Development and application of a model-plant mismatch expression for linear time-invariant systems ☆

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Abstract

When a plant and its controller are sufficiently linear and time-invariant so that they can be represented by transfer functions, and this plant is under classical control (meaning the controller can also be represented by a transfer function), the model-plant mismatch (MPM) that often plagues industrial processes can be written as a closed-form expression. This includes a variety of controllers, among which the ubiquitous Proportional, Integral and Derivative (PID) controller. The MPM expression can then be used to identify a representative transfer function of the "true plant" from the currently available plant model. The MPM expression works for single-input single-output as well as multiple-input multiple-output systems. The closed-loop data required for application of the expression has to be sufficiently exciting. If significant disturbances perturb the plant their values need to be avail-

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able. In this article the expression is applied to industrial data to show its applicability.
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1. Introduction

The situation where only poor process models are available for control is a common one. When there is a notable difference between a process and the available model of the process, it is said that model-plant mismatch (MPM) is present. This situation is not only common, but will usually contribute to deteriorated controller performance. The availability of poor process models is known to be a source of poor control performance, in fact this is listed as one of the most significant reasons for poor control performance in the minerals-processing industry by [1]. The fact that MPM is however not limited to the minerals-processing industry is a reason why research into this area has received some focus in the recent past [2].

For processes where only poor models are available, [1] states that the peripheral control tools are as important as the controller itself. Peripheral control tools constitute all the elements in the control loop, other than the controller itself, that function to improve controller performance. These include fault detection and isolation, data reconciliation, observers, soft sensors, optimisers and model parameter tuners. Some of these peripheral tools are addressed for an ore grinding mill circuit in [3, 4, 5].

Many controller design methods make use of a plant model. A good plant model usually helps to improve controller performance, but the dynamics of industrial processes can change significantly over time (as is shown for the example of a milling circuit in [6]). As soon as the plant dynamics change, MPM is present and the controller designed based on the original model will produce sub-optimal control moves. Examples of the sources of changes in plant dynamics are maintenance or equipment changes as well as changes in operating conditions or parameters. In order to restore the controller performance the process needs to be re-identified and the controller redesigned, which is a costly and time-consuming exercise [7]. Apart from the formerly mentioned problems, process re-identification also often involves intrusive plant tests that disturb the normal operation of the plant [2].

An alternative to full process re-identification is to firstly identify the elements in the process transfer function matrix that contain significant mismatch and to only re-identify these. Algorithms for MPM detection have been proposed by [2] and [8]. These algorithms identify the transfer function matrix elements that contain mismatch as well as the significance of the mismatch. This is useful information that can be used to help assess the need for process re-identification. These algorithms do however not supply any additional information about the "true plant", hence there is still a need for process re-identification (although not as expensive as full process re-identification) and ad-hoc controller re-tuning.

Model identification techniques that make use of closed-loop data have been introduced some time ago (see for example [9] and [10]). A good overview of closed-loop identification is given by [11] in which different closed-loop identification techniques are discussed and their characteristic properties are compared. The methods described by [11] are mostly based on statistical approaches and do not make explicit use of the transfer functions representing the system, unlike the method presented in this article. A more recent approach to on-line closed loop identification is given in [12]. Here the joint plant and controller model is identified using subspace model identification, and thereafter the plant model is separated assuming the controller model is known *a priori*.

This paper presents a closed-form expression for the model-plant mismatch (as first derived in [13]), which can be used to update the model such that it may be representative of the actual plant. This expression is shown to work for multiple-input multiple-output (MIMO) systems. The main difference between this article and [13] is the application of the MPM expression to industrial data.

Although this method is related to closed-loop identification, it does make use of the explicit expression for the mismatch to identify the representative plant model. This implies that the model structure is known *a priori* and can simply be updated through the mismatch expression.

The most common form of advanced control in the process industry is linear model predictive control [14]. Implementing a linear MPC requires a linear process model, typically in the form of an LTI transfer function. Most plants that use advanced control will therefore at some point have a good, representative model of the process. Making the previously known transfer function the starting point for the method is therefore a justified decision, as this is commonly available.

The sources of mismatch mentioned earlier are either due to discrete events (such as plant shut-downs) or e.g. slowly degrading instruments that cause the model to slowly drift over time. It is therefore sufficient to make use of this method after such events (depending on their frequency) or at certain times when the control performance has deteriorated. This supervised approach is preferable for processes where this is the case, rather than on-line model tracking, which would be preferable in processes where the model may change drastically at a high frequency.

The newly identified model may then be used to update the controller, such that it can perform in an optimal manner. The expression is however only valid for systems that contain a controller and plant model that can be expressed by means of transfer functions. This does include an array of controllers, but probably most importantly it includes PID controllers.

PID control is still very predominant in the processing industry. An industrial survey on grinding mill circuits by [15] found that more than 60% of the respondents make use of PID control, which implies a large scope for implementation of the presented expression.

Another limitation on the expression is that it requires the input signals to be sufficiently exciting in order to make the implementation sensible. This limitation is however also present for the MPM detection algorithms presented by [2] and [8], and also for most plant identification methods.

The requirement for sufficient excitation means that either sufficiently large (and known) changes are required for the independent variables (such as achieved with sizeable set-point changes) or sufficiently large (and known) disturbances should be present, or both. The expression handles known disturbances directly, but does not handle unknown disturbances. If it is unavoidable to use data without the presence of large unmeasured disturbances their values should first be estimated for example by making use of a Kalman filter ([16]). If this is not possible, the MPM expression described in this paper may not yield the desired results.

Identifying the mismatch in the manner proposed in this paper is equivalent to identifying the additive uncertainty in the model [17], where the additive model uncertainty is also expressed as the difference between the plant and the model. Another possibility is presented by [18] where the output multiplicative uncertainty is explicitly defined by matching the output of the uncertainty model to the outputs of a set of known models.

