

What Constitutes Effective Mathematics Teaching? Perceptions of Teachers

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Beliefs help shape how teachers perceive effective mathematics teaching. Providers of professional development, be they local or from other countries, need to be cognisant of such perceptions. This paper seeks to answer the question, 'What do South African teachers perceive as effective and ineffective teaching for developing conceptual understanding of mathematics?' A sample of 46 mathematics teachers was shown vignettes from eight different classrooms where the lesson dealt with some aspect of teaching fractions, and were then asked to comment on the strengths and weakness of what they observed. The comments were classified into seven themes with 18 sub-themes or categories. The majority of the comments focused on two themes, use of materials and modes of instruction. The various mathematical approaches for developing the concept of fractions received little attention. Perceptions of which vignette was considered to be the most effective approach to teaching mathematics resulted in a wide variety of responses. Finally implications for professional development are explored. It is suggested that in-service courses should be geared to what teachers themselves consider best practice, and that reflection on practice should play a more significant role in professional development.

Keywords: Mathematics education; teacher perceptions; curriculum implementation; professional development

Introduction

During the past 20 years, the Japan International Cooperation Agency (JICA) has sponsored educational projects in developing countries, with a particular emphasis on mathematics and science education. While JICA trainers want to provide the best possible professional development, their perceptions are strongly influenced by how mathematics is taught in Japan. As a result some of them began to question whether their perception of good mathematics teaching coincided with that of teachers in the host countries.

Perceptions of what constitutes 'effective' teaching will influence what teachers do or attempt to do in their classrooms, and should therefore be a critical aspect of any intervention. Borko and Putnam (1996, p. 675) hold that '[b]ecause teachers' knowledge and beliefs—about teaching, about subject matter, about learners—are major determinants of what they do in the classroom, any efforts to help teachers make significant changes in their teaching practices must help them to acquire new knowledge and beliefs'. In order to determine teachers' perceptions of effective mathematics teaching, a study was conducted to explore what mathematics teachers from South Africa, Zambia, Cambodia and Japan regard as effective mathematics teaching. This paper is, however, confined to the data

collected in South Africa, and seeks to explore the perceptions of 46 mathematics teachers with respect to the characteristics of effective teaching. The research question is: What do South African teachers perceive as effective and ineffective teaching for developing conceptual understanding of mathematics?

Literature on the Perceptions about Effective Teaching of Mathematics

The goal of Outcomes-based Education (OBE) in South Africa was to move away from the apartheid curriculum and to promote important problem-solving and critical-thinking skills the country needed. OBE focused on learner-centred, self-discovery learning and de-emphasised content (Horn, 2009). However the implementation of OBE proved to be problematic because it requires well-trained teachers, small class sizes and resources. The Curriculum and Assessment Policy Statement (CAPS) hence replaced OBE. The first phase of the CAPS implementation started in 2012 while the last phase was implemented in 2014. The CAPS document (Department of Basic Education, 2011) does not explicitly focus on how mathematics should be taught but provides details on what content should be covered. However, the general aims in the document provide several statements suggesting teaching approaches that are learner-centred:

- 'learners should be able to identify and solve problems and make decisions using critical and creative thinking' (Department of Basic Education, 2011, p. 5);
- 'organise and manage themselves and their activities responsibly and effectively' (Department of Basic Education, 2011, p. 5);
- 'To provide the opportunity to develop in learners the ability to be methodical, to generalize, make conjectures and try to justify or prove them' (Department of Basic Education, 2011, p. 8)..

A learner-centred teaching approach is embodied in South African policy documents, but is it realised in practice? A study done by Webb and Webb (2004) in the Eastern Cape about teachers' beliefs suggests that, although the participating teachers' espoused beliefs that included innovative views of teaching and learning, these views were often not reflected in their practice. This gap between belief and practice was also evident in another study that compared the beliefs of six final year teacher students with their actual classroom practice (Van Putten, Stols & Howie, 2014). They found that all six participants espoused learner centeredness in theory, but did not seem to have the same understanding of what it means in practice. For two of the six participants, learner centeredness meant that the learners answer questions they pose, while for another it meant learners do mathematical problems and explain their work on the board. Van Putten et al. (2014, p. 388) also found that all of the participants 'give little opportunity for learner discovery: they teach and explain, answer what questions there are and give exercises to be done as classwork'. These findings are also supported by Morar's (2000) observation that, despite South African teachers' beliefs about learner-centred teaching approaches, they use traditional approaches in their classrooms. The idea of learner centeredness comes from policy documents and officials from education departments advocating this approach, rather than from teachers' own beliefs. This contradiction may provide an explanation for the gap between teachers' verbalised beliefs about learner centeredness and their classroom practices.

