Switched Model Predictive Control for Energy Dispatching of a Photovoltaic-Diesel-Battery Hybrid Power System

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***Abstract*—A new adaptive switched model predictive control (MPC) strategy is designed in this paper for energy dispatching of a photovoltaic-diesel-battery (PDB) hybrid power system, where the battery is unpermitted to charge and discharge simultaneously. The distinguishing feature of the proposed switched MPC is that, new switched constraints are constructed to describe the different modes (charging and discharging) of the battery, such that the burden of using a switched multiple-input-multiple-output (MIMO) state-space model could be circumvented. Parameters of the battery are unknown constants, and are estimated online with an adaptive updating law. In the switched MPC algorithm, predictive horizon and control horizon vary according to the predefined switching schedule. Based on optimization with the switched constraints, receding horizon control is utilized to obtain the dispatching strategy for the hybrid power system. Performances of the closed-loop system with the proposed switched MPC are verified by simulation results.**

***Index Terms*—Energy dispatch, model predictive control, hybrid power system, optimization, receding horizon control.**

I. Introduction

Independent power systems are of great significance for remote areas locating out of national power grids, and several new independent power system models were proposed in recent years. Among those models, photovoltaic (PV) generators are typically used, because of environmental concerns (less greenhouse gas emission and fuel consumption, etc. [1]). Solar PV energy has been adopted in many countries as a complement for national power grids [2]. However, the main disadvantage of PV generator is that, solar energy is subject to daily and seasonal variations [3]. Batteries are usually used for storing surplus solar energy [4] in case of sufficient sunshine, and supplying shortages in case of insufficient solar energy. Sometimes the daily demand for energy might be so large that it cannot be satisfied by solar energy and battery altogether. In this situation, the imbalance is required to be covered by some other devices, such as diesel generator [2], [5].

Photovoltaic-diesel-battery (PDB) hybrid power system has been proposed in a previous research [6] to satisfy the daily requirements of power in a rural Zimbabwean public clinic. In this hybrid system, the battery is used to store surplus energy generated by PV arrays. The diesel generator is used to cover the imbalance whenever load demands cannot be satisfied by the PV arrays and the battery. During working process of the PDB hybrid power system, PV and battery usage are preferred because of environmental and economic concerns, and the diesel generator is the last choice since it consumes expensive fossil fuels and emits greenhouse gases. A dispatching problem arises on scheduling uses of different components of the PDB hybrid system, such that load demands are supplied, and fuel consumption can be reduced as much as possible.

To solve dispatching problems for hybrid power systems, an optimization control strategy [6] is proposed; however, in case of disturbances in load demands and PV power, performance of the optimization strategy in [6] can be significantly deteriorated. Another effective approach for energy dispatching is model predictive control (MPC) [7]. One advantage of MPC for energy dispatching over the optimization control is that, MPC is a closed-loop approach achieving better performances during a relatively long period when disturbances would possibly occur. MPC approaches have been applied previously to dispatching problems, such as optimal dynamic resource allocation [8], cost-optimal operation of water pump station [9], fuel cost minimization of power generation [10], and current management for hybrid fuel cell power system [11]. Besides, there exist other approaches for energy dispatching of hybrid power systems (genetic algorithm [12], for example).

For the PDB hybrid power system, a practical concern is that the battery might be unpermitted to charge and discharge simultaneously. A typical route to solve the dispatching problem for this situation is to design MPC for a switched model. Switched MPC approaches have been studied extensively [13], [14], [15]; however, most of the existing researches are based on complicated switched predictive models, which may potentially render the optimization unsolvable in MPC design. Consequently, if simultaneous charging and discharging are unpermitted in the PDB hybrid power system, it is beneficial to search for less complicated MPC strategies.

In this paper, a new adaptive switched MPC is proposed for the PDB hybrid power system in case that simultaneous charging and discharging are unpermitted. The main *contributions* of this paper include that, 1) the optimal dispatching problem is modeled into a control problem and solved by the approach of MPC, so that the closed-loop system could benefit from advantages such as feedback and prediction; 2) switched modes (charge and discharge) of the battery are described by switched constraints (instead of switched state-space model), such that a unified linear MIMO state-space model could be used to design a simple predictive model; and 3) adaptive parameters with updating law are employed to estimate uncertain constant parameters of the battery. Simulation results demonstrate that, with the proposed switched MPC strategy, energy efficiency of the closed-loop system are satisfactory.

The configuration of this paper is arranged as following: the mathematical model of the PDB hybrid system is proposed in Section II; the detailed switched MPC design for energy dispatch of the PDB hybrid power system is presented in Section III; simulation results are displayed in Section IV; and the conclusion is drawn in the final section.