The paper firstly presents the derivation of the MPM expression and shows how the representative transfer function model of the true plant may be obtained from it. Thereafter the expression is used in a MIMO application example to show its usefulness. Finally the expression is applied to industrial data and the representative plant transfer function is calculated by means of the MPM expression.

2. Model-plant mismatch expression

Consider the one degree of freedom, negative feedback control loop shown in Fig. 1 in which all signals and transfer functions are represented in the Laplace domain. G is the plant that generates the true output $y(s)$, $\hat{G}$ is the model of the plant that generates the model output $\hat{y}(s)$, Q is the controller, $v(s)$ is any disturbance that may be present and $r(s)$ is the reference signal (set-point).

The derivation of the MPM expression which follows is done for a general MIMO system in which all signals may be vectors and all transfer functions

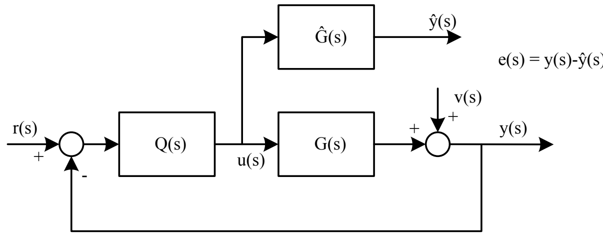

Figure 1: Block diagram of a control loop with model outputs being generated.

may be matrices. G, $\hat{G}$ and Q are all continuous-time, linear time-invariant (LTI) systems, represented in the Laplace domain. G and $\hat{G}$ have the dimensions $n_y \times n_x$ and Q has the dimensions $n_x \times n_y$. y, $\hat{y}$, r and v are $n_y \times 1$ vectors and u is an $n_x \times 1$ vector. For this derivation the number of manipulated variables in the controller must equal the number of controlled variables in the plant, and consequently $n_x = n_y$.

The reference to the Laplace operator (s) will be dropped for ease of representation. Let the residual (e) be the difference between the actual output and the model output as

$$e = y - \hat{y}, \tag{1}$$

$$e = Gu + v - \hat{G}u, \tag{2}$$

$$e = \Delta_M u + v, \tag{3}$$

where $\Delta_M = G - \hat{G}$ is the mismatch. This definition for the mismatch is equivalent to the definition for additive uncertainty presented by [17, p.293]. During this derivation however Δ_M is used to represent uncertainty of any magnitude, as opposed to the weighted uncertainty with a restriction on the maximum singular value in [17] $(\bar{\sigma}(\Delta(j\omega)) \leq 1)$. The control signal $(u(s))$ is given by

$$
\begin{aligned}
u &= Q\left(r-y\right), && (4)\\
u &= Q\left(r-\left[Gu+v\right]\right), && (5)\\
u &= Qr-QGu-Qv, && (6)\\
\left(I+QG\right)u &= Qr-Qv, && (7)\\
u &= \left(I+QG\right)^{-1}Q\left(r-v\right), && (8)\\
u &= Q\left(I+GQ\right)^{-1}\left(r-v\right), && (9)
\end{aligned}
$$

where the push-through rule for matrix manipulation [17, p.68] was used to go from (8) to (9). The matrix inverse operation of (8) requires that the product QG be strictly proper. Substitution of (9) into (3) then gives

$$
\begin{aligned}
e &= \Delta_M Q\left(I+GQ\right)^{-1}\left(r-v\right)+v, && (10)\\
e &= \Delta_M Q\left(I+\{\Delta_M+\hat{G}\}Q\right)^{-1}\left(r-v\right)+v, && (11)\\
e &= \Delta_M Q\left(I+\Delta_M Q+\hat{G}Q\right)^{-1}\left(r-v\right)+v. && (12)
\end{aligned}
$$

The expression $G=\Delta_M+\hat{G}$ is used to go from (10) to (11). After this substitution all the terms in (11) are known, save for the disturbance if it is unmeasured. Further matrix algebra leads to

$$
\left(e-v\right)\left(r-v\right)^{-1}=\Delta_M Q\left(I+\Delta_M Q+\hat{G}Q\right)^{-1}, \qquad (13)
$$

$$
\left(e-v\right)\left(r-v\right)^{-1}\left(I+\Delta_M Q+\hat{G}Q\right)=\Delta_M Q, \qquad (14)
$$

$$
\begin{aligned}
\left(e-v\right)\left(r-v\right)^{-1}\left(I+\hat{G}Q\right) &= \Delta_M Q\\
&\quad -\left(e-v\right)\left(r-v\right)^{-1}\Delta_M Q,
\end{aligned}
\qquad (15)
$$

$$
\begin{aligned}
&(e-v)(r-v)^{-1}\left(I+\hat{G}Q\right) = \\
&\left[I-(e-v)(r-v)^{-1}\right]\Delta_M Q.
\end{aligned} \tag{16}
$$

Rewriting the equation with Δ_M isolated on the left-hand side gives the closed-form mismatch expression as:

$$
\begin{aligned}
&\Delta_M = \left[I-(e-v)(r-v)^{-1}\right]^{-1}\cdot \\
&(e-v)(r-v)^{-1}\left(I+\hat{G}Q\right)Q^{-1}.
\end{aligned} \tag{17}
$$

This expression may be used to derive the mismatch if the disturbances are known. If the disturbances are however unmeasured or even unknown, data from a period of operation free from significant disturbances can be used (if this is possible), and with $v = 0$, (17) becomes

$$
\Delta_M = \left[I-er^{-1}\right]^{-1} er^{-1}\left(I+\hat{G}Q\right)Q^{-1}. \tag{18}
$$

If however unmeasured disturbances cannot be ignored, disturbance estimation techniques (see for example [19]) may be used to account for their values.

The problem with large unmeasured disturbances also plagues classical system identification techniques. This is because the output error (the difference between the measured output and the model output) can be large in the presence of large disturbances, even if the model is perfect [20].

Usually signals (such as $r(s)$) will not be square for MIMO systems and will consequently not have an inverse in the true sense. This issue is discussed further in Appendix A. Sufficient excitation (see [21]) is required in either the disturbance or the reference signal in order for the application of (17) to be sensible. Without sufficient excitation $\nexists\ (r-v)^{-1}$.

