RESEARCH ARTICLE

Modelling of coal trade process for the logistics enterprise and its optimization with stochastic predictive control

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In the paper, a typical coal trade process is described and modelled, where one logistics enterprise with blending equipments lies in the core and two types of common contracts are elucidated to define constraints. A mixed-integer model is built and featured by addressing contract violation, blending operation, real time price information and arbitrarily distributed stochastic demands. To deal with the stochastic demands, probabilistic constraints are formed. Accordingly, stochastic model predictive control (SMPC) strategy with both receding horizon and decreasing horizon formulations is developed to handle the probabilistic constraints and exploit the value of newest price information. By solving a series of mixed-integer linear programs, optimal coal trade decisions for the logistics enterprise can be obtained, including procurement decision, selling decision and operational decision of the blending equipments. Thorough simulation experiments are carried out and compared under three different strategies, which interpret the effectiveness of the proposed strategy.

Keywords: coal trade decisions; stochastic demands; blending operation; stochastic model predictive control

1. Introduction

Coal is one major source for total energy supply in the world. The percentage of coal among all fuels is 19.5% in 2012, following after oil with the percentage of 36.1% (according to data from International Energy Agency; IEA (2013)). And coal takes up the largest part in electricity generation, the ratio of which is 41.3% in 2011. Coal trade both in the international market and domestic market is very active. For example, coal production and net imports for China reached $3,549$ Mt (million tonnes) and 278 Mt respectively in 2012.

Coal trade process usually involves three sides, namely coal suppliers, logistics enterprises and coal consumers, among which the role of logistics enterprises is becoming more and more predominant. Generally, the logistics enterprise is in charge of searching for demands from different consumers such as power companies, cement companies and steel companies, and then formulating a set of coal trade decisions, including procuring coals from suppliers, transporting coals and blending different types of coals to render them suitable for different plants. Coal blending is desired by more consumers at the current moment, since there are more and more stringent environmental regulations which require that contents of some elements/attributes of coals should be within certain upper or lower limits. Moreover, different burning plants may have different preferences on coal qualities. For example, the sulfur oxide content of coals to enter the burning plants of power companies cannot be greater than 0.7% across Guangdong Province of China since 2014. To meet this regulation, different coals with sulfur oxide content above and below 0.7% can be

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mixed together through the blending equipments, otherwise to only consider coals with sulfur oxide content below 0.7% can be very expensive due to insufficient supply in the market. Nowadays, many consumers only propose their requirements to the logistics enterprise and all the jobs are due to the responsibility of the logistics enterprise. Thus, blending capability is one key role that the logistics enterprise can play. This paper will focus on the modelling and optimization for a typical coal trade problem, where one logistics enterprise equipped with blending facilities lies in the core of the trade process. As can be expected, this research will explore more profits for the logistics enterprise. More significantly, optimizing coal trade decisions can bring many benefits to the whole trade system, such as increasing the circulation and blending of coals to meet various demands, reducing costs for consumers, storing coals as a buffer to deal with possible emergencies which may threaten the supply market.

Little research has been done on this type of coal trade optimization, which incorporates contract violation, blending operation, newest information on price forecast and stochastic nature of demands. In this paper, we extend previous research for the following four aspects: 1) The research is carried out from the view of the logistics enterprise, which locates at the core of coal trade process. As stated above, shaping optimal decisions will make the whole trade process more efficient. 2) The stochastic properties of demands in the future are taken into account explicitly, where a series of probabilistic constraints are formulated. 3) The problem of satisfaction or breach of the trade contract is addressed in the modelling and optimization. 4) Stochastic model predictive control strategy, with both receding horizon and decreasing horizon formulations, is devised to achieve optimal performance.

The paper is proceeded as follows. Related literature is reviewed in §2. The details of modelling, i.e., assumptions, objective formulation and various constraints, are presented in §3. In §4, stochastic model predictive control strategy is developed. An illustrative example is studied thoroughly in §5 and we conclude the paper in §6.

2. Literature Review

This research is in connection with three types of problems in existing literatures, i.e., coal logistics optimization, commodity trade problem and supply chain optimization. The connections and differences are set forth below.

2.1 *Coal Logistics Optimization*

Coal blending and distribution problem has drawn a lot of attentions, which can be found in (Sherali and Puri 1993; Shih and Frey 1995; Cao, Lin, and Yan 2006; Liu 2008; Yabin 2010; Yücekaya 2013, and references therein). Coal blending cost is minimised in Shih and Frey (1995) by incorporating the uncertainty of coal elements/attributes. A total cost of the logistics system is taken into account in Cao, Lin, and Yan (2006), including railway transportation cost, procurement cost, ordering cost and holding cost. The blending issue is treated as well in Cao, Lin, and Yan (2006), however, it does not take into account the stochastic variation of both price and demands. In Liu (2008), the blending and inter-modal transportation problem is sufficiently addressed, yet neither demand variation nor price update is accounted for. The price change for procurement and transportation and demand change have been accommodated in the optimization problem of Yabin (2010), while it assumes that both price change and demand change are fixed, which is not realistic by neglecting the stochastic nature of these changes. A multi-objective optimization, considering multiple suppliers, multiple routes, multiple products and the coal quality constraints, is formulated in Yücekaya (2013), yet without treatment of the price and demand variation.

Our research differentiates significantly from the literatures above in two points. Firstly, we not only consider the quality requirements by blending different types of coals, but more specifically, we optimize the operational decisions of blending equipments for the logistics enterprise. To the best knowledge of us, this problem has not been handled before. Secondly, we make use of the information on the stochastic distributions of demands explicitly, where probabilistic constraints are posed and treated. Real time information on price forecast is also incorporated in our model, which is facilitated by the stochastic model predictive control strategy.

2.2 *Commodity Trade Problem*

There are abundant literatures on commodity trade problem, such as Berling and Martínez-de Albéniz (2011); Xie, Park, and Zheng (2013), and their references. In Berling and Martínez-de Albéniz (2011), an inventory control model for spot market procurement is devised by taking both purchase price and demand to be stochastic. It investigates a one-factor price model, and assumes that the arrival process of demand is Poisson, which is stochastic and limited. The replenishment capacity is regarded as infinite. A crude oil procurement strategy for Chinese oil refineries is developed in Xie, Park, and Zheng (2013), with deterministic demands and independent uncertain purchase prices. A Bayesian learning method is utilized to actively assimilate real time price information. In the coal trade process with the logistics enterprise, contracts generally exist to ensure demand predictions more accurate and the procurement planning more realistic, thus neither replenishment nor demand can be infinite. Moreover, there are physical constraints on the storage capacity and blending capacity, and the trade process concerns blending of multiple products (coals), which both complicate the problem. Model predictive control is employed in our study to make use of the newest information regarding price and demand, which has a certain degree of inherent robustness against external disturbances, model uncertainty and model mismatch (Maciejowski 2002; van Staden, Zhang, and Xia 2011).

2.3 *Supply Chain Optimization*

Various control strategies have been successfully applied in supply chain optimization problem, e.g., Perea-López, Ydstie, and Grossmann (2003); Seferlis and Giannelos (2004); Alessandri, Gaggero, and Tonelli (2011); Sarimveis et al. (2008). In Perea-López, Ydstie, and Grossmann (2003); Seferlis and Giannelos (2004); Alessandri, Gaggero, and Tonelli (2011), the model predictive control formulations are discussed in detail corresponding to their respective problems, with the aim of minimising cost (maximising profit) for the multi-product, multi-echelon supply chain networks. And demand variations have been investigated. Sarimveis et al. (2008) presents a comparative review of classical control, dynamic programming and optimal control, model predictive control, robust control and approximate dynamic programming applied in supply chain systems. The coal trade structure belongs to multi-product, multi-echelon systems, hence there are many similarities as in those supply chain systems, such as the inventory dynamics, capacity constraints and procuring/transportation/holding cost functions. In Perea-López, Ydstie, and Grossmann (2003); Seferlis and Giannelos (2004); Alessandri, Gaggero, and Tonelli (2011); Sarimveis et al. (2008), costs of different echelons are summed up and then optimized, thus certain control strategies can be employed to coordinate the behaviors of different echelons. Whereas in the coal trade problem, the profits of different echelons (suppliers, logistics enterprise, consumers) may be contradictory, hence two types of common contracts are utilized to specify obligations of different echelons, i.e., define constraints in the model. It should be noted that there exist a plethora of results in the investigation of supply chain contracts, such as Cachon (2003); Corbett, Zhou, and Tang (2004). However, our research contributes to the rigorous modelling of the coal trade process with the use of common contracts and copes with the case of contract violation throughout the optimization. An approximate method is also proposed to optimize the choices of contractors within our framework in the appendix. Furthermore, our formulation can handle *arbitrary distributed* stochastic demands, provided that their density distributions are known.

3. Modelling

3.1 *Assumptions*

Figure 1 presents the schematic diagram of the coal trade process investigated in the paper. The logistics enterprise receives demand information from different power companies (cases are similar to other consumers like cement/steel companies) and then decides its procurement policies from different suppliers and the operating agenda for its blending equipments, with the aim of minimising the total cost occurred across the trade process. We integrate the whole process, i.e., procurement, transport, blending, storage, selling and contract issue, to tackle the problem in a centralised way.