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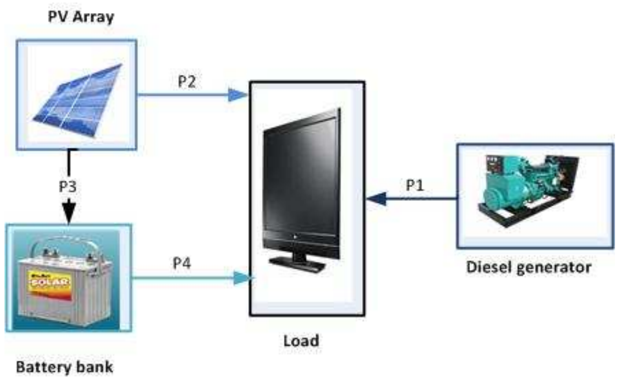

Fig. 1. Configuration of the PDB hybrid system

TABLE I
LOAD DEMAND ($P_L(k)$, KW) OF A ZIMBABWE RURAL CLINIC

time	summer	winter	time	summer	winter
00:30	1.5	1.5	12:30	1.25	1.95
01:30	1.5	1.5	13:30	1.32	1.95
02:30	1.85	1.5	14:30	1.35	1.95
03:30	1.95	1.5	15:30	1.35	1.95
04:30	1.85	1.5	16:30	1.45	1.65
05:30	1.5	1.65	17:30	2.15	1.65
06:30	1.15	1.65	18:30	2.31	3.25
07:30	1.25	1.35	19:30	3.25	3.25
08:30	1.3	1.35	20:30	3.25	2.31
09:30	1.32	3.0	21:30	2.0	2.15
10:30	1.35	3.0	22:30	1.95	2.15
11:30	1.32	1.95	23:30	1.65	1.35

II. PROBLEM STATEMENT

A. Overall structure of the hybrid system

As is displayed in Fig. 1, the PDB hybrid power system is proposed by combining a diesel generator, a photovoltaic array (PV), and a battery bank [6]. The proposed hybrid system supplies the daily requirements of a remote off-grid area.

In this paper, the mathematical model of the PDB hybrid power system is based on a previous version [6]. The difference lies in that simultaneous charging and discharging are unpermitted. The new model is more practical than the previously proposed one.

The hourly load profile used in this paper is displayed in Table I. The data are obtained from a Zimbabwe rural community clinic [6] locating in an off-grid remote area.

B. Photovoltaic array

In this paper, data of energy output from the PV array are given for each hour in a single summer or winter weekday, as shown in Table II. The PV data are obtained from the aforementioned Zimbabwe rural community clinic, by using the methodology and solar radiation data proposed in [16]. As is shown in Fig. 1, energy from PV is used for supplying the load demand and charging the battery.

***Remark** 1:* Data from the aforementioned Zimbabwe rural community clinic are qualified for this research, because 1) this clinic is a typical energy consuming unit locating at an off-grid remote area where there exists energy shortage, and 2) the weather condition and solar radiation in this area are relatively stable, indicating that daily PV energy data are representative.

TABLE II
ENERGY PROVIDED BY THE PV ARRAY ($P_{pv}(k)$, KW)

time	summer	winter	time	summer	winter
00:30	0.00	0.00	12:30	6.59	5.14
01:30	0.00	0.00	13:30	5.84	4.50
02:30	0.00	0.00	14:30	4.84	3.56
03:30	0.00	0.00	15:30	3.47	2.33
04:30	0.00	0.00	16:30	2.07	1.11
05:30	0.00	0.00	17:30	0.09	0.13
06:30	0.09	0.19	18:30	0.00	0.00
07:30	2.30	1.21	19:30	0.00	0.00
08:30	3.98	2.66	20:30	0.00	0.00
09:30	5.42	3.95	21:30	0.00	0.00
10:30	6.45	4.89	22:30	0.00	0.00
11:30	6.75	5.25	23:30	0.00	0.00

PV powers for supplying the load demand and charging the battery are denoted by P_2 and P_3, respectively. They should be subject to the following constraints:

$$0 \leq P_2(k) \leq P_2^{max}, \; 0 \leq P_3(k) \leq P_3^{max},$$
$$0 \leq P_2(k) + P_3(k) \leq P_{pv}(k),$$

where P_2^{max} denotes the maximum amount of power that can be directly transmitted to the load from PV array, and P_3^{max} is the maximum amount of power allowed to charge the battery during one hour.

C. Battery bank

Charging and discharging of battery bank can be described by a simplified discrete dynamic equation:

$$S_{oc}(k+1) = S_{oc}(k) + \eta_c P_3(k) - \eta_d P_4(k), \quad (1)$$

where $S_{oc}(k)$ denotes the state of charge at sampling time k; P_3 and P_4 are charged power and discharged power, respectively; η_c and η_d are charging efficiency and discharging efficiency, respectively. In this paper, it is considered that η_c and η_d are uncertain constant parameters, and are estimated online in the MPC design.

***Remark** 2:* The simplified discrete model of state of charge (1) originates from the continuous model proposed in [11], where variation of the state of charge is proportional to the (charging and discharging) current.

It follows from (1) that the state of charge at a given time τ could be expressed by

$$S_{oc}(\tau) = S_{oc}(0) + \eta_c \sum_{k=0}^{\tau} P_3(k) - \eta_d \sum_{k=0}^{\tau} P_4(k).$$

State of charge of the battery bank is subject to the constraint:

$$B_C^{min} \leq S_{oc}(k) \leq B_C^{max},$$

where B_C^{min} and B_C^{max} are its upper and lower limits. The discharged power of the battery P_4 should satisfy the constraint:

$$0 \leq P_4(k) \leq P_4^{max},$$

where P_4^{max} is the maximum hourly discharging.