The expression $G = \Delta_M + \hat{G}$ may now again be used to obtain the transfer function of the actual plant as

$$\begin{aligned} G = \left[I - (e - v)\,(r - v)^{-1}\right]^{-1} \cdot \\ (e - v)\,(r - v)^{-1}\left(I + \hat{G}Q\right)Q^{-1} + \hat{G}. \end{aligned} \tag{19}$$

If (18) is used as the mismatch expression, the plant transfer function is given by

$$G = \left[I - er^{-1}\right]^{-1} er^{-1}\left(I + \hat{G}Q\right)Q^{-1} + \hat{G}. \tag{20}$$

Notice from the derivation that there is no mathematical limit on the size of Δ_M. The limit on how large Δ_M may be is therefore only based on the usable data that can practically be extracted, e.g. without control valves saturating.

3. MIMO application example

The application of the MPM expression to a SISO system is straightforward, as presented in [13]. Some provisions are however suggested in Appendix A for when the expression is applied to a MIMO system.

In order to illustrate the working of the MPM expression in the MIMO case, the algorithm is applied to a 2×2 ball mill grinding circuit for which MPM is introduced. Consider the ball mill grinding circuit of Fig. 2 which is described in [22].

The manipulated variables are the fresh ore feed rate [u_1 (t/h)] and the dilution water flow rate [u_2 (m^3/h)]. The controlled variables are the product particle size [y_1 (% $-$ 200 mesh)] and the circulating load [y_2 (t/h)]. The nominal values and constraints for the manipulated and controlled variables are given in Table 1. Care should be taken when using the method to not

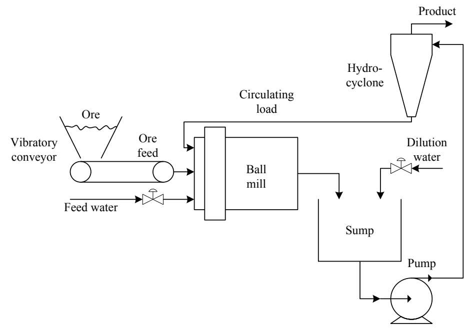

Figure 2: Ball mill grinding (reproduced from [22]).

Table 1: Nominal values and constraints for the 2x2 ball mill grinding circuit variables

Var.	Description	Nom	Min	Max	Unit
u_1	Fresh ore feed rate	65	60	70	t/h
u_2	Dilution water flow rate	45	40	50	m^3/h
y_1	Product particle size	70	68	72	%
y_2	Circulating load	150	140	170	t/h

use data where the output or control variable values are saturated against the limits. This is because the saturation function is not linear and therefore not compatible with the MPM expression.

The MIMO transfer function model of the milling circuit is given by

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}, \tag{21}$$

where

$$g_{11}(s) = \frac{-0.58}{2.5s+1}e^{-0.68s}, \tag{22}$$

$$g_{12}(s) = \frac{4(1-0.9938e^{-0.47s})}{(2s+1)(6s+1)}e^{-0.2s}, \tag{23}$$

$$g_{21}(s) = \frac{2.2}{6s+1}e^{-0.6s}, \tag{24}$$

$$g_{22}(s) = \frac{2.83}{3.5s+1}e^{-0.13s}. \tag{25}$$

Milling circuits are often controlled by decentralized PI(D) controllers [1, 15] as was also implemented for this circuit by [22]. The diagonal PI controller is in the form

$$Q(s) = \begin{bmatrix} K_{c_1}\left(1+\frac{1}{\tau_{I_1}s}\right) & 0 \\ 0 & K_{c_2}\left(1+\frac{1}{\tau_{I_2}s}\right) \end{bmatrix}, \tag{26}$$

with $K_{c_1} = -2$, $\tau_{I_1} = 3.3$ min, $K_{c_2} = 0.42$ and $\tau_{I_2} = 5.2$ min. Next this plant will be perturbed up to the point that robust performance is not achieved with the current controller. This gives a good indication of the point at which process re-identification may be necessary. The robust performance test is carried out as described by [17]. The first step of this test is to represent the uncertainty in each channel in the model through an uncertainty weight of the form

$$w_I(s) = \frac{\tau s + r_0}{(\tau/r_\infty)s+1} \tag{27}$$

where r_0 is the relative uncertainty at steady-state, $1/\tau$ is the approximate frequency where the uncertainty reaches 100% and r_∞ is the magnitude of the weight at higher frequencies. The performance weight is specified as

$$w_P(s) = \frac{s/M + \omega_B}{s + \omega_B A} \tag{28}$$

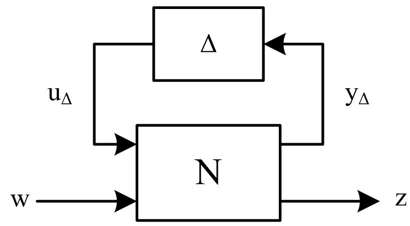

Figure 3: General block diagram with uncertainty included (reproduced from [17, p.114]).

where ω_B is the required bandwidth and A and M are respectively the upper bounds on the sensitivity function at low and high frequencies. Typically $A \approx 0$ and $M \geq 1$. Next the generalized control configuration for representing uncertainty in the plant is derived (see [17, p.113]) as is illustrated in Fig. 3 where w are the exogenous inputs and z the outputs. Nominal stability is achieved if N is internally stable. The tests for robust stability and robust performance make use of the structured singular value μ.

