Optimisation of **S**pecies **S**election at

***P**rinslust **G**ame **F**arm*

FINAL PROJECT REPORT

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Abstract

During the final year of all Industrial Engineering students attending the University of Pretoria, a compulsory module called Project (BPJ 410) needs to be completed. This module consists of a large industrial engineering oriented project to be performed. This project document contains an introduction section, a literature study section, a model formulation section and a conclusion.

Time and money will be invested to ultimately deliver a proposed optimum solution to convert Prinslust, a plains game hunting operation into a scarce game breeding entity, containing only top quality animals. Within the farming communities all around the world, farmers follow different strategies to achieve their main objectives. To be able to conduct this transformation, intensive research regarding all constraints contributing to achieving the objective must be performed.

Within this study, alternative ways used by other farmers who completed a similar transformation successfully, as well as previous studies utilising various techniques to solve closely related problems, are examined. The technique that has the highest probability of delivering an optimal solution to the farmer's objective, is identified.

All the constraints present within this project are defined as tasks to be performed and research regarding each task is conducted to be able to construct a model. The constraints are transformed into mathematical equations that, at the end, delivered and provided a proposed optimal solution after the completion of the validation of the model.

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CHAPTER 1: Introduction

1.1 Background

Prinslust is a game farm situated on the north-west side of the Limpopo Province, South Africa, approximately 13 kilometers west of Alldays and covers an area of 1 070 hectares. Emile Bouwer, the owner of Prinslust, (registered as Royal Albatross Properties 20 (Pty) Ltd), bought the property in April of 2005 with the intention of transforming the property into a hunting and game lodge containing plains game (Bouwer, 2014).

The previous owner of *Prinslust* overgrased the farm with cattle and therefore the farm had to go through a rehabilitation phase that took almost four years (Bouwer, 2014). During this phase, the owner invested large sums of capital into the farm by upgrading the cattle fence to a 2.4 meter high game fence, constructing 12 new watering holes, re-establishing the old farm roads, creating new roads where necessary, removing the old cattle camps, relocating the slaughterhouse, upgrading water pumps, constructing a large shed and upgrading the bushcamp. After procuring specifically selected game (Figure 1) at local game auctions, the owner started scheduling hunting groups to come and enjoy a few days in the bushveld.

Figure 1 Plains Game at Prinslust

Initially the operation generated income from hunters and tourists visiting the farm for hunting and sightseeing but the revenue gradually started to decrease over the recent years (Bouwer, 2014). During 2011 and 2012, the Alldays region experienced an extremely dry season (Figure 2), resulting in the increase of animal feed prices and higher feeding volumes required to keep the game alive. Ultimately the farmer had no choice other than to sell some of his animals at local auctions at extremely low prices.

Figure 2 Alldays' Drought

The draught forced the farmer to reconsider his current model for *Prinslust* and from a financial perspective had to consider alternative ways and means to utilise and manage his farm if he wanted to continue farming. The farmer conducted research on alternative strategies to earn income on his farm. His research resulted in a proposed strategy of transforming *Prinslust* into a scarce game breeding operation that breeds selectively with top quality scarce game (Bouwer, 2014).

This transformation from plains game farming to scarce game farming requires a in-depth research on a number of aspects (Bouwer, 2014). These aspects should include but are not limited to, the type of species to breed with, breeding rate, social structure, future demand for and thus future pricing for the selected species, predator impact, feeding habitat and supplement requirements, infrastructure improvements such as additional fencing including electrified fencing, water cribs, additional roads, etc. In order to finalise his strategy, as to how to proceed going forward, the farmer requires the input and resultant scientific proposal from an outsider which will form the basis for his final decision.

1.2 Problem Statement

The owner of *Prinslust* is forced to convert his plains game hunting operation into a scarce game breeding facility that will deliver top quality scarce game going forward to ensure profitable sustainability of the farming operation within the next five years. He decided to consider the following five game species to selectively farm with, namely Njala, Bushbuck, Kudu, Gold Oryx, and Sable Antelope as depicted in Figure 3.

The core question remains: “How can the best combination of these five game species be found that will deliver the ultimate maximum profit and ensure sustainability over the next five years?”.

Figure 3 Njala, Bushbuck, Kudu, Gold Oryx and Sable Antelope

1.3 Project Aim

In order to determine the optimum combination of the five game species mentioned above, a number of aspects needs to be investigated, quantified and analysed. The core objective of this project is to compile an optimisation model which will provide an ultimate solution for the optimal combination of the different game species to meet the farmer’s objective.

1.4 Project Approach

Operations research consists of four core activities as indicated in Figure 4 (Rardin, 1998). Firstly the problem needs to be identified appropriately to be able to transform the requirements of the problem into a model. The model will most likely infer to a useful solution upon which a decision can be made.

The implementation phase of the model obtained will most likely result in other problems and complications that will at that stage require a solution. This implementation phase will not be dealt with by the student since a facility layout plan of *Prinslust* is required before this phase can be done. The facility layout plan is another project on its own since *Prinslust* will need to be analysed in terms of habitat, water availability and accessibility before facility layout plans can be drawn up and compared. The decision supporting tool explained in Figure 4 is commonly used to help solve problems that require modelling.

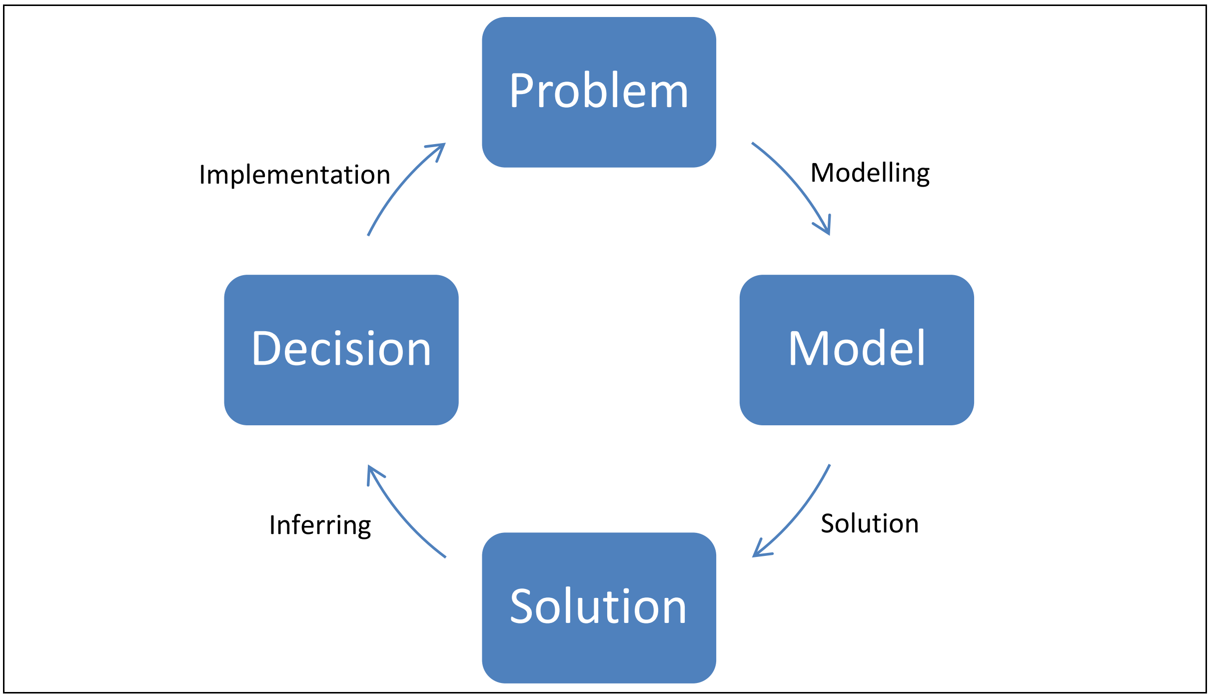


Figure 4 Fundamentals of Operations Research (Rardin, 1998)

In order to be able to use this approach in an attempt to solve the Alldays farmer's problem, the problem must be expressed in terms of constraints to be able to conduct the modelling step. The Alldays farmer is considering different scenarios to implement on his farm. These different scenarios include different combinations of species and also the quantity of the individual species together with the unique requirements for each species. These requirements are included in the following tasks that must be performed to be able to define the constraints of this project:

- → An analysis of the historical statistics of these five animal species' auction prices;
- → A forecast for each species' auction prices;
- → A study of the identified species' habitat;
- → A study on the available habitat on *Prinslust*;
- → A study of the breeding rate of the mentioned species;
- → A study of the infrastructure required to accommodate each species;
- → A study of maintenance required on the infrastructure considered;
- → A study of each species' predators and if they inhabit *Prinslust*; and
- → A study of the risks that may be involved in breeding with each of these scarce game species.

These requirements need to be transformed into constraints and specifications to be able to encode this real world problem into a model world problem to be able to construct an optimisation model. After modelling the problem in optimisation software, the solution obtained must be verified by evaluating the results through testing the sensitivity of various constraints.

After the model has been evaluated and validated, an optimal solution can be obtained from the optimisation model. A sensitivity analysis must then be done to investigate the effect certain parameter changes have on the optimal solution. The results must be analysed and summarised to be able to deliver the findings of the model to the owner of *Prinslust*. Recommendations regarding the results obtained can be made to the farmer but the final decision will be that of the farmer.

1.5 Project Plan

To be able to convert this real world problem at *Prinslust* into a model world problem, tasks necessary to determine the model's boundaries, must be identified together with the activities required to successfully complete each task as shown in Table 1.

Table 1 Tasks and their Activities

Tasks	Activities
Historic statistical analysis	Gathering of data
	Analysing of data
	Summarising and concluding the data
Forecast for auction prices	Identify forecasting method
	Conduct a forecast
Species' habitat study	Research concerning the species' habitat
Available habitat study	Consulting the owner of *Prinslust* farm about available habitat
	Determining the difference between required and available habitat
Breeding rate study	Research concerning species' breeding rate
Social structure study	Research concerning species' social structure
Infrastructure requirement study	Research on infrastructure required and the costs associated

Tasks	Activities
Maintenance requirement study	Research concerning the maintenance on required infrastructure and the costs associated
Predator study	Research on predators of identified species
	Consulting the owner of *Prinslust* farm concerning predators living on the farm or in the area
	Summarising similarities between predators of the identified species and predators inhabiting *Prinslust*
Risk analysis	Research on various risks involved in farming with each of the species
Formulation of optimisation model	Determine all decision variables present in the model
	Define the objective function
	Define all parameters
	Model the problem in optimisation software package
	Evaluate and validate the model
	Analyse results obtained from model and conduct a sensitivity analysis
Consulting the owner of *Prinslust* farm regarding model results	Delivering optimal solution to the farmer
	Compile recommendations for *Prinslust*

Statistical data required for the project will be obtained from various publications including *Mpatamacha Game Capture*, *Soutpansberg Skaarswildstudiegroep*, and *Vleissentraal,* while LINGO optimisation software will be used for modelling the final model obtained.

CHAPTER 2: Literature Study

The promulgation of the Game Theft Action in South Africa in the nineties, that settled ownership of game on exempted farms, resulted in the expansion in game farming (Van der Walt, 2011). Cattle and sheep farmers, as well as new entrants from other businesses and occupations, changed to game farming because of the enormous commercial value of game.

Within game farming, there are two core approaches followed by farmers (Bouwer, 2014). The first approach is called plains game farming where the animals walk freely on a piece of land and are farmed for the purpose of hunting (meat, skin and horn off-take) as well as tourist viewing. The animals feed from the land and in the dry seasons, farmers provide food and water as and when necessary. Income is generated by means of hunters and tourists visiting the farm and hunting the animals during the winter months.

The second approach involves the selective breeding of scarce game to be able to sell the animals at game auctions for record prices. The scarce game is usually detained in specific sized camps mainly for protection against predators and ease of management. The habitat present in each camp must be suitable for the scarce game species living in the camp (Bouwer, 2014).

The fencing of the camps may differ from species to species but usually consists of a 2.4 meter high game fence with additional electric fences added to the normal game fence (Bouwer, 2014). Some farmers prefer to add an additional half meter deep wall underneath the fences to prevent predators from crawling under the fences and entering the scarce game camps (Bouwer, 2014).

When considering to convert the method of farming from plains game farming to scarce game farming, there is a large number of economic considerations to keep in mind and to compare to one another. Such a transformation is complex and requires a lot of research and knowledge to be able to complete successfully.

More recently the outbreak of foot and mouth decease in Namibia, the unstable political situation in Zimbabwe, the banning of hunting in Botswana, the depletion of trophy quality game in the rest of Africa and the extensive poaching of game during the wet season in Zambia has all contributed to a high demand for quality trophy game in South Africa (Bouwer, 2014). Great emphasis has been placed on the breeding of quality animals with the ultimate aim of breeding game which will, on a sustainable basis, deliver animals that can challenge the record books as trophy animals. South Africa has become the preferred destination for overseas hunters purely because of its reputation to constantly provide quality animals for which they are prepared to pay a premium. This has resulted in a huge growth in the game farm industry overall (Bouwer, 2014).

A significant number of different farmers have already transformed their farming methods from plains game farming to scarce game farming. The *Mortons, Naude, Du Toit*, and *SA Stud Game's* stories will be studied to get background on how they started farming, what they changed, the strategies they followed and how they completed the transformation successfully.

2.1 Previous Farmers

2.1.1 Tony and Richard Morton

Originally, Tony Morton farmed with cows in KwaZulu-Natal and in the Eastern Cape (Gouws, 2011). Approximately 19 years ago he bought a farm against the Limpopo River next to the Botswana border south of Rooibokkraal with the intention of giving up his cow-farming to start farming with scarce game. He started with eight Sable heifers and one bull that he bought from Sable Ranch. As time went by, he bought "bastergemsbokke" (Roan) and Njalas to accompany the Sables (Gouws, 2011).

According to the *Mortons*, the best strategy to follow when farming with scarce game is to buy only the best genes game at auctions and not to settle for anything less. A good balance between an animal's horn length, horn form, body structure and appearance must be present before the animal will even be considered as an option to bid on at an auction (Gouws, 2011).

In winter times, when the field is exhausted, these farmers produce their own food on the farm to keep their game in top condition. A nutritionist helps them to construct their food mix to ensure that it has the ability to satisfy the optimal protein and energy requirements of the game.

Each animal also has its own feeding container, assuring that each animal receives a fixed amount of food each day. To minimise bullying between the animals for food and to assure that each animal has a fair chance to attain its portion, these feeding containers are placed at least five meters away from each other (Gouws, 2011).

The strategy of buying only the best genes game when starting to acquire scarce game, will definitely be a good recommendation to the Alldays farmer. Providing each animal with its own feeding container as done by the *Mortons*, may result in slightly higher expenses but in the long run the animals will benefit from it.

2.1.2 Jaco Naude and Piet du Toit

Approximately 50 km from Sun City and only 20 km from Lindleyspoort in South Africa lies the mixed bushveld paradise, *SJ Naudé Boerdery*, next to the picture-perfect Pilanesberg. Since 2007, the farmer Jaco Naudé practiced the breeding of scarce game together with the farm manager as well as two veterinarians. Only the best breeding material is used on the farm to improve horn lengths, body structure and the colour of the animals' fur (Naudé, 2014).

The *Piet du Toit Wildbedryf* emerged on the farm Hoogenboomen, situated on the north-west side of Rustenburg, South Africa (Du Toit, 2014). It originally started as a cow and lucern farming operation with some game on the property that were naturally found in the area. The cow and lucern farming did not, in the long run, guarantee sustainability and was quickly transformed into a scarce game operation. According to Piet, genetics are the core to achieve success in this industry and that's why, from the beginning, he bought game containing only the best genes. The starting point is to have scarce game with the best possible genetics available and then, with confidence and anticipation, let the animals develop (Du Toit, 2014).

From these two farmers, it is clear that the most important aspect when farming with scarce game is to breed with only the best, top quality genes. This will assure top quality calves, resulting in probabilities of getting high prices when selling the animals at game auctions.

2.1.3 SA Stud Game

The scarce game stud farm, *SA Stud Game*, originated in 2008 and is situated approximately 30 km west of Rustenburg in South Africa (Henning, 2014). During 2008, the farm was analysed and by the end of 2008, a thorough facility plan was completed and the development and construction started. A large amount of capital was spent on building new infrastructure including thirty camps surrounded by fences. Concrete slabs were inserted underneath almost 300 hectares of the fences to protect the game against predators, whilst the rest of the camps' fences were electrified. A large portion of bush had to be removed to be able to establish grass camps. SA Stud Game is not focused on tourism or hunting but rather on producing top quality scarce game (Henning, 2014).