In this paper, it is considered that simultaneous charging and discharging are unpermitted, that is

$$P_3(k)P_4(k) = 0. \tag{2}$$

Switches between charging and discharging are arranged in a heuristic manner according to data in Table I and Table II. When the load demand exceeds PV power (between sunset and sunrise), the battery is set in discharging state; and when the PV power exceeds the load demand (during day time), the battery is set in charging state.

***Remark** 3:* It should be acknowledged that the heuristic manner of switching time could possibly brings some drawbacks. For example, there might be disturbances in PV power and load demand. The disturbances could possibly result in lower PV power and higher load demand, such that charging would happen when PV power is insufficient (and discharging would happen when PV power is sufficient). However, in another aspect, the performance of MPC with respect to disturbances can be well tested.

D. Diesel generator

The diesel generator is used to cover the imbalance, when the load demands cannot be satisfied by the PV array and the battery altogether. It is the final choice, because 1) the fuel is expensive, and 2) it generates greenhouse gases such as Carbon Dioxide (CO_2). The advantage of using diesel generator is that it can be operated at any time according to demands.

The energy from diesel generator is subject to the constraint:

$$0 \leq P_1(k) \leq P_1^{max},$$

where P_1^{max} is the maximum amount of power that can be generated by the diesel generator during one hour.

As is mentioned before, the diesel generator, photovoltaic array and battery bank should supply the daily requirements of power cooperatively:

$$P_1(k) + P_2(k) + P_4(k) = P_L(k).$$

E. Objective

The *objective* of this paper is to design the scheduling of P_2, P_3 and P_4 for the PDB hybrid power system in case of uncertain constant charging and discharging coefficients (η_c and η_d), such that the usage of diesel generator P_1 can be minimized. It is also under consideration that the battery should not be excessively used, such that its deterioration can be prevented.

III. Switched model predictive control design

In this section, design procedures of the switched model predictive control are presented, including parameter estimation, system model transformation, objective function design, constraints treatment, and MPC algorithm.

A. Online estimation for battery parameters

Define $u(k) \triangleq [P_2(k), P_3(k), P_4(k)]^T$. The dynamic process of the battery bank can be expressed by

$$S_{oc}(k) = S_{oc}(k-1) + b_m u(k-1), \tag{3}$$

where $b_m \triangleq [0, \eta_c, -\eta_d]$. or equivalently,

$$S_{oc}(k) = S_{oc}(k-1) + b_b u_b(k-1), \tag{4}$$

where $b_b \triangleq [\eta_c, -\eta_d]$, and $u_b \triangleq [P_3(k), P_4(k)]^T$.

The estimated battery dynamic system can be given by

$$\hat{S}_{oc}(k) = S_{oc}(k-1) + \hat{b}_b(k-1)u_b(k-1), \tag{5}$$

where $\hat{S}_{oc}(k)$ is the estimated state of charge, and $\hat{b}_b \triangleq [\hat{\eta}_c, -\hat{\eta}_d]$ denotes estimated parameters.

The cost function for online identification is designed by

$$\begin{aligned} J_p =& \frac{1}{2}\tilde{S}_{oc}(k)^2 \\ =& \frac{1}{2}\left(S_{oc}(k) - S_{oc}(k-1) - \hat{b}_b(k-1)u_b(k-1)\right)^2. \end{aligned}$$

where $\tilde{S}_{oc}(k) \triangleq S_{oc}(k) - \hat{S}_{oc}(k)$. Its gradient with respect to $\hat{b}_b$ can be calculated by

$$\nabla J_p = -\left(\Delta S_{oc}(k) - \hat{b}_b(k-1)u_b(k-1)\right)u_b(k-1),$$

where $\Delta S_{oc}(k) \triangleq S_{oc}(k) - S_{oc}(k-1)$. Consequently, the updating law for $\hat{b}_b$ can be designed by

$$\hat{b}_b(k) = \hat{b}_b(k-1) - \lambda \nabla J_p$$

and the updating law for $\hat{b}_m$ is designed by

$$\hat{b}_m(k) = [0, \hat{\eta}_c(k), -\hat{\eta}_d(k)] = [0, \hat{b}_b(k)]. \tag{6}$$

It can be proved that [18], with the proposed updating law (6), estimated parameters converge to their actual values, if the control u_b is persistently exciting (PE [18]), and satisfies

$$u_b(k)^T u_b(k) < \frac{2-\alpha}{\lambda}, \tag{7}$$

where $0 < \alpha < 2$. In this paper, (7) is treated as an additional constraint. PE of $u_b(k)$ is explained in Appendix.

B. MIMO linear state-space modeling

In this section, the model of the hybrid system is transformed into a linear state-space form to facilitate MPC design.

Define outputs

$$\begin{aligned} y_m(k) \triangleq& c_1\left(P_L(k) - P_1(k)\right) = c_1\left(P_2(k) + P_4(k)\right), \\ y_a(k) \triangleq& c_3\left(P_2(k) + P_3(k)\right), \\ y_b(k) \triangleq& c_2\left(P_3(k) + P_4(k)\right), \end{aligned}$$

where c_1, c_2 and c_3 are positive weight coefficients. It can be seen that minimizing $\sum c_1^2 P_1(k)^2$ is equal to minimizing $\sum\left(c_1 P_L(k) - y_m(k)\right)^2$; the usage of PV can be encouraged by minimizing $\sum\left(c_3 P_{pv}(k) - y_a(k)\right)^2$; and the usage of the battery can be minimized by penalizing $\sum y_b(k)^2$.