Effective teaching is not only concerned with learner centeredness but also with the educational activities that will best bring about some desired learning (Kyriacou, 2007; Roussouw, Rhodes & Christiansen, 1998). Certain instructional characteristics are associated with effective mathematics teaching. According to Protheroe (2007), the characteristics of effective teaching should include learners that are actively engaged in doing mathematics, work on challenging problems, make interdisciplinary connections, communicate mathematically by sharing ideas and use manipulatives. Lovitt and Clarke (2011, p. 1) identified the following features of what they called a 'rich and balanced mathematics lesson':

- it draws on a range of important mathematical content;
- it is engaging for students;
- all students are able to make a start, as it caters for a range of levels of understanding;

- it can be successfully undertaken using a range of methods or approaches;
- it provides a measure of choice or openness, leading to a sense of student ownership;
- it involves students actively in their own learning;
- it shows the way in which mathematics can help to make sense of the world;
- it makes appropriate and effective use of technology;
- it allows students to show connections they are able to make between the concepts they have learned;
- it draws the attention of students to important aspects of mathematical activity; and
- it helps teachers to decide what specific help students may require in the relevant content areas, or ways in which students might be extended.

An analysis of the study by Fajet, Bello, Leftwich, Mesler and Shaver (2005) about teachers' perceptions of 'good' teaching shows that they fall into two categories, professional competence and affective qualities. With respect to professional competence they identified the following: sufficient content knowledge; the ability to clearly convey their knowledge; the ability to spark interest; good classroom management skills; being fair; being well organised; encouragement of learners; and the use of hands-on activities. Some of the affective qualities of good teachers include the fact that they are optimistic, supportive, patient, kind, caring and enthusiastic.

The use of learners' hands-on activities is generally associated with learner centeredness and effective teaching. Roussouw et al. (1998) hold that many learning theories, including Piaget's findings, support the idea that learners should be active in the classroom. Teachers use concrete materials as a way to involve learners (Furner, Yahya & Duffy, 2005). Mutodi and Ngirande (2014) investigated the perceptions of 30 mathematics teachers in Limpopo province on the use of concrete materials in constructing mathematical meaning. The study found that 96.7% of the teachers believed that the use of concrete material 'bridge[s] the gap that separates how mathematics is taught and how mathematics is learned' (p. 449). The important consideration, however, is whether these activities with hands-on material lead to mathematical knowledge (Roussouw et al., 1998).

Methodology

Overview of Possible Methodologies

Over the years, a number of methodologies have been utilised to gauge teacher perceptions on a variety of topics (Fajet et al., 2005). One popular methodology is a questionnaire with its variations and permutations. A second methodology is to use a more open-ended approach based on interviews, personal journals observations or concept mapping (Roussouw et al., 1998; Özgün-Koca & Şen, 2006). Typically respondents are asked to supply answers in their own words to fairly general questions or scenarios. A third methodology is to show respondents vignettes of video-recorded classroom episodes and ask them to identify what they believe to be examples of either effective or ineffective teaching. Such an approach is less restrictive than a questionnaire as it does not provide statements about teaching practice. It is also more focused than the second methodology as it calls for comments on specific examples and not just general statements about teaching. However it, too, has its shortcomings. The examples of practices selected, even if justified and realistic, are limited to what is seen on the screen. Respondents' perceptions of good teaching may not be evoked by what they see in the vignette. Nevertheless, it was decided that, of the three methodologies, the third one was most likely to yield the most reliable picture of perceived good mathematics teaching. People are prone to view events through the lens of their own worldviews, and hence offering comments on the practices of others reveals something of one's own beliefs (Pajares, 1992).