Prinslust will require proper fencing with the necessary protection in place, similar to *SA Stud Game*. This will contribute to securing the scarce game from predators and the catching of illnesses from other animal species, as well as fighting among other game species.

The common denominator of the people mentioned above is proper genetics. Top quality genetics are the core and the most important factor to keep in mind when considering the transformation to farm with scarce game. The genetics of an animal can be evaluated by comparing horn lengths, horn form, horn growth, body structure, body fur and appearance.

The success of the studied farmers indicate that the transformation from plains game to scarce game is economically a wise decision, if it is done correctly. This transformation can become very complicated and intense and there are more than one way to complete this transformation. There is a large number of constraints that need to be analised and kept in mind while attempting to transform *Prinslust* from a plains game farm to a scarce game farm.

To formulate the best possible solution for *Prinslust*, the following section of the literature study consists of an in-depth research section regarding previous studies and investigations and the techniques used and methods followed to obtain the ultimate goal. These previous investigations have been done on the optimisation of available resources to be able to maximise the desired outcome, which is mainly profit or return on investments. By comparing these different techniques used, an appropriate method, or combination of methods, can be identified to solve the Alldays farmer's problem.

2.2 Previous Studies

2.2.1 Game Cropping and Wildlife Conservation in Kenya

A study regarding game cropping and wildlife conservation in Kenya was done through the development of a dynamic simulation model with adaptive control (Van Kooten, et al, 1996). This study consists of the development of a simulation model for examining economic incentives and alternative institutional arrangements for accomplishing the allocation of range resources in such a way that the conservation goal of the Wildlife Conservation and Management Act is accomplished together with the sustainable development of the local economy.

A dynamic stochastic simulation model was then developed with adaptive management of herbivores, predators and domestic livestock to address policies regarding the multiple use of rangeland resources (Van Kooten, et al, 1996). Included in this dynamic stochastic simulation model is the analysis of the implications of a switch from a traditional pastoral regime to game cropping.

Firstly all the constraints were identified and the necessary research was done regarding the constraints, including the interactions between forage, herbivores and predators. Variables under the control of the operator were identified. Objectives were determined and constraints regarding herbivores, predators and domestic livestock were modelled. Results obtained from the stochastic simulation models were tested and indicate that dynamic analytical tools can be suitably applied to gain insights into multiple-use resource allocation and policy analysis problems that face wildlife conservationists and ranch managers (Van Kooten, et al, 1996).

This Kenya study has many similarities with the Prinslust farm problem. Both have many important factors and constraints which need to be recognised, and both are regarding a transformation from an older traditional method to something more modern. Although a simulation model was the best method to improve this problem in Kenya, the Alldays farmer's problem require some method that will be able to deliver an optimal solution and not just a simulation of the problem. A simulation model can help one understand the process, steps and constraints of a problem much clearer but when an optimal solution is required, this method of modelling will not be sufficient.

2.2.2 Optimising Game Production in a New Era

At *Grootfontein Agricultural Development Institute* a study was conducted on the change from livestock to integrated game farming as the result of the changes in the socio-economic environment in the southern part of Africa (Furstenburg, 2010). After a 10-year study done on integrated game or livestock production in the Succulent Valley Bushveld in the Eastern Cape, a game production optimisation strategy was developed.

Land size and habitat requirements, spatial separation and interaction, social behavioral needs and the performance potential of game species are all limitations that are poorly understood by landowners and scientists (Furstenburg, 2010). Intensive research was done on these limitations mentioned to be able to manipulate the game in order to increase and optimise production.

Production optimisation was performed by developing nine key steps compiled from the research performed on the limiting constraints mentioned above (Furstenburg, 2010). The following nine key steps can be followed when transforming from livestock to game farming:

Step 1

A ranch needs to be divided into units according to the lands' vegetation structure, florist and landscape, covering far more that the superficial descriptions of biome or vegetation type. These units consist of the different habitat requirements for different animal species.

Step 2

Every unit on the ranch needs to be ranked according to the percentage suitability and quantified with respect to *inter alia* forage production and cover for each animal species.

Step 3

The social and spatial needs of each game species have proven to have an even greater influence on game production than feeding so the game animals should be ranked accordingly.

Step 4

Every potential game species for which the habitat may be suitable, described by some quantified measure, must then be allocated to the game herd at an appropriate rate of stocking.

Step 5

Stocking rate of the animal species can be calculated by the factor most limiting to the animals' performance potential, which is expressed either by large stock unit carrying capacity, browser unit carrying capacity, minimal social roaming space, or minimal feeding space.

Step 6

The proportion of browse and grass required by each animal species need to be distinguished, quantified and treated separately with regard to the forage potential of the habitat.

Step 7

Determine the maximum number of animals per species that can be run sustainably as a single species on the land unit by dividing the most limiting parameter with the sum of the allocated percentage suitabilities of all the potential habitats.

Step 8

Define the different ratios of feeding structure of the different animals that are to be run on the farm; i.e. the ratios of highly selective short grass grazers to bulk grazers to browsers to intermediate low-selective grazers.

Step 9

The species composition can be selected with due regard to animal performance potential, long-term vision, objectives and the market to be entered once the maximum load for every suitable game species had been calculated.

Although *Prinslust* has no livestock and already contains game on the farm, the process of transforming from a plains game farm to an intensive scarce game farming operation consists of almost the same aspects. The same research tactics can be used when conducting research on the same contributing factors as well as the placement of scarce game within camps.

Unfortunately, the Alldays farmer’s transformation is not that simple when choosing the specific game species to farm with. The constraints are more dependent on each other as well as on other factors such as animal prices at auctions and various other risks. A more intense method of identifying a combination of specific scarce game species that will deliver an optimal maximum profit, is required. Operations research is one technique that can be used to model scenarios and problems that will deliver an optimal solution for specific constraints.

2.2.3 Operations Research

Advanced analytical methods can be used to simplify decision making by using scientific approaches that best design and operate a system under specified required conditions (Venkataramanan & Winston, 2003). A system refers to an organisation of interdependent components working together to be able to achieve and accomplish the core goal of the system. Mathematical models are often used when a decision making process is approached scientifically.

When a problem exists in reality, it is an indication that an opportunity for improvement is present. A real world problem may consists of one or many scenarios that must be able to be encoded into specific constraints and specifications (Figure 5). The problem can be transformed into a modelled problem by preparing and encoding the data present in the problem into specifications. This problem can then be modelled by using specifications identified and decoded, to evaluate how the solution of the modelled problem solves the problem in the real world. Figure 5 illustrates this concept.

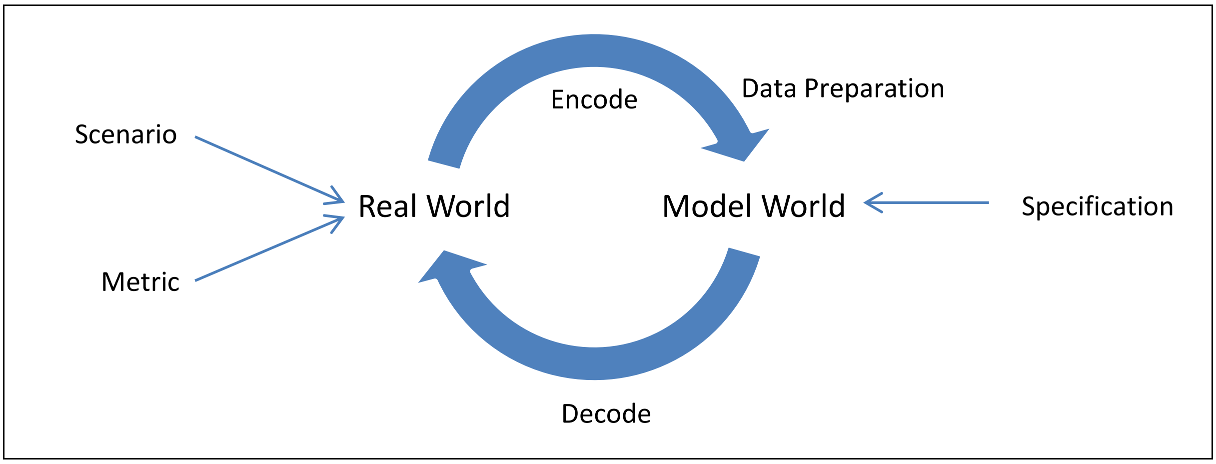


Figure 5 Model World and Real World Relationship

In 1974, ranch managers also experienced a problem which demanded specific optimal answers. It was a real problem, yet they found their solution in a mathematical optimisation model that was built based on the limiting constraints present at that time.

2.2.4 A Serial Optimisation Model for Ranch Management

Managers in business and industry all around the world are continuously challenged with making decisions regarding the efficient allocation of scarce resources (Bartlett, Evans, & Bement, 1974). Farmers' economic existence is based on efficiency and they must find a way to determine when and how to use the available resources. Linear programming, a mathematical optimisation technique, is used to allocate scarce and fixed resources to various management alternatives.

The optimisation technique used in solving this problem consists of a set of linear equations which clearly express the limited resources, the management alternatives, and the decision-maker's objective in mathematical terms (Bartlett, Evans, & Bement, 1974). The available resources were allocated among alternative range products, including cows and calves or yearlings. Then, the various different ways of how and when to use the resources were identified.

The objective of any linear programming model is usually to minimise or maximise a core function subject to a number of linear constraints (Bartlett, Evans, & Bement, 1974). Therefore, the objective function was identified. The results obtained from the optimal solution of the model yielded a set of computer decisions guiding ranch managers in managing the available resources as optimal as possible together with other non-quantified variables (Bartlett, et al, 1974).

This is one optimisation model that is developed to be able to solve a variety of different basic ranch management problems. A basic structure of the model can be applied to different scenarios. Four examples of how the basic structure of the model can be applied are given below to be able to indicate how the basic model can solve various types of problems (Bartlett, Evans, & Bement, 1974).

I. Within the first problem, the water resource in a reservoir are managed by using the following decision variables:

   $S_t \triangleq$ reservoir storage at time t, t ∈ {1, 2, 3, 4}

   $I_t \triangleq$ inflow during time t, t ∈ {1, 2, 3, 4}

   $R_t \triangleq$ release during time t, t ∈ {1, 2, 3, 4}

   The objective function is defined as follows:

$$\boldsymbol{S_t + I_t - R_t = S_{t+1}} \quad \text{[continuity equation for a reservoir]} \quad (1)$$

II. The reservoir continuity equation can also be applied to manage the forage availability on a farm by using the following decision variables:

SC_t ≜ standing crop at time t, t ∈ {1, 2, 3, 4}

G_t ≜ growth during time t, t ∈ {1, 2, 3, 4}

C_t ≜ amount of forage used during season t, t ∈ {1, 2, 3, 4}

The objective function can then be rewritten as the following:

$\boldsymbol{SC_t + G_t - C_t = SC_{t+1}}$ [continuity equation for usable forage] (2)

From applying equation (2), the following linear equations are the result:

- $\boldsymbol{SC_1 + G_1 - C_1 = SC_2}$ (3)
- $SC_2 + G_2 - C_2 = SC_3$
- $SC_3 + G_3 - C_3 = SC_4$
- $SC_4 + G_4 - C_4 \geq M$

[M=minimum amount of forage that must be left at the end of each grasing year], M ≥ 0

III. This relationship can rapidly also be adapted to the flow of livestock during the year by rewriting the basic equation (equation 1) in terms of the following decision variables:

H_t ≜ the herd size at the start of season t, t ∈ {1, 2, 3, 4}

B_t ≜ number of animals bought at the start of season t, t ∈ {1, 2, 3, 4}

NS_t ≜ number of animals sold at the end of season t, t ∈ {1, 2, 3, 4}

The objective function will look as follows:

$\boldsymbol{H_t + B_t - NS_t = H_{t+1}}$ [continuity function for flow of livestock during the year] (4)

From applying equation (4), the following linear equations are the result:

- $\boldsymbol{H_1 + B_1 - NS_1 = H_2}$ (5)
- $H_2 + B_2 - NS_2 = H_3$
- $H_3 + B_3 - NS_3 = H_4$
- $H_4 + B_4 - NS_4 \geq O$

[M=minimum amount of forage that must be left at the end of each grasing year], M ≥ 0

IV. The forth example illustrates a very important aspect in any business or operation all over the world namely cash flow. This is also the one example most related to the core objective of the Prinslust farm problem. The decision variables are defined as follows:

A_t ≜ the amount of cash available for investment at start of season t, t ∈ {1, 2, 3, 4}

c_{it} ≜ the cost of *i* expenditures during season t, *i* ∈ R, t ∈ {1, 2, 3, 4}

r_{jt} ≜ the revenue from j products sold during season t, j ∈ R , t ∈ {1, 2, 3, 4}

The objective function are the following:

$$\boldsymbol{A_t - \sum_{i=1}^{M} c_{it} + \sum_{j=1}^{P} r_{jt} = A_{t+1}} \qquad \text{[cash flow]} \qquad (6)$$

Within any of these four examples based the basic model, additional constraints can easily be added when building the model in optimisation software. This model framework can also contribute to a much larger model aiming to obtain an optimal solution for a problem with a much larger scope. Hence this basic structure model can be used for determining the number of each game specie to be bought, the amount of adjustment to be made to the habitat of each camp when breeding with a specific game specie and the number of water cribs to be built additionally when solving the problem at *Prinslust*. The flow of livestock example, example three, is the example most related to *Prinslust*.

The work by Bartlett, et al (1974) is an excellent example of how linear programming can be applied to support farming decisions. At *Prinslust*, this method of determining an optimal solution will be very appropriate since the resources and the different alternatives are known and can be expressed in mathematical terms. The key question is what combination of resources and alternatives will deliver the highest income.

Even though this journal completed on ranch management is 40 years old, it is still a very helpful tool to use when attempting to resolve *Prinslust's* problem.

2.2.5 An Optimisation Model for the Management of a South African Game Ranch

Farmers in South Africa have a commercial intent to acquire, keep and dispose of wildlife animals through three main methods including breeding, tourism and hunting (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006). The importance of game ranch management is realised as many part-time owners of game farms fund their farms from other sources of income, since some farms operate on meagre profit or some farms even at a loss (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006).

The backbone of sustainable development includes the rational use of natural resources without endangering the needs of future generations (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006). Therefor the effective management of natural resources is of utmost importance and needs more and more attention. The general issue of resource management within game ranching in a South African context are addressed. It include facets such as biology, ecology and eco-tourism that all have financial implications (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006).

The modelling of this problem was completed by making use of a multi-objective integer linear program (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006). The constraints included within this model are the quantity of animal feed available, budget constraints, minimum viable group size for each species and certain preference-governed specifications regarding the percentage of each species to the total number of animals on the farm. Within the optimization model, multi-objective characteristics are used to obtain a somewhat exact picture of reality including the incorporation of random annual rainfall parameters.

Within the model formulation, $i \in \{1,2, \dots, I\}$ represents a specific species while $k \in \{1,2, \dots, K\}$ represents the planning horizon. The parameters are defined as follow:

m_i ≜ minimum viable group size for species i, where i = {1, 2, . . . , I}.

l_i ≜ large animal units required per animal of species i, where i = {1, 2, . . . , I}.

b_i ≜ browsing units required per animal of species i, where i = {1, 2, . . . , I}.

g_i ≜ expected growth for species i, where i = {1, 2, . . . , I}.

a_i ≜ acquisition cost per animal of species i, where i = {1, 2, . . . , I}.

r_i ≜ tourism rating per animal of species i, where i = {1, 2, . . . , I}.

h_i ≜ hunting income generated per animal of species i that is hunted, where i = {1, 2, . . . , I}.

μ_i ≜ minimum fraction of the total number of animals to consist of animals of species i, where i = {1, 2, . . . , I}.

v_i ≜ maximum fraction of the total number of animals to consist of animals of species i, where i = {1, 2, . . . , I}.

K ≜ the interest rate used as a discounting factor to account for the time value of money.

C_k ≜ budget constraint for purchases during year k, where k = {1, 2, . . . ,K}.

ζ_k ≜ the random variable describing rainfall during year k where k = {1, 2, . . . ,K}.