Define the augmented system states

$$x(k) \triangleq [S_{oc}(k), y_m(k-1), y_a(k-1), y_b(k-1)]^T,$$

and the augmented output

$$y(k) \triangleq [y_m(k-1), y_a(k-1), y_b(k-1)]^T,$$

such that a linear state-space model can be obtained:

$$\begin{cases} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{cases} \tag{8}$$

where

$$A = \begin{bmatrix} 1 & 0_{1\times 3} \\ 0_{3\times 1} & 0_{3\times 3} \end{bmatrix},\ B = \begin{bmatrix} 0 & \eta_c & -\eta_d \\ c_1 & 0 & c_1 \\ c_3 & c_3 & 0 \\ 0 & c_2 & c_2 \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{3\times 1} & I_{3\times 3} \end{bmatrix}.$$

The linear state-space system (8) is considered as the plant to be controlled via the MPC approach.

C. Objective function for MPC

The objective function comprises of following three items:

1) $\min J_1(k) = \min \sum_k^{k+N_p} (c_1 P_L(k) - y_m(k))^2$, which indicates that usage of the diesel generator should be minimized;
2) $\min J_2(k) = \min \sum_k^{k+N_p} y_b(k)^2$, which penalizes the use of the battery bank;
3) $\min J_3(k) = \min \sum_k^{k+N_p} (c_3 P_{pv}(k) - y_a(k))^2$, which implies that usage of PV generator is encouraged.

Here, N_p represents the predictive horizon for MPC design.

Define $Y(k) \triangleq [y^T(k), y^T(k+1|k), \ldots, y^T(k+N_p-1|k)]^T$, where $y(k+i|k)$ denotes the predicted value of y at step i $(i = 1, \ldots, N_p)$ from sampling time k. Define the reference value $R(k) \triangleq [c_1 P_L(k), c_3 P_{pv}(k), 0, c_1 P_L(k+1), c_3 P_{pv}(k+1), 0, \ldots, c_1 P_L(k+N_p-1), c_3 P_{pv}(k+N_p-1), 0]^T$. The overall objective function is then given by

$$\begin{aligned} \min J(k) &= \min(J_1(k) + J_2(k) + J_3(k)) \\ &= \min (Y(k) - R(k))^T (Y(k) - R(k)). \end{aligned} \tag{9}$$

D. Constraints for the MIMO linear system

In this paper, switched constraints are used to describe different modes (charging or discharging) of the battery, such that the plant to be controlled could be expressed by a unified linear MIMO state-space model without switching parts.

Constraints can be categorized as following:

(a) Energy flows from generators and battery are non-negative values and are subjected to their maximum values: $0 \le P_1(k) = P_L(k) - y_m(k) \le P_1^{max}$, $0 \le P_i(k) \le P_i^{max}$ $(i = 2, 3, 4)$, where P_i^{max} $(i = 1, 2, 3, 4)$ denote the maximum values of energy flows.

(b) Energy flow from the PV generator ($P_{pv}(k)$) is no less than the sum of PV energy directly used on the load ($P_2(k)$) and the battery charge rate ($P_3(k)$), that is

$$P_{pv}(k) \ge P_2(k) + P_3(k).$$

(c) State of charge of the battery is restricted between its minimum and maximum values:

$$B_C^{min} \le S_{oc}(k) \le B_C^{max}.$$

(d) $P_3(k)$, and $P_4(k)$ should satisfy the additional constraint (7), which can be rewrite as following:

$$P_3(k) + P_4(k) < \sqrt{\frac{2-\alpha}{\lambda}}.$$

(e) Charging and discharging cannot happen at the same time, as is implied by (2).

Constraints (a)-(d) are convex, while constraint (e) is non-convex. To achieve convex optimization in MPC design, we divide constraint (e) into two switched cases: 1) charging ($P_4 = 0$), and 2) discharging ($P_3 = 0$).

*1) **Charging**:* The constraint (e) can be rewritten by

$$\begin{cases} P_4(k) &\le 0, \\ P_4(k) &\ge 0. \end{cases}$$

Constraints (a), (b), (d) and (e) can be compactly rewritten by

$$M_{11} u(k) \le \gamma_{11}, \tag{10}$$

where

$$M_{11} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix},\ \gamma_{11} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_L(k) \\ P_{pv}(k) \\ P_2^{max} \\ P_3^{max} \\ P_4^{max} \\ P_1^{max} - P_L(k) \\ \sqrt{\frac{2-\alpha}{\lambda}} \end{bmatrix}.$$

Define the predictive control vector:

$$U(k) \triangleq [u^T(k), u^T(k+1|k), \cdots, u^T(k+N_c-1|k)]^T,$$

where $u(k+i|k)$ is the predicted value of u from the sampling time k, and N_c denotes the control horizon. Since each $u(k+i|k)$ in the predictive control vector $U(k)$ should satisfy (10), it follows that $U(k)$ should satisfy

$$\bar{M}_{11} U(k) \le \bar{\gamma}_{11}, \tag{11}$$

where

$$\bar{M}_{11} = \underbrace{\begin{bmatrix} M_{11} & & \\ & \ddots & \\ & & M_{11} \end{bmatrix}}_{N_c},\ \bar{\gamma}_{11} = \begin{bmatrix} \gamma_{11} \\ \vdots \\ \gamma_{11} \end{bmatrix}.$$

