and adjustment and depreciation costs. The first-order condition for profit maximization thus gives,

$$R_t = (1-\alpha)\, A\left(G_t\right)^{\alpha}\left(k_t\right)^{-\alpha} \tag{9}$$

And, the zero-profit condition in the competitive economy leads to the wage rate,

$$w_t = \alpha A\left(G_t\right)^{\alpha}\left(k_t\right)^{1-\alpha} \tag{10}$$

Competitive equilibrium

The representative household of period t solves the following problem, obtained by substituting (2) and (3) into (1),

$$\max_{s_t^k} \left\{ \ln\left((1-\psi)\, w_t - s_t^k \right) + \beta \ln\left(1 + (1-\psi) r_t\right) s_t^k \right\} \tag{11}$$

taking prices as given. The optimization yields,

$$s_t^k = (1-\psi) w_t \beta / (1+\beta) \tag{12}$$

Eq. (12) shows the agent's optimal saving as a function of her wage income. Dividing both sides by (y_t), and using (5) and (10), one obtains

$$s_t^k / y_t = (1 - s_t^g / y_t) \alpha \beta / (1+\beta) \tag{13}$$

Thus, eqs. (12) and (13) capture the crowding-out effect of the public variable through taxation. Using logarithmic preference and production functions and exogenous labour supply implies a non-distortionary transfer of income from the private to the public sector of the economy.[12]

Capital dynamics and growth

We get the dynamics of the private capital stock, first by substituting eq. (12) into eq. (4), and using (10),

$$k_{t+1} = k_t \left(1 - \delta + A(1-\psi) \chi \left(G_t / k_t\right)^{\alpha}\right)^{\kappa} \tag{14}$$

where $\chi \equiv \beta\alpha / (1+\beta)$.

The difference equation for the public capital stock is computed, by substituting (5) into

(6), and using (7), as:

$$G_{t+1} = G_t \left(1 - \delta + A\psi \left(G_t/k_t\right)^{\alpha-1}\right)^{\kappa} \tag{15}$$

Equations (14) and (15) characterize the dynamics of the economy during the transition. They explicitly demonstrate complementarities among public and private capital. On the other hand, (14) captures the crowding-out effect of public investment, through a negative relationship between taxation (ψ) and private capital accumulation (k_{t+1}).

From (14) and (15), we obtain the following difference equation for the public-private capital ratio,

$$G_{t+1}/k_{t+1} = \left(G_t/k_t\right) \left(\left(1 - \delta + A\psi \left(G_t/k_t\right)^{\alpha-1}\right) / \left(1 - \delta + A\chi(1-\psi) \left(G_t/k_t\right)^{\alpha}\right)\right)^{\kappa} \tag{16}$$

The log-linearized version of eq. (16) is shown to be stable in Appendix A.

On the balanced growth path, considering (16), the public-private capital stock ratio is constant:

$$G/k = \psi / \left((1-\psi)\chi\right) \tag{17}$$

Also, from (7), y/k is constant. Therefore, the capital stocks and output grow at the same rate γ_y:

$$\gamma_y \equiv \ln\left(G_{t+1}/G_t\right) = \ln\left(k_{t+1}/k_t\right) = \ln\left(y_{t+1}/y_t\right) \tag{18}$$

Growth maximizing public capital stock and flow

Using (14), γ_y is easily computed,

$$\gamma_y = \kappa \ln\left(1 - \delta + A\chi(1-\psi)\left(G_t/k_t\right)^{\alpha}\right) \tag{19}$$

Solving for ψ from (17) and substituting the result into (19), we obtain

$$\gamma_y = \kappa \ln\left(1 - \delta + A\chi\left(G/k\right)^{\alpha} / \left(\chi\left(G/k\right) + 1\right)\right) \tag{20}$$

Eq. (20) represents the growth rate of the economy as a function of the steady-state public-private capital *stock* ratio G/k. The last term captures the nonlinear relationship between economic growth and the public-private capital ratio.

The public-private capital stock ratio ($(G/k)^*$) that maximizes the growth rate (20) is,

$$(G/k)^* = (1+\beta) / (\beta(1-\alpha)) \tag{21}$$

With regard to the *flow* of public capital (public investment), we substitute (17) into (19), and use (5) to replace the tax rate, and obtain

$$\gamma_y = \kappa \ln\left(1 - \delta + A\chi^{1-\alpha}(1 - s_t^g/y)^{1-\alpha}\left(s_t^g/y\right)^{\alpha}\right) \tag{22}$$

Eq. (22) shows the growth rate of the economy as a function of the public investment-output ratio (s^g/y). Maximizing it with respect to s^g/y, we get the following familiar result,

$$(s^g/y)^* = \alpha \tag{23}$$

Therefore, (23) is the growth-maximizing productive government expenditure, which balances the negative taxation and the positive productive effects of public investment on the economy, as does the stock of public capital in eq. (20). This is also the optimal public investment when $\kappa = 1$ and $\delta = 1$ (see, for e.g., Barro, 1990 and Futagami et al., 1993).

Both (22) and (23) will be referred to in the next section for empirical estimation. Particularly, we estimate (22) with standard control variables and determine the optimal public investment ratio (23), using data from SSA. We then compare this result with the average of the actual public investment-GDP ratio that these countries have.

3. Estimation

This section empirically examines the nonlinear relationship between the *flow* of public capital (public investment) and output growth using panel data from SSA countries, as data on public capital stock are often limited and unreliable.[13] It also analyzes complementarities and crowding-out effects between public and private investment. We estimate not only the growth model of Section 2 but also a system of difference equations involving population growth and economic growth, as well as public investment and private investment. Estimations of equations are conducted both separately and simultaneously using nonlinear System GMM.

The first estimation equation is a growth equation, based on (22), that regresses per capita GDP growth on public investment and other control variables (lagged dependent variable, private investment, and population growth). The second and third estimation equations characterize the dynamics of the private and public capital flows. The fourth is a population growth equation that regresses population growth on lagged population growth and GDP per capita variables. These four equations constitute a system of macroeconomic dynamics that captures the mutual interaction among public investment, private investment, population growth and output growth.[14] The estimation of the growth equation yields the growth-maximizing level of public investment. We compare this estimate with the optimal level of public investment –, which is loosely defined as the constant level of public investment that maximizes the sum of discounted consumption gain –,[15] obtained from simulating the system of equations.

3.1. Data

The panel data used in the study cover 33 SSA countries, for the period 1967 to 2011.[16] The data for GDP per capita are obtained from PWT 8.0 (Feenstra et al., 2013) while the data for the public and private investment variables are extracted from the African Development Indicators (World Bank, 2012). Data for population and world income are from World Development Indicators (World Bank, 2012). Public investment includes only fixed capital investment by governments and non-financial enterprises. We use the output version of GDP panel data from PWT 8.0, which has no terms of trade effects (in contrast to the expenditure version) and is consistent with growth rates over changing benchmarking (Feenstra et al., 2013) . For world income, we use world GDP in constant 2005 US$.

Table 1 provides summary statistics, definitions and data sources of the variables used in the estimation. The average public investment of these countries over the sample period is 6.06 percent of real GDP but it is 7.26% at the end of the sample period, 2008 to 2011. The average growth rate of real GDP per capita is 0.6 percent for the sample periods. This rises to 2.3% by the end of the period.

TABLE 1 ABOUT HERE

3.2. Econometric Methods

We estimate the dynamic panel equations, first, separately and, second, together, as a simultaneous equations system using System GMM.[17] All methods include cross-section fixed effects (FE) and time dummies. As we are dealing with a dynamic problem we want to emphasize the time dimension rather than the cross-section dimension. We thus make three choices: First, we use yearly data rather than 5-year averages, making the time dimension longer.[18] Second, we use FE methods rather than OLS and random effects (RE), based on the Hausman test that rejects RE. Finally, we use system GMM, which adds an equation in first differences to one in levels thereby emphasizing growth rates rather than levels.