$L_k(\zeta_k)$ ≜ large animal units (LAUs) available in year k, as a function of the realization of the random rainfall during year k, ζ_k , where k = {1, 2, . . . ,K}.

$B_k(\zeta_k)$ ≜ browsing animal units (BAUs) available in year k, as a function of the realization of the random rainfall during year k, ζ_k , where k = {1, 2, . . . ,K}.

The decision variables included within this model are defined as the following:

x_{ik} ≜ number of animals of species i on the ranch at the beginning of year k, where i = {1, 2, . . . , I}, k = {1, 2, . . . ,K}.

p_{ik} ≜ number of animals of species i to be purchased during year k, i = {1, 2, . . . , I}, k = {1, 2, . . . ,K}.

y_{ik} ≜ number of animals of species i to be hunted during year k, i = {1, 2, . . . , I}, k = {1, 2, . . . ,K}.

We then have the following mathematical program:

$$\max z_1 = \sum_{i=1}^{I} \sum_{k=1}^{K} r_i(x_{ik} + p_{ik} - y_{ik}) \tag{7}$$

$$\max z_2 = \sum_{i=1}^{I} \sum_{k=1}^{K} (y_{ik}h_i - p_{ik}a_i)(1-K)^{k-1} \tag{8}$$

subject to

$$x_{ik} + p_{ik} - y_{ik} \geq m_i \qquad \forall i, k \tag{9}$$

$$x_{i1} = s_i \qquad \forall i \tag{10}$$

$$\frac{x_{ik}}{(1+g_i)} = (x_{i,k-1} + p_{i,k-1} - y_{i,k-1}) \qquad \forall i, \forall k \in \{2,3,\ldots,K\} \tag{11}$$

$$u_i \leq \frac{(x_{ik}+p_{ik}-y_{ik})}{\sum_{i=1}^{I}(x_{ik}+p_{ik}-y_{ik})} \qquad \forall i,k \qquad (12)$$

$$v_i \geq \frac{(x_{ik}+p_{ik}-y_{ik})}{\sum_{i=1}^{I}(x_{ik}+p_{ik}-y_{ik})} \qquad \forall i,k \qquad (13)$$

$$\sum_{i=1}^{I}(a_i p_{ik}) \leq C_k \qquad \forall k \qquad (14)$$

$$\sum_{i=1}^{I} l_i(x_{ik}+p_{ik}) \leq L_k(\zeta_k) \qquad \forall k \qquad (15)$$

$$\sum_{i=1}^{I} b_i(x_{ik}+p_{ik}) \leq B_k(\zeta_k) \qquad \forall k \qquad (16)$$

$$x_{ik}, p_{ik}, y_{ik} \geq 0 \text{ and integer} \qquad \forall i,k \qquad (17)$$

A modified goal programming approach is used to transform and simplify this original problem explained above. The multriple objective functions is partially conflicting since the main objective (7) of this model is eco-tourism that attempts to reduce the quantity of hunting on the game ranch. While the secondary objective (8) of this model is the income generated through hunting. This hunting costs can be related directly to the attractiveness and the procurement cost (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006).

Function (19) and (20) introduces the two goal constraints respectively. For each of the K years within the planning horizon, ϕ_p indicates the annual goal rating for eco-tourism, while d_p^k indicates the deficiency variable. For the secondary goal, ϕ_s indicates the annual income from hunting and d_s^k indicates the deficiency variable. This deficiency variable represents the quantity by which the main goal is not achieved. The minimisation of the sum of d_p^k and d_s^k over the K year planning horizon is the main purpose of the single objective function (18) (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006).

Stochastic rainfall causes an uncertainty in the availability of food leading to possible shortages in either or both of the feeding types. Thus, the variable $\gamma_k(\zeta_k)$ indicates the difference between available and required grasing units (Joubert, Luhandjula, Ncube, le Roux, & de Wet, 2006). A distinction must also be made between the positive and negative differences since the tracking of any food shortage is the only interest. The following two expressions are thus also necessary:

$$\gamma_k(\zeta_k) = \gamma_k^+(\zeta_k) - \gamma_k^-(\zeta_k) \qquad \forall k$$

$$\gamma_k^+(\zeta_k), \gamma_k^-(\zeta_k) \geq 0 \qquad \forall k$$

The same principle is present for the browsing units, indicated through the variable $\beta_k(\zeta_k)$. This results in the following two equations indicating the difference between the required and available browsing units:

$$\beta_k(\zeta_k) = \beta_k^+(\zeta_k) - \beta_k^-(\zeta_k) \qquad \forall k$$

$$\beta_k^+(\zeta_k), \beta_k^-(\zeta_k) \geq 0 \qquad \forall k$$

When additional food sources are required, it can be purchased at a cost of c_γ per grasing unit and c_β per browsing unit. The parameter ζ_k^j indicates the jth realisation of ζ_k such that $\{(\zeta_k^j, \rho_j), j = 1,2, \dots, Q\}$ with $\rho_j > 0, \forall j$. An ordinary deterministic optimization model is the result when determining the stochastic rainfall in terms of a two-stage fixed resource approach. The following ordinary mixed integer problem is formulated

$$\min z = \sum_{k=1}^{K}\left(d_p^k + d_s^k\right) - 0.01[\sum_{i=1}^{I}\sum_{k=1}^{K} r_i(x_{ik} + p_{ik} - y_{ik})] - 0.001[\sum_{i=1}^{I}\sum_{k=1}^{K}(y_{ik}h_i - p_{ik}a_i)(1-K)^{k-1}] + \sum_{k=1}^{K}\sum_{j=1}^{J}\rho_j[c_\gamma \cdot \gamma_k^-(\zeta_k^j) + c_\beta \cdot \beta_k^-(\zeta_k^j)] \tag{18}$$

subject to

$$\textstyle\sum_{i=1}^{I} r_i(x_{ik} + p_{ik} - y_{ik}) \geq \phi_p - d_p^k \qquad \forall k \tag{19}$$

$$\textstyle\sum_{i=1}^{I}(y_{ik}\, h_i - p_{ik}a_i) \geq \phi_s - d_s^k \qquad \forall k \tag{20}$$

$$x_{ik} + p_{ik} - y_{ik} \geq \mathrm{m}_i \qquad \forall i, k \tag{21}$$

$$x_{i1} = \mathrm{s}_i \qquad \forall i \tag{22}$$

$$\frac{x_{ik}}{(1+g_i)} = (x_{i,k-1} + p_{i,k-1} - y_{i,k-1}) \qquad \forall i, \forall k \in \{2,3, \dots, K\} \tag{23}$$

$$u_i \leq \frac{(x_{ik}+p_{ik}-y_{ik})}{\sum_{i=1}^{I}(x_{ik}+p_{ik}-y_{ik})} \qquad \forall i, k \tag{24}$$

$$v_i \geq \frac{(x_{ik}+p_{ik}-y_{ik})}{\sum_{i=1}^{I}(x_{ik}+p_{ik}-y_{ik})} \qquad \forall i, k \tag{25}$$

$$\textstyle\sum_{i=1}^{I}(a_i p_{ik}) \leq \mathrm{C}_k \qquad \forall k \tag{26}$$

$$\textstyle\sum_{i=1}^{I} l_i(x_{ik} + p_{ik}) + \gamma_k^+(\zeta_k^j) = \mathrm{L}_k(\zeta_k) + \gamma_k^-(\zeta_k^j) \qquad \forall j, k \tag{27}$$

$$\textstyle\sum_{i=1}^{I} b_i(x_{ik} + p_{ik}) + \beta_k^+(\zeta_k^j) \leq \mathrm{B}_k(\zeta_k) + \beta_k^-(\zeta_k^j) \qquad \forall j, k \tag{28}$$

$$\gamma_k^+(\zeta_k^j), \gamma_k^-(\zeta_k^j) \geq 0 \qquad \forall k \qquad (29)$$

$$\beta_k^+(\zeta_k^j), \beta_k^-(\zeta_k^j) \geq 0 \qquad \forall k \qquad (30)$$

$$d_s^k, d_p^k \geq 0 \qquad \forall k \qquad (31)$$

$$x_{ik} + p_{ik} - y_{ik} \geq 0 \text{ and integer} \qquad \forall i, k \qquad (32)$$

This integer linear programming model is, to a large extent, related to *Prinslust's* problem. The same basic principle of generating maximum profit is present in both problems. The applicable problems both have a number of similar constraints including minimum viable group size for each species, feeding supplements and the available habitat. This model can have the purpose of being a guideline when starting to model *Prinslust's* problem in optimisation software.

The time gap between the publication of this journal and today can be seen as perfect opportunity for solving a problem that are most likely experienced by other farmers. For farmers and within almost any industry today, the managing of a large number of constraints can be extremely complex, especially when the constraints are dependent on each other in multiple ways.

Attempting to solve *Prinslust's* problem through using the knowledge and guidance gathered from the completion of the literature study, other farmers attempting the same transformation from plains game farming to scarce game farming may also benefit from the end-model. This benefit other farmers can gain from using the end-model that will be developed from the literature study may be the difference between failure and success. The increase in successful farmers in the scarce game industry will not only increase the scarce game market and industry but will also increase job-creation and influence the South African economy is a positive way.

2.3 Concluding Remarks

The farming of scarce game species in South Africa is currently a market that is rapidly increasing over a short period of time. It indicates great numbers of return on investment (ROI) for farmers and even people in other industries as mentioned above by some previous farmers. A good indication of how to enter the scarce game market was obtained from the previous farmers' research section.

Through studying and investigating previous work done to attempt similar transformations, a great amount of insight on various industrial engineering techniques were obtained. It, however, became clear that the development of an optimization model will be the best industrial engineering technique suited for the transformation process of *Prinslust*.

CHAPTER 3: Model Formulation

3.1 Constraints

Constraints are those factors limiting the outcome of the project by socially, physically or financially restricting the decision variables' values present in the problem (Venkataramanan & Winston, 2003). Decision variables are those variables whose values are controlled by the modeler and their values influence the performance of a system or model. To be able to construct an optimisation model to solve a problem, it is critical that all constraints are properly identified and understood.

When developing an optimisation model, social, physical and financial constraints must be considered when finding an optimal solution for the problem.

Only one social constraint is present within this model which is the breeding rate of the animals.

The physical constraint firstly includes the location of the farm that restricts the model to specific types of feeding material available for the game and other additional feeding material is expensive. Secondly, the available habitat on *Prinslust* farm is a restriction because species requiring other types of habitat will result in an increase in expenses.

The implementation of new infrastructure, maintenance thereof, the purchase of top quality game at forecasted prices, the animal feed and the protection of the game against other risks, represent the financial constraints. Predators of the five selected scarce game species resident at *Prinslust* and the area around *Prinslust* have the ability to cause great losses in breeding rates if proper fencing is not established. Additional protection methods to shelter the scarce game against these predators may result in significant expenses included in the financial constraints.

A good balanced combination between all these constraints is required to satisfy the farmer's expectations. To be able to determine the best combination of scarce game species that will satisfy the farmer's objective the best, research on each of the aspects mentioned is required to be able to make an informed decision.

3.2 Tasks and Activities

Tasks, each consisting of one or more activities, were generated based on the constraints identified. Some tasks require research and the gathering of information of certain aspects regarding the intensive breeding of scarce game

3.2.1 Historic Statistical Analysis

An analysis of the five identified scarce game species' previous auction prices is essential for conducting a forecast to determine the future value of the specific animals. The forecasted value will play a big role when determining which combination of scarce game will most likely deliver the highest return on investment.

The mean prices paid at *Vleissentraal* auctions for specific game species were studied and analysed to serve as a basis to forecast the prices for the game species for the next five years (Vleissentraal, 2010). *Vleissentraal* has undisputedly been the biggest auctioneer of game over the last decade and therefore the statistics will be on target (Bouwer, 2014). The mean auction prices for Bushbucks, Nyalas, Sable Antelopes, Kudus and Golden Oryxes (Figure 6) are shown for comparison purposes. The Sable Antelopes are significantly more expensive than the Nyalas, Kudus and Bushbucks. Even though the Golden Oryx specie is new in this market, it's price exceeds even the Sable Antelopes' price by more than double within the first year of entering the market.

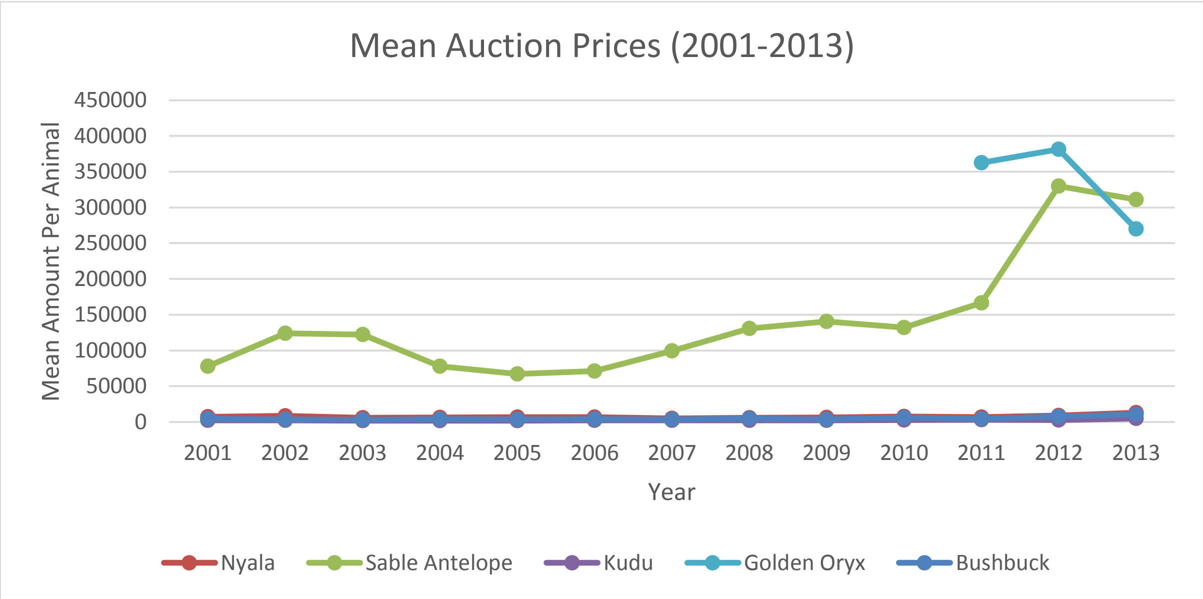


Figure 6 Mean Auction Prices

When comparing the Bushbuck breeding groups prices against the prices of Bushbuck rams, there is not a significant difference between them, although both tend to have large variances between one year and the next (Figure 7). From Figure 7 it seems as if the Bushbuck breeding group's prices are increasing against a higher rate than that of the rams alone.

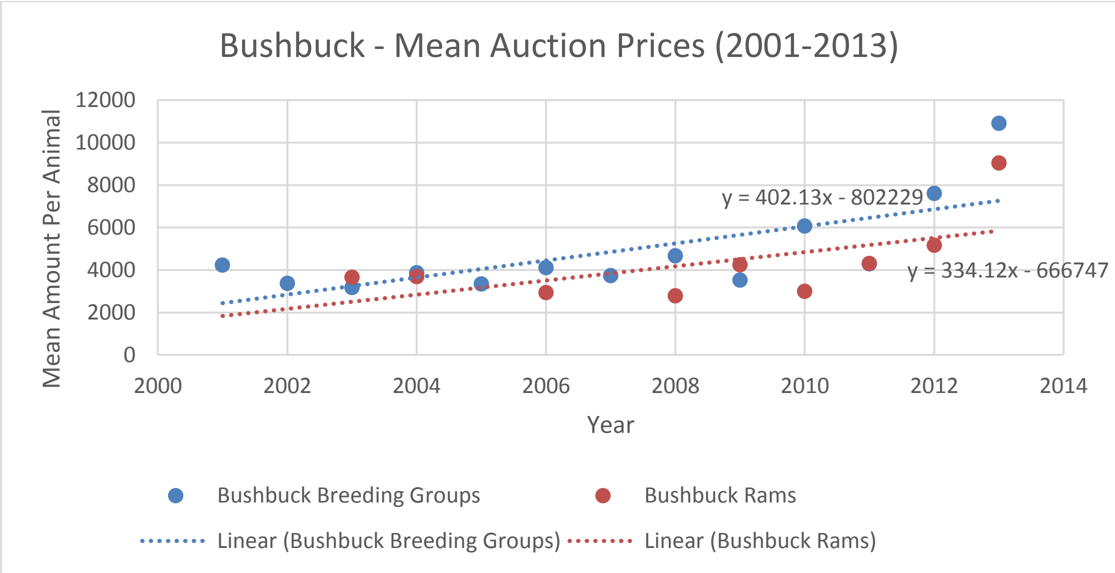


Figure 7 Bushbuck Auction Prices

From Figure 8 it becomes clear that the prices for Nyala bulls are higher than that of Nyala breeding groups but recently the gap became smaller. The total prices of this game specie tended to increase over the past few years, indicating promising investing opportunities.