Constraint (c) is expressed with respect to state of charge; to facilitate the MPC design, it should be transformed into a form with respect to predictive control vector $U(k)$. After some derivations (presented in Appendix), Constraint (c) can be transformed into a compact form:

$$\bar{M}_2 U(k) \le \bar{\gamma}_2, \tag{12}$$

where

$$\bar{M}_2 = \begin{bmatrix} -B_m \\ B_m \end{bmatrix}, \bar{\gamma}_2 = \begin{bmatrix} \left(S_{oc}(k) - B_C^{min}\right)[1, 1, \cdots, 1]^T \\ \left(B_C^{max} - S_{oc}(k)\right)[1, 1, \cdots, 1]^T \end{bmatrix},$$

$$B_m(\hat{b}_m) = \underbrace{\begin{bmatrix} \hat{b}_m & 0 & \cdots & 0 \\ \hat{b}_m & \hat{b}_m & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ \hat{b}_m & \hat{b}_m & \cdots & \hat{b}_m \end{bmatrix}}_{N_c}.$$

In (12), $S_{oc}(k)$ can be obtained in real-time, and the constraint is expressed with respect to the predictive control vector $U(k)$.

Combining constraints (11) and (12) yields constraints for MPC design in charging state:

$$\bar{M}_c U(k) \leq \bar{\gamma}_c \tag{13}$$

where $\bar{M}_c = [\bar{M}_{11}^T, \bar{M}_2^T]^T$, $\bar{\gamma}_c = [\bar{\gamma}_{11}^T, \bar{\gamma}_2^T]^T$.

*2) **Discharging**:* The constraint (e) is equivalent to

$$\begin{cases} P_3(k) & \leq 0, \\ P_3(k) & \geq 0. \end{cases}$$

Constraints (a), (b), (d) and (e) can be compactly written by $M_{12}u(k) \leq \gamma_{12}$, where

$$M_{12} = \begin{bmatrix} -1 & 0 & 0 \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ 0 & 0 & -1 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}, \ \gamma_{12} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ P_L(k) \\ P_{pv}(k) \\ P_2^{max} \\ P_3^{max} \\ P_4^{max} \\ P_1^{max} - P_L(k) \\ \sqrt{\frac{2-\alpha}{\lambda}} \end{bmatrix}.$$

It follows that the predictive control vector $U(k)$ should satisfy $\bar{M}_{12}U(k) \leq \bar{\gamma}_{12}$, where

$$\bar{M}_{12} = \begin{bmatrix} M_{12} & & \\ & \ddots & \\ & & M_{12} \end{bmatrix}, \ \bar{\gamma}_{12} = \begin{bmatrix} \gamma_{12} \\ \vdots \\ \gamma_{12} \end{bmatrix}.$$

Consequently, constraints for MPC design in discharging state can be expressed by:

$$\bar{M}_d U(k) \leq \bar{\gamma}_d \tag{14}$$

where $\bar{M}_d = [\bar{M}_{12}^T, \bar{M}_2^T]^T$, $\bar{\gamma}_d = [\bar{\gamma}_{12}^T, \bar{\gamma}_2^T]^T$; $\bar{M}_2$ and $\bar{\gamma}_2$ are give by (12) to satisfy the constraint (c).

E. Control horizon

Switching time is predefined in this paper according to load demand profile and PV power supply. The battery is in the state of charging, when the PV power supply exceeds the load demand in a certain hour; the battery is in the state of discharging, when the PV provides insufficient power for the load demand in a certain hour. In summer weekdays, for example, the battery switches from discharging to charging at 7:30, and from charging to discharging at 17:30.

In this paper, control horizon and predictive horizon are varying according to switching times. At a given time k, find the nearest switching time $T_k > k$. The control horizon and predictive horizon for time k are given by

$$N_c(k) = N_p(k) = T_k - k. \tag{15}$$

F. Switched MPC algorithm

Standard linear MPC algorithm can be referred to [7]. With the linear state-space equations (8), the objective function (9) and the constraints (13) or (14), a switched MIMO MPC algorithm can be designed for the PDB hybrid system:

i. For time k, find the control horizon $N_c(k)$ and the predictive horizon $N_p(k)$ through (15).
ii. Optimization: find optimal $U(k)$, such that the following optimization problem is solved:

$$\begin{aligned} \min \quad & \left(U(k)^T E U(k) + 2HU(k)\right), \\ \text{s.t.:} \quad & \text{(13) for the case of charging,} \\ & \text{or (14) for the case of discharging,} \end{aligned}$$

where E and H are MPC gains calculated based on the objective function (9). Detailed calculation of E and H is presented in Appendix.

v. Receding horizon control:

$$u(k) = [I_{3\times 3}, 0, \cdots, 0]\, U(k).$$

vi. Update the estimated parameters $\hat{\eta}_c$ and $\hat{\eta}_d$ by using the proposed updating law (6).
vii. Set $k = k + 1$, and update system states, inputs and outputs with control $u(k)$ and state-space equations (8). Repeat steps i-vi until k reaches its predefined value.