Although FE estimations of dynamic panel data are biased, the bias approaches zero for a large time-dimension sample size (Bond, 2002). As a general rule, this bias is of order $1/T$, where T represents time-dimension. Thus, it is sufficiently small for $T = 30$ or more (see, for e.g., Judson and Owen, 1999, Baltagi, 2008, Ch.8). In our data, the time dimension T is on average 25 years, based on the average of 848 observations for 33 countries, leading to a 4% bias in the coefficient of the lagged dependent variable. Hence the fixed effects estimates could suffer from a downward bias in the coefficient of the lagged dependent variable, which may in turn affect the coefficient of the public investment variable.

We, therefore, present nonlinear estimates based on the System GMM method. The System GMM version uses one equation in first differences with lagged levels as instruments and one within-groups estimator equation in levels using lagged first differences as instruments.[19] The coefficients of the two equations then are restricted to be the same for the level variables and their counterparts in the first difference equation. Alternatively, the first difference equation could be replaced by the Arellano and Bover (1995) method of orthogonal deviations. Implementation of non-linear items is more easily tractable in the first difference version of System GMM given the complexity of the orthogonal deviation model. On the other hand, the orthogonal deviation method has the advantage of losing fewer observations in case of missing values (Roodman, 2006).[20]

We apply the Durbin-Wu-Hausman test for the endogeneity of regressors other than the lagged dependent variable (see Appendix B). The basic principle is to run the first stage regressions, save the residuals and add them to the first stage regression. This is then estimated in least-squares, because adding residuals turns it into an IV regression according to the Frisch-Waugh-Lovell theorem. If the residuals are significant the corresponding variables are endogenous (see Wooldridge, 2002); if not they are exogenous or predetermined. We do the first stage regression for both equations of the system GMM method separately, applying the FE estimator with cross-section GLS weights. Then we add the saved residuals to the GMMSYS estimations and estimate them using the SUR method because the difference equation has a residuals difference, *u(0)-u(-1)*, and the level equation its first term. In the special case where the level equation endogeneity is rejected but the difference equation indicates endogeneity we can conclude that the level of the regressor is not correlated with its current residual and its lagged value not with that of the lagged residual. By implication the correlation of *x-x(-1)* with *u(0)-u(-1)* can stem either from a correlation of *x(-1)* with *u(0)*, implying forward endogeneity, or of *x* with *u(-1)*, implying that *x(-1)* is predetermined. In the latter case *x(-2)* should be used as an instrument (Baltagi, 2008). A similar procedure has been suggested by Yontcheva and Masud (2005).

Instruments should not only reduce the standard error but also increase the J-statistic through the impact on over-identifying constraints. On the other hand, the J-statistic should not go too high. By implication the p-value of the difference in the J-statistic should be reasonably far away from both zero and unity according to the difference in Sargan-Hansen test. However, in the final version we have only one instrument per regressor in line with Okui (2009) and no such test is necessary and the Hansen-Sargan J-statistic is close to zero (or its p-value is close to unity) because of absence of over-identifying constraints up to the number of constraints on the coefficients.

First-order serial correlation should be limited in general but is inevitable in the orthogonal deviation version of system GMM, which is thus not a problem. Second-order serial correlation should be limited in order not to undermine the effect of the instruments on the J-statistic. It is tested in terms of differences of the residuals. The Arellano-Bond test for the difference GMM is only valid for coefficients above 0.2 (see Roodman, 2009b). The crucial test then is the requirement that the Sargan-Hansen J-statistic should not be too high in order to be in the chi-square distribution and it should not be too low to have effective instruments. The corresponding p-values should not be close to zero or unity (Roodman, 2009a). However, when $1/T$ is small only a small correction is needed and the p-values may be close to unity.

Bun and Windmeijer (2010) have pointed out that the Monte-Carlo studies underlying the system GMM method have assumed a ratio of the variances of the fixed effects and the residuals of unity. If the ratio is much higher system GMM may produce a bias for the coefficient of the lagged dependent variable. Reporting this ratio is therefore a crucial ingredient, in justifying the use of system GMM.

Clemens and Bazzi (2009) show for the case of one additional endogenous regressor that the correlation of the residuals of the model equation and that of the endogenous regressors regressed on its own lagged should be low. As we have this case we also report that correlation (see Appendix B).

3.3. Separate equations estimations with System GMM

3.3.1. Growth equation

We now estimate the possible nonlinear relationship between the flow of public capital and growth using panel data from SSA countries based on equations from the model developed in Section 2. First, we employ eq. (22),[21] with standard control variables - lagged dependent variable, lagged private investment as a share of GDP, population growth rate and the world GDP[22] - to determine whether there exists a nonlinear relationship between public investment and growth. Then, we obtain an estimate for the output elasticity of public capital (α). Finally, we use the estimated value for α and eq. (23), in order to obtain the growth-maximizing rate of public investment, which can then be compared to the existing value of the panel average at the end of the period and results from a simulation analysis.

Rewriting (22) (with no adjustment cost and complete depreciation), including standard control variables and error terms, in a panel form, we have

$$\begin{aligned}\ln(y_{it}) &= a_1 \ln(y_{it-1}) + (1-\alpha)\ln\left(1-(s^g/y)_{it}\right) + \alpha \ln(s^g/y)_{it} \\ &\quad + a_2 \ln\left(s^k/y\right)_{it-1} + a_3\gamma_{p,it} + a_4\gamma_{p,it}^2 + a_5\gamma_{p,it}^3 + a_6\ln(wld) + e_i + \mu_t + u_{it} \qquad (24)\end{aligned}$$

where $\left(s^k/y\right)_{it-1}$, $\gamma_{p,it}$ and wld denote a one period lagged private investment-GDP ratio, population growth[23] and the world income, respectively. e_i, μ_t, and u_{it} are the fixed effects, the time dummy and the error terms, respectively. Eq. (24) thus shows a dynamic panel data model, where we have rewritten (22) with growth expressed difference in log income levels and have specified control variables explicitly.[24]

We first estimate (24) separately using the first difference approach to System GMM.[25] The result is as follows (t-values in parentheses):

$$\begin{aligned}\ln(y_{it}) &= \underset{(72.8)}{.97}\ln(y_{it-1}) + \underset{(14.5)}{.8983}\ln\left(1-(s^g/y)_{it}\right) + \underset{(14.5)}{.1017}\ln(s^g/y)_{it} + \underset{(6.7)}{.066}\ln\left(s^k/y\right)_{it-1} \\ &\quad + \underset{(3.2)}{3.47}\gamma_{p,it} - \underset{(-5.5)}{70.1}\gamma_{p,it}^2 + \underset{(3.5)}{355.5}\gamma_{p,it}^3 + \underset{(4.5)}{.059}\ln(wld) + e_i + \mu_t + u_{it} \qquad (25)\end{aligned}$$

The coefficient for the lagged dependent variable is significant, and at 0.97 it indicates the persistence of output. The fact that the estimated coefficient of the lagged dependent value does not differ significantly from unity suggests that the theoretical model is reasonable for our SSA sample.[26] The coefficient of the world income variable is also positive and significant. The nonlinear coefficient estimate of public investment, – the growth maximizing level of public investment as denoted by α in the theoretical model, – is thus estimated at 0.102, with standard error 0.007. This result suggests, then, the need to increase public investment, as percent of GDP, from its 7.26 percent level at the end of the sample period, for a growth maximizing policy. The coefficient for private investment share is 0.066 percent and is also significant. The population growth rate is significant (in all of its form) and has an asymmetric inverted u-shape with a maximum at 3.3%, a value achieved around $1983-85$. Note that our percent estimate of the optimal level of public investment is, in general, smaller than most of those in the recent literature (see Section 1).