$\mu(N)$ is defined as: Find the smallest structured Δ which makes the matrix $I - N\Delta$ singular, then $\mu(N) = 1/\bar{\sigma}(\Delta)$, where $\bar{\sigma}$ is the maximum singular value. For robust stability:

$$\mu_{\Delta}\left(N_{11}\right) < 1, \forall\omega \tag{29}$$

and N must be nominally stable. For robust performance:

$$\mu_{\hat{\Delta}}\left(N\right) < 1, \forall\omega, \hat{\Delta} = \begin{bmatrix} \Delta & 0 \\ 0 & \Delta_P \end{bmatrix} \tag{30}$$

and N is still required to be nominally stable. The test for robust performance is carried out for 10% gain uncertainty with

$$W_I = \frac{0.21s + 0.1}{0.1s + 1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{31}$$

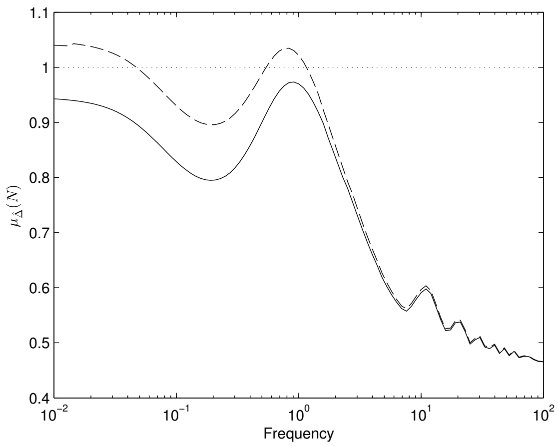

Figure 4: Robust performance test results for 10% uncertainty (solid line) and for the perturbed plant (dashed line).

and

$$W_P = \frac{0.45s + 0.05}{s}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{32}$$

The performance weight specifies integral action and a closed-loop bandwidth of 0.05. This test shows that the performance specification is achieved for 10% uncertainty (Fig. 4 shows the structured singular value $\mu_{\hat{\Delta}}(N)$ for this test [solid line]).

The plant is then perturbed to be

$$g_{11}(s) = \frac{-0.464}{2s+1}e^{-0.68s}, \tag{33}$$

$$g_{12}(s) = \frac{4(1-1.1014e^{-0.47s})}{(2s+1)(6s+1)}e^{-0.2s}, \tag{34}$$

$$g_{21}(s) = \frac{2.2}{6.6s+1}e^{-0.6s}, \tag{35}$$

$$g_{22}(s) = \frac{2.547}{3.5s+1}e^{-0.13s}, \tag{36}$$

which is less severe than the mismatch introduced into the system by [22] but

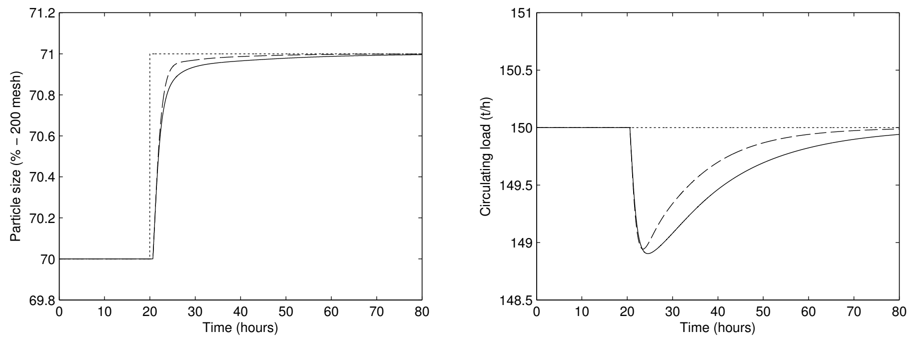

Figure 5: Response of controlled variables for a step in the particle size showing the set-point (dotted line), the nominal response (dashed line) and the perturbed response (solid line).

more severe than allowed by the robust performance analysis weight. Now robust performance is not achieved as illustrated in Fig. 4 (the structured singular value is now shown by the dashed line). The uncertainty (and also the changed plant model) should now be calculated. The nominal and perturbed responses for a step in the particle size set-point are shown in Fig. 5 and Fig. 6. The nominal and perturbed responses for a step in the circulating load set-point are shown in Fig. 7 and Fig. 8.

Once the output signals have been generated the mismatch can be identified. The mismatch is calculated using (18) to be

$$\Delta_M = G - \hat{G} = \begin{pmatrix} \frac{0.0232\mathrm{e}^{-0.68s}}{s^2+0.9\,s+0.2} & \frac{-0.0066\mathrm{e}^{-0.67\,s}}{s^2+0.667\,s+0.0833} \\ \frac{-0.0333\mathrm{e}^{-0.6s}\,s}{s^2+0.3182\,s+0.0253} & \frac{-0.0809\mathrm{e}^{-0.13s}}{s+0.2857} \end{pmatrix}, \tag{37}$$

from which the actual transfer function is calculated to be exactly the same as the original transfer function as given in (22) - (25).

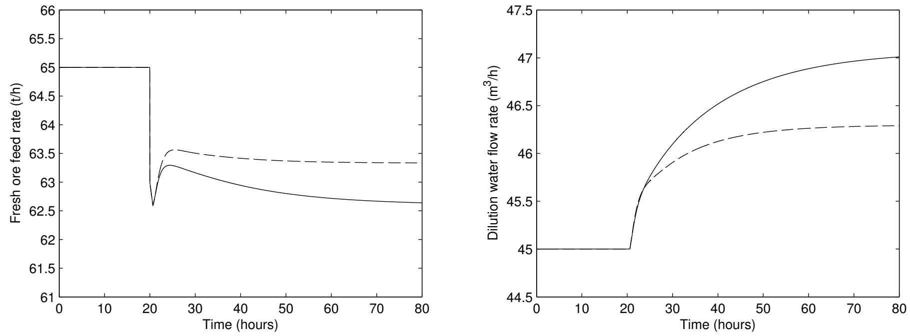

Figure 6: Response of manipulated variables for a step in the particle size showing the nominal response (dashed line) and the perturbed response (solid line).