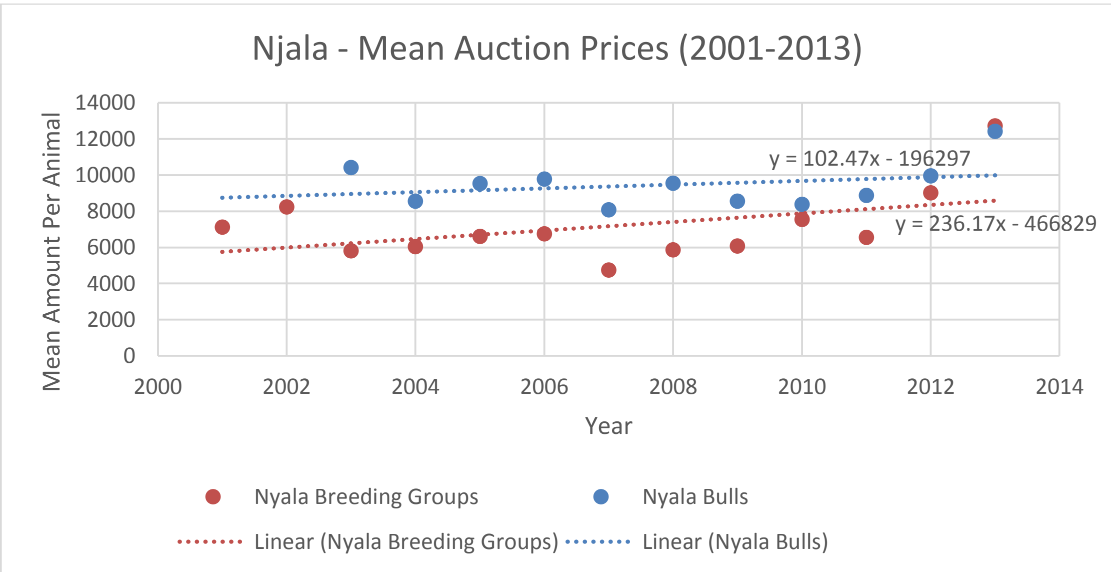


Figure 8 Nyala Auction Prices

The Sable Antelopes' auction prices also increased dramatically over the past few years (Figure 9). The breeding groups tend to be sold for much lower prices than the Sable Antelope bulls. It looks as if the most promising return on investment lies within the Sable Antelope bulls.

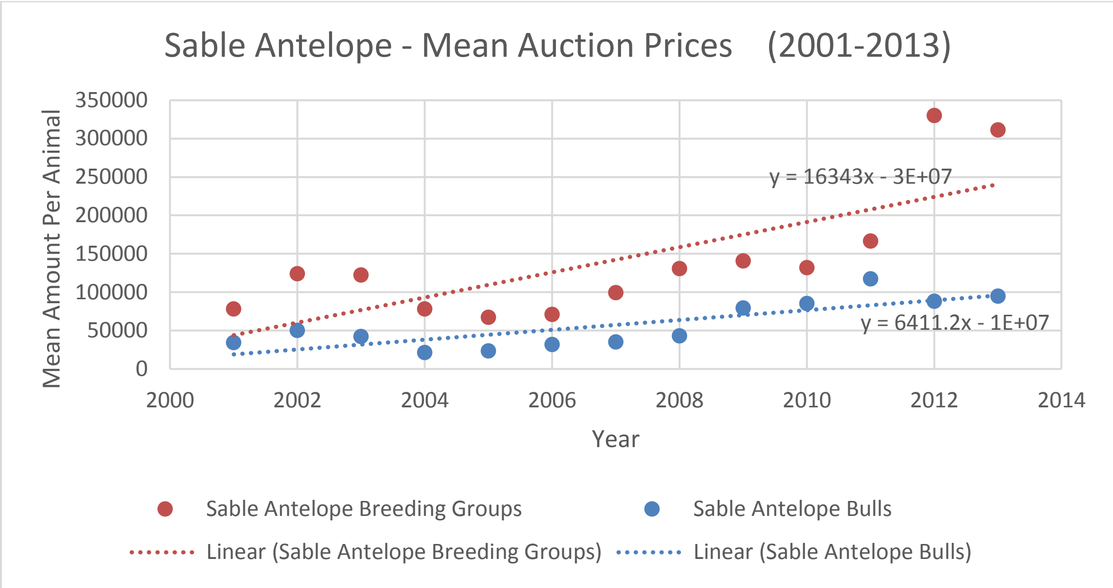


Figure 9 Sable Antelope Auction Prices

The Kudu specie's auction prices in Figure 10 look very promising with a dramatic increase in the bulls' prices over the last couple of years. The family breeding groups' prices also increased over the years indicating safe investment opportunities.

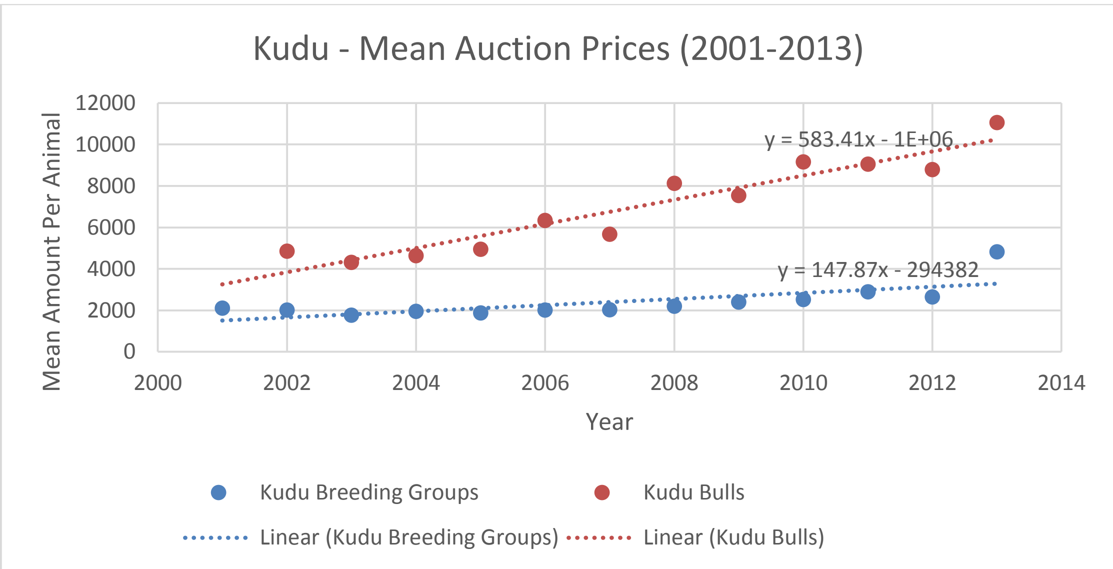


Figure 10 Kudu Auction Prices

The Golden Oryx species does not have much historical statistics because they are relatively new in the market (Figure 11). They came on auction during 2011 but reached extremely high prices and therefore can be strongly recommended. Even though it decreased over the first three years, their prices are still much higher than that of the other scarce game species.

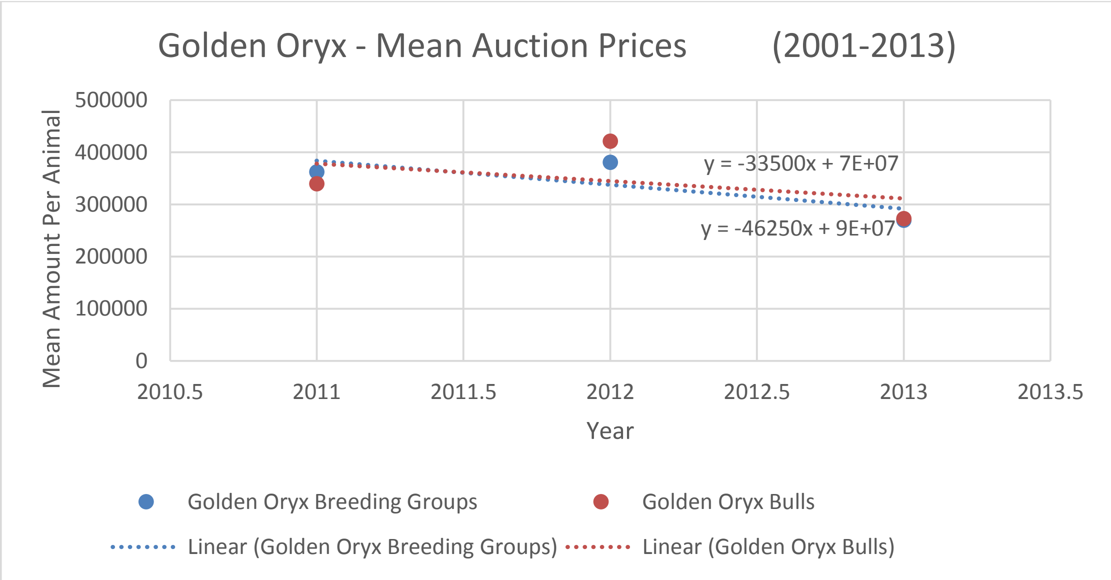


Figure 11 Golden Oryx Prices

3.2.2 Forecast Method Identification for Auction Prices

Four basic types of forecasting techniques can be used to help determine the direction of future trends by using historical data (Jacobs, 2011). This four forecasting types include qualitative, time series analysis, causal relationships and simulation. Qualitative techniques are based on estimates and opinions and are therefore subjective or judgmental. Causal forecasting refer to the relationship between factors while simulation models allow the forecaster to use various assumptions about the conditions of the forecast (Jacobs, 2011).

Time series analysis refer to the idea that data linked to past demand give a good indication and can be used to predict demand in the near future (Jacobs, 2011). The mean prices paid for the specific scarce game species at auctions for the past 13 years are time series and have the potential to deliver predicted auction prices for the future.

The basic linear regression forecasting method, also referred to as the straight line method, is used for forecasting the auction prices for the next five years. A guide that assists the process of selecting an appropriate forecasting method is used to determine this (Jacobs, 2011).

Table 2 Forecast Guide

Forecasting Method	Amount of Historical Data	Data Pattern	Forecast Horison
Linear regression	10 to 20 observations for seasonality at least	Stationary, trend and seasonality	Short to medium
Simple moving average	6 to 12 months, weekly data are often used	Data should be stationary	Short
Weighted moving average and simple exponential smoothing	5 to 10 observations needed to start	Data should be stationary	Short
Exponential smoothing with trend	5 to 10 observations needed to start	Stationary and trend	Short

The straight line method is assumed to deliver the best forecasted solutions because of a short to medium forecasting horizon present as well as the observations, the number of years being taken into account, being more than 10. This forecast method identified will be used in conducting the forecast at a later stage of the project.

3.2.3 Species' Habitat Study

When a scarce game species is kept in enclosures, it is essential that the camps consist of that specific species' habitat. It may be the case that *Prinslust* already possesses a specific species' habitat, possesses certain components of a specific species' habitat or may not contain any components necessary in a specific species' habitat at all.

In order to determine the capital required to ensure that the required habitat for each scarce game species in this project is available, it is necessary to conduct a study on the desired habitat of each scarce game species, the available habitat and ultimately determine the difference between the two. The degree of the difference between the required and available habitat will determine the capital required to accommodate each scarce game species.

3.2.3.1 Required Habitat Study

Habitat requirements are a very important aspect since animals can't survive without it. The quality of the habitat is directly related to the condition of the animals (Bouwer, 2014). When an animal species is forced to live in an area that is not consistent with their natural habitat, the chances of survival decreases. When breeding intensively with scarce game, it is therefore of utmost importance that each game species is living in an enclosure that represents its natural habitat. The following section consists of an in-depth study containing and describing the unique characteristics of each of the five scarce game species' habitat.

Table 3 Required Habitat Study

Specie	Topic	Description
Bushbuck	Diet	The Bushbuck, *Tragelaphus Scriptus*, is a highly selective leaf eater, having a diet consisting of 90% of leaves, small branch pieces, flowers and wild fruits (Bothma, 2011). The other 10% of its diet consists of grass, but not young grass. When no juicy food is available, the Bushbuck is dependent on water and can't survive without it.
	Flora	The habitat preferred by the Bushbuck is thick shrubs, forest or thick bushveld, preferably close to water (Bothma, 2011). The Bushbuck is not territorial, but inhabitable areas ranging from 3 hectares to 175 hectares, depending on the quality of the habitat and the season.
Nyala	Diet	The Nyala, *Tragelaphus Scriptus Angasi*, species can transform from a 90% leaf-diet to a 70% grass-diet but grass normally includes 12% to 30% of the Nyala's diet (Furstenburg, 2002). Important additions to the Nyala's diet are fruit, pods and flowers. During dry seasons, wet and dried lusern are good additions to the diet.
	Flora	The Nyalas' habitat must consist of thick bushes covering a minimum of 15% of the total habitat (Furstenburg, 2002). The essentialities in a Nyala's habitat include a large component of shaded areas with thick bushes, a large number of trees associated with water, freely accessible drinking water, high quality leaves as well as intermediate to short, sweet grass. Nyalas inhabit the exact same habitat than the Bushbuck and are therefore in competition with the Bushbuck (Furstenburg D. , 2002).

Sable Antelope	Diet	The Sable Antelope species, *Hippotragus Niger*, is more suited for tropical and subtropical regions with an abundance of water (Bothma J. , 2014). The optimal habitat for Sable Antelopes is frost-free, open woodland intermingled with tropical grassland near water. Additional food supplements such as lucerne and antelope cubes will be required during the dry seasons (Bothma, 2014).
	Flora	The core component in a Sable Antelope's diet is medium to tall grass, approximately 200 mm tall, while 10 % of the diet consists of leaves and another five percent consists of shrubs(Bothma, 2014). The grasses include guinea or buffalo grass, ginger grass, black-footed grass, red grass and spear grass. The leaves included in the diet contains those of raisin bushes, black thorn and sweet thorn. Lusern is a good addition to their diet during the dry seasons.
Kudu	Diet	Kudus, *Tragelaphus*, are not selective eaters and feed mainly on leaves, shoots, pods or fruit of a wide range of shrubs, trees, dicot forbs and succulents (Wildlife Ranching, 2009). Their diet consists of 18% of grass, 20% dicot broad leaf forbs and 60% trees and shrub browse. A dietary fibre intake of between 9% and 11% and protein intake of between 19% and 23% should be maintained throughout the year.
	Flora	Open woodland with scattered thicket bush, broken bushveld and savannah, both on plains and mountain slopes are the Kudu's ideal habitat (Wildlife Ranching, 2009).
Golden Oryx	Diet	Oryxes, *Oryx Gazelle Beisa*, are considered grasers or browsers and can therefore survive on dry grass that can be supplemented with foliage (Burchell Golden Oryx, 2013). When water is not available, water-storing fruits and vegetables will be consumed as replacement.
	Flora	They have adapted to a large number of places where most large mammals are unable to live in such as the dry Sahara desert (Burchell Golden Oryx, 2013).

To be able to determine the required expenses realistically, it is of utmost importance that inflation must also be taken into account for the investigation period of this project. A quick overview of the latest inflation rates delivered the results as seen in Figure 12 (Inflation.eu, 2010). The equation obtained in Figure 12 from the past 10 years' inflation rates will deliver realistic and applicable inflation rates for the optimisation model. The determined inflation values will also be applicable when determining the required infrastructure expenses and maintenance expenses, revealed at a later stage in Chapter 3.