3.3.2. Private investment equation

Public investment is believed to have both complementary and crowding-out effects on private investment. In the growth model, eqs. (12) and (13) show that public investment crowds out private investment. Eqs. (14) and (15), on the other hand, capture complementarities between the stock variables.[27]

Our second estimation equation is a regression of private investment on public investment, both as shares of GDP. We set up the model intended to empirically determine the net effects of crowding-in and crowding-out of public investment. We include six year lagged public investment in a cubic specification.[28]

For estimation, we use System GMM in its orthogonal deviation variant by Arellano and Bover (1995):

$$\begin{aligned}\ln\left(s^k/y\right)_{it} = {} & \underset{(11.9)}{.59}\ln\left(s^k/y\right)_{it-1} + \underset{3.83}{.17}\ln\left(s^k/y\right)_{it-2} + \underset{(1.3)}{.2}\left(\gamma_y\right)_{it-1} - \underset{(-2.1)}{.14}\left(\ln\left(s^g/y\right)_{it-6}\right) \\ & + \underset{(2.9)}{.21}\left(\ln\left(s^g/y\right)_{it-6}\right)^2 - \underset{(-2.56)}{.054}\left(\ln\left(s^g/y\right)_{it-6}\right)^3 + e_i + \mu_t + u_{it} \qquad (26)\end{aligned}$$

The first term of equation (26) denotes lagged private investment while the second represents two-periods lagged private investment. The third term, one-period lagged GDP per capita growth rate, captures the accelerator mechanism; higher lagged growth is expected to lead to a higher level of current investment. The fourth to sixth term is the cubic public investment variable having a maximum at 9.5%. This value is fairly close to that of the growth equation.[29]

3.3.3. Public investment equation

Our third estimation equation treats public investment as the dependent variable where the lags of public, changes in private investment and growth rates are the independent variables. This formulation is in considers policy responses that policy makers often react to changes in macroeconomic variables. For instance, an increase in private investment or stronger growth may lead to a change in public investment policy:

$$
\begin{aligned}
\ln\left(s^g/y\right)_{it} &= \underset{(13.5)}{.8}\ln\left(s^g/y\right)_{it-1} + \underset{(3.3)}{.076}\left(\ln\left(s^k/y\right)_{it-1} - \ln\left(s^k/y\right)_{it-2}\right) \\
&\quad + \underset{(4.76)}{1.55}\left(\gamma_y\right)_{it-2} + e_i + \mu_t + u_{it} \qquad (27)
\end{aligned}
$$

We estimate equation (27) using the orthogonal deviation method of Arellano and Bover (1995) for System GMM. Government action is self-perpetuating, as indicated by the coefficient for the lagged dependent variable of 0.8. Changes in lagged private investment have a net positive effect on public investment. Finally, current GDP per capita growth has a positive impact as it could increase the revenues available for public spending.

3.3.4. Population growth equation

Our fourth estimation equation is a population growth equation. Recall that we want to run simulations of a system that characterizes the macroeconomic dynamics of the economy in order to further examine the optimal public investment, and also analyze its effects on the economy. So far we have three equations (eqs. (24), (26) and (27)) but four endogenous variables (income, public and private investment and population growth).

The fourth equation is:

$$
\begin{aligned}
\gamma_{p,it} &= \underset{(2.2)}{.003} + \underset{(21.3)}{2.29}\gamma_{p,it-1} \underset{(-5.2)}{-1.6}\gamma_{p,it-2} - \underset{(-0.75)}{.28}\gamma_{p,it-3} + \underset{(4.02)}{1.03}\gamma_{p,it-4} \underset{(-6.3)}{-8}\gamma_{p,it-5} + \underset{(5.0)}{.73}\gamma_{p,it-6} \\
&\quad - \underset{(-4.1)}{.55}\gamma_{p,it-7} \underset{(3.55)}{+.17}\gamma_{p,it-8} \underset{(2.05)}{+.0013}\ln y_{it-4} \underset{(-2.24)}{-.0014}\ln y_{it-5} + e_i + \mu_t + u_{it} \qquad (28)
\end{aligned}
$$

The data used for estimating (28) have more than thirty observations per country. Thus, the FE bias is sufficiently small. Therefore, we estimate it with fixed effects, using lagged levels as instruments, while taking into account the period-SUR version of panel corrected standard errors (PCSE) similar to (26) and (27).[30] The coefficients of lagged values of population growth sum up to about 0.95. The adjusted R^2 is 0.98. The sum of all lagged income coefficients is negative in line with the standard demographic transition.

3.4. A simultaneous equation system with System GMM

We estimate eqs. (24), (26), (27) and (28) as a simultaneous equations system as well.[31] Using System GMM enables us to deal with both endogeneity and contemporaneous correlation. We set up the system in which we write each of the first three equations as a System GMM estimator model, once in first-differences and once in levels, subtracting the country-specific averages of each variable (the within estimator). The fourth equation is written only in levels as a within-groups estimator (fixed-effects) model. This approach combines the strength of the SUR estimator, taking into account relations between the residuals of the equations, and that of the System GMM estimator, taking into account fixed effects and endogeneity without imposing a normality assumption on the residuals. The following are the results from the simultaneous estimations (t-ratios in parentheses):

$$\begin{aligned}
\ln(y_{it}) &= \underset{(96.5)}{.97}\ln(y_{it-1}) + \underset{(18.7)}{.898}\ln(1-(s^g/y)_{it}) + \underset{(18.7)}{.103}\ln(s^g/y)_{it} + \underset{(9.9)}{.069}\ln\left(s^k/y\right)_{it-1} \\
&\quad + \underset{(4.4)}{3.55}\gamma_{p,it} - \underset{(-6.2)}{71}\gamma_{p,it}^2 + \underset{(3.8)}{349}\gamma_{p,it}^3 + \underset{(6.6)}{.062}\ln(wld) + e_i + \mu_t + u_{it} \qquad (29)\\
\ln\left(s^k/y\right)_{it} &= \underset{(17.6)}{.47}\ln\left(s^k/y\right)_{it-1} + \underset{(16.4)}{.24}\ln\left(s^k/y\right)_{it-2} + \underset{(3.2)}{.16}\left(\gamma_y\right)_{it-1} + \underset{(8.0)}{.15}\left(\ln(s^g/y)_{it-6}\right) \\
&\quad + \underset{(5.8)}{.025}\left(\ln(s^g/y)_{it-6}\right)^2 - \underset{(-9.6)}{.027}\left(\ln(s^g/y)_{it-6}\right)^3 + e_i + \mu_t + u_{it} \qquad (30)\\
\ln(s^g/y)_{it} &= \underset{(32.5)}{.63}\ln(s^g/y)_{it-1} + \underset{(9.66)}{.115}\left(\ln\left(s^k/y\right)_{it-1} - \ln\left(s^k/y\right)_{it-2}\right) + \underset{(5.6)}{.24}\left(\gamma_y\right)_{it-2} \\
&\quad + e_i + \mu_t + u_{it} \qquad (31)\\
\gamma_{p,it} &= \underset{(43.4)}{2.88}\gamma_{p,it-1} \underset{(-18.4)}{-3.37}\gamma_{p,it-2} + \underset{(9.1)}{1.8}\gamma_{p,it-3} - \underset{(-3.1)}{.32}\gamma_{p,it-4} - \underset{(-2.2)}{.049}\gamma_{p,it-5} \\
&\quad - \underset{(-2.8)}{.0004}\ln y_{it-2} + \underset{(2.5)}{.0005}\ln y_{it-3} - \underset{(-2.5)}{.00049}\ln y_{it-6} + e_i + \mu_t + u_{it} \qquad (32)
\end{aligned}$$

Across the two approaches (of separate and simultaneous), coefficients have the same sign except where collinearity is prominent in the cubic polynomials of public investment, in the private investment equations and the lags of the population growth equation. They differ slightly in magnitude, though; and, the coefficient of the growth rate in the public investment equation differs strongly. Significance (as measured by $t-$values) is higher throughout in the simultaneous equation estimate as is usually the case when using SUR or GMM methods in systems. The two insignificant regressors of the separate estimation – growth in the private investment equation and a lag in the population growth equation – become significant now. In the separate estimation significance is basically always better than the 5% level. In the simultaneous estimation it is always better than the 3% level and mostly zero until the fourth digit. Comparing the GMM estimates, in the growth equation, most coefficients are larger in absolute terms in the simultaneous GMM estimations. In the private investment equation, the opposite is the case (ignoring the difference in the public investment polynomial).[32]

4. Simulations

In this section, we simulate the system of four equations and conduct policy experiments in order to determine the public investment GDP share that maximizes sum of discounted consumption gain and assess the effects on investment, net income, and consumption. Initial values are constructed from regressing the variables on linear-quadratic time trends, in the first five-to-ten-year period. First, we simulate a benchmark economy with values that (roughly) match with the panel average of real economies of SSA, particularly during the end of the sample period.[33] Then, we examine the effect of an increase to a certain constant level of public investment.