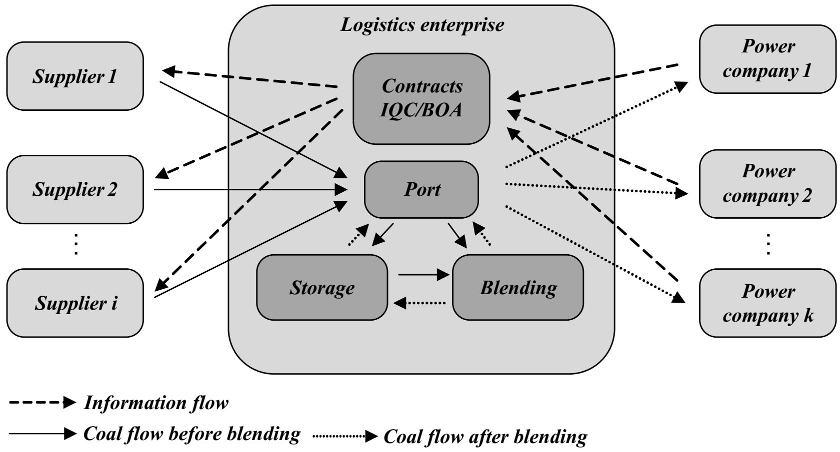


Figure 1. Schematic diagram of the coal trade process

To send final products to power companies, the services provided by the logistics enterprise include procuring coals from suppliers, organizing coal transportation from suppliers to its working port and from its working port to power companies, blending different types of coals to satisfy the requirements for different power companies. One of the core abilities of the logistics enterprise comes from the blending service, which the logistics enterprise distinguishes itself from other common trade companies.

Assumption 1. The logistics enterprise here will only trade with power companies who have no blending facilities but request blending services for their demands due to environmental regulations or operational considerations.

Assumption 2. The logistics enterprise will have constant unit revenue[1] during one sampling period from all the services if offers to power companies.

Thus in the coal trade process, the enterprise will try to reduce all related costs to increase its competitiveness, i.e., the procurement cost, the transport cost, the holding cost, the backlogging penalty cost, the operational cost of blending equipments and the contract violation cost. This constructs a basis for the formulation of the objective function.

We consider two common types of contracts between suppliers and the logistics enterprise, i.e., indefinite quantity contract (IQC) and basic ordering agreement (BOA) (Ingrao 1976). For both IQC and BOA, a maximum limit for each type of coal that suppliers can afford during each period is specified. In order to arrange transportation, a minimum amount of coals for each trade is also specified. If lower than the amount, the trade will generally not happen. For the IQC, another lower limit of the trading amount is imposed and the price is discounted by a certain percentage of the market price. There is an ordering fee for each period, which should be provided to suppliers when signing the IQC. The ordering fee should be returned to the logistics enterprise when the actual trading amount for the current period is inside the IQC limit, otherwise the ordering fee will be charged by suppliers as a punishment to the enterprise due to breach of the IQC. While in the BOA, a written understanding for the interests of both contractors will be indicated. However, the BOA is not so strict as the IQC, and there is no ordering fee for BOA. And breach of BOA will have no effects on both sides.

[1]The revenue plus the transportation cost from the logistics enterprise to power company k is the amount that power company k should pay. Since this transportation cost is small and always regarded as fixed, we do not take into account the transportation costs from the logistics enterprise to power companies in the paper.

Assumption 3. Considering the IQC's between suppliers and the logistics enterprise, the enterprise will violate the contracts and suppliers will not.

Assumption 3 is reasonable by taking into account the fact that suppliers, who sell products, would not like to lose a long term customer (the logistics enterprise). Thus in the paper, we would only explore the possibility that the enterprise breaches the IQC.

When considering the contracts between the logistics enterprise and its customers, i.e., power companies, only IQC's are within the scope of the paper. The reason is that the logistics enterprise would not like to sign BOA's with any power companies, since BOA will render the logistics enterprise difficult to predict the customers' demands and thus difficult to devise realistic and profitable coal trade plans. In terms of power companies, the case when they violate IQC's will not be treated here since power companies will not easily break the contracts with the logistics enterprise which provides final products to them, including all the services.

Assumption 4. Power companies are rational in the sense that the demand predictions provided by them are accountable and credible.

Assumption 5. Transportation capacity is regarded as sufficient enough.

The justification for Assumption 5 is that only one logistics enterprise is studied in the paper so that the transportation capacity constraint is not considered in the modelling. The enterprise can always find its transportation solutions from the market, which are reflected by the transportation cost parameters in our model.

Assumption 6. There are equal sampling periods. During each sampling period, the prices and demands are seen as invariant. And in one sampling period, there is enough time to make decisions and implement the decisions, such as transporting coals.

3.2 *Objective function*

The notations for the modelling and optimization are listed here.

Indices

i	index to denote supplier i
j	index to denote coal type j
k	index to denote power company k
t	index to denote the discrete time instant t
Δt	index to denote difference between time instant t and the current time t_0
l	index to denote element/attribute l representing coal quality

Index Sets

I_h	subset of suppliers that have IQC's with the logistics enterprise
I_s	subset of suppliers that have BOA's with the logistics enterprise
I	set of all suppliers $= I_h \cup I_s$
J	set of all coal types j
K	set of all power companies k
L_a	set of elements/atributes that employ additive property when blending different coals
L_n	set of elements/attributes that are not additive when blending different coals
L	set of all elements/attributes $= L_a \cup L_n$

Decision Variables

u_{ijt}	the amount of coal type j bought from supplier i to the logistics enterprise during $[t,\ t+1)$
y_{jkt}	the amount of coal type j planned to send to power company k from the logistics enterprise during $[t,\ t+1)$
B_{kt}	the amount of coals blended for power company k during $[t,\ t+1)$
$\tilde{z}_{it}$	binary variable used to denote whether the logistics enterprise follows or breaches the IQC with supplier $i \in I_h$, which is equal to 1 when following the contract and 0 otherwise
z_{it}	binary variable used to denote whether the logistics enterprise trades with supplier $i \in I$, which is equal to 1 when buying coals and 0 otherwise

Parameters

ρP_{ijt}	unit procuring cost corresponding to u_{ijt}
ρT_{it}	unit transportation cost from supplier i to the logistics enterprise at time instant t
ρH_t	unit holding cost for the logistics enterprise at time instant t
ρb_k	unit cost for backlogging related to power company k at each time instant
ρO_t	unit operational cost of the blending equipments for the logistics enterprise at time t
ρS_t	unit revenue by providing all the services to power companies at time instant t
D_{kt}	total demand of coals of power company k during sampling period $[t,\ t+1)$
$\overline{D_{kt}}/\underline{D_{kt}}$	upper/lower limit of D_{kt} specified in contracts
$\mathcal{D}_{kt\|t_0}$	the distribution which D_{kt} follows forecast at the current time t_0
$\overline{p_k}/\underline{p_k}$	maximum/minimum probability that the logistics enterprise plans to meet the future demand of power company k
$\overline{\gamma_{t\|t_0}}/\underline{\gamma_{t\|t_0}}$	minimum upper bound of the $\overline{p_k}/\underline{p_k}$ confidence level of the distribution $\mathcal{D}_{kt\|t_0}$
V_{jt}	inventory level of coal type j at the logistics enterprise at time instant t
$V_t/\overline{V}$	the total inventory level of all coal types at time instant t/the upper limit of V_t
B_M	maximum blending capacity for each sampling period
e_{jl}	value of element/attribute l for coal type j
$\overline{E_{kl}}/\underline{E_{kl}}$	maximum/minimum limit of index l allowed in power company k
$\overline{u_{ijt}}$	maximum limit of u_{ijt} specified in the contracts
$\underline{u_{it}}$	minimum amount of coals for each trade specified in the contracts
$\tilde{u}_{it}$	lower limit of coals that the logistics enterprise should procure by the IQC
OF_{it}	ordering fee specified in the IQC with supplier $i \in I_h$ for time instant t
η_i	discount rate from supplier $i \in I_h$ defined in the IQC for each sampling period
r	inflation rate for each sampling period
BC_t	cost of buying coals for the logistics enterprise during sampling period $[t,\ t+1)$
CVC_t	contract (IQC) violation cost during sampling period $[t,\ t+1)$
TC_t	transportation cost from all suppliers to the logistics enterprise during $[t,\ t+1)$
HC_t	holding cost of coals for the logistics enterprise during sampling period $[t,\ t+1)$
PC_t	backlogging cost due to unsatisfied demand during sampling period $[t,\ t+1)$
OC_t	operational cost of the blending equipments for the logistics enterprise during $[t,\ t+1)$
PS_t	revenue due to all services provided by the logistics enterprise during $[t,\ t+1)$
T	length of the prediction horizon
CT	total cost for the logistics enterprise during horizon T

The unit cost parameters $\rho P_{ijt},\ \rho T_{it},\ \rho H_t, \rho S_t$ at current time instant $\Delta t = 0$ $(t = t_0)$ are known exactly from the market, while for $\Delta t = 1, 2, \ldots, T-1$ they can only be forecast and take on stochastic properties. In the paper, the notations represent expected values of these cost parameters for the future time instants when $\Delta t = 1, 2, \ldots, T-1$.