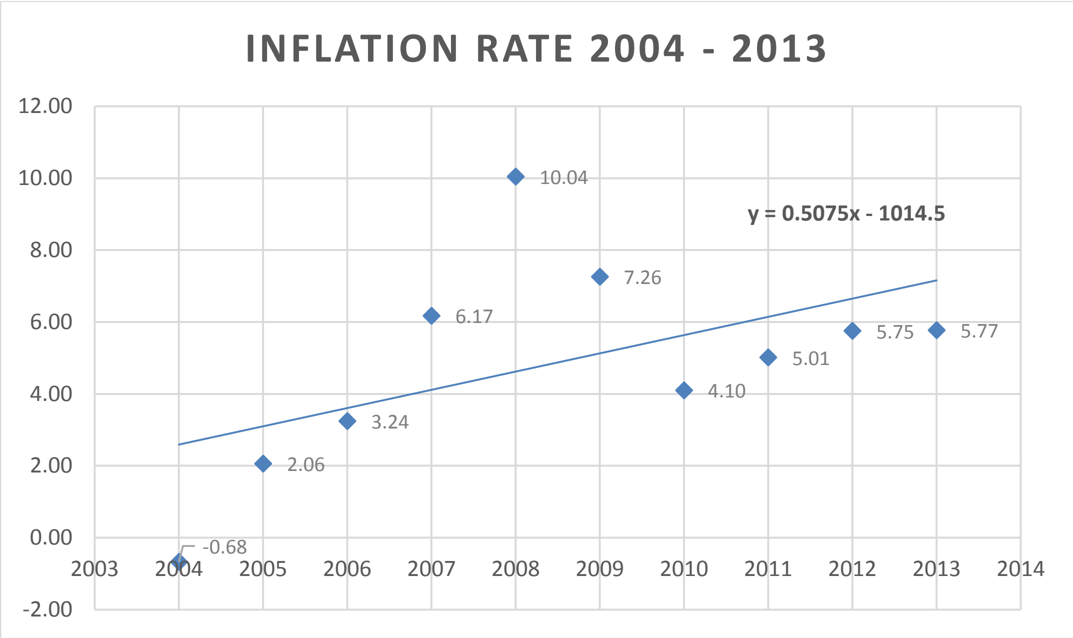


Figure 12Inflation Rate

3.2.3.2 Consulting the owner of *Prinslust* farm about available habitat

A consultation with the owner of *Prinslust* revealed that Bushbuck, Kudu and Oryx occur naturally on the farm. Exceptional Kudu bulls have been hunted on *Prinslust* thus indicating optimal habitat for the Kudu (Bouwer, 2014). Bushbuck occur naturally on the farm and since the habitat requirements for Bushbuck and Njala are the same, he foresees no problem with the Njala adapting to the conditions on the farm.

Oryx also thrive on the farm, to the extent that Golden Oryx and split Golden Oryx have naturally bred on *Prinslust* (Bouwer, 2014). Older inhabitants of the area have told the owner that the Sable Antilope species occurred naturally on his farm many years ago and he is thus confident that they will adapt and survive.

3.2.3.3 Determining the difference between required and available habitat

There is no significant gap between the required and available habitat. Habitat distribution is however not the same all over the farm and care must be taken with the allocation of habitat amongst the different species. This will impact on the infrastructure layout. Waterholes also need to be established in each camp since no natural waterholes are present on *Prinslust*.

The Alldays area can regularly experiences very dry seasons, usually between September and December and therefore additional feeding supplements will most likely be compulsory during these dry seasons.

3.2.4 Breeding Rate Study

It is important to keep the breeding rate of each species in mind because it is a direct measurement of the rate at which each scarce game species increases. The increase rate will then have a direct influence on the total income earned since the more animals sold at auctions, the higher the income generated.

Table 4 Breeding Rate Study

Specie	Breeding rates
Bushbuck	A Bushbuck eve mates for the first time at the age of 16 months and a ram at the age of 10 to 12 months (Bothma, 2011). The Bushbuck mates during the whole year but has peak seasons during spring and autumn. Pregnancy takes between 180 and 200 days. A Bushbuck's average life-expectancy is 11 years for males and females.
Nyala	Mating is not restricted to a specific season for Nyalas and can occur throughout the year (Furstenburg D. , 2002). Pregnancy for a Njala eve takes approximately 220 days and a Njala eve can give birth every nine to ten months. An eve will achieve sexual maturity after 18 months while a bull can only start mating after three years from date of birth. The maximum life-expectancy for an eve is 8 years and for a bull is 11 years (Furstenburg D. , 2002).
Sable Antelope	For the Sable Antelope, mating season varies from region to region with a peak from January to March in South Africa (Bothma, 2014). A cow will carry an unborn calf for a period of 266 days. Nursery herds are formed where up to eight calves walk with a single cow. Sexual maturity is reached at the age of 32 months in a bull and 24 months in a cow. The herd bull evicts the young bulls when they become three years old. A

	herd bull only breeds until the age of ten years while a cow breeds until they are between 10 and 13 years old (Bothma, 2014).
Kudu	Rutting season for Kudus are from April to June and calving season between December and May (Wildlife Ranching, 2009). The mating ratio for adult Kudus in intensive breeding groups is one bull with 2.5 to 4.2 cows and cows carry an unborn calf for between 250 and 260 days. Social adulthood are reached at the age of five years for a bull and three years for a cow. Although bulls can live up to the age of 16 years, bulls and cows can mate only up to the age of nine years.
Golden Oryx	The first mating for calves can occur at the age of five years for bulls and two years for cows (Wildlife Ranching, 2009). A cow will carry an unborn calf for between 261 and 275 days. Bulls can mate up to the age of 12 years but cows can mate up to the age of 16 years.

3.2.5 Social Structure Study

The social structure of a species is a perfect measurement of the number of animals that can be farmed with within a specific camp. Putting too many animals in one camp might result in the animals not breeding at the optimal breeding rate, however putting too little animals in a camp, potential breeding opportunities are lost. Therefore it is important to determine the exact maximum number of a specific scarce game species that can be bred within one camp.

Nyalas and Bushbucks are in competition for food when living in the same area since these two species inhabit the same habitat. This can result in the Nyalas reducing the Bushbuck species by 20% per year and eventually replacing the Bushbuck species in areas where they occur together. Therefore Bushbucks and Nyalas cannot be bred within the same camp (Bothma,2011).

Table 5 Social Structure Study

Species	Social Structure
Bushbuck	Small troops consisting of one adult ram, two or three adult eves and their lambs form regularly (Bothma, 2011). When breeding intensively with Bushbucks in camps, no more than five rams can be kept in a 2.5 hectare camp that must be divided into five smaller camps with solid walls between them (Bothma, 2011). The solid walls will prevent the rams from fighting, as can be done through fences and injuring themselves and other animals.
Nyala	The Nyala is semi-social resulting in unstable temporary groups of animals changing individuals constantly between groups (Furstenburg D. , 2002). For optimal production, the adult relationship can be between five and seven eves per adult bull.
Sable Antelope	When breeding extensively with the Sable Antelope species, family groups of between six and ten individuals form (Wildlife Ranching, 2009). These family groups usually consist of several adult cows, which are older than three years old and their young offspring of both sexes, some heifers and a dominant bull older than six years.
Kudu	Kudus form breeding groups of between one and two socially mature bulls, between two and four adult cows with one to three youngsters (Wildlife Ranching, 2009).
Golden Oryx	The Golden Oryx form mixed groups of between five and forty individuals, including several territorial adult bulls, adult non-lactating cows and sub-adult cows (Wildlife Ranching, 2009). When farming intensively with Golden Oryxes, 12 cows can walk with a single bull.

3.2.6 Infrastructure Requirement Study

The infrastructure required for the five scarce game species may differ from one species to the next. A large contribution to the expenses, especially in the beginning of the transformation process from plains game farming to scarce game farming, are the infrastructure. Therefore it is important to construct the exact requirements of the scarce game species to be farmed with at *Prinslust*.

Table 6 Infrastructure Requirement Study

Species	Type	Infrastructure Requirements
Bushbuck	Fencing	The bottom part of the fence must be covered with mesh fencing to keep the Bushbucks inside and animals such as bush pigs, warthog and other small animals outside to prevent competition for food, water and shelter (Bothma, 2011). The center of the fence, 750 mm above the ground and 225mm away from the fence, must be covered with another electrified fence, helping to protect the Bushbucks from predators. The fence must be a minimum of 1.8 m high and shelter must be available inside the camp. This shelter can include branches hanging from the fence or from a roof constructed inside the camp (Bothma, 2011).
	Water cribs	For the keeping of Bushbucks in camps, it is important to keep in mind that this species prefers natural water points (Bothma, 2011). This means that the water must be on the ground, not lifted in a container, surrounded with natural elements such as soil, grass and even bushes.
Nyala	Fencing	When kept in camps, Nyalas can jump up to 1.8 m high but have the habit of wanting to hide underneath bushes when feeling threatened (Furstenburg D. , 2002). The specific fencing requirements for the Nyala is an exact replica of those of the Bushbuck and therefore the Bushbucks and Nyalas can rotate between camps when necessary.
	Water cribs	The same water point requirements are present for the Bushbuck and the Nyala (Bothma, 2011).
Sable Antelope	Fencing	The Sable Antelopes are crawlers and therefore a normal game fence will require additional electric fences between 250 mm and 300 mm above the ground and 225 mm away from the original game fence (Bothma, 2014). When breeding intensively with Sable Antelopes, it is of utmost importance to remember that two herds cannot be kept in adjacent camps.

	Water cribs	Sable Antelopes prefer to walk knee-deep into the water or to kneel when drinking (Bothma, 2014) thus larger drinking holes will be necessary.
Kudu	Fencing	The Kudu can jump and go through wildlife-proof wire fences with ease and therefore the fence must be three meters high and electrification are required (Bothma, 2010).
	Water cribs	Kudus prefer to drink from the ground at natural waterholes surrounded by an open area (Bothma, 2010).
Golden Oryx	Fencing	In addition to the regular game fences, the Golden Oryx requires an electrified fence with a base stand of between 250 and 300 mm above the ground and 225 mm away from the regular wire fence (Bothma, 2010).
	Water cribs	The Golden Oryx prefer natural waterholes that are not surrounded by a lot of thick bushes (Bothma, 2010).

3.2.7 Maintenance Requirement Study

Maintaining and sustaining good proper fences are one of the most important aspects in the scarce game business (Bouwer, 2014). Poor fence-keeping may result in scarce game injuring themselves against the fences or even escaping from the camps, different scarce game species mixing between camps, predators entering camps and killing valuable animals, or poachers getting their hands on the game and killing it for food and survival (Bouwer, 2014).

The basic maintenance includes firstly a ground area of approximately half a meter along all game fences that should be kept clean at all times, together with electric fences especially. Secondly, any damaged poles, droppers or wire should be repaired or replaced as soon as a problem is noted (Bouwer, 2014). It is wise to use a voltage tester to check every day if the electric fence line is working and to keep spare insulators on the farm to assure that repairs can be done shortly after a problem arises (Electric Fencing Direct, 2013).

3.2.8 Predator Study

Predators are one of the largest contributors to the mortality rate of most game species (Bouwer, 2014). To be able to protect the five scarce game species against natural predators, it is important to identify the common predators of each scarce game species as well as what predators are present in and around *Prinslust*. As soon as the similarities between the predators of the five scarce game species and the predators present on *Prinslust* are determined, protection methods for the five scarce game species can be identified.

Leopard, Lions, Hyenas, Cheetahs, Hunting Dogs and Crocodiles are the predators that Bushbucks are most vulnerable to (Softschool.com, 2014). The predators of Njalas are very similar as for Bushbucks and includes Leopards, Lions, Hyenas, African Wild Dogs as well as Baboons that kill the calves (The Animal Files, 2006). The most frequent predators for Sable Antelopes also include Lions, Leopards, Hyenas, Hunting Dogs and Crocodiles (Out to Africa, 1996). Kudus have predators that include big cats, namely Lions and Leopards, Wild Dogs, Hyenas and Pythons (Softschool.com, 2014). Typical predators of Golden Oryxes include Lions, Wild Dogs and Hyenas (Softschool.com, 2014).

According to the Alldays farmer, at *Prinslust* the predators present in the area include Hyenas, Cheetah, Leopards, Pythons and Baboons. While farming with plains game, the farmer has lost approximately twenty animal each year because of the predators (Bouwer, 2014). Although the farmer on the farm next to *Prinslust* has been farming with thousands of Crocodiles for the past five years, only once, after a flood, has the carcass of a Crocodile been found on *Prinslust*.

The core method of keeping the scarce game protected from their predators, is by installing proper electric fences as mentioned above. For the protection against crawling predators such as Pythons, Hyenas, Cheetahs and Leopards, a concrete slab half a meter deep can be constructed underneath the fences of the camps containing the scarce game (Bothma, 2011). Mesh fencing can also be added at the bottom parts of the fences containing the smaller game such as Bushbucks and Nyalas to keep Baboons, Cheetahs, Leopards and Hyenas from climbing through the fence if the electric fence fails to do so (Bothma, 2011). According to the owner of *Prinslust* farm, such infrastructure will reduce the fatalities of the game to approximately 2% per year.

3.2.9 Risk Analysis

There are various other risks involved when breeding with scarce game that contribute to the mortality rate of the animals. When breeding with scarce game, the mortality rate needs to be as low as possible since one death can mean thousands of Rands wasted.

To catch wild game, there are two methods commonly followed by farmers, depending on the resources available (Bouwer, 2014). The one common method is by making use of a helicopter to control the game and to lead them where needed. On the ground, a large circle is formed with cables connected by using trees and temporary poles. On the cables, large canvasses are hung that will be able to enclose the circle made by the cables. Workers wait with the piled-up canvasses, usually in thick bushes if possible, where the cables are joined and when the helicopter has directed the animals within the circle, the helicopter activates a siren that tells the workers on the ground to close the circle of canvasses. The animals are then pushed on by dividers of canvasses closing as they pass through. A funnel of canvas direct them (Figure 13) until they are safely in a truck. For game with sharp horns such as Oryxes, black PVC pipes with warm ends are put over their horns for the protection of themselves and other animals in the same truck compartment (Bouwer, 2014).

Figure 13 Game Capture Process

The second method is called passive game capture where the game is lured into certain camps over a period of time by using water and food (Bouwer, 2014). The animals are then moved by hand onto the transportation trucks. Over time, the animals may become tame that will make the moving process much easier.

The risks involved in the catching and transporting process are important and must be taken into account. The whole process of movement is very stressful on all of these wild animals and most of the time, tranquilizers are given to the animals to calm them (Bouwer, 2014). Kudus are usually the most difficult to catch because they can jump very high.

Another very important risk involved when farming with scarce game is poachers killing the very valuable animals for their meat and skin (Bouwer, 2014). The best and most efficient way to reduce and eliminate poachers is by the electrification of all camps containing scarce game or even the electrification of the whole farm. This option will result in extra, unnecessary expenses.

Droughts, which may occur frequently in the Alldays region, will result in extra expenses providing the necessary water and food to the animals (Bouwer, 2014). Should this risk materialize, it may add a large amount of expenses and must be taken into consideration.

3.3 Model

After identifying all the relevant constraints present within this model and gathering all relevant information required to complete all the identified tasks and their activities, the formulation of an optimisation model can take place. The sets are identified as follow:

Set of species $i \in \boldsymbol{I}$ =	1	Bushbuck
	2	Nyala
	3	Sable Antelope
	4	Kudu
	5	Golden Oryx

Set of years $j \in \boldsymbol{J}$ =	1	Year 2015
	2	Year 2016
	3	Year 2017
	4	Year 2018
	5	Year 2019
	6	Year 2020

The following parameters are defined within this model:

f_{ij} ≜ the given forecasted price for an animal of a family of species i in year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

g_{ij} ≜ the given forecasted price for a male of species i in year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$, $j = \{1, 2, \ldots, \boldsymbol{J}\}$.

h_i ≜ the given initial expense required to implement the required habitat for one animal of species i, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

b_i ≜ the given breeding rate for species i per year, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

k_i ≜ the given initial expense required to implement the required infrastructure for one animal of species i, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

m_i ≜ the given maintenance required for the infrastructure for one animal of species i per year, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

d_i ≜ the given annual death rate of species i due to predators, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

r_i ≜ the given risks associated with species i, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

w_j ≜ the given inflation rate in year j, where $j = \{1, 2, \ldots, \boldsymbol{J}\}$.