4.1. The benchmark economy

The result of the benchmark simulation is shown in Figure 1. Population growth first increases and then decreases, consistent with demographic transition. The GDP per capita growth rate has strong ups and downs captured by time dummies. It has a peak in 1978 just before the second oil crisis and a highly negative value in 1983 through the Latin American debt crisis, both of which hit SSA severely and led to a "lost decade" (Greene, 1989, Humphreys and Underwood, 1989). During the 1982 crisis public and private investment grow more quickly than GDP and therefore both investment GDP ratios have a small peak. Part of it goes only into the residuals of our equations because the actual growth rates were slightly lower during the 1982 crisis. After the crisis, growth resumes (with ups and downs), and more strongly so after 1990.

FIGURE 1 OVER HERE

The public and private investment/GDP shares are about 6.9 and 10.5 percent, respectively, at the end of the simulation period based on the simultaneous-equations estimation. When using the estimates from the separate regressions, the public and private investment/GDP shares increase to 10 and 11 percent, respectively.[34] For all variables the simulation values at the end of the sample period are quite close to those of the actual panel average for 2008-2011 presented in the last column of Table 1.

4.2. Counterfactual analysis: Is public investment optimal in SSA countries?

From Table 1, the actual panel-average of public investment is about 6.1 percent of real GDP. At the end of the sample period, $2008 - 2011$, the value is 7.26 percent. Meanwhile the level of public investment that maximizes the growth rate from the nonlinear growth regressions, is 10.2 percent of GDP. These results imply that on average the public investment share of output in SSA countries is sub-optimal.

From the simulation and policy experiment, the constant level of the public investment that maximizes the discounted sum of per capita consumption gain until 2050 is 8.4%, at three different discount rates, using estimates from the simultaneous equations regression (Table 2). Figure 2 plots the effects from increasing public investment to this optimal level. While public investment increases by about 20% from its benchmark value, consumption increases, from its benchmark value, by about 3.5%. However, there is a slight decline in private investment due to taxation and crowding-out effects.

TABLE 2 OVER HERE

FIGURE 2 OVER HERE

4.2.1. Sensitivity analysis

We also run the simulation using the estimates from the separate equations estimation. In this case, on average about 9.3 percent of public investment (as a share of GDP) maximizes the sum of discounted per capita consumption gain until the year 2050 (see Table 2). The simulation results are shown in Figure 3. Public investment first goes down but picks up later on. This is followed by a consumption gain, at first, due to a reduction in tax but, later on, due to an increase in income, which, in turn, increases due to an increase in public investment.

FIGURE 3 OVER HERE

The difference in the simulations' outcomes is apparently due to differences in the estimates of the variables, which in turn depend on the estimation methods employed. Both methods have their own merits. The advantages of the simultaneous equations estimation vis-à-vis the separate is similar to that of a SUR estimation. It takes into account the contemporaneous correlation. However, in general, the orthogonal deviation method used in the separate equation estimation has the advantage of losing fewer observations than first-differences. But this is less important in our case, as there are hardly missing observations in the sample.

However, note that, although the values for optimal public investment (Table 2) differ from each other to some extent, they are all larger than the typically observed value of 7.26 percent.[35] In addition, they are much smaller than the values, which were reported, by earlier works, for other areas. For instance, Aschauer's (2000a) estimate of the growth maximizing level of public capital for the US is about 30 percent; Miller and Tsoukis's (2001) for a wide range of low and middle income countries is 18 percent; Kamps's (2005) for European and OECD countries is 20 percent. On the other hand, they are quite close to Luoto's (2011) 10 percent estimate for Finland.

5. Conclusion

Economists have long acknowledged the importance of public investment. Many believe public investment enhances productivity and complements private investment, with a positive impact on long-run growth and welfare. Others argue that the higher taxation, for instance, resulting from the larger public investment, lowers growth and welfare as it distorts private saving and efforts, thus crowding out private investment. Hence, the relationship between long-run growth and public investment could be non-monotonic, with the likelihood of an optimal level of public investment.

The present paper first developed an endogenous growth model that posited nonlinearity in the public capital and growth relationship in SSA countries. Using panel data from SSA countries, from 1967 to 2011, and applying various econometric techniques, the paper identified the growth-maximizing level of public investment in the region. It found that not only does public investment highly matter for economic growth but also that the current level prevailing in SSA is, on average, sub-optimal. Applying separate and simultaneous equations estimation methodologies, we found growth maximizing public investment GDP share of about 10.2 percent.

An important aspect of public investment is its indirect impact on growth through private investment, and conversely. To shed light on this phenomenon, we formulated a system of difference equations that captured the relationships among output growth, public and private investment and population growth, and conducted estimation both separately and simultaneously. Both complementarities and crowding-out effects were detected between public and private investments while accelerator and net complementarity effects were found to be stronger under the simultaneous equations estimation. Applying the estimates from these regressions we then ran simulations to determine the level of public investment that maximizes the sum of discounted per capita consumption gain. The optimal value was computed to be between 8.1 percent and 9.6 percent, when using discount rates ranging from 4 percent to 12 percent, and various econometric techniques, respectively. All estimates are larger than the observed value of 7.26 percent at the end of the sample period. The present findings are, therefore, not in concert with the previous finding of public over-investment in the region (e.g., Devarajan et al., 2001, 2003). Our estimates are, nevertheless, generally much lower than those for other regions and country groups.

Appendix

A. Stability of the capital ratio dynamics

To examine the stability of (16), first rewrite it, using (17), as:

$$\begin{aligned}&(G_{t+1}/k_{t+1})^{\frac{1}{\kappa}}(1-\delta)/(A\psi)+(G_{t+1}/k_{t+1})^{\frac{1}{\kappa}}(G_t/k_t)^{\alpha}(G/k)^{-1}\\ =\;&(1-\delta)/(A\psi)(G_t/k_t)^{\frac{1}{\kappa}}+(G_t/k_t)^{\alpha-1+\frac{1}{\kappa}}\end{aligned}\tag{A.1}$$

Then, log-linearize (A.1) near the steady-state capital ratio (G/k), (see Novales, et al. 2009), to obtain

$$z_{t+1}\approx\Theta z_t\tag{A.2}$$

where $z_t\equiv\ln(G_t/k_t)-\ln(G/k)$ and

$$\Theta\equiv 1-\frac{(\chi(1-\psi)/\psi)^{1-\alpha}}{\frac{1}{\kappa}\left((1-\delta)/(A\psi)+(\chi(1-\psi)/\psi)^{1-\alpha}\right)}\tag{A.3}$$

Thus, the root of the log-linearized eq. (A.2) is stable as long as $0<\Theta<1$, which is the case since the denominator of the second term of (A.3) is greater than the nominator while both are positive.

B. Endogeneity and GMMSYS

B.1. Durbin-Wu-Hausman test

The results for the Durbin-Wu-Hausman regarding endogeneity described in the econometrics section of the main text are presented in Table B.1.

TABLE B.1. ABOUT HERE

Note that we cannot rely on the normality assumption we have to be a bit flexible here. In particular, with a GDP variable in the denominator exogeneity is not very plausible. Assuming exogeneity leads to a public investment coefficient of 0.13.

B.2. Bias comparison: GMMSYS and FELS (Fixed effects least squares)

Table B.2 shows the coefficients of the lagged dependent variables for GMMSYS and FELS. In all cases, the standard result that FELS underestimates the coefficients is confirmed. In the simultaneous estimation case, the difference is larger than the usual 1/T bias for the investment equations. For the other equations, it is in this order of magnitude and the difference could be due to having other exogenous and endogenous or predetermined regressors (see Bruno, 2005 and Clemens and Bazzi, 2009, respectively). If we use the SUR method in simultaneous equation estimation the coefficients are equal to or larger than those of GMMSYS. This point may be worth further (econometric) research.