The cost of buying coals for the logistics enterprise during sampling period $[t,\ t+1)$ can be written as

$$BC_t = \sum_{i \in I_h} \eta_i \cdot \sum_{j \in J} \rho P_{ijt} \cdot u_{ijt} + \sum_{i \in I_s} \sum_{j \in J} \rho P_{ijt} \cdot u_{ijt} \tag{1}$$

The first term calculates the buying cost from suppliers which have IQC's with the logistics enterprise, thus it contains a discount rate η_i. And the second term accounts for the buying cost from suppliers with BOA's.

$$CVC_t = \sum_{i \in I_h} OF_{it} \cdot (1 - \tilde{z}_{it}) \tag{2}$$

Equation (2) gives the contract (IQC) violation cost. If the IQC is followed, then $\tilde{z}_{it} = 1$ and the corresponding CVC_t will be zero. On the contrary, ordering fee OF_{it} will be charged by supplier i

if the IQC is breached, when $\tilde{z}_{it} = 0$.

$$TC_t = \sum_{i \in I} \rho T_{it} \cdot \sum_{j \in J} u_{ijt} \tag{3}$$

The equation above represents the transportation cost during sampling period $[t,\ t+1)$, where different coal types are not distinguished and the cost is weighed by the total amount (quantity) of all types.

$$HC_t = \rho H_t \cdot \frac{V_t + V_{t+1}}{2} \tag{4}$$

Equation (4) considers the total holding cost during $[t,\ t+1)$, including storage cost, taxes, administrative cost, the potential investment gain from the money stored on the commodities (coals), etc. Since the inventory is changing during the sampling period $[t,\ t+1)$, the average inventory is approximated by the mean of V_t and V_{t+1}.

$$PC_t = \sum_{k \in K} \rho b_k \cdot \Big(D_{kt} - \sum_{j \in J} y_{jkt} \Big) \tag{5}$$

In equation (5), D_{kt} denotes the demand of power company k during the period $[t,\ t+1)$. Therefore, $\big(D_{kt} - \sum_{j \in J} y_{jkt}\big)$ refers to the amount of unsatisfactory demand during $[t,\ t+1)$, and this unsatisfactory demand will become lost sales. ρb_k is the unit cost due to backlogging for each sampling period. It is set by the logistics enterprise and is generally much larger than the corresponding buying cost, which implies that the unsatisfactory demand is not expected in most cases. Different values of ρb_k indicate different priorities of the power companies to the logistics enterprise.

$$OC_t = \rho O_t \cdot \sum_{k \in K} B_{kt} \tag{6}$$

Equation (6) gives the operational cost of the blending equipments, where B_{kt} are online decision variables to denote how many coals should be blended for power company k during sampling period $[t,\ t+1)$. More and more power companies ask for blending services, since there are restrictions on the coal elements/attributes. These restrictions arise from either the environmental regulations or the operational requirements for different burning plants.

$$PS_t = -\rho S_t \cdot \sum_{k \in K} \sum_{j \in J} y_{jkt} \tag{7}$$

Equation (7) computes the predicted revenue by the services with which the logistics enterprise provides power companies, according to Assumption 2. The services consist of many contents, such as coal procurement, organizing transportation and the blending service.

PS_t is negative in (7), since the objective function is interpreted as the total cost for a prediction horizon T in (8). And the inflation factor with a rate r is included in the formulation.

$$CT = \sum_{\Delta t=0}^{T-1} \frac{BC_t + CVC_t + TC_t + HC_t + OC_t + PS_t}{(1+r)^{\Delta t}} + PC_{t_0}, \quad \text{where} \quad \Delta t = t - t_0 \tag{8}$$

In (8), the backlogging cost is computed only for the current time t_0. The reason is that total demands D_{kt} are exactly known at t_0 but are of stochastic nature for the future time instants $\Delta t = 1, 2, \ldots, T-1$ by means of predictions. The stochastic issue will be addressed later.

3.3 *Constraints*

The dynamics of the inventory level for each coal type j at the logistics enterprise is as follows:

$$V_{j(t+1)} = V_{jt} + \sum_{i\in I} u_{ijt} - \sum_{k\in K} y_{jkt}, \quad \forall j \in J, \quad \text{for} \quad \Delta t = 0, 1, \ldots, T-1. \tag{9}$$

The term $\sum_{i\in I} u_{ijt}$ considers the total amount of coal type j bought at time t, and the term $\sum_{k\in K} y_{jkt}$ is the total amount of coal type j planned to send to all power companies. The logistics enterprise expects the inventory level to be within a range rather than a fixed level, so that the inventory can be adjusted to increase its profit (e.g., dealing with the price fluctuations) and reduce the risks caused by some emergency accidents (e.g., political intensions may influence the supply market of coals).

$$0 \leq \sum_{j\in J} V_{jt} \leq \overline{V}, \quad \Delta t = 0, 1, \ldots, T-1. \tag{10}$$

For both IQC and BOA, an upper limit of each product that supplier i can provide is specified:

$$u_{ijt} \leq \overline{u_{ijt}}, \quad \forall i \in I,\ j \in J,\ \Delta t = 0, 1, \ldots, T-1. \tag{11}$$

For ease of transportation, each supplier will set up a minimum amount of coals for each trade, otherwise the trade will not take place.

$$\underline{u_{it}} \leq \sum_{j\in J} u_{ijt} \leq \sum_{j\in J} \overline{u_{ijt}}, \quad \text{or} \quad u_{ijt} = 0, \quad \forall i \in I, \ \Delta t = 0, 1, \ldots, T-1.$$

The equation above can be further formulated as a mixed-integer linear constraint:

$$z_{it}\underline{u_{it}} \leq \sum_{j\in J} u_{ijt} \leq z_{it} \sum_{j\in J} \overline{u_{ijt}}, \quad z_{it} \in \{0,\, 1\}, \quad \forall i \in I, \ \Delta t = 0, 1, \ldots, T-1. \tag{12}$$

For contract belonging to IQC, there will be another lower limit $\tilde{u}_{it}$ specified ($\tilde{u}_{it} > \underline{u_{it}}$), which the logistics enterprise should buy from supplier i. If the amount of coals that the logistics enterprise buy at time instant t is less than $\tilde{u}_{it}$ (the price is discounted as well), then the IQC is violated.

$$\begin{cases} \tilde{u}_{it} \leq \sum_{j\in J} u_{ijt}, & \text{if IQC is followed,} \\ \sum_{j\in J} u_{ijt} < \tilde{u}_{it}, & \text{if IQC is violated,} \end{cases} \qquad \forall i \in I_h, \ \Delta t = 0, 1, \ldots, T-1.$$

Use is made of the 'Big M' technique to transfer the above logic constraint into linear constraint (13) with binary variables $\tilde{z}_{it}$, where M is a large positive number compared with the scale of $\tilde{u}_{it}$. Hence, breach of IQC implies $\tilde{z}_{it} = 0$ in (13). This is in accordance with (2), where $\tilde{z}_{it} = 0$ gives that the corresponding ordering fee will be charged as a punishment.

$$\begin{cases} \tilde{u}_{it} \leq \sum_{j\in J} u_{ijt} + M(1 - \tilde{z}_{it}), \\ \sum_{j\in J} u_{ijt} < \tilde{u}_{it} + M\tilde{z}_{it}, \end{cases} \qquad \tilde{z}_{it} \in \{0,\, 1\}, \quad \forall i \in I_h, \ \Delta t = 0, 1, \ldots, T-1. \tag{13}$$

Total demands D_{kt_0} are requested exactly from different power companies at current time instant $\Delta t = 0$, however D_{kt} ($t > t_0$) will be stochastic variables for the future time t and can only be predicted. Accordingly, $\sum_{j\in J} y_{jkt_0}$ denotes the actual amounts sold to power company k at current time t_0, while $\sum_{j\in J} y_{jkt}$ $(t > t_0)$ signifies the amounts planned for power company k for the future time t by the logistics enterprise. Yet $\sum_{j\in J} y_{jkt}$ $(t > t_0)$ may probably differ from the real trading amounts in the future due to the stochastic property of D_{kt} $(t > t_0)$. For the current time step, the maximum amounts of $\sum_{j\in J} y_{jkt_0}$ can certainly not exceed the deterministic demands, D_{kt_0}, while the minimization of $PC_{t_0} = \sum_{k\in K} \rho b_k \cdot \left(D_{kt_0} - \sum_{j\in J} y_{jkt_0}\right)$ in (8) yields that the former value will tend to approach the latter value. Thus constraint (14) must hold.

$$\sum_{j\in J} y_{jkt} \leq D_{kt}, \quad \forall k \in K, \quad \Delta t = 0 \tag{14}$$

Constraint (15) represents the minimum amount of coals that power companies have to buy, and constraint (16) represents the maximum amount of coals that the logistics enterprise can afford, which are both stipulated in contracts.

$$\sum_{j\in J} y_{jkt} \geq \underline{D_{kt}}, \quad \forall k \in K, \quad \Delta t = 0, 1, 2, \ldots, T-1. \tag{15}$$

$$\sum_{j\in J} y_{jkt} \leq \overline{D_{kt}}, \quad \forall k \in K, \quad \Delta t = 0, 1, 2, \ldots, T-1. \tag{16}$$

According to (14), (15) and (16), obviously for any current time instant $t = t_0$, the following inequality holds:

$$\underline{D_{kt}} \leq D_{kt} \leq \overline{D_{kt}}, \quad \forall k \in K, \quad \Delta t = 0 \tag{17}$$

Constraint (18) denotes that the probability that the logistics enterprise plans to satisfy the future demands of power company k should be no smaller than $\underline{p_k}$. This implies that the expected counts to be able to meet demands of power company k during the future $T-1$ periods are at least $\underline{p_k}(T-1)$.

$$\Pr\left\{\sum_{j\in J} y_{jkt} \geq D_{kt}\right\} \geq \underline{p_k}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1. \tag{18}$$

Constraint (19) means that the logistics enterprise cannot be too optimistic about the markets in case that there will be many coals overstocked in the future. For example, if for some future instant t, D_{kt} is a random variable of which the upper bound is unlimited or a very large value, to satisfy all possible demands at time t, denoted by $\Pr\left\{\sum_{j\in J} y_{jkt} \geq D_{kt}\right\} = 1$, will either be impossible or too conservative. That is why a maximum probability $\overline{p_k}$ needs to be enforced as well.

$$\Pr\left\{\sum_{j\in J} y_{jkt} \geq D_{kt}\right\} \leq \overline{p_k}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1. \tag{19}$$

It is noted that (18) and (19) are not deterministic but probabilistic constraints, which will be handled in next section.