Variables are defined as follow:

$x_{i,j}$ $\triangleq$ the number of males of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

$y_{i,j}$ $\triangleq$ the number of females of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

Two decision variables are defined as follow:

$p_{i,j}$ $\triangleq$ the number of males of species i to be purchase during year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$, $j = \{1, 2, \ldots, \boldsymbol{J}\}$.

$q_{i,j}$ $\triangleq$ the number of animals in a family of species i to be purchase during year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

The basic formulation of the objective functions include the following:

$$\max z_1 = \sum_{i=1}^{5} \left[\left(x_{i,6} \times g_{i,6} \right) + \left(y_{i,6} \times f_{i,6} \right) \right] \tag{33}$$

$$\min z_2 = \sum_{i}^{5} (\{ \left(x_{i,1} + y_{i,1} \right) + \sum_{j=2}^{5} \left[p_{ij} + q_{ij} \right] \} \times (h_i + k_i + m_i + w_j)]) \tag{34}$$

Subject to

$$(q_{ij} + \{y_{ij} \times b_i\}) \times (1 - s_i) \times (1 - r_i) = y_{i,j+1} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{35}$$

$$(p_{ij} + x_{ij}) \times (1 - s_i) \times (1 - r_i) = x_{i,j+1} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{36}$$

$$q_{ij}, y_{ij}, p_{ij}, x_{ij} \geq 0 \text{ and integer} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{37}$$

The primary objective function (33) of the model illustrates the potential maximum income that can be generated after farming selectively with only scarce game for a period of five years. Scarce game are usually sold either separately as male animals or in family groups containing females and young bulls. When sold at an auction, the selling price is always per animal. The number of males and animals per family present on *Prinslust* farm at the beginning of the sixth year are multiplied with its calculated auction value respectively. These auction values are forecasted from historic data through making use of an appropriate forecasting method as illustrated earlier.

In the secondary objective function (34), the expenses required to provide the necessary habitat and protection through fencing for the specific number of animal species present of *Prinslust*, are minimised. Included in the secondary objective is an inflation rate that ensure that a realistic model solution can be obtained.

Constraint (35) ensures that the number of females of each species can be calculated with precision. According to the owner of *Prinslust* farm, one quarter of the animals in a family of animals sold at auctions are usually young males. Therefore, the calculation of the number of males of each species present can be observed in constraint (36). To be able to limit the decision variable values to non-negative integer values, constraint (37) is present.

3.4 Solution Process

The multiple objective function can be compiled into a single objective function to simplify the final model. Since the goal of the primary objective is to maximise the total income, and the goal of the secondary objective is to minimise the total expenses, a single objective function (38) can be obtained by subtracting the secondary objective from the primary objective.

The process of validating the model can be completed through the act of consulting the owner of *Prinslust* farm. The owner has great knowledge of the farm, the surrounding area, and the community and therefor will be of great help when validating the model. During the consultation with the owner, it became clear that more constraints are required within the model. These constraints include

- ✓ A limiting factor which states that not more that R 5 000 000 may be spent during each year for the entire period of five years;
- ✓ One quarter of the male animals together with all female animals will be used to form family groups when determining the value of the animals on the farm;
- ✓ When purchasing a specific species at an auction, a minimum of at least one male of that species must be bought;
- ✓ When purchasing a specific species at an auction, a minimum of at least three Bushbuck, three Nyalas, five Sable Antelopes, five Kudus and three Golden Oryxes must be bought;
- ✓ The maximum available habitat present on *Prinslust* farm is 1200 hectares; and
- ✓ The direct relationship between the number of males and females present on *Prinslust* farm must always be at least 1:2.

After consulting the owner of *Prinslust* farm for the purpose of validating the developed model, the two new parameters must be added to the model. These seven parameters are:

$a_{i,j}$ ≜ the total amount of money spent on species i during of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$, $j = \{1, 2, \ldots, \boldsymbol{J}\}$.

$v_{i,j}$ ≜ the total value of the animals of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

$t_{i,j}$ ≜ the number of animals of species i bred during year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$, $j = \{1, 2, \ldots, \boldsymbol{J}\}$.

$l_{i,j}$ ≜ the total amount of money spent on infrastructure, maintenance and habitat per animals of species i for the year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

e_i ≜ the given hectare space required in breeding camps per animal of species i, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

u_i ≜ the given hectare space required on the farm per animal of species i, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

s_i ≜ the given social structure of species i, where $i = \{1, 2, \ldots, \boldsymbol{I}\}$.

The basic formulation of the objective functions include the following:

$$\max z = \sum_{i=1}^{5} [v_{i,6} - \sum_{j=1}^{5} a_{i,j}] \tag{38}$$

Subject to

$$y_{i,j} \times (b_i - 1) = t_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{39}$$

$$y_{1,1} = 4 \tag{40}$$

$$y_{2,1} = 1 \tag{41}$$

$$y_{3,1} = 0 \tag{42}$$

$$y_{4,1} = 46 \tag{43}$$

$$y_{5,1} = 0 \tag{44}$$

$$(y_{i,j-1} + \left(0.5 \times t_{i,j-1}\right) + q_{i,j-1} \times (1 - d_i) \times (1 - r_i) = y_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \{2, \ldots, 6\} \tag{45}$$

$$x_{1,1} = 6 \tag{46}$$

$$x_{2,1} = 2 \tag{47}$$

$$x_{3,1} = 0 \tag{48}$$

$$x_{4,1} = 57 \tag{49}$$

$$x_{5,1} = 0 \tag{50}$$

$$(x_{i,j-1} + \left(0.5 \times t_{i,j-1}\right) + p_{i,j-1} \times (1 - d_i) \times (1 - r_i) = y_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \{2, \ldots, 6\} \tag{51}$$

$$a_{1,j} + a_{2,j} + a_{3,j} + a_{4,j} + a_{5,j} \leq 5\,000\,000 \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{52}$$

$$\left(0.5075 \times (2014 + j)\right) - 1014.5 = w_j \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{53}$$

$$65 \times o_i \times 365 \times (1 + \left(\frac{w_1}{100}\right)) = h_{i1} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \tag{54}$$

$$h_{i,j-1} \times (1 + \left(\frac{w_1}{100}\right)) = h_{ij} \qquad \forall i \in \boldsymbol{I}, j \in \{2, \ldots, 6\} \tag{55}$$

$$\left(\frac{115000}{2.5}\right) \times e_i \times (1 + \left(\frac{w_j}{100}\right)) = k_{ij} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (56)$$

$$\left(\frac{10000}{2.5}\right) \times e_i \times \left(1 + \left(\frac{w_j}{100}\right)\right) = m_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (57)$$

$$h_{i,j} + k_{i,j} + m_{i,j} = l_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (58)$$

$$\left(\left(x_{i,1} + y_{i,1}\right) \times l_{i,1}\right) + \left(2000 \times 36 \times \left(1 + \left(\frac{w_j}{100}\right)\right)\right) + \left(0.75 \times p_{i,1} \times g_{i,1}\right) +$$
$$(\left(\left(0.25 \times p_{i,1}\right) + q_{i,1}\right) \times f_{i,1}) = a_{i,1} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (59)$$

$$(\left(t_{i,j} + p_{i,j} + q_{i,j}\right) \times l_{i,j}) + \left(2000 \times 36 \times \left(1 + \left(\frac{w_j}{100}\right)\right)\right) + \left(0.75 \times p_{i,j} \times g_{i,j}\right) +$$
$$(\left(\left(0.25 \times p_{i,j}\right) + q_{i,j}\right) \times f_{i,j}) = a_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \{2, \ldots, 6\} \qquad (60)$$

$$(236.17 \times (2014 + j) - 466820) = f_{1,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (61)$$

$$(334.12 \times (2014 + j) - 666747) = f_{2,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (62)$$

$$(6411.18 \times (2014 + j) - 12809845.42 = f_{3,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (63)$$

$$(147.87 \times (2014 + j) - 294382 = f_{4,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (64)$$

$$(-27750 \times (2014 + j) - 56183250 = f_{5,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (65)$$

$$(102.47 \times (2014 + j) - 196297 = g_{1,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (66)$$

$$(402.13 \times (2014 + j) - 802229 = g_{2,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (67)$$

$$(16342.62 \times (2014 + j) - 32657316.92 = g_{3,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (68)$$

$$(583.41 \times (2014 + j) - 32819666.67 = g_{4,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (69)$$

$$(16500 \times (2014 + j) - 32819666.67 = g_{5,j} \qquad \forall\, j \in \boldsymbol{J} \qquad (70)$$

$$((y_{i,j} + (0.25 \times x_{i,j})) \times f_{i,j}) + (0.75 \times x_{i,j} \times g_{i,j}) = v_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (71)$$

$$s_i \times x_{i,j} \geq y_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (72)$$

$$0.5 \times x_{i,j} \leq y_{i,j} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (73)$$

$$(x_{i,j} + y_{i,j}) \times (e_i + u_i) \leq 1200 \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (74)$$

$$q_{ij}, y_{ij}, p_{ij}, x_{ij} \geq 0 \text{ and integer} \qquad \forall i \in \boldsymbol{I}, j \in \boldsymbol{J} \qquad (75)$$

The number of animals bred during each year can be obtained from constraint (39). The input values of the total number of female animals present on the farm at the beginning of year one can be seen in constraints (40) to (44) while constraint (45) determines the number of female animals present for each consecutive year. Similarly, the input values of the total number of male animals present on the farm at the beginning of year one can be seen in constraints (46) to (50) while constraint (51) determines the number of male animals present on *Prinslust* for each consecutive year.

To be able to limit the allowable amount of money to be spent during each year, constraints (52) must be added to the model. Constraint (53) illustrates the inflation rate for each of the years present in the model. The expenses regarding habitat adjustment are determined for year one and for all the other years in constraint (54) and (55) respectively. Constraint (56) and (57) illustrates the infrastructure implementation and maintenance cost of the infrastructure respectively. The amount of money spent per animal on the farm is determined through constraint (58).

Constraint (59) and (60) determines the total amount of money spent on each species during the course of each year. These two constraints were added for the purpose of simplifying the objective function and to attain better insight into the model. The previously determined forecasting calculations for male animals and family group animals are represented by constraint (61) to (65) and (66) to (70) respectively.

The value of each animal is determined through constraint (71) while constraint (72) illustrates the specific social structure related to each scarce game species. To account for the number of male animals against the number of female animals present on *Prinslust* farm, constraint (75) are added to the model. Constraint (76) ensures that the maximum carrying capacity of the farm will not be exceeded. The final and last constraint makes sure that the variables to be determined by the model must always be positive integers or zero.

3.5 Numerical Illustration and Model Validation

The most effective way to illustrate the results obtained from the model are by converting the outcome tables, obtained from the solution report delivered by LINGO, into usable information. It is also of utmost importance to make sure that the model reacts as expected when changing certain constraints.

For the purpose of validating the model, the model was run for a variety of iterations. The three major input constraints determined by the farmer of *Prinslust*, were changed to enable the validation process. These three constraints are:

- The number of animals currently present on the farm;
- The available budget per year; and
- The project time period of five years.

3.5.1 Current Number of Animals

A total number of four different combinations of input values for the number of animals currently present on the farm are going to be evaluated. Each of the four different combination can be seen as a model run, delivering a new set of output values.

Within the figures present is each run, year one is represented is dark blue, year two in red, year three in green, year four in purple, year five in light blue and year six in orange.

3.5.1.1 Run 1:

The input values for the first run are as follows:

$x_{i,j}$ ≜ the number of males of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

- $x_{1,1} = 0$
- $x_{2,1} = 0$
- $x_{3,1} = 0$
- $x_{4,1} = 0$
- $x_{5,1} = 0$

$y_{i,j} \triangleq$ the number of females of species i present on the farm at the beginning of year j, where $i = \{1, 2, \dots, \boldsymbol{I}\}, j = \{1, 2, \dots, \boldsymbol{J}\}$.

- $y_{1,1} = 0$
- $y_{2,1} = 0$
- $y_{3,1} = 0$
- $y_{4,1} = 0$
- $y_{5,1} = 0$

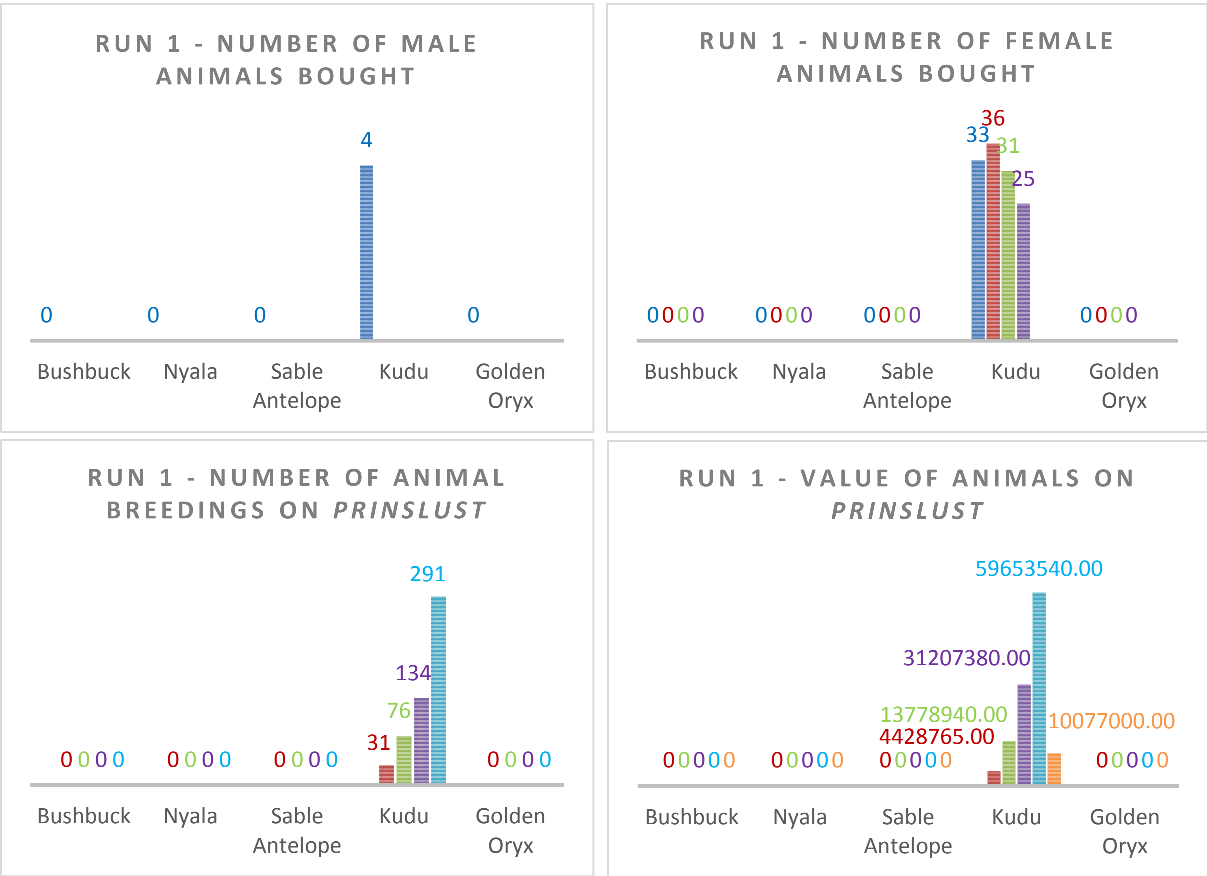


Figure 14Changing Number - Run 1 Results

The results of the first run can be observed in Figure 14. Since there are no animals of any species present on the farm in year one, as can be seen in the input values above, the model will buy the species or a combination of the species with the highest rate of return (ROR) on its own. With a maximum budget of R 5 000 000 per year, the total value of the animals at the end of the fifth year is R 59 653 540.

Only the kudu species is bought within this set of input values since the kudu species indicate the best investment growth over a period of five years, taking into account the breeding rates, risks involved, predators, eating and living habitats, captivity, and death rates.