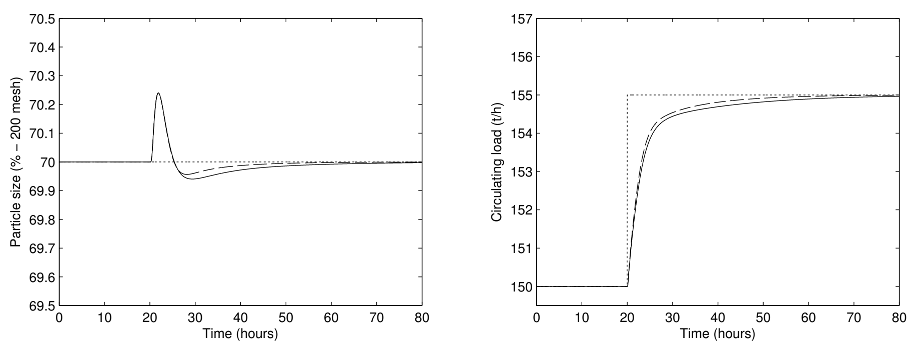

Figure 7: Response of controlled variables for a step in the circulating load showing the set-point (dotted line), the nominal response (dashed line) and the perturbed response (solid line).

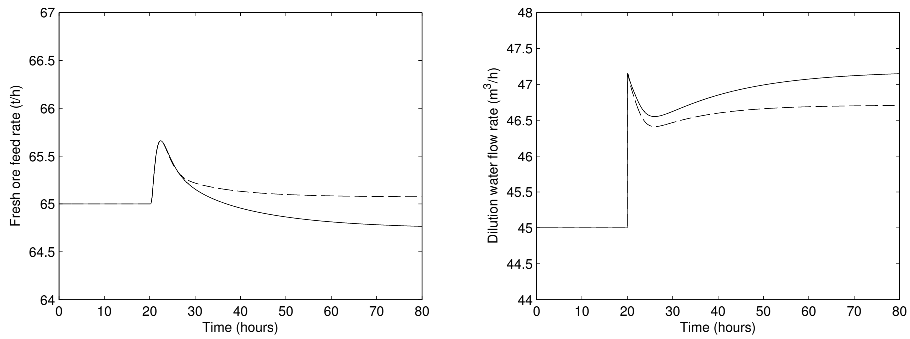

Figure 8: Response of manipulated variables for a step in the circulating load showing the nominal response (dashed line) and the perturbed response (solid line).

4. Industrial case study

To show the application of the method on industrial data, a case study is presented in this section for a splitter column which is part of a Polymer Hydrotreating unit. The data presented was collected during a step testing campaign conducted in 2014.

The purpose of the Polymer Hydrotreater is to convert olefins to the corresponding paraffins to produce a slate of petrol and diesel or jet fuel. After the hydrotreating reaction has taken place, the material is sent to a stripper column (which mainly removes unwanted components) and finally into the splitter column. The splitter column (see Fig. 9[1] for a simplified process diagram) separates the lighter petrol cut from the heavier diesel or jet fuel cut. The unit can either produce diesel of jet fuel depending on the flashpoint[2] of the material in the bottoms of the splitter.

[1]TIC - temperature indicating controller; FIC - flow indicating controller; PDC - differential pressure controller; PIC - pressure indicating controller

[2]The flashpoint is the lowest temperature at which the material will vaporize to form

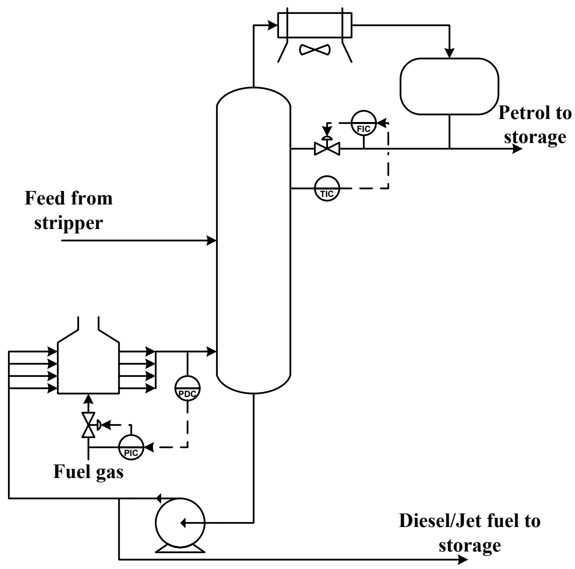

Figure 9: Polymer Hydrotreater splitter column.

The main variables to be controlled in this splitter column are:

- the temperature near the top of the column (on the tray just below where the reflux is added),
- and the percentage vaporization of fluid leaving the heater (this is a good indication of the temperature).

Together these variables largely define the operation of the column. The top temperature controller cascades to a reflux flow controller and the percentage vaporization controller cascades to a fuel gas pressure controller. Both control loops are in cascade configurations and the slave loops are sufficiently fast to ensure that the bandwidth of the slave loops are much larger than the

an ignitable mixture in air.

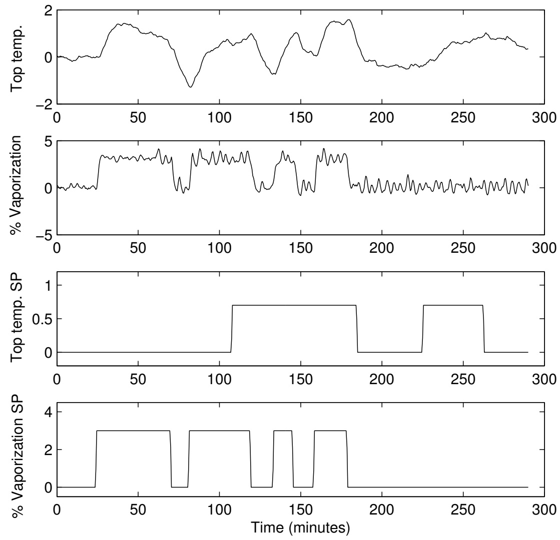

Figure 10: Step test data for a period of 5 hours.

bandwidth of the respective master control loops. The closed-loop transfer function of the inner loop is therefore approximately one ($T_{slave} \approx 1$) [23]. With this approximation the focus can shift to the master control loops.