TABLE B.2. ABOUT HERE

B.3. GMM properties for separately estimated equations

The issues discussed in this section are only raised for single equation estimation in the econometric literature. Therefore we discuss them only for the separate estimation case. The information related to discussions of GMM is provided in Tables B.2 and B.3 below. The bias is corrected at a reasonable percentage close to 1/T, which is 4% for the growth and public investment regressions and 5% for the private investment equation, respectively (Table B.2).

A mild negative first-order serial correlation is still present in the public investment equation, but it is too small to cause a bias. The standard remedy of adding more lags of the dependent variable is not applied here as it increases the standard error of the regression. If we add the residual of the regression as in the Breusch-Godfrey test and carry out a GMM-SYS regression, the result for serial correlation of the growth equation is even stronger and for the two investment equations becomes the opposite in sign. However, as Roodman (2009a, 2009b) point out, under orthogonal deviations (which is used in the investment equations) first-order serial correlation is built in automatically and inevitably and, therefore, there should not be a problem econometrically.

The absence of second-order serial correlation, which is important for the effectiveness of the instruments, can be assured at least at the 5% level (Table B.3). The variance ratio, the square of the standard deviation of the fixed effects and the standard error of the estimation is not unity as in the Monte Carlo studies, but rather below it. According to Bun and Windmeijer (2010), this is not critical whereas values above two are problematic in the absence of other regressors as they might cause a bias in the coefficient of the lagged dependent variables.

TABLE B.3. ABOUT HERE

The endogenous or predetermined regressors of our growth equation are represented through investment and population equations. Endogeneity should be weak and system GMM works better if the correlation of the residuals of all these equations are low (Clemens and Bazzi, 2009). As shown in Table B.4, the correlation coefficients are very low indeed, which indicates that the endogeneity is not overly strong.

TABLE B.4. ABOUT HERE

B.4. Instruments used and more

B.4.1. Separate estimation

- Growth, difference equation: Obs.: 831
  - LNRGDPCH(-2), LOG(1-GPBI(-1)/100), LOG(GPBI(-1)/100), LNGPRI(-2), C, (D(LNPOP))^2, D(LNPOP), (D(LNPOP))^3, LOG(WLD(-2)), time dummies 1967-2009 (other redundant).[36]
- Growth, level equation: Obs.: 787

  - D(LNRGDPCH(-5)), D(LOG(1-GPBI(-1)/100)), D(LOG(GPBI(-1)/100)) D(LNGPRI(-2)), D((D(LNPOP(-6)))^2), D((D(LNPOP(-6)))), D((D(LNPOP(-6)))^3), C, D(LOG(WLD(-2))), time dummies 1968-2008 (others redundant).

- Private investment equation (orthogonal deviations; first and last lag indicated): Obs.: 653
  - (LOG(GPRI),-2,-3), (LOG(GPBI(-7))), (LOG(GPBI(-7)))^2, (LOG(GPBI(-7)))^3, (D(LNRGDPCH),-2,-2), c, time dummies.
- Public investment equation (orthogonal deviations; first and last lag indicated): Obs.: 828
  - (LOG(GPBI),-4,-4), LOG(GPRI(-1))-LOG(GPRI(-2)), D(LNRGDPCH(-2)), time dummies, c.
- Population growth equation: Obs.: 1407
  - None (least squares with time and cross-section dummies).

B.4.2. Simultaneous estimation

(J-statistic 0.01985; ivs L= 328, coefficients K=189, L- K=139, p(nJ)=1)

- Growth, difference equation: Obs.: 831; S.E. of regression: 0.147228
  - LNRGDPCH(-5), LOG(1-GPBI(-1)/100), LOG(GPBI(-1)/100), LNGPRI(-2), C, (D(LNPOP))^2, D(LNPOP), (D(LNPOP))^3, LOG(WLD(-5)), time dummies 1967-2008.
- Growth, level equations: Obs.: 787; S.E. of regression: 0.109108
  - D(LNRGDPCH(-5)), D(LOG(1-GPBI(-1)/100)), D(LOG(GPBI(-1)/100)), D(LNGPRI(-2)), D((D(LNPOP(-6)))^2), D((D(LNPOP(-6)))), D((D(LNPOP(-6)))^3), C, D(LOG(WLD(-2))), times dummies 1968-2008.

- Private investment, difference equation: Obs.: 644; S.E. of regression: 0.432295
  - LNGPRI(-2), LNGPRI(-3), LOG(GPBI(-7)), LOG(GPBI(-7))^2, LOG(GPBI(-7))^3, D(LNRGDPCH(-2)), time dummies 1972-2010.
- Private investment, level equation: Obs.: 600; S.E. of regression: 0.329645
  - D(LNGPRI(-2)), D(LNGPRI(-3)), D(LOG(GPBI(-7))), D(LOG(GPBI(-7))^2), D(LOG(GPBI(-7))^3), D(D(LNRGDPCH(-2))), time dummies 1973-2010.
- Public investment, difference equation: Obs.: 787; S.E. of regression: 0.440612
  - LOG(GPBI(-2)), LNGPRI(-1)-LNGPRI(-2), D(LNRGDPCH(-1)), time dummies 1968-2010
- Public investment, level equation: Obs.: 792; S.E. of regression: 0.330517
  - D(LOG(GPBI(-2))), LNGPRI(-1)-LNGPRI(-2), D(D(LNRGDPCH(-1))), time dummies 1968-2010.
- Population growth equation: Obs.: 1464; S.E. of regression: 0.000872
  - D(LNPOP(-1)), D(LNPOP(-2)), D(LNPOP(-3)), D(LNPOP(-4)), D(LNPOP(-5)), LOG(RGDPCH(-2)), LOG(RGDPCH(-3)), LOG(RGDPCH(-6)), time dummies 1967-2010, C.

B.5. Autoregressive process for world income

The autoregressive process for world income, which we use for the simulation is:

$$\begin{aligned} \ln(wld) &= \underset{(3.73)}{3.38} + \underset{(8.2)}{1.1} \ln\left(wld(-1)\right) - \underset{(-2.5)}{0.47} \ln\left(wld(-2)\right) \\ &\quad + \underset{(1.9)}{0.24} \ln\left(wld(-3)\right) + \underset{(3.03)}{0.003t} \end{aligned}$$

The adjusted R^2 is 0.999. The Durbin-Watson is 2.02 and gets worse when adding the fifth and six lags, (the fourth one is insignificant). Also, we use only three lags in this auxiliary regression since adding a moving average term could make only a 2% difference in simulations, in hundreds years.[37]

Notes

[1]Based on Penn World Table 6.3 (Heston et al., 2009).

[2]For instance, using cross country data, Canning (1999), Aschauer (2000b), and Demetriades and Mamuneas (2000) estimate elasticity of output for public capital to be as large as that for private capital; though Miller and Tsoukis (2001) and Kamps (2005) estimate lower values for public capital. On the other hand, Milbourne et al. (2003) report insignificant effects of public investment. Using country specific data, Everaert and Heylen (2004) and Fedderke et al. (2006) estimate elasticity values of public capital, for Belgium and South Africa, respectively, from 0.3 to 0.5. Luoto (2011) estimates about 0.1 for Finland.

[3]For instance, Fedderke et al. (2006) and Fedderke and Bogetic (2009) study the impact of infrastructure on growth for South Africa, and Ayogu (1999) for Nigeria. Calderon and Serven (2008) and Calderon (2009), using large panel data sets covering over 100 countries, examine the relationship between infrastructure assets and growth in SSA; Estache et al. (2005), applying an augmented Solow model with infrastructure variables, and using pooled OLS study the relationship for 41 SSA countries. Boopen (2006) studies the impact of transport infrastructure on economic growth for SSA countries, using a dynamic panel model of Difference GMM method. Devarajan et al. (2001, 2003) examine the productivity of public investment in Africa employing 2SLS for a cross section of countries.

[4]For example, Aschauer (2000a) and Kamps (2005) examined the optimality of public capital in the United States and European countries, respectively, while Miller and Tsoukis (2001) was on a set of low and middle-income countries. Kalaitzidakis and Tzouvelekas (2011) studied a set of developing and developed countries with a particular emphasis on military spending.