Procurement of Videos

The major requirement for the selection of vignettes of mathematics teaching was that they should not reflect the race or culture of any of the potential respondents. For example the principal investigators did not want to include video clips of Asian teachers and show them to respondents from both Asia and Africa to preclude a possible pro-Asian bias from the former group. This requirement proved to be very

restrictive as almost all vignettes at their disposal were from either Asian or African countries. Consequently they opted for a set of eight vignettes of teaching fractions at the grade 4 and 5 levels in American schools, and used them in all four countries involved in the project. The selected vignettes depicted a variety of classroom interactions and teaching approaches:

- Teacher 1—learners use hands-on activities with sweets (Skittles) and the teacher asks students recall questions to develop the idea of a numerator and denominator.
- Teacher 2—the teacher uses the learners as objects, as well as blocks, to explain what numerators and denominators are and involves learners by asking them recall questions.
- Teacher 3—learners use paper strips as a manipulative and the teacher asks them recall questions to develop the need for equivalence in the addition of fractions.
- Teacher 4—the teacher explains on the blackboard the addition of fractions and asks learners many recall questions.
- Teacher 5—learners fold paper circles while the teacher asks many questions (and gives them time to think) to develop their understanding of equivalent fractions.
- Teacher 6—the teacher shows a pizza on a TV screen to explain the concept of numerator and denominator. The learners respond in a chorus to the recall questions asked by the teacher.
- Teacher 7—the teacher explains what a numerator and denominator are by drawing five circles on whiteboard (two shaded). He does not ask any questions or involve the learners in any way.
- Teacher 8—the teacher asks many recall questions to activate students' prior knowledge to explain how fractions should be added.

Sample Selection

KwaZulu–Natal was selected as the sample site because it was able to provide a cross-section of urban and rural schools in a reasonably small geographical area. With the help of contacts in the local universities, a request for volunteers was circulated. The final sample consisted of 46 mathematics educators from rural schools, former model C schools and one private school. Although the classrooms depicted in the vignettes were all in primary schools, the sample comprised secondary school teachers of mathematics. The rationale for this choice was based on the assumption that secondary school mathematics teachers would be able to provide deeper insights into the mathematical aspects teaching of fractions.

Data Collection

Each participant attended a session during which each of the eight 3–5 minute vignettes were shown. After each vignette, participants were given the opportunity to respond in writing to two questions:

- What was the best aspect of this lesson segment in terms of teaching fractions? Explain.
- What was the weakest aspect of this lesson segment in terms of teaching fractions? Explain.

After showing all eight vignettes, three further questions were posed:

- Pick the one lesson segment that you regard to be the *most effective* in terms of the pupils learning about fractions, and explain your choice.
- Pick the one lesson segment that you regard to be the *least effective* in terms of the pupils learning about fractions, and explain your choice.
- Pick the one lesson segment that you think most resembles the teaching of mathematics in your area (not just your school), and explain what the lesson has in common with local practice.

Analysis of Data

The purpose of our research was not to analyse the lessons, but rather to use the vignettes as prompts to elicit teachers' perceptions as to what constitutes effective teaching. In this study we used an inductive approach to generate categories of perceptions of effective teaching. An advantage of an inductive analysis process is that it may reveal new patterns, themes, categories and ways of seeing familiar situations (Edwards & Talbot, 1999), which can possibly contribute towards theory building.

We individually analysed the data and compared and refined the emerging categories. Altogether 18 categories were grouped into seven broader themes. The final themes and categories that were used are shown in the [Appendix](#), along with explanations of each category. These categories were used to code the transcripts of all of the comments made by the 46 teachers in response to the eight vignettes. The analysis of the transcripts followed a procedure similar to that followed by Ward and McCotter (2004). We first ‘fractured’ the data by dividing each transcript into segments or ‘chunks’ that expressed a single idea. These segments could consist of an entire paragraph, or be as short as a single sentence or even phrase. Each of the three authors then individually coded the segments. Where the coding differed, the individual authors met face-to-face to discuss the reasoning behind his or her choice and to come to a final resolution.

Results

Perceptions of Effective and Ineffective Teaching

The number of segments generated by the comments of the 46 teachers on both the effective and ineffective aspects of the eight vignettes came to 2138. The frequencies and percentages of these segments per theme and category are shown in [Table 1](#).

Two themes, ‘use of materials’ (theme 4) and ‘modes of instruction’ (theme 3), together accounted for more than half of the comments. In the eyes of many of the respondents, the use of materials is what contributed most to the effective teaching of mathematics. Indeed most of the teachers in the vignettes did use some kind of teaching aid. These varied from coloured Skittles (a kind of sweet), to strips of paper, to pictures of pizza, to audio-visual apparatus. Conversely the poor use of materials, or the lack thereof, drew the highest percentage of comments when it came to perceptions of ineffective teaching.