As is delineated before, due to environmental regulations or operational requirements of certain burning plants, there are strict restrictions on the elements/attributes that reflect coal quality when consuming coals in fired power companies. For example, coals with different sulfur oxide contents can be blended to make sure the final products satisfy the corresponding regulation, e.g., no greater than 0.7% in Guangdong Province of China. Such attributes/elements are *additive* since their values can be changed through blending. Other additive elements/attributes include ash content, volatile matter, nitrous oxide, calorific value, etc. Constraints for additive elements/attributes can be formed as:

$$\underline{E_{kl}} \leq \frac{\sum_{j\in J} y_{jkt} e_{jl}}{\sum_{j\in J} y_{jkt}} \leq \overline{E_{kl}}, \quad \forall l \in L_a, \quad \text{for } \Delta t = 0, 1, \ldots, T-1. \tag{20}$$

There are other elements/attributes that are not additive. It suggests that coal of that type is not allowed to use in the burning plants, if the values of those elements/attributes are out of the specifications. This cannot be solved by blending since the coal may be harmful to the plants. These non-additive elements/attributes comprise of the grindability size and the moisture content. The constraints are thus written as:

$$y_{jkt} = 0, \quad \text{if } \ e_{jl} \notin [\underline{E_{kl}}, \ \ \overline{E_{kl}}], \quad \forall l \in L_n, \quad \text{for } \Delta t = 0, 1, \ldots, T-1. \tag{21}$$

Constraint (22) illustrates that the *cumulative* coals planned for power company k should not be greater than those blended, according to Assumption 1. This formulation is less stringent than $\sum_{j\in J} y_{jkt} \leq B_{kt}$ for each t, which requires that the coals planned to send to power company k, *at each time instant*, should not exceed those blended. It provides more flexibility for the logistics enterprise to operate the blending equipments to minimise the cumulative costs in a prediction horizon T, according to the predictions of future price and demands. (22a) is for the case when $t_0 = 0$. For $t_0 \geq 1$, the term in the bracket of the right hand side of (22b) expresses what have happened in reality by time t_0, which denotes the amount of coals that have been blended and stored but not sold yet. (22) is applied for all the prediction instants $\Delta t = 0, 1, \ldots, T-1$.

$$\sum_{t_0}^{t_0+\Delta t} \sum_{j\in J} y_{jkt} \leq \sum_{t_0}^{t_0+\Delta t} B_{kt}, \quad \forall k \in K, \qquad \text{when } \ t_0 = 0, \tag{22a}$$

$$\sum_{t_0}^{t_0+\Delta t} \sum_{j\in J} y_{jkt} \leq \left(\sum_{0}^{t_0-1} B_{kt} - \sum_{0}^{t_0-1} \sum_{j\in J} y_{jkt} \right) + \sum_{t_0}^{t_0+\Delta t} B_{kt}, \quad \forall k \in K, \quad \text{when } \ t_0 \geq 1. \tag{22b}$$

And the total amount of coals blended for all power companies cannot surplus the maximum blending capacity:

$$\sum_{k\in K} B_{kt} \leq B_M, \quad \Delta t = 0, 1, \ldots, T-1. \tag{23}$$

Finally, there are non-negativity constraints on the decision variables and inventory level to guarantee that the results are realistic.

$$u_{ijt} \geq 0, \quad y_{jkt} \geq 0, \quad B_{kt} \geq 0, \quad \forall j \in J, \quad k \in K, \quad \text{for } \Delta t = 0, 1, \ldots, T-1. \tag{24}$$

In the above formulations, some parameters are prescribed in contracts, while others are defined by the logistics enterprise, which are shown in Figure 2 for clarity.

4. Stochastic model predictive control (SMPC)

4.1 *Introduction of SMPC*

The idea of model predictive control (MPC) has been well applied in diverse optimization problems (van Staden, Zhang, and Xia 2011; Perea-López, Ydstie, and Grossmann 2003; Seferlis and

Giannelos 2004; Alessandri, Gaggero, and Tonelli 2011; Elaiw, Xia, and Shehata 2012; Zhang and Xia 2011). MPC takes into account not only the current behaviour of the system but also the future behaviour within a certain prediction horizon, thus it optimizes a predicted cost. It has a strong ability to handle diverse constraints. Moreover, MPC is implemented using a receding horizon manner such that it always makes use of the newest measurements and thus achieves optimal performance.

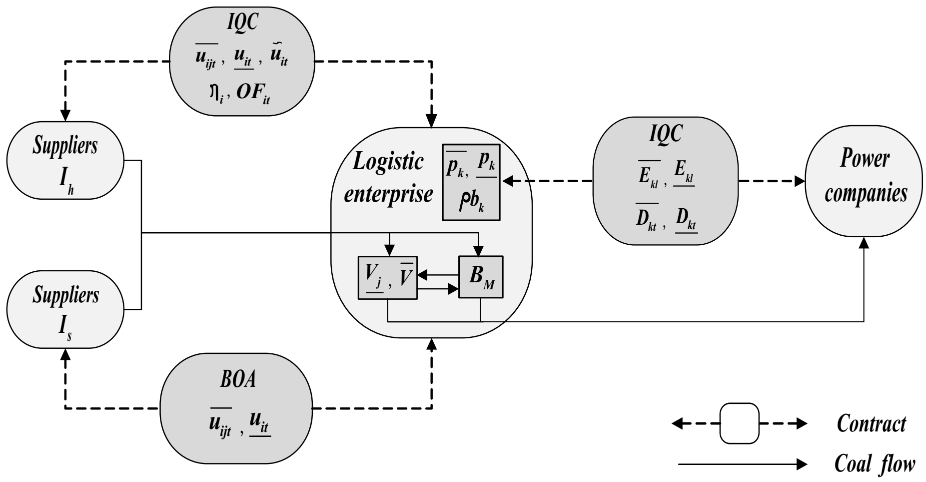


Figure 2. Parameters specified in contracts and chosen by the logistics enterprise

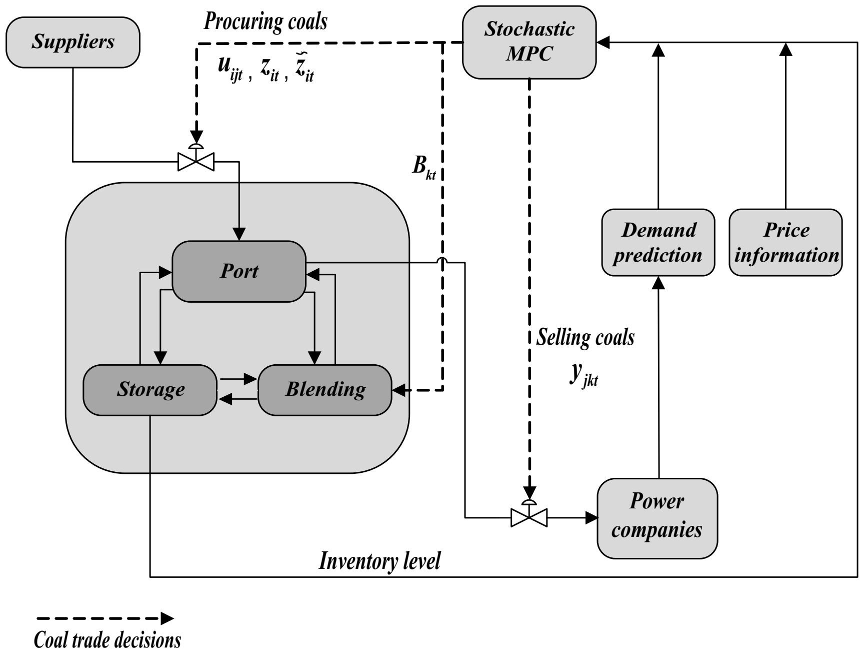


Figure 3. Stochastic MPC applied into the optimization of coal trade decisions

Stochastic model predictive control (SMPC) is a newly developed MPC methodology that is currently a hot topic in both theoretical and applied research. SMPC is proposed to handle random system uncertainty which is in fact omnipresent in the modelling of most real systems. Due to the random uncertainty, constraints are not necessarily to be deterministic. Instead they can be probabilistic, which means that constraints are only required to be satisfied with a minimum probability. The occurrence of probabilistic constraints mainly arises from two reasons: one is that random uncertainty may have infinitely support such that constraints will be impossible to be obeyed with a probability of 100%, and the other is that probabilistic constraints can lead to considerably better performance when the corresponding deterministic constraints are over conservative. There are several interesting progresses on the research of the theory of SMPC recently (Cannon et al. 2011; Oldewurtel et al. 2013; Calafiore and Fagiano 2013; Kouvaritakis et al. 2010; Cannon et al. 2012), which readers can refer to.

According to Section 3, since the logistics enterprise aims at minimising the cumulative cost during a prediction horizon subject to a set of random variables ($\rho P_{ijt},\ \rho T_{it},\ \rho H_t,\ \rho S_t$ and D_{kt} for $\Delta t = 1, 2, \ldots, T-1$), the discrete time SMPC strategy can be employed by Assumption 6, as depicted in Figure 3. At each time instant, we can get new estimations of those random variables, which need updating throughout the optimization. By implementing SMPC, the optimal set of coal trade decisions, i.e., the procurement decision, the selling decision and the operational decision of blending equipments, can be obtained. And it is long enough to implement these decisions during one sampling period, according to Assumption 6.