[5]The main purpose of the theoretical model is to capture the non-linearity between public investment and growth. Thus the model may be viewed as a simplifying one; the detailed variables are subsequently specified under the empirical modelling.

[6]For the entire period, the observed mean value for public investment/GDP share is lower than 6.1 percent.

[7]See Kocherlakota and Yi (1996), Aschauer (2000a), Bleaney, et al. (2001) and Kalaitzidakis and Tzouvelekas (2011) for the application of endogenous growth models.

[8]For instance, in the case of public capital, the existing aggregate capital stock consists of past investment in electricity, telecommunication, roads, etc.

[9]However, these bodies of literature do not focus on public capital and growth.

[10]The model is kept simple for sake of tractability and technicality. For instance, population growth is set to zero, as it could result in scaling effects in growth, though the general results hold under nonzero population growth. The applications of log-linear preference and production function, and fixed flat-rate taxes on income and capital (in contrast to alternative financing methods) also serve to obtain a tractable solution. See Angyridis (2014) for models with progressive taxation and heterogenous agents.

[11]We set similar technological parameters for public and private capital in order to avoid unnecessarily complicating the model.

[12]In a logarithmic utility function, the inter-temporal elasticity of substitution is unity, and consequently the income effect of a wage increase exactly compensates the substitution effect (de la Croix and Michel, 2002, p. 13–14). We abstract from other tax distortions while we keep the model simple, for our purpose.

[13]Construction of public capital stock data depends on rather arbitrary assumptions about depreciation and initial capital stock.

[14]We focus on these variables due to, both, their particular importance in growth theories and more readily available data. While the GMM estimation properly accounts for any time or country specific unobserved heterogeneity, we don't expect major institutional differences (unlike the case of developing countries versus OECD countries, for instance) within the current sample of African countries.

[15]The standard definition of optimality is the maximization of the sum of discounted utility given the resource constraints in the economy. But, we estimate here the constant level of public investment that maximizes the sum of discounted consumption gains due to its simplicity and better applicability to panel of countries.

[16]Countries are included in the study based on the availability of relatively reliable data. These are: Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Congo, Dem. Rep., Congo, Rep., Cote d'Ivoire, Equatorial Guinea, Ethiopia, The Gambia, Ghana, Guinea, Kenya, Malawi, Mali, Mauritania, Mozambique, Namibia, Niger, Rwanda, Senegal, Sierra Leone, South Africa, Sudan, Tanzania, Togo, Uganda, Zambia, and Zimbabwe.

[17]In the working paper version of the paper, we compared the results to weighted least square (WLS) and seemingly unrelated regression (SUR).

[18]See also Attanasio et al. (2000) for the advantages of annual data (vis-a-vis n-year averages), such as providing information that is lost when averaging.

[19]All regressors are thus treated as endogenous, which are instrumented internally using their own lagged

values. Alternatively, external instruments could also be applied (e.g., Dincecco and Prado, 2012), depending on data availability and the estimation method used.

[20]We add time dummies to deal with cross-sectional dependence (see Roodman, 2009b, Smith and Fuertes, 2010).

[21]We only consider the case when there is complete depreciation of capital and zero adjustment cost, $\kappa = 1$ and $\delta = 1$.

[22]One may also use a time trend to get similar results. However, note that both population growth and world income growth show similar downward trends. Thus, omitting the latter may yield a bias in the coefficient of the former.

[23]Population growth is highly significant in its nonlinear forms.

[24]Absence of the control variables (and $a_1 = 1$), (24) reduces to the special case (22) with $\kappa = 1$ and $\delta = 1$. We see below the coefficient estimate of a_1 is close to unity.

[25]We use GMM-HAC (GMM heteroscedasticity and autocorrelation consistent standard errors). The HAC mechanism uses the Bartlett kernel and the variable bandwidth of Newey-West.

[26]Note that one can rewrite (22) as having a level variable on the left-hand side and a unit coefficient of a lagged dependent variable on the right-hand side.

[27]By construction, eq. (7) implies that increasing public capital (for a given private capital) enhances the productivity of private capital and conversely.

[28]This is in line with the fact that firms often need sufficient time to relocate to new roads and railways, for instance, in their reinvestment decisions. An interacting variable (between public and private investment) turns out not to be significant in this longer data set. The growth model (in Section 2) does not feature such phenomena due to the particular production functional form adopted. However, note that the application of standard production functions is justified technically, as they are well-behaved and, often, provide tractable solutions.

[29]Cavallo and Daude (2011) find a negative effect of public investment, but they do not use any non-linear specification.

[30]We use the residuals from regression of equation (28) to run panel unit root tests. The unit root hypothesis is always rejected, indicating co-integration of the variables in the equation. Using the Breusch-Godfrey test for serial correlation in the presence of endogeneity, we have also re-run the regression with lagged residuals added to the regression. The lagged residuals turn out to be insignificant, indicating an absence of serial correlation and of the corresponding potential bias in the coefficients.

[31]A world income equation is not part of the simultaneous equation estimation as it leads to a ‘near singular matrix’ result. However, a separately estimated auxiliary equation for the world income growth is

used for the simulations. See Appendix B.5.

[32]Standard errors of regression are reported in Appendix B.

[33]The simulation starts in 1960 when the earliest data are available for the estimation of the quadratic time trends.

[34]There is no steady state as population growth rate keeps changing.

[35]Even at a discount rate of 12 percent the optimal public investment rate in both simulations is at least 8.4 percent, which is still higher than the actual end-of-panel average value.

[36]The variables RGDPCH, GPBI, GPRI, POP, WLD stand for real GDP per capita level, public investment/GDP percentage, private investment/GDP percentage, population, and world income respectively.

[37]Note that more observations are lost from the relatively good 1960*s*, thus giving more weight to the weak growth years afterwards.

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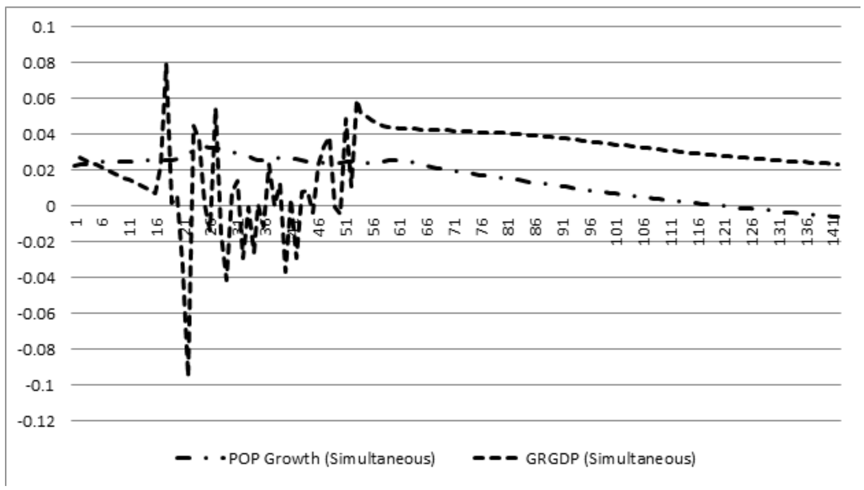
Tables and Figures

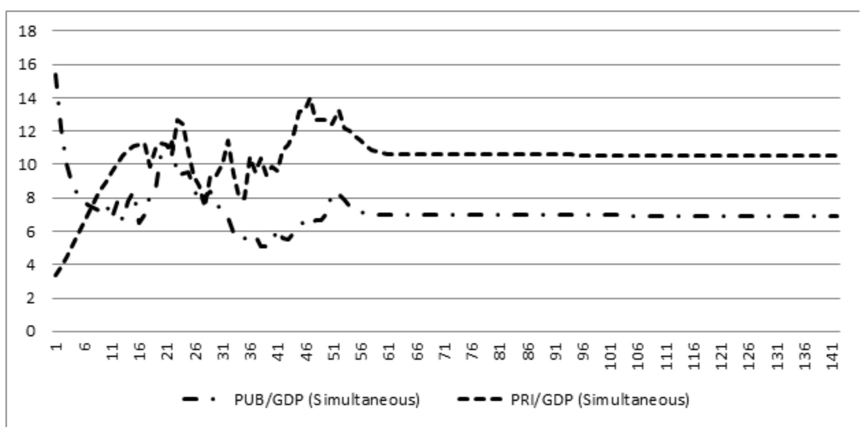
(A. K. Fosu)