***Remark** 4:* It should be noted that, although MPC is applied, the dispatching of the hybrid power system is fundamentally an optimization problem (instead of a control design problem); consequently, stability of the closed-loop system is not assured by the proposed adaptive switched MPC. However, states of the closed-loop system are guaranteed bounded, since the optimization in MPC is processed with constraints on P_1, P_2, P_3, P_4, and S_{oc}.

IV. SIMULATION AND DISCUSSION

In this section, simulation results of the PDB hybrid system with the proposed switched MPC are presented. The daily demand and PV power supply in a Zimbabwe clinic are listed in Table I and II, respectively. To test the performances of the closed-loop system with disturbances, it is assumed that actual demands are 20% larger than expected, and PV provides 20% less power than expected. Values of system parameters and control parameters are listed in Table III and Table IV, respectively. Initial values of $P_i(k)(i = 1, 2, 3, 4)$ are set to zeros. Initial values of state of charge is set to $x_m(1) = 0.7B_C^{max}$.

TABLE III
VALUES OF SYSTEM PARAMETERS

Notations	Values	Notations	Values
P_1^{max}	5 kW	B_c^{max}	54.5 kWh
P_2^{max}	5 kW	B_c^{min}	27.25 kWh
P_3^{max}	5 kW	η_c	0.8
P_4^{max}	5 kW	η_d	1.2

TABLE IV
VALUES OF CONTROL PARAMETERS

Notations	Values	Notations	Values
c_1	1.0	λ (summer)	0.1
c_2	0.2	λ (winter)	0.2
c_3	0.8	α	0.01

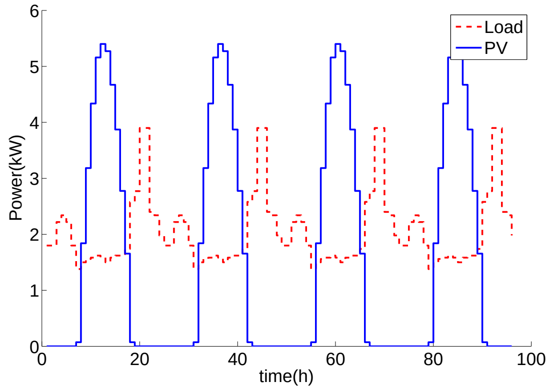

Fig. 2. Load demand and PV power in summer

A. Simulation results of the proposed switched MPC

For the proposed switched MPC with online estimation for uncertain parameters, initial values of estimated parameters are given by $\hat{\eta}_c(0) = 1.0$ and $\hat{\eta}_d(0) = 1.0$. "Interior Point" [17] is used as the numerical approach for solving optimization problem at each MPC cycle. The time spans of simulation cases are assigned to 4 days (96 hours).

In summer days, switching times are 7:30 (from discharging to charging) and 17:30 (from charging to discharging) everyday. Results are displayed in Fig. 2 and 3. As can be seen from Fig. 2 and 3, whenever the PV power is sufficient for demands, it is used directly to satisfy demands, and the battery is in charging state. In case that PV power is insufficient, the battery switches into discharging state to satisfy the load demands. In summer days, it seems that PV power is quite sufficient, so that the diesel generator is only used for covering the imbalance resulted from disturbances. It is illustrated by Fig. 4 that estimated parameters converge to actual values of the uncertain parameters ultimately.

In winter days, switching times are 8:30 (from discharging to charging) and 16:30 (from charging to discharging) everyday. Results are shown in Fig. 5 and 6, where principles are similar with those in summer. The main difference in summer is that, PV power is relatively insufficient, and the diesel generator covers the imbalance resulted from both insufficient PV power and external disturbances. As can be seen from

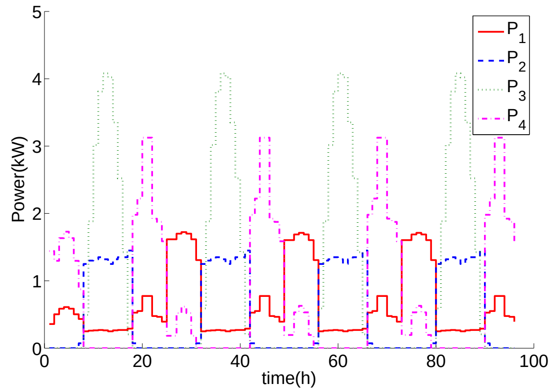

Fig. 3. Energy flows of the closed-loop system (summer)

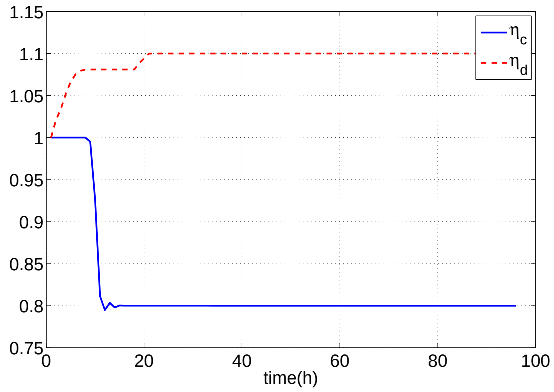

Fig. 4. Estimated parameters (summer)

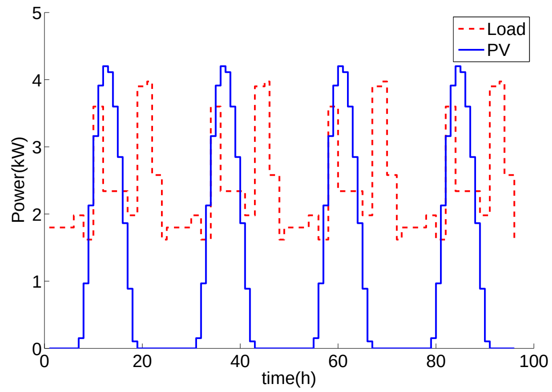

Fig. 5. Load demand and PV power in winter

Fig. 7, estimated parameters converge to actual values of the uncertain parameters ultimately.