Definition 1. $\underline{\gamma_{t|t_0}}$ and $\overline{\gamma_{t|t_0}}$ are the minimum values such that

$$\Pr\left\{D_{kt} \le \underline{\gamma_{t|t_0}}\right\} = \underline{p_k}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1 \tag{25a}$$

$$\Pr\left\{D_{kt} \le \overline{\gamma_{t|t_0}}\right\} = \overline{p_k}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1 \tag{25b}$$

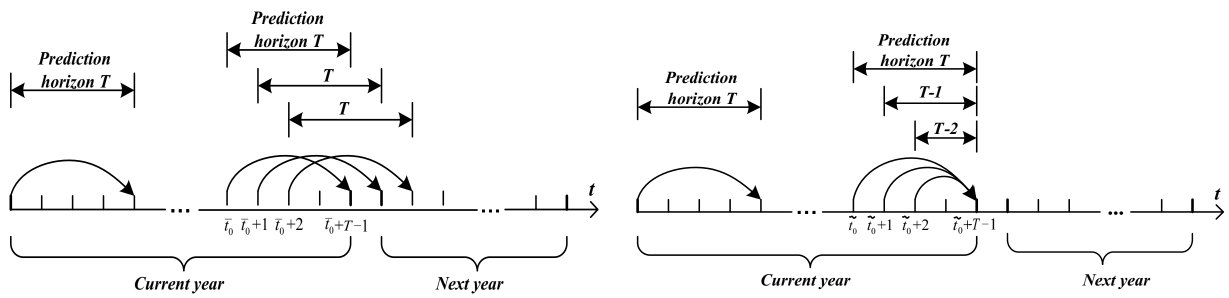
with D_{kt} subject to a distribution $\mathcal{D}_{kt|t_0}$ forecast at $t = t_0$.

According to Definition 1, probabilistic constraints (18) and (19), i.e., $\Pr\left\{\sum_{j\in J} y_{jkt} \ge D_{kt}\right\} \ge \underline{p_k}$ and $\Pr\left\{\sum_{j\in J} y_{jkt} \ge D_{kt}\right\} \le \overline{p_k}$, can then be transferred to the following deterministic constraints:

$$\sum_{j\in J} y_{jkt} \ge \underline{\gamma_{t|t_0}}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1 \tag{26a}$$

$$\sum_{j\in J} y_{jkt} \le \overline{\gamma_{t|t_0}}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1. \tag{26b}$$

It is worth to mention that the distribution $\mathcal{D}_{kt|t_0}$ can be arbitrary rather than take on a particular form (Kouvaritakis et al. 2010). Only if the probability density function (*pdf*) of the distribution $\mathcal{D}_{kt|t_0}$ is provided, $\underline{\gamma_{t|t_0}}$ and $\overline{\gamma_{t|t_0}}$ can be computed, either analytically with the use of the inverse cumulative distribution function or numerically by discretizing the interval of the *pdf*.



(a) Receding horizon implementation (b) Decreasing horizon implementation

Figure 4. Receding horizon and decreasing horizon model predictive control

4.2 *Receding horizon implementation*

Generally in practice, contracts between suppliers and the logistics enterprise and between power companies and the logistics enterprise are negotiated and signed every year.

Definition 2. Contracts are regarded as *continuous*, in the sense that, if for some time instant $t = \bar{t}_0$, contracts for next year have been signed, and the end of the prediction horizon $t = \bar{t}_0 + T - 1$ is within the time when this year's contracts are valid.

If all contracts are continuous, then constraints in Section 3.3 for the horizon T can always be defined for any initial time t_0. Correspondingly, the cumulative costs are minimised continuously with a receding horizon manner (see Figure 4(a)), which is the most common way in MPC.

Algorithm 1. (*SMPC algorithm*)
At the current time $t = t_0$ ($t_0 = 0, 1, 2, \ldots,$),

Step 1. With the newest estimations of the parameters $\rho P_{ijt},\ \rho T_{it},\ \rho H_t, \rho S_t$ and D_{kt}, substitute the expected values into objective function (8) and compute $\underline{\gamma_{t|t_0}},\ \overline{\gamma_{t|t_0}}$ in (25); V_{jt_0} is assumed to be known at the first beginning when $t_0 = 0$, and can be updated in Step 3 for $t_0 = 1, 2, \ldots$.

Step 2. Perform the following optimization:

$$\min_{\{u_{ijt},\ y_{jkt},\ B_{kt},\ \tilde{z}_{it},\ z_{it}\}} \quad (8)$$

$$\text{subject to} \quad \Big\{\ (9), (10), (11), (12), (13), (14), (15), (16), (20), (21), \\ (22a) \text{ or } (22b), (23), (24) \text{ and } (26a, b).\ \Big\} \tag{27}$$

Step 3. Using the first element of the optimal solutions of Step 2, $\{u_{ijt_0}, y_{jkt_0}, B_{kt_0}, \tilde{z}_{it_0}, z_{it_0}\}$, decide the coal trade pattern at the current time t_0, calculate the inventory level $V_{j(t_0+1)}$ in (9). Move to the next step $t = t_0 + 1$, update constraint (22b), then repeat Step 1, 2 and 3.

It is noted that the objective function (8) is linear in the decision variables $\{u_{ijt},\ y_{jkt},\ B_{kt},\ \tilde{z}_{it},\ z_{it}\}$ and all constraints in (27) are linear. Thus, the optimization problem (27) is a mixed-integer linear program, which can be solved by many optimization packages available for commercial/academic usages. We use the well known package, GUROBI 5.6, to figure out the optimal solutions in the simulation experiments.

4.3 *Decreasing horizon implementation*

According to Definition 2, contracts are not continuous, if for some instant $\tilde{t}_0$, by the end of the prediction horizon $\tilde{t}_0 + T - 1$ that is also the end of this contract year, contracts for next year have not been available. In this case, the costs, only for the current year, are the attention of the logistics enterprise, we will then implement the decreasing horizon MPC since $\tilde{t}_0$. Consequently, at the time step $t = t_0$ (when $t_0 \geq \tilde{t}_0$), the objective function (8) is modified to be

$$\widetilde{CT} = \sum_{\Delta t=0}^{\tilde{t}_0+T-1-t_0} \frac{BC_t + CVC_t + TC_t + HC_t + OC_t + PS_t}{(1+r)^{\Delta t}} + PC_{t_0}, \quad \text{where} \quad \Delta t = t - t_0 \tag{28}$$

From equation (28), it is revealed that the prediction horizon $\tilde{t}_0 + T - t_0$ is no greater than T, which decreases as t_0 increases. The algorithm for decreasing horizon MPC is the same as in Algorithm 1, except that the prediction horizon changes to $\tilde{t}_0 + T - t_0$ (see Figure 4(b)). Thus constraints in the optimization problem (27) should be revised to $\Delta t = 0, 1, \ldots, \tilde{t}_0 + T - 1 - t_0$, as the objective function (28).

5. Simulation experiments

An illustrative example with 8 suppliers are considered here, among which 4 suppliers are of IQC's and the other 4 are with BOA's. In total, 23 types of coal will be provided by the suppliers, and 4 additive elements/attributes and 1 non-additive element are handled. 4 power companies are served by the logistics enterprise. Contracts last for one year and decisions are to be made every half a month, thus there are 24 sampling periods altogether. The size of this example is comparable to a real problem and all parameters are given in Table 2−6. We assume that the predictions of the future price and demands can be made with a certain accuracy, though the way to do the predictions are outside the purview of this paper. In this example, the future uncertain cost in Yuan (the Chinese currency) and demands take on the forms in Table 7 and 3 respectively. From Table 3, the demands at each sampling period are assumed to be normally distributed variables, with mean μ_{kt} to be another random variables but standard deviations σ_{kt} to be fixed. The initial inventory level is assumed to be 0 and the prediction horizon T is set to be 10. Only the profit of current year is focused.

To illustrate the effectiveness of the proposed stochastic model predictive control strategy, simulations and comparisons are conducted for three different strategies.

(i) The proposed model predictive control strategy is applied. For $t_0 = 0, 1, \ldots, 13$, (27) is solved repeatedly with a receding horizon manner with $T = 10$, and for $t_0 = 14, 15, \ldots, 23$ ($\tilde{t}_0 = 14$), the decreasing horizon SMPC is implemented.