The theme ‘modes of instruction’ (theme 3) was also perceived as an important aspect of effective teaching of mathematics. These comments reflected the ways in which the teachers observed in the vignettes structured the lesson in terms of learning experiences. These modes included teacher

Table 1. Frequencies and percentages of segments per theme and category, arranged from most to least frequent

Theme	Percentage	Code and category	Effective <i>n</i>	Ineffective <i>n</i>	Total <i>N</i>
Use of materials	27.83	4a Improvised material	224	123	347
		4b Audio-visual technologies	145	103	248
Modes of instruction	27.36	3a Approaches which encourage student activity	155	78	233
		3b Explanations	195	127	322
		3c Links to real life	12	11	23
		3d Individual work	4	3	7
Presentation of content	16.23	2a Conceptual development	96	67	163
		2b Mathematical method	109	75	184
Discourse	14.97	1a Teacher–student interaction	202	118	320
Teacher attributes	5.94	6a Dress, age, experience, confidence, communication, voice	64	45	109
		6b Classroom atmosphere	9	7	16
		6c Teacher preparation	3	2	5
		5a Logistics	18	15	33
Lesson attributes	4.54	5b Clarification of objective	9	6	15
		5c Review of previous lessons	20	4	24
		5d Pace/structure	22	21	43
Assessment	3.13	7a Formative/informal	38	29	67
Total	100		1304	834	2138

Table 2. Examples of comments

Theme	Effective	Ineffective
Presentation of content includes comments related to conceptual development and mathematical methods	<ul style="list-style-type: none"> • Linking to learners' experience. • He allows the learners to contribute in their learning, using their prior knowledge. • She starts with simple fractions in order to build up the learners' understanding 	<ul style="list-style-type: none"> • He needed to explain the method of converting fractions to equivalent fractions after the practical exercise in order to give the lesson a meaning to the learners. • Doesn't incorporate a definition of fractions. • He may have needed to give a little more guidance towards the idea of $2/4=1/2$ as most battled to grasp it
Modes of instruction include comments about approaches that encourage or discourage student activity (hands-on), explanations, links to real life, and individual work	<ul style="list-style-type: none"> • Children could explore the topic and discover for themselves • She wrote down on the board as she explains to the learners. • It is expected of teachers to relate the concept to real-life situations or familiar examples 	<ul style="list-style-type: none"> • No input from learners, educator talking continuously. • There is no time for learners to work the problem—she is telling the answers. • Teacher did a lot of talking instead of learners. Examples were done by the teacher instead of learners
Use of materials includes comments about audio-visual technologies	<ul style="list-style-type: none"> • The real-life situation was an interesting approach but limited the number you could use. • She uses simple examples and what they experience every day to explain. • Technological aids—use of computer to aid understanding 	<ul style="list-style-type: none"> • The colour she used on her overhead projector is not clear enough and the size should have been larger. • The fraction they found could have been written on board so class could see what they had done. • Visibility —writing on the board is not clear for all learners to read

explanations, hands-on manipulation of materials by the learners and real-life applications. This theme drew the second highest number of comments both for effective and ineffective practices.

The theme with the third highest frequency, 'presentation of content' (theme 2), included comments on conceptual development in general and more specifically on how important mathematical concepts were developed—in this case fractions. The former category included comments on how, for example, new concepts were linked to existing knowledge. The second category included comments on the ways in which the concept of fractions was approached.

For all themes, the number of comments on effective teaching outnumbered those on ineffective teaching. Participants apparently were more interested in describing what was effective than finding fault. This difference is particularly noticeable for the themes 'discourse', 'modes of instruction' and 'use of materials'. Table 2 shows some examples of comments made about the three themes with highest frequency.

Characteristics of the Most and Least Effective Lesson

The data presented thus far provide an overall view of the frequency of comments on the eight vignettes as well as some examples of these comments. In this section, we select the one lesson that was deemed by the largest number of participants to be most effective and describe its characteristics drawing on the perceptions of the participants. We then do the same for the lesson that was deemed to be the least effective.