A 5-hour excerpt of step test data is shown in Fig. 10[3]. Note that the step test data have been standardized to start from zero for intellectual property reasons. This will however not have any effect on subsequent modelling as the constant bias is usually removed from all signals before system identification [21].

The tuning parameters for the controllers are shown in Table 2. All time constants shown are in minutes and the controllers have the form

$$Q = K_c \left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha \tau_D s + 1} \right), \tag{38}$$

[3]SP - set-point

Table 2: Tuning parameters for splitter column controllers.

Loop	K_c	τ_I	τ_D
Top temp.	1	8	0.5
% Vaporization	0.5	2	0

with $\alpha = 0.1$.

It is usually possible to sufficiently represent a binary distillation column with a linear model if both products are of high purity and the reflux is large [24], which is the case in our example. The operating point also does not change significantly during the step testing campaign. These reasons along with the fact that it is possible to obtain a representative linear model for the plant from the data[4], allows us to consider this process to be sufficiently linear.

In order to apply the model-plant mismatch expression an initial model is needed ($\hat{G}$ in (19)). Hereafter the plant has to be perturbed, and operating data from the perturbed plant should be captured for use. As it is not possible for production reasons to perturb this industrial plant, the closed-loop operating data available are therefore assumed to be for the perturbed plant (G in (19)).

Model identification of the splitter column was done using third party vendor software. Selected step-test campaign data were used, and the resulting 2×2 transfer function matrix is shown in (40) (with time in minutes). Note again that this model is denoted as G because it is considered to be

[4]This will be shown later in this section, see Table 5.

Table 3: Summary of the referenced models

Model	TF	Description	How obtained
G	(40)	Perturbed plant model. Assumed unknown for the application of the MPM expression	SID using 3rd party software
$\hat{G}$	(41)	Assumed original model from which controller was designed	Adapted G based on plant changes
G^*	(48)	Calculated model	Obtained from (17) with plant data

the perturbed plant. This model is assumed to be unknown and is simply shown here for the comparison that will be made once the MPM expression has been applied. A summary of the models referred to in this section is given in Table 3.

$$\begin{bmatrix} \text{Top temp.} \\ \text{\% Vapor.} \end{bmatrix} = G \cdot \begin{bmatrix} \text{Reflux} \\ \text{Fuel gas} \end{bmatrix}, \tag{39}$$

$$G = \begin{bmatrix} \frac{-0.58}{10\,s+1} & \frac{4}{15\,s+1}\mathrm{e}^{-3\,s} \\ 0 & \frac{3.8}{2.8\,s+1}\mathrm{e}^{-0.5\,s} \end{bmatrix}. \tag{40}$$

Consider now the scenario where there was a plant shut-down during which changes were made to the original plant (represented by $\hat{G}$). Suppose that during the shut-down the thermowell housing the element sensing the top temperature in the column was cleaned of a build-up of residue. This removes some lag when measuring the top temperature. Suppose also that the transmitter was re-calibrated for a smaller range. These two changes will cause the time constants of $G_{1,1}$ and $G_{1,2}$ to decrease by similar amounts when compared to $\hat{G}_{1,1}$ and $\hat{G}_{1,2}$, as well as causing the gains of both these transfer function elements to increase by similar amounts. Suppose that the changes are 20% in either case. This value is chosen large enough to have a significant impact on the output responses as can be seen in Fig. 11 and Fig. 12. This means the original model of the plant was:

$$\hat{G} = \begin{bmatrix} \frac{-0.464}{12\,s+1} & \frac{3.2}{18\,s+1}\mathrm{e}^{-3\,s} \\ 0 & \frac{3.8}{2.8\,s+1}\mathrm{e}^{-0.5\,s} \end{bmatrix}. \tag{41}$$

To reiterate, $\hat{G}$ is considered to have been the original model of the plant. The plant model is then assumed to have changed to G. The plant data available are assumed to be obtained from the closed-loop system where the plant is represented by G. These data will now be used to apply the MPM expression.

Q and $\hat{G}$ are known; r, u, and y are determined from the data. There are no measured disturbances that greatly affect the process. A section of data is therefore selected for which no significant unmeasured disturbances seem to affect the plant so that (19) can be applied. Such an excerpt of data is shown in Fig. 11 for the top temperature when a set-point change is made. Note that $\hat{y}$ is generated by propagating the measured u through the known system $\hat{G}$ as

$$\hat{y} = \hat{G}u. \tag{42}$$

A similar section of data is shown in Fig. 12 for when a step change in the percent vaporization is made.

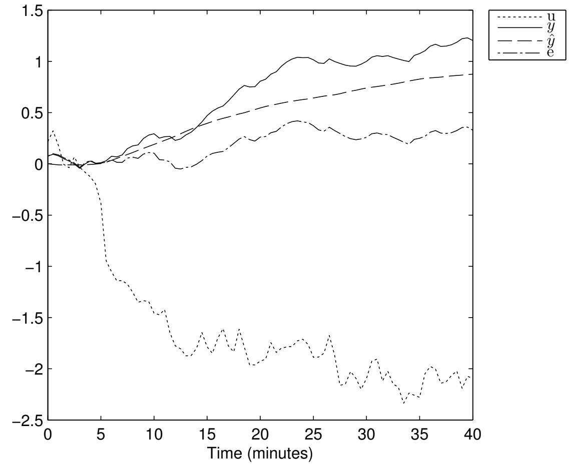

Figure 11: Data for the top temperature showing the plant input (dotted line), plant output (solid line), model output (dashed line), and the error (dash-dot line).