(ii) A closed loop model predictive control strategy is employed but the significant difference with (i) is without treatment of probabilistic constraints. Although the actual demands will evolve as in Table 3, only the expected future demands μ_{kt} ($t > t_0$) are utilized and no consideration is given to the stochastic distributions of the future demands. Thus, constraints (26a, b) of the optimization problem in (27) are changed into

$$\sum_{j \in J} y_{jkt} \le \mu_{kt}, \quad \forall k \in K, \quad \Delta t = 1, 2, \ldots, T-1.$$

And the objective function incorporates the expected future demands as follows:

$$\widehat{CT} = CT + \sum_{\Delta t=1}^{T-1} \sum_{k} \frac{\rho b_k \cdot \left(\mu_{kt} - \sum_{j \in J} y_{jkt}\right)}{(1+r)^{\Delta t}},$$

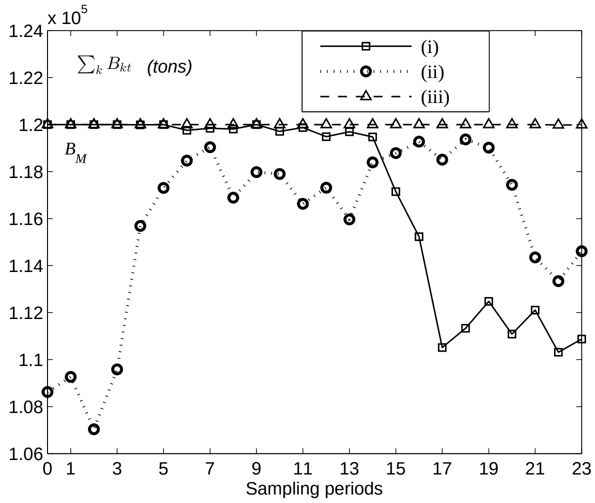
where CT is defined in (8). For $t_0 = 0, 1, \ldots, 13$, the receding horizon MPC with $T = 10$ is implemented, and for $t_0 = 14, 15, \ldots, 23$ ($\tilde{t}_0 = 14$), the decreasing horizon MPC is applied.

(iii) An open loop optimization is implemented with a planning horizon $T = 24$ considering probabilistic constraints, i.e., solving (27) *only* at $t_0 = 0$. This means that all decision variables across one year are computed at $t_0 = 0$ and will be implemented in reality afterwards. An adjustment is made at each sampling period according to the actual demands from power companies. $\widetilde{V}_{k(t+1)} = \max\left\{\widetilde{V}_{kt} + \sum_j y_{jkt} - D_{kt},\ 0\right\}$ with $\widetilde{V}_{kt_0} = 0$, where $\widetilde{V}_{kt}$ represents the coal inventory for power company k at time instant t. When $\widetilde{V}_{k(t+1)} = \widetilde{V}_{kt} + \sum_j y_{jkt} - D_{kt}$, it means that demands of power company k at t can be met, but otherwise not.

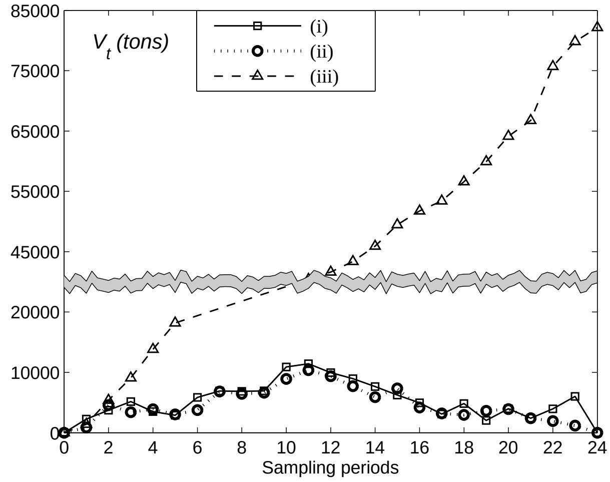
Table 1 lists the size of the optimization problem at the initial time $t_0 = 0$ for different strategies, on a CPU Intel Core i5, 1.70 GHz with a 4GB of RAM.

Table 1. Size of the optimization problem at $t_0 = 0$ for different strategies

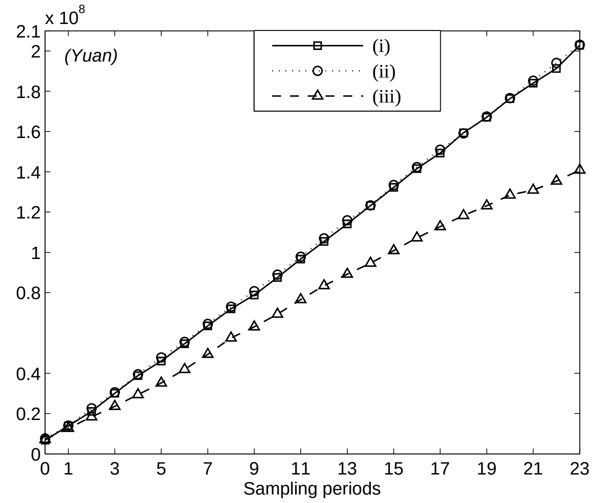
Strategy	Continuous variables	Binary variables	Horizon T	CPUs
(i)	1190	120	10	5
(ii)	1190	120	10	5
(iii)	2856	288	24	618



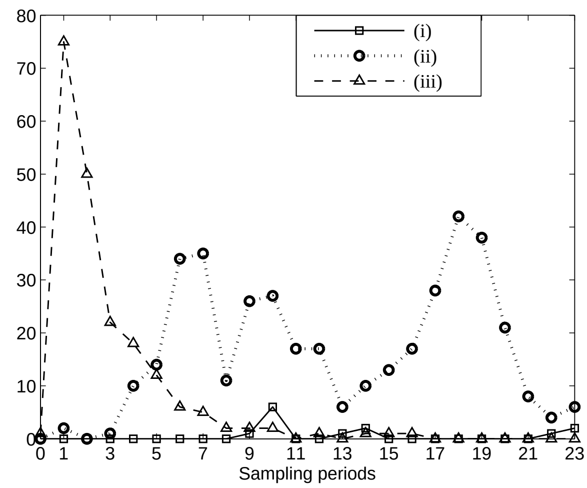
(a) Coals blended for all power companies



(b) Inventory level



(c) Cumulative profits



(d) Frequencies of unsatisfied demands

Figure 5. Comparison of three strategies

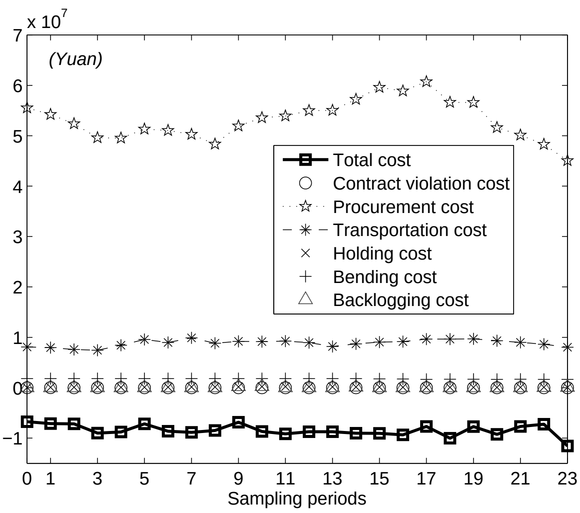


Figure 6. Closed loop objective costs by strategy (i)

Due to the existence of stochastic variables, 50 times trials are carried out for the above three strategies. In the following figures except 5(d), the y-axis values take the average of the 50 times simulations. Figure 5 compares the results of different strategies. The amounts of coals blended for all power companies, i.e., $\sum_k B_{kt}$, are plotted in Figure 5(a). It is shown that the second strategy requires the smallest amount of coals for blending, while the open loop optimization asks for the largest. The reason is that strategy (ii) takes the expected future demands as the actual demands, which are smaller than the probabilistic demands defined for strategy (i) ($\underline{p_k}$ in Table 5 are all greater than 50%). The open loop strategy needs full blending operations at all time instants since it incorporates a larger horizon 24. This brings more uncertainty in the decision making process, thus more coals needs to be prepared.

The inventory level for all types of coals is depicted in Figure 5(b). Strategy (i) has a slightly bigger inventory than strategy (ii), again this is because strategy (i) prepares for more possible realizations of demands. However, the open loop strategy will result in a much and much larger inventory since that it manages the uncertainty along a very long horizon at time $t_0 = 0$ and does not make use of the information available at the future time instants. This demonstrates the benefits of the MPC strategy clearly.

The absolute values of *real cumulative costs*, denoted by $\sum_{t_0=0}^{t_n} \big(BC_{t_0} + CVC_{t_0} + TC_{t_0} + HC_{t_0} + OC_{t_0} + PS_{t_0}\big)$ for $t_n = 0, 1, \ldots, 23$, are also referred to as the cumulative profits, shown in Figure 5(c). The open loop strategy leads to the lowest profits, the reason for which is without updating decisions according to the latest information. To be strange at first glance, strategy (ii) will even produce more profits than strategy (i), though the difference is trivial (0.107%). Figure 5(b) partially hints us since strategy (i) incurs a larger holding cost due to a larger inventory. The advantage of strategy (i) is then manifestly displayed in Figure 5(d), which shows the counts of unsatisfied demands for all power companies out of 50 times simulations at each sampling time. The total number for strategy (i), (ii), (iii) is 13, 387, 199 respectively. Use of probabilistic constraints attests to its powerful value, since even the open loop strategy (probabilistic constraints considered) generates less number of lost sales. This index is vital since the logistics enterprise must keep the frequencies of unsatisfied demands at a very low level, otherwise loyal customers may be lost and the reputation will be degraded.

Figure 6 shows the closed loop objective costs produced by strategy (i). Total cost in the figure means the total profit at each sampling period t_0, i.e., $\big\{BC_{t_0} + CVC_{t_0} + TC_{t_0} + HC_{t_0} + OC_{t_0} + PS_{t_0}\big\}$. In the setting of this example, the procurement cost and transportation cost dominate the whole costs. That is why the importance of the employment of MPC strategy is highlighted, since it always exploit the newest forecast on the corresponding price information.