When it came to the selection of the most effective lesson, the votes were dispersed across all eight vignettes, although two of them received only one vote each. The most highly endorsed lesson (teacher 5) received 17 of the votes cast, with the next highest coming in with 10. The aim of the lesson receiving the most votes was to develop the concept of equivalent fractions and the mathematical approach was to treat fractions as part of a whole. Paper circles were folded by the learners themselves to discover that $\frac{1}{2} = \frac{2}{4}$. A feature of the lesson was that learners were allowed to discover the concept for themselves, and at their own pace. Probing questions were posed and the learners were given enough time to think through and justify their answers. Some of the reasons for choosing this lesson are given below.

[The teacher's] method of teaching the kids without giving them the answer empowers them, and the practical hands on approach as well as the time taken for them all to have the 'aa-ha' moment makes interesting learning and development of own thinking style.

[The teacher] tells the children what to expect from the lesson. He asks the kids many questions—letting them come up with answers (he didn't provide them). He lets them use a physical shape to fold [to] discover fractions and equivalent fractions. So multiple senses are being used.

When it came to selecting the least effective lesson, two garnered the majority of the votes, receiving 17 and 15, respectively. The next highest lesson received only four votes. The one receiving 17 votes (teacher 4) dealt with the addition of fractions, and the approach was purely procedural: $\frac{5}{8} + \frac{2}{3} = \frac{?}{24} + \frac{?}{24} = \frac{?}{24}$. Some of the reasons for selecting this lesson are given below.

[The teacher] was unable to motivate learners. He seemed disinterested/did not display enthusiasm—lacked explanation—did not introduce new concepts e.g. LCF. Neither did he explain to learners why it was necessary to arrive at a LCF.

By giving them "rules" to follow they are not discovering the methods for themselves thus they will not know the 'why' behind what they are doing and hence not learn anything.

The second least effective lesson receiving 15 votes (teacher 7) dealt with notation (numerator and denominator) and introduced fractions as part of a subset—two shaded circles from a set of five. Reasons for selecting this lesson as ineffective dealt more with the presentation than with the mathematical approach.

There was no class involvement during the lesson. No planning on how to introduce fractions to the kids. Sometimes learners need to own the lesson so that they enjoy being in class.

Although good in theory the children did not get to participate at all. There is no evidence to suggest that the learners learnt in this lesson as we only heard the teacher.

It is of interest to note that the three vignettes chosen overall as least effective between them also received 10 votes as being the most effective. Clearly, participants held very different notions as to what constitutes effective mathematical teaching.

Finally, participants were asked to identify the one lesson that most closely resembled the way in which mathematics was taught in schools in their area, not just their own school. As it turned out, the same three lessons that had been identified as 'least effective' were also chosen as the lessons most resembling local practice.

A lot of teachers teach mathematics without really showing the learner how the fraction came about, what does it mean. They just teach how to solve the fractions mathematically. They don't give learners examples to show that fractions apply in real life situation[s] as well.

Because there were no resources and also she had a way of trying to remind learners about the denominator and numerator in future by saying 'denominator is down'. [Both begin with the letter d.] This is what most teachers do.

The teacher's abstract methods and work purely in mathematical terms with little reference to the outside world. Focus on concept rather than doing things. Locally we have a lot more teacher talk and less of the hands on approach.

Discussion

The results mirror the findings of Fajet et al. (2005), that is, that teachers' perceptions of 'good' teaching fell mainly into two categories: professional competence and affective qualities. In our case the former was dominant with a large number of comments on effective or ineffective instructional techniques (theme 3) and on the use of materials (theme 4). Indeed many of the attributes of an effective mathematics lesson listed by Lovitt and Clarke (2011), such as involving students actively in their own learning and making connections with prior knowledge, were touched upon by the respondents in their comments. Affective qualities (themes 1 and 6) also garnered their fair share of comments. It would appear then that general pedagogical skills are perceived to be most important when it comes to effective teaching and learning of mathematics.

The way in which the teaching of a mathematics concept was approached (category 2b) received relatively little attention. Fractions present a major challenge in the primary school curriculum but can be approached in different ways, for example as parts of regions and sets, as ratios, or as quotients. Kilpatrick, Swafford and Findell (2001, p. 7) suggest ways of building on learners' informal knowledge:

Students' informal notions of partitioning, sharing, and measuring provide a starting point for building the concept of rational number. Young children appreciate the idea of 'fair shares,' and they can use that understanding to partition quantities into equal parts. In some ways, sharing can play the role for rational numbers that counting does for whole numbers.