B. Comparisons and discussions

To better illustrate performances of the proposed adaptive switched MPC, Comparisons with other techniques are presented. Diesel consumptions of systems with different techniques are listed in Table V.

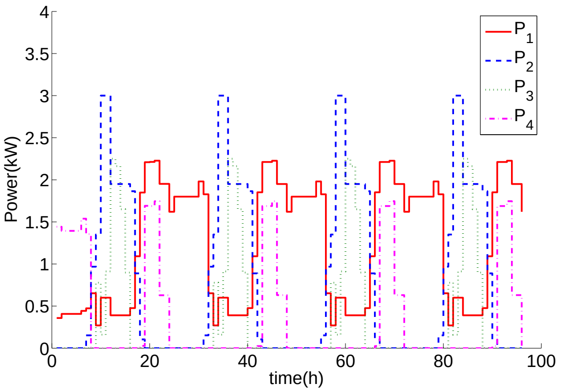

Fig. 6. Energy flows of the closed-loop system (winter)

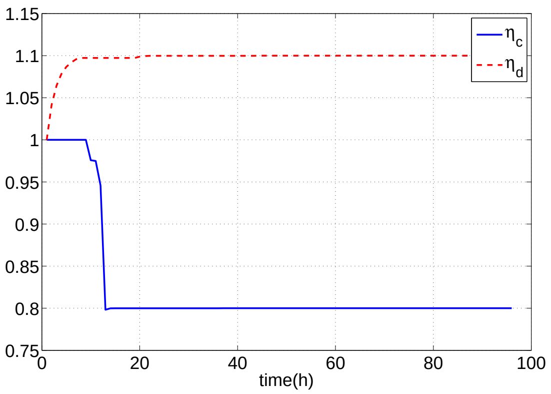

Fig. 7. Estimated parameters (winter)

TABLE V
DIESEL ENERGY CONSUMPTIONS (KWH) OF PDB HYBRID SYSTEM WITH DIFFERENT STRATEGIES

	Summer	Winter
Adaptive switched MPC	63.7	118.4
switched MPC	63.9	118.2
Intuitive strategy	68.0	125.3
open loop optimal control	81.4	140.2

The switched MPC without online estimation for uncertain parameters is designed by setting their nominal values to $\bar{\eta}_c = 1.0$ and $\bar{\eta}_d = 1.0$. As can be seen from Table V, the result of the switched MPC without online estimation is fairly satisfactory. It is quite similar with that of the proposed switched MPC with online estimation, implying that the inherent robustness of MPC with respect to certain degree of parametric uncertainties is acceptable.

The intuitive strategy can be described as following. if the sunlight is sufficient, PV array is used (P_2) to satisfy the load demand (P_L) as a priority. The PV array is also used for charging the battery bank (P_3) in case of sufficient supply for the load demand. When the sunlight is insufficient, the battery bank discharges to satisfy the load demand as the second choice, since battery power is cheaper than the diesel power. Finally, if the load demand is too large for PV array and battery bank to supply, the diesel generator is operated to cover the imbalance. It can be indicated by the result listed in Table V that, the robustness of the proposed MPC with online estimation is superior over that of the intuitive strategy.

TABLE VI
DIESEL ENERGY CONSUMPTIONS (KWH) OF THE CLOSED-LOOP SYSTEM WITH DIFFERENT BATTERY CAPACITIES

B_C^{max}	43.6	49.1	54.5	60.0	65.4
Summer	81.3	68.1	63.9	60.2	56.7
Winter	125.3	121.8	118.4	114.9	111.5

The open loop optimal control is designed by using the objective function and constraints of the proposed switched MPC, but without receding horizon control and online estimation. As can be seen from Table V, the diesel consumption with open loop optimal control is the largest. The reason is that, only one optimization is conducted at the very beginning of operation, without consideration of parametric uncertainties and external disturbances. In contrast, with the proposed switched MPC, optimization is conducted in each sampling time with feedback of system states.

Another comparison can be conducted among systems with different battery capacities. As can be seen from results displayed in Table VI, total diesel energy consumption decreases, if the capacity of battery bank can be increased. The reason is that, in case of larger battery capacity, the proposed MPC strategy would choose to store more PV power for future consumption rather than abandoning surplus PV power due to limit of battery capacity, such that less diesel energy would be used for covering the imbalances.