6. Conclusions

A typical coal trade process with the logistics enterprise as the key node is investigated. Several crucial issues have been addressed in the modelling of the process, i.e., contract violation, unsatisfied demands, operations of the blending equipments, update of the predictions of unit cost information and arbitrarily distributed stochastic demands. A mixed-integer linear model is formulated. A stochastic model predictive control strategy, with both receding horizon and decreasing horizon manner, is devised for the optimization. The proposed strategy is compared with two other strategies through an illustrative case, which clearly demonstrates its strong capability either in aiming at high profits or reducing the possibilities that unsatisfied demands happen. Thus, the model fits the process as simulated rather well, and the SMPC strategy constructs a powerful optimization and decision tool for the problem. Additionally, our results will also generate implications on the trade decision optimization of similar commodities, such as soybean, wheat and corn.

Although newest predictions of the *average* unit cost are incorporated in the MPC strategy, they are stochastic in nature as well and the stochastic properties have not been treated explicitly. Future research will focus on investigating the stochastic issue of the unit cost information in modelling and optimization techniques, and perhaps the theory of stochastic model predictive control needs to be developed accordingly.

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Table 2. Trading limits, ordering fees and discounts

Supplier	Coal type j	$\overline{u_{ijt}}$ (tons)	$\tilde{u}_{it}$ (tons)	$\underline{u_{it}}$ (tons)	OF_{it} (Yuan)	η_i
IQC 1	Qtype11	8000	17000	12000	467500	0.96
	Qtype12	5000				
	Qtype13	6000				
	Qtype14	7000				
IQC 2	Qtype21	8000	12000	8000	330000	0.93
	Qtype22	7000				
IQC 3	Qtype31	8000	10000	5000	275000	0.94
	Qtype32	7000				
IQC 4	Qtype41	9000	25000	20000	825000	0.96
	Qtype42	13000				
	Qtype43	7000				
	Qtype44	8000				
BOA 1	Btype11	8000		15000		
	Btype12	11000				
	Btype13	7000				
	Btype14	10000				
BOA 2	Btype21	10000		15000		
	Btype22	10000				
BOA 3	Btype31	7000		10000		
	Btype32	8000				
BOA 4	Btype41	5000		10000		
	Btype42	6000				
	Btype43	7000				

Table 3. Values of $\overline{\mu_{kt}}$ (thousand tons); $\underline{D_{kt}} = 0.85 \cdot \overline{\mu_{kt}}$ and $\overline{D_{kt}} = 1.15 \cdot \overline{\mu_{kt}}$; D_{kt} are normally distributed: $\mathcal{N}(\mu_{kt},\ \sigma_{kt})$ with $\mu_{kt} = \overline{\mu_{kt}} + 0.06 \cdot \overline{\mu_{kt}} \cdot \varepsilon_{kt}$ and $\sigma_{kt} = 0.08 \cdot \overline{\mu_{kt}}$, where ε_{kt} are uniformly distributed random variables on $[-1,\ 1]$

t	0	1	2	3	4	5	6	7	8	9	10	11
P1	15.5	15.4	15.1	15.5	16.0	16.1	17.8	17.6	16.8	16.8	17.5	16.2
P2	22.0	22.8	22.2	22.5	23.0	25.5	26.5	26.9	26.0	25.6	26.0	25.5
P3	32.5	32.6	32.6	32.6	32.5	33.0	33.6	33.5	30.0	33.7	34.8	33.3
P4	38.7	38.5	38.5	39.7	42.0	43.0	43.5	43.0	43.5	42.7	44.0	43.5
t	12	13	14	15	16	17	18	19	20	21	22	23
P1	16.7	16.7	17.0	18.0	18.5	20.0	19.5	18.0	17.0	16.5	15.0	14.8
P2	24.4	23.6	24.5	25.0	24.5	27.5	27.5	26.0	26.5	25.0	24.5	25.3
P3	33.7	32.5	36.0	38.0	37.0	36.5	35.0	35.0	32.2	32.0	32.5	32.7
P4	42.5	41.6	39.5	39.0	40.5	40.0	41.5	42.5	42.5	41.5	41.5	41.8

Table 4. Values of both additive and non-additive coal elements/attributes

Supplier	Coal type j	Additive: Calorific value (Mcal/kg)	Additive: Volatile matter (%)	Additive: Ash content (%)	Additive: Sulfur content (%)	Non-additive: Moisture content (%)
IQC 1	Qtype11	6.0	30	8	0.8	10
	Qtype12	5.5	27	17	0.3	10
	Qtype13	5.0	28	20	0.3	12
	Qtype14	4.5	27	28	0.9	9
IQC 2	Qtype21	6.0	36	12	0.8	15
	Qtype22	5.5	28	15	0.8	6
IQC 3	Qtype31	5.3	27	7	0.5	10
	Qtype32	5.5	27	7	0.4	14
IQC 4	Qtype41	5.0	15	23	1.5	8
	Qtype42	5.8	14	12	0.8	8
	Qtype43	6.0	20	22	1	10
	Qtype44	5.5	16	26	1.2	17
BOA 1	Btype11	3.8	38	4	0.6	33
	Btype12	4.0	40	4	0.6	33
	Btype13	4.7	42	5	0.6	22
	Btype14	5.4	33	5	0.6	16
BOA 2	Btype21	4.6	22	8	0.4	23
	Btype22	5.5	40	6	0.3	13
BOA 3	Btype31	6.5	30	16	1.6	6
	Btype32	5.5	26	18	0.6	3
BOA 4	Btype41	5.5	40	22	1.2	12
	Btype42	4.8	34	28	0.8	19
	Btype43	5.8	36	16	1	8

Table 5. Parameters related to power companies

Power company	$\underline{p_k}$	$\overline{p_k}$	ρb_k (×620 Yuan)	Calorific value (Mcal/kg) $\underline{E_{kl}}$	Calorific value (Mcal/kg) $\overline{E_{kl}}$	Volatile matter (%) $\underline{E_{kl}}$	Volatile matter (%) $\overline{E_{kl}}$	Ash content (%) $\underline{E_{kl}}$	Ash content (%) $\overline{E_{kl}}$	Sulfur content (%) $\underline{E_{kl}}$	Sulfur content (%) $\overline{E_{kl}}$	Non-additive: Moisture content (%) $\underline{E_{kl}}$	Non-additive: Moisture content (%) $\overline{E_{kl}}$
P1	0.65	0.9	2.2	5.3	6.0	19	32	10	19	0	0.7	3	33
P2	0.58	0.85	2	5.5	6.5	20	35	12	22	0	0.7	3	34
P3	0.61	0.86	2.1	5.1	6.0	22	36	9	23	0	0.7	2	33
P4	0.54	0.83	1.9	5.6	6.5	18	30	14	25	0	0.7	2	35

Table 6. Other parameter values for the simulations

$\rho S_t = 620 + 40\cos(\frac{\pi}{9}t) + 5\varepsilon_t$ Yuan/ton	$r = 0.004$
$\rho O_t = 15$ Yuan/ton	$B_M = 120000$ tons
$\rho H_t = 20$ Yuan/ton	$\overline{V} = 130000$ tons

Table 7. Simulations of ρP_{ijt} and ρT_{it} with the form $\mu + a \cdot \cos(\alpha t + r) + b \cdot \varepsilon_t$ (Yuan/ton), where $\varepsilon_0 = 0$ and ε_t $(t \geq 1)$ are uniformly distributed random variables on $[-1,\ 1]$

Supplier	Coal type j	Parameters of ρP_{ijt}					Parameters of ρT_{it}				
		μ	a	b	α	r	μ	a	b	α	r
IQC 1	Qtype11	565	45	5	$\pi/8$	0	37	2	0.25	$\pi/24$	π
	Qtype12	536	44	5	$\pi/8$	0					
	Qtype13	499	41	5	$\pi/8$	0					
	Qtype14	420	40	5	$\pi/8$	0					
IQC 2	Qtype21	514	46	5	$\pi/8$	0	120.4	2.4	0.25	$\pi/24$	π
	Qtype22	492	43	5	$\pi/8$	0					
IQC 3	Qtype31	438	42	5	$\pi/8$	0	128.4	2.4	0.25	$\pi/24$	π
	Qtype32	455	45	5	$\pi/8$	0					
IQC 4	Qtype41	437	48	5	$\pi/8$	0	42	2	0.25	$\pi/24$	π
	Qtype42	520	50	5	$\pi/8$	0					
	Qtype43	520	50	5	$\pi/8$	0					
	Qtype44	439	46	5	$\pi/8$	0					
BOA 1	Btype11	392	38	5	$\pi/8$	0	65.5	2.5	0.25	$\pi/24$	π
	Btype12	384	36	5	$\pi/8$	0					
	Btype13	492	38	5	$\pi/8$	0					
	Btype14	520	40	5	$\pi/8$	0					
BOA 2	Btype21	407	43	5	$\pi/8$	0	84.5	2.5	0.25	$\pi/24$	π
	Btype22	479	44	5	$\pi/8$	0					
BOA 3	Btype31	407	38	5	$\pi/8$	0	200.7	2.7	0.25	$\pi/24$	π
	Btype32	378	37	5	$\pi/8$	0					
BOA 4	Btype41	427	43	5	$\pi/8$	0	117.4	2.4	0.25	$\pi/24$	π
	Btype42	376	39	5	$\pi/8$	0					
	Btype43	441	44	5	$\pi/8$	0					

References

Alessandri, A., M. Gaggero, and F. Tonelli. 2011. "Min-max and predictive control for the management of distribution in supply chains." *IEEE Transactions on Control Systems Technology* 19 (5): 1075–1089.

Berling, P., and V. Martínez-de Albéniz. 2011. "Optimal inventory policies when purchase price and demand are stochastic." *Operations Research* 59 (1): 109–124.