Figure 1a. Benchmark simulation, using estimates from the Simultaneous equations estimation

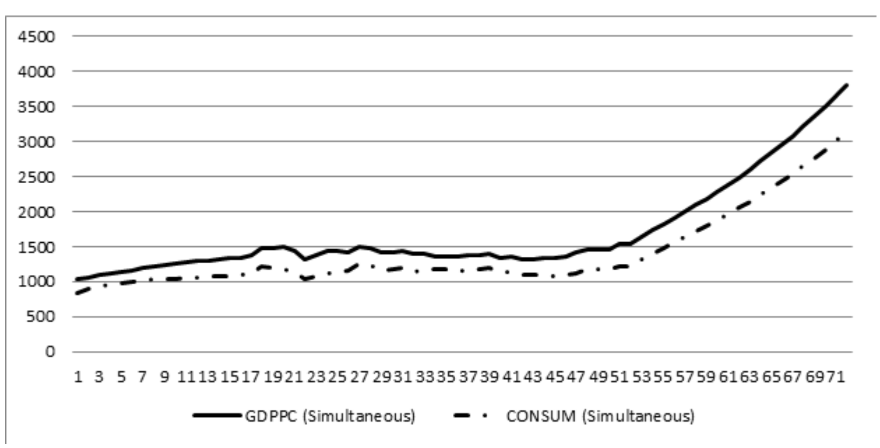
(i) Population and GDP per capita growth rates[a]



(ii) Public and private investment (%GDP)[a]



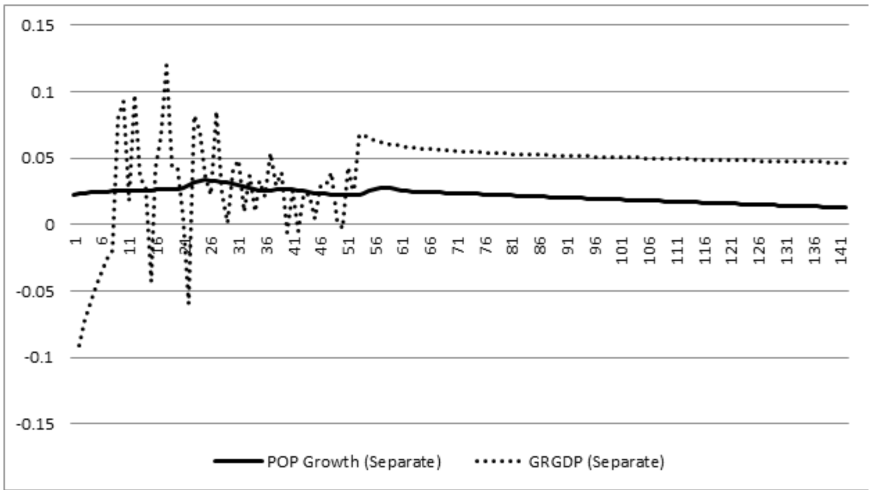
(iii) GDP and consumption per capita[b]



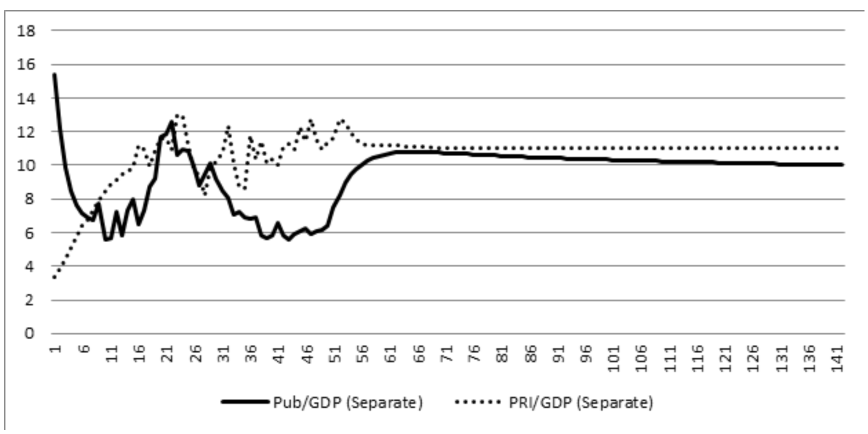
Notes:
CONSUM - Consumption per capita. NETINC - Net income per capita. See Table 1 for the rest of variables' definition.
[a]Simulation time horizon covers 1960 to 2100.
[b] Simulation time horizon covers1960 to 2030.

Figure 1b. Benchmark simulation, using estimates from the Separate equations estimation

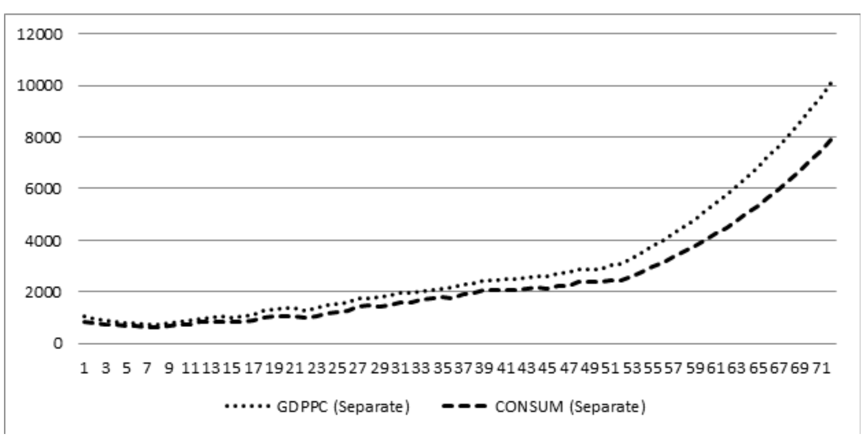
(i) Population and GDP per capita growth rates[a]



(ii) Public and private investment (%GDP)[a]



(iii) GDP and consumption per capita[b]



Notes:
See Table 1 for variables' definition.
[a]Simulation time horizon covers 1960 to 2100.
[b] Simulation time horizon covers1960 to 2030.

Figure 2. Effects of raising public investment to 8.4% – the optimal value under the simultaneous equations estimation

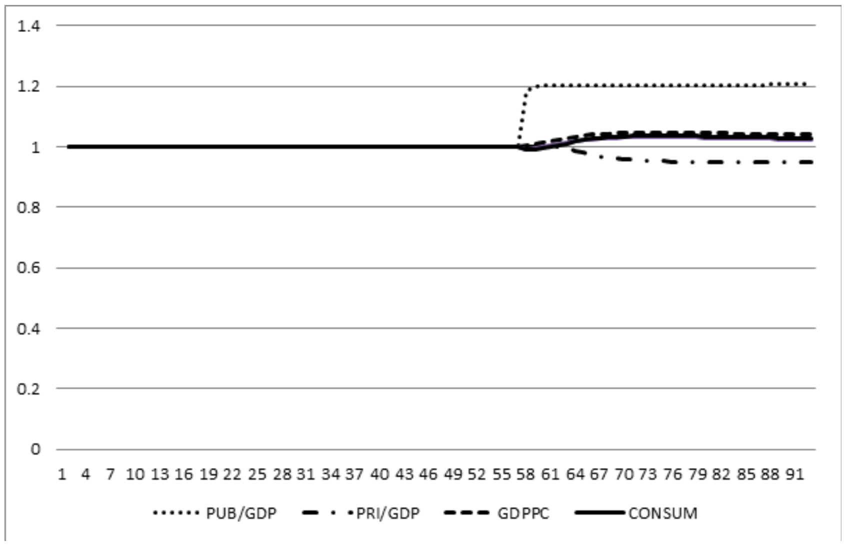


Figure 3. Effects of raising public investment to 9.3% – the optimal value under the separate equations estimation