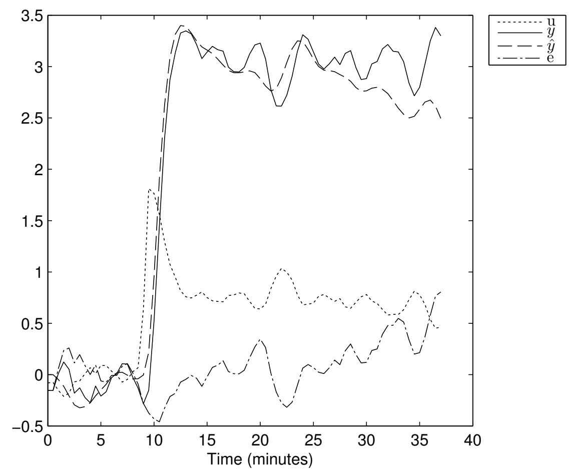

Figure 12: Data for the heater flue gas % vaporization showing the plant input (dotted line), plant output (solid line), model output (dashed line), and the error (dash-dot line).

The last signal needed for the application of the MPM expression is e in the Laplace domain. There are many ways in which this signal may be obtained, see for example [25] for an overview of such methods. The method however used here is via a direct transfer function estimation method (described in [26]). The error model driven by the reference signal is defined as

$$er^{-1} = e_{M_r}(s) = \frac{B_0(s)}{A_0(s)} \tag{43}$$

with

$$y - \hat{y} = e_{M_r} r, \tag{44}$$

$B_0(s)$ and $A_0(s)$ are polynomials in s defined as ([26]):

$$B_0(s) = \sum_{i=0}^{m} b_i s^i \tag{45}$$

$$A_0(s) = \sum_{i=0}^{n-1} a_i s^i + s^n \tag{46}$$

and $n \geq m$, where n is the number of poles and m is the number of zeros of $e_{M_r}(s)$. The method makes use of the equation error to fit a continuous-time transfer function model to discrete-time data, and was recently included in the continuous time system identification toolbox in Matlab. The equation error is a linear algebraic function of the model parameters in the form [27]:

$$\varepsilon_{EE}(t) = A_0 y(t) - B_0 u(t). \tag{47}$$

This method supplies the signal in the Laplace domain, which means that all the signals needed to apply the MPM expression are now available.

Table 4: Percentage difference between identified model G and calculated model G^*

Model	$G(1,1)$	$G(1,2)$	$G(2,2)$
Gain K	16.7 %	16.3 %	0.3 %
Time constant τ	4.0 %	5.8 %	5.7 %
Time delay θ	-	0 %	0 %

Once the MPM expression of (20) has been applied the calculated plant transfer function (denoted as G^*) is obtained to be

$$G^* = \begin{bmatrix} \frac{-0.677}{10.40\,s+1} & \frac{4.65}{15.87\,s+1}\mathrm{e}^{-3\,s} \\ 0 & \frac{3.81}{2.96\,s+1}\mathrm{e}^{-0.5\,s} \end{bmatrix}. \tag{48}$$

It can be seen that the calculated transfer function G^* is not significantly different from the identified model for the actual plant transfer function G in (40). The relative differences between the transfer function element parameters is shown in Table 4. The main reason for the presence of any difference here is because of imperfect model identification.

The question remains whether the calculated transfer function is truly a better reflection of the plant than the model $\hat{G}$ derived previously. To illustrate this the model outputs of $\hat{G}$ and G^* are compared to the "actual" plant output in Fig. 13, Fig. 14, and Fig. 15 for the three non-trivial transfer function elements of G. To make the MPM detection worthwhile, the fit between the output related to $G^*_{1,1}$ and $G^*_{1,2}$ and the plant data should be much higher than for the output related to $\hat{G}_{1,1}$ and $\hat{G}_{1,2}$. This is indeed the case as can be seen from Table 5 in which the normalized root mean square errors (NRMSE) for these comparisons are shown.

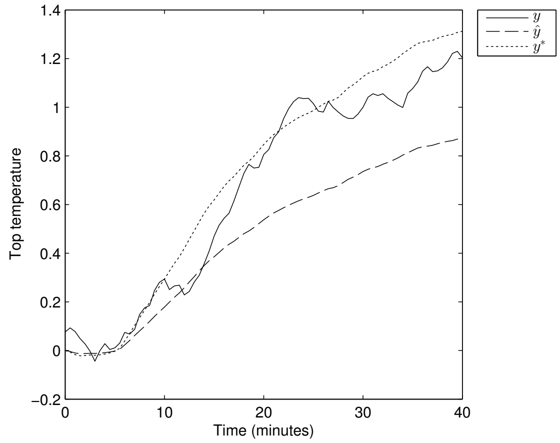

Figure 13: Model comparison for $G_{1,1}$ showing the plant output (solid line), original model $\hat{G}$ output (dashed line), and the calculated model G^* output (dotted line).

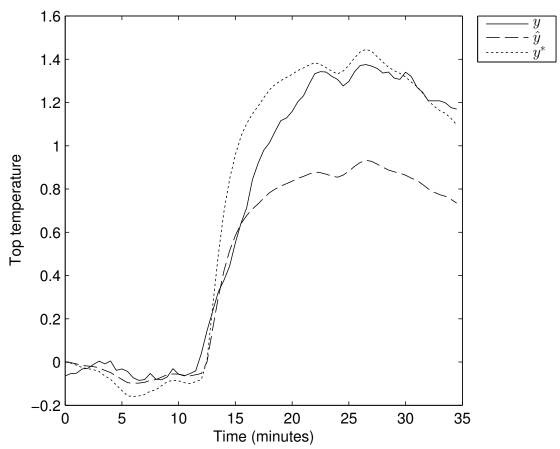

Figure 14: Model comparison for $G_{1,2}$ showing the plant output (solid line), original model $\hat{G}$ output (dashed line), and the calculated model G^* output (dotted line).