3.5.1.2 Run 2:

For the second run, the input values are as follows:

$x_{i,j}$ ≜ the number of males of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

- $x_{1,1} = 10$
- $x_{2,1} = 10$
- $x_{3,1} = 10$
- $x_{4,1} = 10$
- $x_{5,1} = 10$

$y_{i,j}$ ≜ the number of females of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

- $y_{1,1} = 5$
- $y_{2,1} = 5$
- $y_{3,1} = 5$
- $y_{4,1} = 5$
- $y_{5,1} = 5$

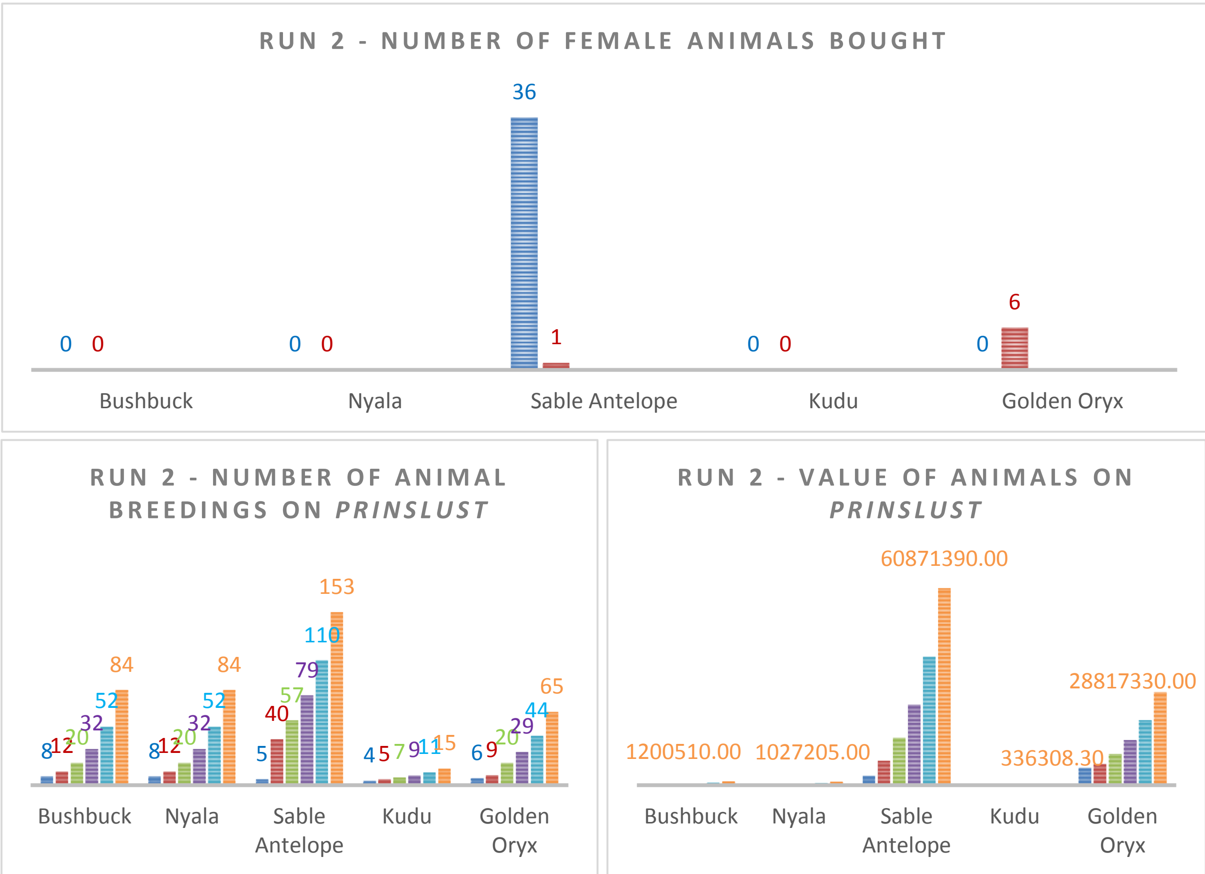


Figure 15Changing Number - Run 2 Results

Since there are 15 animals of each species already present on the farm for this run, it is clear that each species will continue breeding according to it respective breeding rate (the Kudu breeding rate being the smallest). From the graphs present in Figure 15, it is clear that a total of 37 female Sable Antelopes and six Golden Oryxes were bought but no male animals were bought. This is appropriate results since Sable Antelopes and Golden Oryxes have the largest social structure, meaning that a large amount of female animals can walk with a single male animal. Since the male animals are more expensive than family groups, a higher social structure relationship is a big advantage.

Although the Golden Oryxes are bought only during the second year, it is still a very good investing opportunity because of the fact that this species is relatively new on the market and had a very high entry price. Even though the Golden Oryx species currently has a decreasing forecasted value per year, it is still much higher than that of the other three scarce game species (Bushbuck, Nyala and Kudu).

3.5.1.3 Run 3:

The input values for the third run are as follows:

$x_{i,j} \triangleq$ the number of males of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

- $x_{1,1} = 6$
- $x_{2,1} = 2$
- $x_{3,1} = 0$
- $x_{4,1} = 57$
- $x_{5,1} = 0$

$y_{i,j} \triangleq$ the number of females of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

- $y_{1,1} = 4$
- $y_{2,1} = 1$
- $y_{3,1} = 0$
- $y_{4,1} = 46$
- $y_{5,1} = 0$

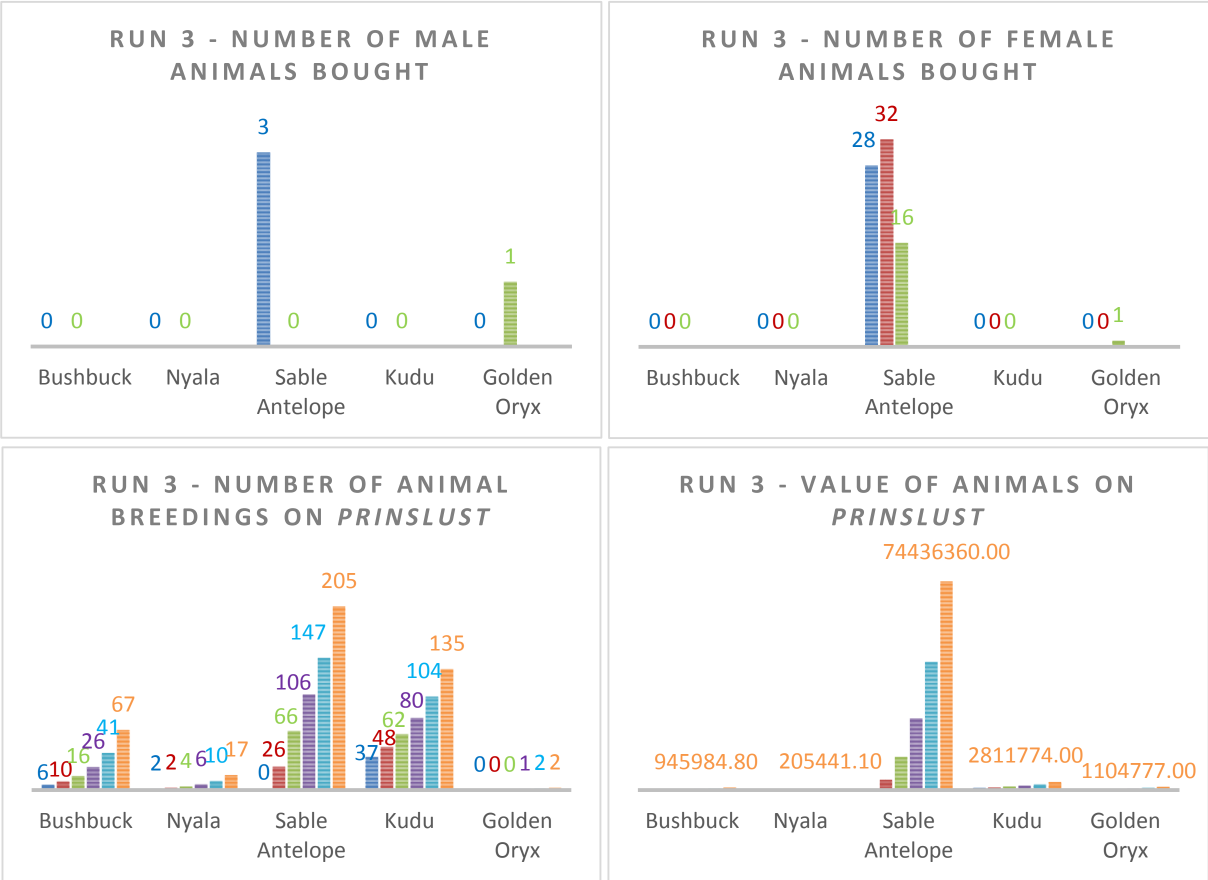


Figure 16Changing Number - Run 3 Results

Figure 16 illustrates the results obtained from the model of the actual number of animals currently present on *Prinslust*. Yet again, Sable Antelopes is the best selection of scarce game species because of the fact that this species have a large social structure, even though no Sable Antelopes are currently present on *Prinslust*. The Golden Oryxes addition during the third year can be expected since this species are a better investment opportunity as described above.

3.5.1.4 Run 4:

For the fourth and final run, the input values are as follows:

$x_{i,j} \triangleq$ the number of males of species i present on the farm at the beginning of year j, where $i = \{1, 2, \ldots, \boldsymbol{I}\}, j = \{1, 2, \ldots, \boldsymbol{J}\}$.

- $x_{1,1} = 0$
- $x_{2,1} = 0$
- $x_{3,1} = 6$
- $x_{4,1} = 0$
- $x_{5,1} = 6$

$y_{i,j} \triangleq$ the number of females of species i present on the farm at the beginning of year j, where $i = \{1, 2, ..., \boldsymbol{I}\}, j = \{1, 2, ..., \boldsymbol{J}\}$.

- $y_{1,1} = 0$
- $y_{2,1} = 0$
- $y_{3,1} = 4$
- $y_{4,1} = 0$
- $y_{5,1} = 4$

For the last run of changing the initial number of animals of the various scarce game species present on *Prinslust*, the scarce game species delivering the best potential investment were give values. As expected, none of the other three scarce game species (Bushbuck, Nyala and Kudu) were bought.

Since the Golden Oryxes' current forecasted values are decreasing over the years, it is anticipated that too much Golden Oryxes will lead to a decrease in value over time. Therefor no further addition of this animal species is expected as seen in the results in Figure 17. It is expected that the model will only buy more Sable Antelopes because of the species high social structure rate and high forecasted values as described earlier.

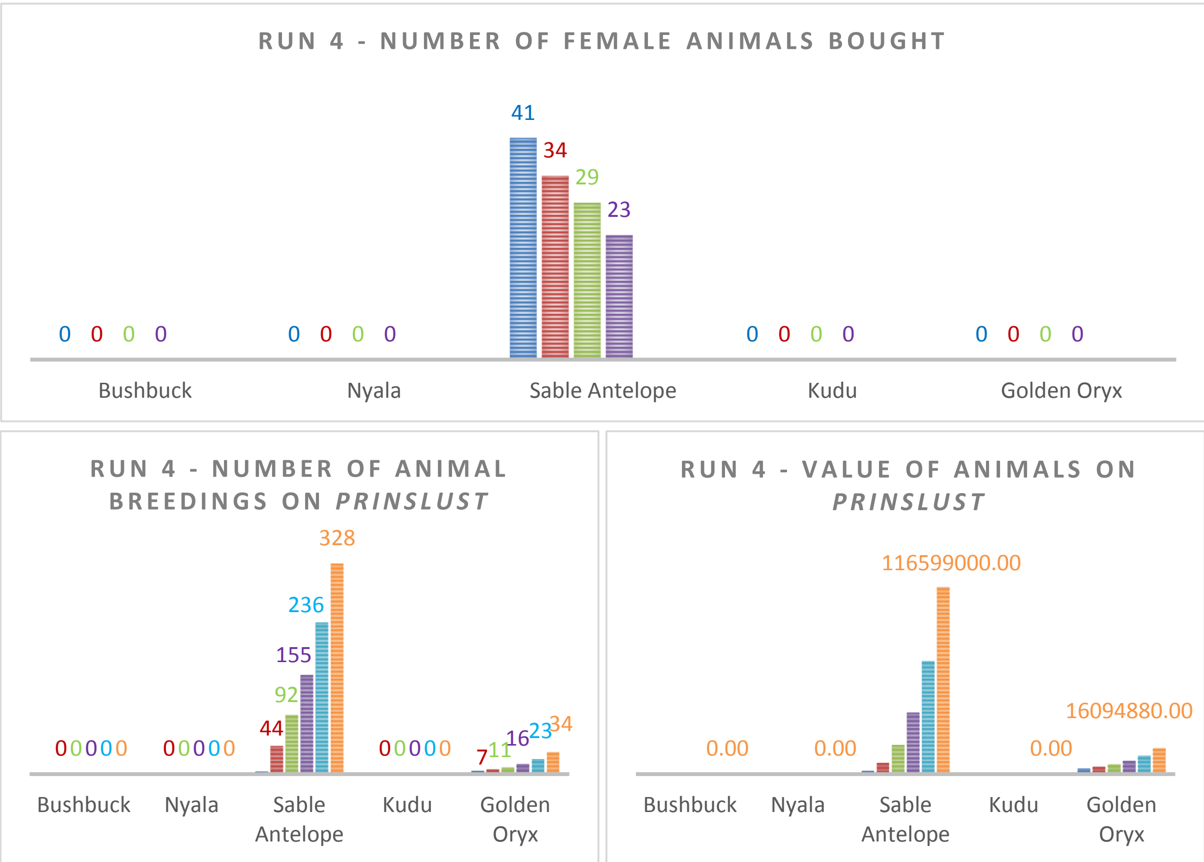


Figure 17Changing Number - Run 4 Results

3.5.2 Budget

When changing the budget of the optimisation model, a clear indication of the relationship among the different investment opportunities can be obtained. A total number of five different runs were investigated, each containing a different budget input value as follow:

- Run 1: R 3 000 000
- Run 2: R 4 000 000
- Run 3: R 5 000 000
- Run 4: R 10 000 000
- Run 5: R 15 000 000

The results containing the total value of the animals present on Prinslust, is concluded in Figure 18. It is clear that there is an exponential growth of the value of the animals present over time. This is expected since the growth of the value of the scarce game species is much higher than the inflation rate over the same time period. For the farmer of *Prinslust*, this results illustrate that the higher budget the farmer can put into farming with scarce game, the quicker is will increase over time.

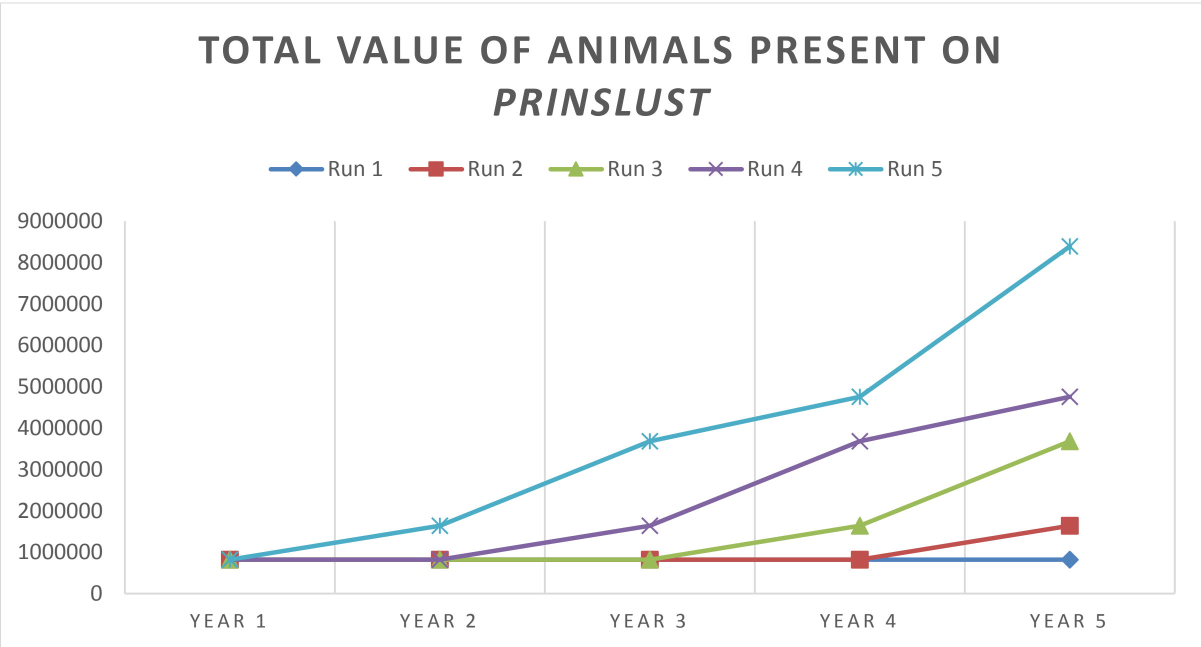


Figure 18Changing Budget - Value

Total profit may result in an even better indication of the importance of the available budget. Allowing the right budget for a project of this scale is of utmost importance since is have an enormous impact on the return of investment (ROI).