Some teachers in the vignettes used coloured sweets to introduce fractions as part of set (serving), others introduced fractions as part of a whole pizza or class, and some followed a procedural approach, for example, $5/8 + 2/3 = 15/24 + 16/24 = 31/24$. Given that high-school teachers are often critical about the poor mathematical understanding of primary school graduates, it would have been reasonable to expect a far greater percentage of comments on the mathematical approach to the teaching of fractions as observed in the vignettes. However this category was conspicuous for its dearth of comments.

Returning to the rationale for the multi-country project, it is interesting to note that, when critiquing a mathematics lesson, Japanese educators tend to place more emphasis on aspects of the mathematical approach. For example, 24% of the reasons for choosing the most effective lesson given by the Japanese respondents focused on the mathematical approach (2b), whereas the same figure for the South African sample was 8% (Ono, Maeda, Nakamura & Chikamori, 2013). This difference became obvious during the JICA supported Mpumalanga Secondary Science Initiative. The Japanese facilitators tended to focus on how to teach specific mathematical concepts effectively—how to teach a mathematical topic by invoking learners' thinking processes, or utilising content specific effective strategies of teaching. On the other hand, their South African counterparts, in the belief that they were being faithful to the new curriculum, tended to focus more on pedagogical or generic aspects of teaching—how teachers interact with learners and facilitate group work.

Identification of Effective Lessons

The respondents' perceptions of what constituted effective and ineffective mathematical teaching add an interesting dimension to the findings. The one vignette identified as most effective is the one that most closely reflects the intentions of the South African CAPS document to ensure that learners are able to identify and solve problems. On the other hand, those vignettes deemed to represent the least effective method of teaching tended to be more teacher-centred, and in one case straightforward 'chalk-and-talk'. Hence there appears to be an encouraging congruence between what the majority of the respondents deemed to be effective and what the policy documents advocate. It may be useful to differentiate between verbalised and internalised or espoused beliefs or perceptions. Van Putten (2014, p. iii) concludes that, although six teachers 'espouse the theory of learner-centred classrooms ... only two of them truly put this theory into practice in their own teaching, thus demonstrating that espoused theory and theory in action are not necessarily the same'. The policy documents define the official curriculum, including statements on ideal teaching strategies, but changing practices is a complex process. Ball (1996, p. 2) explains that 'mathematics reforms challenge culturally embedded views of mathematics, ... and of what is entailed in teaching and learning it, we will find that realising the reform visions will

require profound and extensive societal and individual learning—and unlearning—not just by teachers, but also by players across the system.’

The results suggest a diversity of opinion on which vignette represented the most effective mode of teaching and learning. A policy document might attempt to define ‘effectiveness’, but what is actually implemented will be modified by what is feasible and consistent with cultural norms. For example, an ‘effective’ discovery-type lesson in a well-resourced private school with 24 learners in the room may not be feasible in a classroom with 60-plus learners crammed in that is designed to accommodate half that number, and with no resources other than chalk. A well-planned and delivered chalk-and-talk lesson might well produce the most optimal learning that can be expected in this situation. Given the diversity of schools, there is no one effective type lesson. The sample in this study reflected the diversity of South African schools; one private, and former model-C and rural schools were all represented. Hence it is to be expected that there would be a diversity of opinion on which vignette represented the most effective mode of teaching and learning. It is likely that, in responding to this question, teachers were inclined to think about which of the observed vignettes depicted might be most effective in their own situation.

Implications for Professional Development

The results of this study have implications for professional development, whether provided by outside agencies such as JICA, or by local trainers such as subject advisers. The results show that teachers are more predisposed to offer comments about positive aspects of a lesson than to find fault. It has also been shown in Japan that reflecting on a lesson, and building on its strengths—a practice known as lesson study—can be a powerful form of professional development (Ono and Ferreira, 2010). In-service courses could be enriched by focusing on a lesson (either live or taped), identifying its effective features and building on them to further improve it. In other words, reflection is both a powerful form of professional development and a mode of learning that complements teachers’ natural disposition to look for the positive.