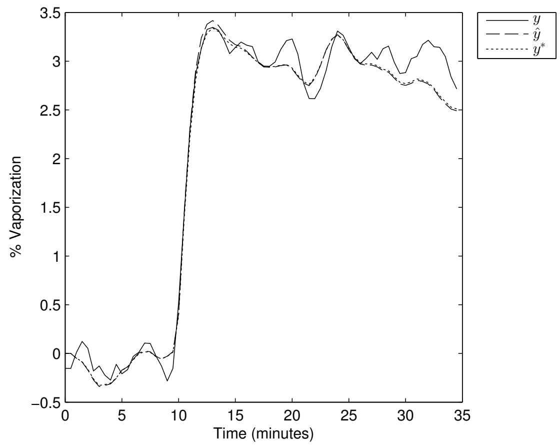

Figure 15: Model comparison for $G_{2,2}$ showing the plant output (solid line), original model $\hat{G}$ output (dashed line), and the calculated model G^* output (dotted line).

Table 5: NRMSE for the fit between models and plant output data

Model	$G(1,1)$	$G(1,2)$	$G(2,2)$
$\hat{G}$	44.14 %	51.48 %	87.15 %
G^*	75.01 %	76.53 %	87.63 %

5. Application to controller design

Once the mismatch (Δ_M) has been identified correctly, the expression $G = \Delta_M + \hat{G}$ may be used to obtain the representative transfer function of the plant. The representative transfer function of the plant may then be used to redesign the controller. In the case for the MPM identified in Section 4, this would mean that the PID controller (38) for the output "Top Temperature", should be redesigned.

There are many ways of tuning PI(D) controllers. Some of the more common methods include the Ziegler-Nichols method, the Cohen-Coon method, the IMC tuning relations [28], the SIMC expansion thereof [29], Lambda tuning (which is a specific case of the IMC relations), tuning based on the minimization of the integral error, pole placement, and loop shaping (see [30] for more examples). Most of these methods however make explicit use either of the plant transfer function or of the model parameters that characterize the transfer function.

There are also many other controller design methods that do not specifically lead to PI(D) controllers but do produce controllers representable by means of transfer functions. These methods include linear quadratic Gaussian (LQG) control, as well as H_2 and H_∞ control.

The method chosen for controller design is not so important, what is however important is that the reader appreciates how commonly the plant transfer function is used in controller design. In these cases MPM will cause the controller to perform outside of its original design intent. If the MPM is severe enough this could lead to the dynamic performance specifications not being met or even instability. In such a case the MPM expression presented in this article may be used to update the available plant model and the controller design procedure may be repeated. This will lead to better adherence to the control specifications.

6. Conclusion

This paper presents a closed-form expression for the MPM that may be present in a feedback control system where the controller is representable by means of a transfer function. The expression may be used to identify a representative plant transfer function from closed-loop operational data. The expression is directly applicable for SISO systems where the plant is easily identified. In the MIMO case some provisions are needed to ensure correct results. The plant model was correctly identified in an example with a MIMO plant. The main contribution of this paper as opposed to [13] was to show how this same expression was successfully applied to industrial data. The updated plant transfer function can then be used to redefine the controller.

This expression does however need sufficiently exciting signals to make its application sensible. By using industrial data containing noise, Section 4 shows that the expression may also be applied when measurement and/or process noise is present. The requirement for sufficient excitation however includes the need for the amplitude of step changes to be significantly larger than the noise amplitude.

Because this is a data driven method the fidelity of the data is important. Deviations in the data such as may be caused for example by sensor failures or valves getting stuck in place are not handled directly by the method. The same care taken when selecting data for system identification should be taken when selecting data when applying this method.

The MPM expression also handles measured disturbances, but unmeasured disturbances may affect the accuracy of the identified model. Care should therefore be taken to use plant data that do not contain significant unmeasured disturbances.

Appendix A. MIMO application provisions

It was stated in Section 2 that signals such as $r(s)$ are usually not square, which is a problem for MIMO applications. This is because a non-square matrix does not have an inverse in the traditional sense. Say for example the output ($y(s)$) is $n \times 1$ generated by applying an $n \times 1$ input signal ($u(s)$) to an $n \times n$ plant ($G(s)$) as

$$y = Gu, \tag{A.1}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}, \tag{A.2}$$

from which $y(s)$ is calculated to be

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11}u_1 + \cdots + g_{1n}u_n \\ \vdots \\ g_{n1}u_1 + \cdots + g_{nn}u_n \end{bmatrix}. \tag{A.3}$$

The MPM method determines a transfer function from the "inverse" of the input signal as,

$$G = yu^{-1}. \tag{A.4}$$

In the SISO case this is not a problem as G, $y(s)$ and $u(s)$ are scalars. In the MIMO case however the expression cannot be applied directly as the non-square signal $u(s)$ does not have an inverse. If however the input signal is rewritten as the diagonal matrix

$$U = \begin{bmatrix} u_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & u_n \end{bmatrix}, \tag{A.5}$$

the output becomes

$$Y = \begin{bmatrix} g_{11}u_1 & \cdots & g_{1n}u_n \\ \vdots & \ddots & \vdots \\ g_{n1}u_1 & \cdots & g_{nn}u_n \end{bmatrix}. \tag{A.6}$$

Now U is square and does have a matrix inverse. Applying equation (A.4) now gives

$$\begin{aligned} G &= YU^{-1}, && \text{(A.7)} \\ G &= \begin{bmatrix} g_{11}u_1 & \cdots & g_{1n}u_n \\ \vdots & \ddots & \vdots \\ g_{n1}u_1 & \cdots & g_{nn}u_n \end{bmatrix} \begin{bmatrix} u_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & u_2 \end{bmatrix}^{-1}, && \text{(A.8)} \\ &= \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix}, && \text{(A.9)} \end{aligned}$$

which is equal to the original transfer function.

The input signal can easily be written in the form of a square matrix as in (A.5). The output is however not usually available as a square matrix. It is however apparent that the first entry of (A.6) is equal to the first output in (A.3) if $u_2 \cdots u_n = 0$. This means that a portion of the output signal generated without excitation in $u_2 \cdots u_n$ can be used to calculate the first entry of (A.6). The same argument holds for the calculation of the other entries of (A.6).

A similar situation holds true for measured disturbances. If disturbances are however unmeasured, care would need to be taken to use a portion of data that is disturbance free as unmeasured disturbances are not explicitly handled by the expression.

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