V. CONCLUSION

In this paper, a new switched model predictive control strategy is developed for energy dispatching of a photovoltaic-diesel-battery hybrid power system in case that simultaneous charging and discharging of the battery are unpermitted. Different switched states of the battery are described by switched constraints, so that the hybrid system could be expressed by a unified linear MIMO state-space model, and the difficulty of constructing complicated switched predictive state-space model is avoided. Uncertain battery parameters are estimated online with the adaptive updating law. Simulation results and comparisons with other strategies imply that the proposed switched MPC algorithm is satisfactory in dispatching energy usages for the PDB hybrid power system.

Some future works of this research include: 1) to construct an objective function with more factors that may significantly influence the overall cost; and 2) to enhance the MPC design with the consideration of time-varying system parameters.

APPENDIX

PERSISTENT EXCITATION OF $u_b(k)$

PE of $u_b(k)$ can be explained as following:

$$u_b(k)u_b(k)^T = \begin{bmatrix} P_3(k)^2 & P_3(k)P_4(k) \\ P_4(k)P_3(k) & P_4(k)^2 \end{bmatrix}$$

where $P_3(k)P_4(k) = 0$ since simultaneous charging and discharging are unpermitted. Moreover, for this hybrid system, there always exists a time span $(h, h + H_0)$ containing both charging and discharging; it follows that $0 < \alpha_1 I_{2\times 2} \leqslant \Sigma_{k=h}^{h+H_0} u_b(k)u_b(k)^T \leqslant \alpha_2 I_{2\times 2}$, where $\alpha_1 = \min\left[\Sigma_{k=h}^{h+H_0} P_3(k), \Sigma_{k=h}^{h+H_0} P_4(k)\right]$, and $\alpha_2 = \max\left[\Sigma_{k=h}^{h+H_0} P_3(k), \Sigma_{k=h}^{h+H_0} P_4(k)\right]$. Consequently, $u_b(k)$ is persistently exciting in the time span $(h, h + H_0)$.

Transformation of constraint (c)

Based on the battery dynamic equation (3), predicted values of x_m can be calculated by

$$S_{oc}(k+i|k) = S_{oc}(k) + b_m \sum_{j=k}^{j\leq k+i-1} u(j), \tag{16}$$

where $S_{oc}(k+i|k)$ denotes the predicted value of S_{oc} from sampling time k. It follows from (16) that

$$X_m(k) = S_{oc}(k)[1, 1, \cdots, 1]^T + B_m U(k),$$

where

$$X_m(k) \triangleq [S_{oc}(k), S_{oc}(k+1|k), \cdots, S_{oc}(k+N_c-1|k)]^T,$$

$$B_m(b_m) = \underbrace{\begin{bmatrix} b_m & 0 & \cdots & 0 \\ b_m & b_m & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ b_m & b_m & \cdots & b_m \end{bmatrix}}_{N_c}.$$

However, since b_m is uncertain, its estimated value $\hat{b}_m = [0, \hat{\eta}_c, \hat{\eta}_d]$ should be used in calculating B_m.

Each $S_{oc}(k+i|k)$ in the predictive state of charge $X_m(k)$ should satisfy Constraints (c); consequently,

$$B_{min}\underbrace{[1, 1, \cdots, 1]}_{N_c}{}^T \leq X_m(k) \leq B_{max}\underbrace{[1, 1, \cdots, 1]}_{N_c}{}^T,$$

leading to (12).

Calculation of MPC gains

According to classical MPC design [7], MPC gains can be calculated as followings:

$$F(k) = \left[\ (CA)^T\ , (CA^2)^T\ , \cdots\ , (CA^{N_p(k)})^T\ \right]^T,$$

$$\Phi(k) = \begin{bmatrix} \Phi_{11} & 0 & \cdots & 0 \\ \Phi_{21} & \Phi_{22} & & 0 \\ \vdots & & \ddots & \vdots \\ \Phi_{N_p,1} & \Phi_{N_p,2} & \cdots & \Phi_{N_p,N_c} \end{bmatrix},$$

where $\Phi_{ij} = CA^{i-j}\hat{B}$, and $\hat{B}$ is in the form of B with η_c and η_d replaced by $\hat{\eta}_c$ and $\hat{\eta}_d$.

The output vector can be expressed with respect to input vector: $Y(k) = Fx(k) + \Phi U(k)$. It follows that the objective functions can be given by

$$\begin{aligned} J(k) &= (Y(k) - R(k))^T\,(Y(k) - R(k)) \\ &= (Fx(k) - R(k))^T\,(Fx(k) - R(k)) \\ &\quad + 2(Fx(k) - R(k))^T \Phi U(k) + U(k)^T \Phi^T \Phi U(k). \end{aligned}$$

where $(Fx(k) - R(k))^T\,(Fx(k) - R(k))$ is independent on $U(k)$; consequently, according to [7], optimizing $J(k)$ can be transformed as following:

$$\begin{aligned} &\min J(k) = \min\,(Y(k) - R(k))^T\,(Y(k) - R(k)) \\ \Leftrightarrow\ &\min\left[2(Fx(k) - R(k))^T \Phi U(k) + U(k)^T \Phi^T \Phi U(k)\right] \\ \Leftrightarrow\ &\min\left(U(k)^T E U(k) + 2HU(k)\right), \end{aligned}$$

where $E(k) = \Phi(k)^T\Phi(k)$, and $H(k) = (Fx(k) - R(k))^T\Phi$.

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