Cachon, G. P. 2003. "Supply Chain Coordination with Contracts." Handbooks in Operations Research and Management Science: Supply Chain Management 11 (C): 227-239. Edited by Steve Graves and Ton de Ko. Amsterdam: North-Holland.

Calafiore, G. C., and L. Fagiano. 2013. "Stochastic model predictive control of LPV systems via scenario optimization." *Automatica* 49 (6): 1861–1866.

Cannon, M., Q. Cheng, B. Kouvaritakis, and S. V. Raković. 2012. "Stochastic tube MPC with state estimation." *Automatica* 48 (3): 536 – 541.

Cannon, M., B. Kouvaritakis, S.V. Raković, and Q. Cheng. 2011. "Stochastic tubes in model predictive control with probabilistic constraints." *IEEE Transactions on Automatic Control* 56 (1): 194–200.

Cao, X.-M., B.-L. Lin, and H.-X. Yan. 2006. "Integrated coal transportation and inventory model under condition of rail direct transportation." *Beijing Jiaotong Daxue Xuebao/Journal of Beijing Jiaotong University* 30 (6): 27–31.

Corbett, C. J., D. Zhou, and C.S. Tang. 2004. "Designing supply contracts: Contract type and information asymmetry." *Management Science* 50 (4): 550–559.

Elaiw, A. M., X. Xia, and A.M. Shehata. 2012. "Application of model predictive control to optimal dynamic dispatch of generation with emission limitations." *Electric Power Systems Research* 84 (1): 31–44.

IEA. 2013. *International Energy Agency.* http://www.iea.org/newsroomandevents/news/2013/october/key-world-energy-statistics-2013-now-available.html.

Ingrao, A. P. 1976. “Types of government contracts.” *AFL Rev.* 18: 63–86.
Kouvaritakis, B., M. Cannon, S. V. Raković, and Q. Cheng. 2010. “Explicit use of probabilistic distributions in linear predictive control.” *Automatica* 46 (10): 1719–1724.
Liu, C.-M. 2008. “A Blending and Inter-Modal Transportation Model for the Coal Distribution Problem.” *International Journal of Operations Research* 5 (2): 107–116.
Maciejowski, J. M. 2002. *Predictive control: with constraints.* Harlow : Prentice Hall.
Oldewurtel, F., D. Sturzenegger, P. M. Esfahani, G. Andersson, M. Morari, and J. Lygeros. 2013. “Adaptively constrained Stochastic Model Predictive Control for Closed-loop Constraint Satisfaction.” In *Proceedings of the American Control Conference, Washington, DC,* 4674–4681.
Perea-López, E., B. E. Ydstie, and I. E. Grossmann. 2003. “A model predictive control strategy for supply chain optimization.” *Computers and Chemical Engineering* 27 (8-9): 1201–1218.
Sarimveis, H., P. Patrinos, C. D. Tarantilis, and C. T. Kiranoudis. 2008. “Dynamic modeling and control of supply chain systems: A review.” *Computers and Operations Research* 35 (11): 3530–3561.
Seferlis, P., and N. F. Giannelos. 2004. “A two-layered optimisation-based control strategy for multi-echelon supply chain networks.” *Computers and Chemical Engineering* 28 (5): 799–809.
Sherali, H., and R. Puri. 1993. “Models for a coal blending and distribution problem.” *Omega* 21 (2): 235–243.
Shih, J.-S., and H. C. Frey. 1995. “Coal blending optimization under uncertainty.” *European Journal of Operational Research* 83 (3): 452–465.
van Staden, A. J., J. Zhang, and X. Xia. 2011. “A model predictive control strategy for load shifting in a water pumping scheme with maximum demand charges.” *Applied Energy* 88 (12): 4785–4794.
Xie, Z., C. S. Park, and L. Zheng. 2013. “Procurement models under purchase price uncertainty for Chinese oil refineries.” *International Journal of Production Research* 51 (10): 2952–2968.
Yabin, L. 2010. “Research on simulation and optimization of transshipment port operation in a power coal ocean shipping logistics system on the basis of WITNESS.” *Journal of Convergence Information Technology* 5 (2): 84–87.
Yücekaya, A. 2013. “Managing fuel coal supply chains with multiple objectives and multimode transportation.” *EMJ - Engineering Management Journal* 25 (1): 58–70.
Zhang, J., and X. Xia. 2011. “A model predictive control approach to the periodic implementation of the solutions of the optimal dynamic resource allocation problem.” *Automatica* 47 (2): 358–362.

Appendix A. Optimization of the choices of contractors

In the body of the paper, many of the constraints are defined according to contracts. We did not touch upon how to make contracts and choose contractors, since those contents are out of the scope of this research. However, in our framework, a straightforward extension can be made to optimize one set of choices of contractors, which may be viewed as a guide for the logistics enterprise.

Supplementary Parameters

- m_i the number of suppliers to be chosen for IQC's
- m_k the number of power companies to be chosen
- τ_i binary variable to denote whether the logistics enterprise signs the IQC with supplier i, which takes a value of 1 when contract is signed and 0 otherwise
- τ_k binary variable to denote whether the logistics enterprise signs the IQC with power company k, which takes a value of 1 when contract is signed and 0 otherwise

BOA can be handled easily in reality since it is not a strict contract without ordering fee or discount. Only suppliers with IQC's are optimized here. There are totally m_i suppliers and m_k power companies to be selected, with deterministic parameters and current predictions of stochastic parameters provided.

The cost of buying coals and the transportation cost are approximated as follows:

$$BC_t = \sum_i \eta_i \cdot \sum_j \rho P_{ijt} \cdot (1+\alpha_j)\, u_{ijt}, \quad TC_t = \sum_i \rho T_{it} \cdot \sum_j (1+\alpha_j)\, u_{ijt}, \tag{A1}$$

where α_j is a constant that the logistics enterprise predefines, which means the ratio of the amount of coal type j bought from suppliers with BOA's to the amount bought from suppliers with IQC's. Similarly, the dynamics for the inventory level is rewritten as

$$V_{j(t+1)} = V_{jt} + (1+\alpha_j)\sum_i u_{ijt} - \sum_k y_{jkt}, \quad \text{for} \quad \Delta t = 0, 1, \ldots, \widetilde{T} - 1, \tag{A2}$$

where $\widetilde{T}$ denotes the number of sampling periods across one whole year in which contracts will do effect. Correspondingly, objective and constraints here will be considered for the horizon $\widetilde{T}$ as well.

And the total cost of (8) changes into

$$CT = \sum_{\Delta t=0}^{\widetilde{T}-1} \frac{BC_t + TC_t + HC_t + OC_t + PS_t}{(1+r)^{\Delta t}} + \sum_{\Delta t=0}^{\widetilde{T}-1}\sum_i OF_{it}\cdot\tau_i + \sum_{\Delta t=0}^{\widetilde{T}-1}\sum_k (\overline{D_{kt}} - \underline{D_{kt}})\cdot(\rho H_t + \rho O_t)\cdot\tau_k \tag{A3}$$

Comparing the first term of (A3) with (8), the reason to rule out CVC_t is that the case of explicit breach of IQC is not considered here since there are no suppliers with BOA's, but nevertheless the total constraint violation fees are accounted for in the second term. And the third term presents the limit of the uncertainty of the holding and operational costs, which is expected to be as small as possible.

Constraints on u_{ijt} can then be formed as

$$\tau_i \tilde{u}_{it} \le \sum_j u_{ijt} \le \tau_i \sum_{j\in J} \overline{u_{ijt}}, \quad \tau_i \in \{0,\,1\}, \quad \forall i,\, j,\, \Delta t = 0, 1, \ldots, \widetilde{T} - 1. \tag{A4}$$

Constraints to interpret satisfaction of demands of power companies are formulated as

$$\begin{aligned} &\sum_j y_{jkt} \ge \tau_k\, \underline{D_{kt}}, \quad \sum_j y_{jkt} \le \tau_k\, \overline{D_{kt}}, \\ &\sum_j y_{jkt} \ge \tau_k\, \underline{\gamma_t}, \quad \sum_j y_{jkt} \le \tau_k\, \overline{\gamma_t}, \quad \tau_k \in \{0,\,1\}, \end{aligned} \tag{A5}$$

where $\underline{\gamma_t}$, $\overline{\gamma_t}$ are computed using the current estimations of D_{kt} provided by power companies.

Then solve the following optimization problem:

$$\begin{aligned} &\min_{\{u_{ijt},\, y_{jkt},\, B_{kt},\, \tau_i,\, \tau_k\}} \quad \text{(A3)} \\ &\text{subject to} \quad \text{(A2), (10), (11), (A4), (20), (21), (22a), (23), (24) and (A5).} \end{aligned} \tag{A6}$$

It should be noted that all the equations/constraints should be reformulated for $i = 1, 2, \ldots, m_i$, $j \in J$, $k = 1, 2, \ldots, m_k$. Following the results of (A6), choose suppliers and power companies with $\tau_i = 1$ and $\tau_k = 1$ as contractors of the logistics enterprise.

It is pointed out that there are two simplifications, compared to the formulation in the body of the paper.

1. (A1) presents only rough calculations of the costs since it is assumed implicitly that the coals from suppliers with BOA's have the same average price as those from suppliers with IQC's.
2. Explicit breach of contract is not taken into account.

Both of the above points depend explicitly on the real time market information (supply and transportation), which is not yet available at the stage of choosing contractors. This justifies the employment of model predictive control strategy to update the trade decisions at each time period, which will result in better performance.