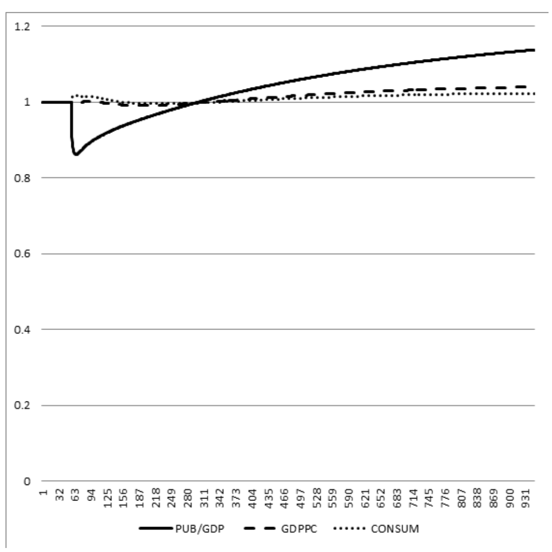


Table 1. Summary Statistics for 33 SSA countries, 1967-2011

		1967-2011		2008-2011	
		Mean	Stdev	Mean	Stdev
GDPPC		1807	2019	2504	3467
GRGDP		0.006	0.101	0.023	0.112
PUB/GDP[a]		6.06	4.47	7.26	5.26
PRI/GDP[a]		9.3	8.8	11.45	5.3
POP Growth		2.66	0.011	2.43	0.0066

Notes:

GDPPC - GDP per capita (PPP); PUB/GDP - Public investment/GDP percentage; GDPGR - GDP per capita growth rate; PRI/GDP - Private investment/GDP percentage; POP Growth - Population growth rate.

[a]Means inferred from the panel average of the natural logarithm in line with the model. This differs from the panel average taken directly, which is one or two percentage points higher

Source: The data for GDP per capita (base year 2005) are obtained from the PWT 8.0. The data for the public and private investment variables are from the African Development Indicators while population data are from the World Development Indicator.

Table 2. Public investment GDP shares that maximize the sum of discounted consumption gains under various discount rates and estimation methods

Time horizons	2050			2100			2900		
Discount rates	0.04	0.08	0.12	0.04	0.08	0.12	0.04	0.08	0.12
Separate[a]	9.50%	9.30%	9.20%	9.60%	9.50%	9.30%	9.60%	9.50%	9.30%
Simultaneous[b]	8.40%	8.40%	8.40%	8.10%	8.30%	8.40%	8.10%	8.30%	8.40%

Notes:

[a]Estimates from the Separate equations estimation are used.

[b]Estimates from the Simultaneous equations estimation are used.

Source: Own calculations.

Table B.1. Endogeneity in the growth regressions[a]

Regressor	*p-val in system*	*IV used*	*Regressor*	*p-val in system*	*IV used*		*remarks*
d(log(1-gpbi/100))	0	log(1-gpbi(-1)/100)	log(1-gpbi/100)	0	d(log(1-gpbi(-1)/100))		endogenous
d(log(gpbi/100))	0	log(1-gpbi(-1)/100)	log(gpbi/100)	0	d(log(gpbi(-1)/100))		endogenous
d(lngpri(-1))	0.6031	lngpri(-2)	lngpri(-1)	0.3419	d(lngpri(-2))		predetermined[b]
d((D(LNPOP))^2)	0.145	(D(LNPOP))^2	D(LNPOP))^2	0.4331	D((D(LNPOP(-6)))^2)		ex./predetermined[f]
d(log(wld))	0.0034	LOG(WLD(-2))	log(wld)	0.7625	D(LOG(WLD(-2)))		predetermined[c]
Private invest. Eq.			Public Invest. Eq.				
d			e	g			
			LOG(GPRI(-1))-LOG(GPRI(-2))		0.2901		

Notes:

[a]Tests analogous to Durbin-Wu-Hausman test; both stages without time dummies.

[b]Assuming exogeneity, using the regressor as instrument and doing the DWH test yields significant effects of the residuals, which should not happen under exogeneity.

[c]As the level is not correlated with the residuals, but the difference is there must be either forward endogeneity or/and correlation with the previous residuals, i.e. predeterminateness.

[d]It follows from the growth equation that GDP per capita growth depends on lagged investment and therefore on the lagged residual of the investment equation; therefore it is predetermined.

[e]As growth depends on public investment without lag it must be correlated here with the residuals and therefore it must be endogenous. The difference of the private investment is exogenous although the levels depend on public investment squared and cubed in the private investment equation with six lags and could lead to predeterminateness. Taking the residuals of the private investment equation and adding the sixth lag to the public investment equation leads to an insignificant result though.

[f]Assuming exogeneity also in the level equation, using the regressor as instrument and doing the DWH yields a significant effect of the residuals. The level variable is indicated as exogenous in both cases.

[g]In spite of the high p-value we consider the variable not as exogenous. It was also endogenous in the previous regression, which is very similar.

Table B.2. Coefficients of lagged dependent variables under GMM and fixed effects

Estimation		Separate			Simultaneous	
Equation	Growth	Priv Inv[a]	Pub Inv	Growth	Priv Inv[a]	Pub inv
GMMSYS	0.97	0.762	0.8	0.971	0.706	0.628
FELS	0.91	0.715	0.733	0.91	0.58	0.477

Table B.3. GMM properties for separately estimated equations

	Growth 1	Priv Inv	Pub Inv
no of coefficients K[g]	51	45	47
instrument rank L	102	139	89
L-K[a]	51	94	42
J	0.023	96.151	51.147
Observations	831 787	653	828
p(nJ)[i]	0.9998	0.419	0.157
1st order ser.cor.[b]			
coeffcients	0.052	-0.060	-0.103
p-values	0.188	0.180	0.007
2nd order ser.cor.[c]			
coeffcients	-0.086; -0.096	0.062	0.071
p-values	0.09; 0.0783[h]	0.251	0.127
s.e.e.[d]	0.109	0.328	0.332
stdev fixed effects[e]	0.053	0.156	0.144
variance ratio[f]	0.241	0.226	0.189

Notes:

[a] Calculated as numbers of variables L-K including time dummies in the growth regressions, but per observation in investment equations.

[b] We save the residuals of the level equation and regress them on their own lag. We use cross-section and time fixed effects, and panel-corrected standard errors of the period SUR type to correct for serial correlation in this regression.

[c] Second-order serial correlation is obtained for regressing the difference of the residuals on their second lag (Roodman 2009b). This can be implemented in two ways. (1) Saving the residuals of the difference equation and regressing it on its own second lag. (2) Savings the residuals of the level equation and regressing its difference on it is own second lag. We run and report both versions.

[d] Standard error of estimation for the level equation.

[e] Standard deviation of fixed effects.

[f] Ratio of stdev of fixed effects and s.e.e., the two cells above this one.

[g] Including time dummies. There is one instrument per regressor, but the growth equations have constraints on the coefficients also for the time dummies.

[h] The first method has 774 observations; the second has 661. Rejection of 2nd order serial correlation seems justified even when allowing a 10% level.

[i] For the growth models and the systems the J-statistic has to be multiplied by the number of observations n. The p-value is close to unity when we use only one instrument per regressor. If time dummies and their related coefficient restrictions were not included in the count we would get p=0.065 although we use only one instrument per regressor and because the coefficient restrictions between the two equations make the J statistic increase away from zero.

Table B.4. Correlation between the residuals of the equations

Correlatio	*Separate equations*					
Probability		growth	priv inv	publ inv	popgr	world gr
RESID Trend Trowth		1				

RESID Priv inv		-0.13391	1			
		0.0009	-----			
RESID Publ inv		0.0021	-0.01119	1		
		0.9586	0.782	-----		
RESID Pop Growth		-0.07661	0.05152	0.026693	1	
		0.0578	0.2024	0.5091	-----	
RESID World Inc model		-0.01818	-0.01033	-0.02008	0.010493	1
		0.653	0.7984	0.6195	0.7953	-----