The results of this study also suggest that the perceptions of teachers, and the reality of their classroom environment, need to be taken into account when reflecting on a lesson, and finding ways in which to improve it. If a reflective approach is adopted, the question of effectiveness, positive aspects of the lesson, could be considered in terms of both various belief systems and practical considerations. Instead of judging a lesson to be either ‘good’ or ‘bad’, the reflection could be developed from more than one perspective. For example, ‘Given that this teacher has opted to introduce the concept by means of a lecture, what are its strengths and how could it be improved? How would I teach this lesson in my classroom?’ There may well be cases, especially in rural schools, where the teachers’ belief systems combined with over-crowded classrooms lacking in resources, result in teacher-centred mathematics lessons. In terms of student learning in such a situation, professional development might be more effective in the short term at any rate, if it seeks to build on that teacher’s ability to deliver a good chalk-and-talk lesson, rather than to attempt conversion to a learner-centred approach. Conversely, teachers predisposed to a more learner-centred, discovery approach should be helped to make their lesson result in more effective learning. Again using the above example, ‘Given that the teacher used a lecture to introduce the concept, could elements of a discovery approach be incorporated? If so how?’ A reflective approach such as this might lead to the incorporation of a few new but feasible techniques which that to address the intent of the curriculum policy documents.

Conclusion

When attempting to identify the strengths and weaknesses of a set of lessons, the teachers in this study tended to focus on generic pedagogical skills rather than the development of understanding of mathematical concepts. The diversity of the sample manifested itself in the selection of the one most effective lesson, as each of the eight vignettes received some support. The notion of effectiveness is a function of teachers’ personal perception of excellence and awareness of the school environment. Given this diversity, what constitutes effective professional development for one teacher may be of little value to another.

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Appendix

Table A1. Categories for the analysis of perceptions of mathematics lessons

Theme	Category	Expansion with examples and key words
1. Discourse: comments on how the teacher interacts either with groups or individuals	1b: Teacher–student interaction	Comments about the quality (or lack thereof) of teacher interaction with students, e.g. questioning skills/dialogue with students/talk and chalk/teacher-centred (i.e. lack of interaction)
2. Presentation of content: presentation of content includes comments related to conceptual development and mathematical methods	2a: Conceptual development (minds-on)	Building on previous concepts, linking concepts to develop understanding, e.g. using pre-knowledge/why questions/thinking/visualisation
	2b: Mathematical method	Comments on how mathematical concepts were treated, e.g. fractions as numbers/unit/set/algorithm/procedure/rules
3. Modes of instruction: mode of instruction includes comments about approaches that encourage or discourage student activity (hands-on), explanations, links to real life and individual work	3a: Approaches which encourage student activity	Referring to students as being active, e.g. explore/discover/students are involved/participate
	3b: Approaches based on explanations	Quality and type of explanations, e.g. clarity of explanation/use of examples
	3c: Approaches that make links to real life	Examples from real life are used, e.g. outside the classroom/at home/applications (however not about the use of materials in class)
	3d: Approaches that use individual work	Reference to students that work individually, e.g. seat work/doing an exercise (alone)
4. Use of materials: use of materials includes comments about improvised teaching aids as well as audio-visual technologies	4a: Improvised material	References to materials used in the lessons, e.g. skittles/pizza/paper strips
	4b: Audio-visual technologies	References to audio-visuals used in the lessons, e.g. blackboard/OHP/TV/computer
5. Lesson attributes: lesson attributes include comments about the structure (introduction, development, etc.) and delivery of the lesson (pace, management of time, etc.)	5a: Logistics	Classroom management, e.g. time management/organisation
	5b: Clarification of aims/objective	Remarks about the teachers stating or clarifying the objective/aims of the lesson, e.g. 'today we will ...'
	5c: Review of previous lessons	Remarks about the review of previous lessons are used to structure the current lesson e.g. 'you will remember what we did ...'
	5d: Pace/structure	Remarks about the pace or structure of the lesson e.g. the explanation was too fast for most students/(however not speaking too fast – see 6a)
6. Teacher attributes: teacher attributes includes comments on the teacher's habits, personality, professionalism, etc.	6a: Dress, age, experience, confidence, communication, voice	Comments about the teacher's attributes, e.g. appearance/level of content knowledge or pedagogy/experience/pace of talking

Theme	Category	Expansion with examples and key words
	6b: Classroom atmosphere	Comments on teacher's ability to create a warm atmosphere
	6c: Teacher preparation	Comments on the perceived level of preparation, e.g. well-prepared lesson/very organised
7. Assessment: assessment includes comments on the monitoring of learning.	7a: Formative/informal	Comments on the formative assessment part of lesson, e.g. monitors student learning/checks for understanding