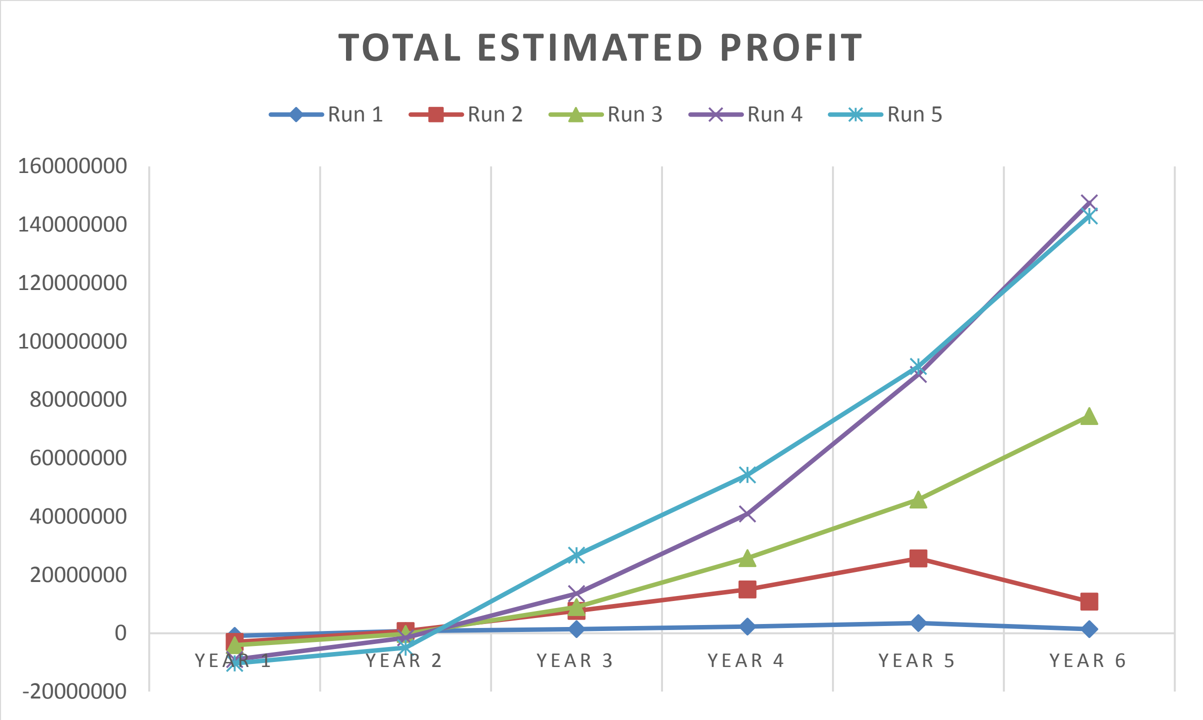


Figure 19Changing Budget - Profit

Figure 19 is a perfect illustration of the impact of a budget on a large scale project. Even though, for a larger budget, a negative profit is present for the first two years, within the first year of delivering a positive profit, the positive number is already exceeding all the other budget options. An exponential budget increase, as can be observed in Figure 19, is expected and appears very logical.

The importance of considering different budgets can be illustrated by using the light blue and purple profit lines. The purpose forecasted studies is to be able to identify when the best selling time for an investment is.

From the two lines identified, it is clear that the blue line is a better investment opportunity for a time period of three, four or even five years. Therefore the blue line is a better choice over a shorter time period between three and five years. When aiming to have a longer investment period, six years or longer, the purple line indicates a much steeper inclination, resulting is much higher profits over time.

The green line is a perfect average between the two high-performing investment budgets and the two lower-performing budgets. This will still be a very good investment returning R 75 million over a period of five years. Because of the two lower budgets' slow increase rates and even decrease over the last year of the current time period, no recommendations would be made in their direction.

3.5.3 Time Period

The evaluation of the optimisation model over different time periods is of utmost importance since it will indicate a variety of alternative considerations, representing even greater profits over longer periods of time. Figure 20 illustrates the total estimated project profit over the following project time periods:

- Run 1: 4 years
- Run 2: 5 years
- Run 3: 6 years
- Run 4: 7 years
- Run 5: 8 years

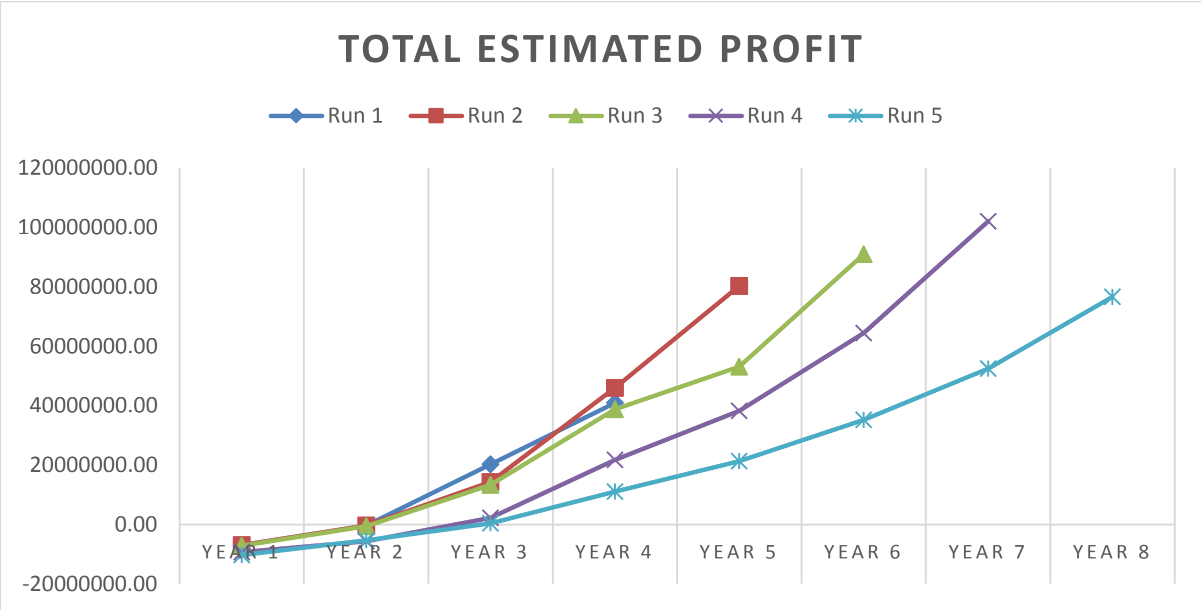


Figure 20Changing Time Period - Profit

It is clear that all the different runs deliver a negative profit for the first year as well as the second year of the project life. It is expected that the shorter project life runs tend to increase in profit much quicker as those of the longer project life runs, as can be observer in Figure 20. It seems as if an exponential trend is present when expanding the project life, indicating much higher potential profits at the end of a larger project life. This illustration is an indication that longer project lives should definitely be considered.

CHAPTER 4: Conclusion and Recommendation

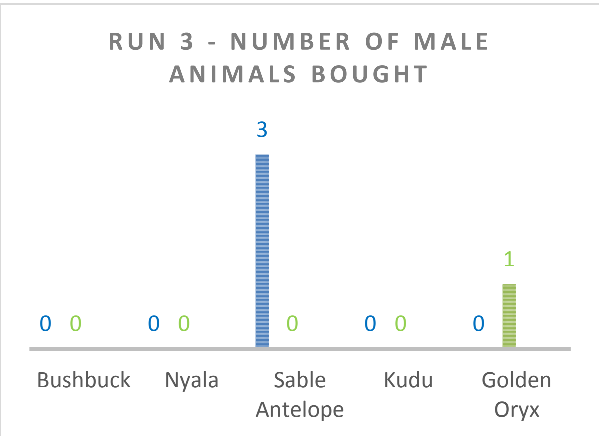
After identifying the core concepts of the Alldays farmer's problem in the introduction of this document, intensive research within the literature study guided the project to operations research and ultimately to a mathematically orientated optimisation model which will deliver a proposed optimum solution. The constraints, rewritten as tasks, required extensive research as set out in the model formulation section.

This research obtained was used to help formulate a mathematical optimisation model within the model section of this document. First, the research was converted into mathematical equations from where it was used within the optimisation model and objective function. Once a basic written model was completed, the formulation of the model in optimisation software was the second step

A first attempt to validate the model was done by consulting the owner of *Prinslust* farm. The model was then adjusted according to the feedback from the farmer. Before delivering the final results obtained from the model to the Alldays farmer, a second and final validation process was conducted to illustrate that the model behaves as expected.

4.1 Recommendations

From the given input values of the farmer of *Prinslust*, the results in terms of the number of male and female animals of each scarce game species, as seen in Figure 20, were obtained from the optimisation model.



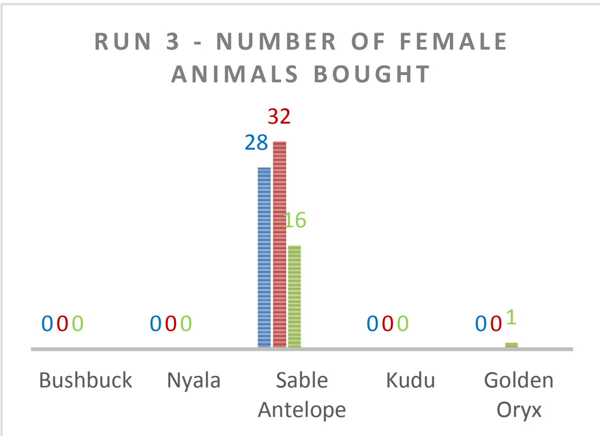


Figure 21 Number of Animals to be Bought

It is recommended that a total number of three Sable Antelope males be bought during the first year together with 28 Sable Antelope females. During the second year, it is recommended that another 32 Sable Antelope females should be purchased. In addition, another 16 Sable Antelope females should be purchased during the third year together with one Golden Oryx male as well as one Golden Oryx female.

If execution takes place as suggested above, Figure 21 can be used as guidance of what can be expected during the course of the first five years. The core question can then be asked again:

"How can the best combination of these five game species be found

that will deliver the ultimate maximum profit

and ensure sustainability over the next five years?".

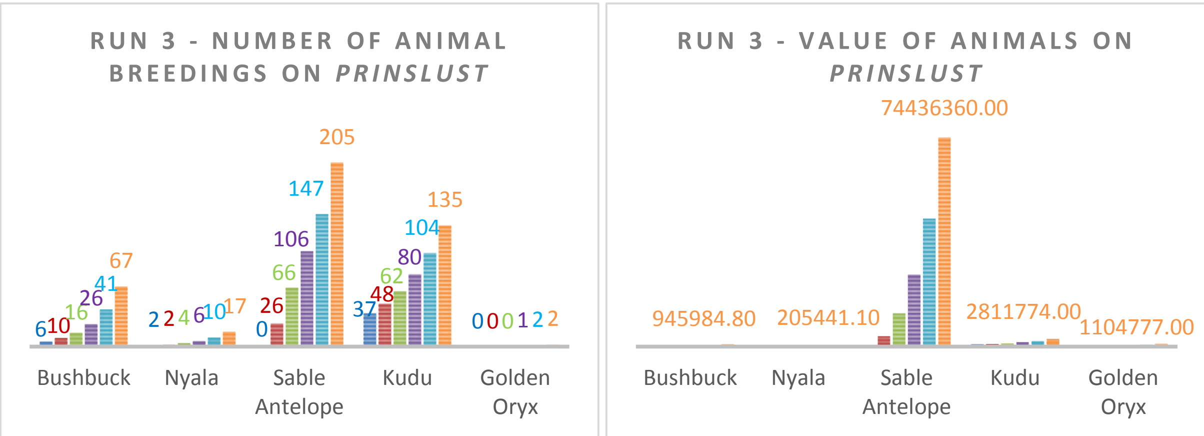


Figure 22 Potential Gain

When following the recommended model solution, the breeding numbers will most likely follow the pattern observed in Figure 21 together with the estimated value of the animals on *Prinslust*. A total maximum profit of R 59 590 920 can be obtained within five years when following the recommended purchases.

Of course the given results must not be followed blindly and other alternatives with larger budgets and longer time periods must also be considered before making a final decision. As time goes on, it is also of utmost importance to update applicable estimations such as the forecasted values for the auction prices and the inflation rate.

4.2 Way Forward

Before any scarce game species can actually be bought at auctions to start intense selective farming on *Prinslust*, numerous steps must first be completed. After completing this first phase, developing an optimisation model, of the project on *Prinslust*, a second and third project, each containing several steps, will ultimately enable the owner of Prinslust to start his selective farming with scarce game.

The second phase to be completed must deliver a very accurate facility layout of the farm, indicating exactly where it is best suited to implement which scarce game species' camps. The steps included in this project will be:

1. Completion of an environmental study on the Alldays region;
2. Completion of an environmental study on *Prinslust*;
3. Analysis of the physical layout of the natural habitat present on *Prinslust*;
4. Development of a layout structure illustrating the natural habitat present on *Prinslust*;
5. Development of a facility layout plan of *Prinslust*; and
6. Validation of the facility layout plan through the help of the owner of *Prinslust*.

The third and final phase of the *Prinslust* project must consist of the physical implementation of the designed facility layout developed in the second phase. Numerous steps will be required within this final phase.

1. Comprehensive time and planning schedule;
2. Comprehensive budget schedule;
3. Sufficient raw material purchasing schedule;
4. Physical implementation of the infrastructure required; and
5. Comprehensive maintenance schedule.

After the completion of all three phases of the Prinslust scarce game intensive breeding operation, this farmer may have the key to succeed in the scarce game market. If it is proven that a physical implementation of this model is effective and efficient, it will most likely, with the necessary updating of constraints and habitat studies, help other farmers to succeed.

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ADDENDUM

Lingo Model

```
Sets:

!Set of species: i
       1=Bushbuck
       2=Nyala
       3=Sable Antelope
       4=Kudu
       5=Golden Oryx;
species/1..5/:b,d,s,r,e,o,u;

!Set of years: j
       1=Year 2015
       2=Year 2016
       3=Year 2017
       4=Year 2018
       5=Year 2019
       6=Year 2020;
year/1..6/:;

comb(species,year):f,g,x,y,p,q,t,a,v,h,w,m,k,l;

endsets


data:

b=2.5 2.5 2 1.8 2.2;!breeding rate per year;

d=0.05 0.05 0.05 0.05 0.05;!deaths due to predatores;

r=0.025 0.025 0.025 0.025 0.025;!deaths due to other risks;

e=0.15 0.15 0 0 0;!hectar space required in camps per animal per species;

u=0 0 0.6 0.45 0.47;!hactar space required per animal per species;

o=0.125 0.125 0.25 0.25 0.25;
                  !additional feeding unit required per animal per species;

s=3 7 10 4 12;!social structure;

enddata


!Objective Function;

max=@sum(species(i):(v(i,6)-@sum(year(j)|j#NE#6:a(i,j))));

!Subject to;

@for(year(j):(a(1,j)+a(2,j)+a(3,j)+a(4,j)+a(5,j))<=3000000);
            !maximum available budget per year for purchasing animals;


x(1,1)=6;   !beginning values of male animals present on farm;
x(2,1)=2;
x(3,1)=0;
x(4,1)=57;
x(5,1)=0;
```

```
y(1,1)=4;    !beginning values of female animals present on farm;
y(2,1)=1;
y(3,1)=0;
y(4,1)=46;
y(5,1)=0;

@for(species(i):
            @for(year(j):w(i,j)=((0.5075*(2014+j))-1014.5)));
                  !inflation equation;

@for(species(i):h(i,1)=(65*o(i))*365*(1+w(i,1)/100));
                  !expences regarding habitat implementation;

@for(species(i):
      @for(year(j)|j#NE#1:h(i,j)=h(i,j-1)*(1+(w(i,j)/100))));
                  !expences regarding habitat implementation;

@for(species(i):
      @for(year(j):k(i,j)=(((115000/2.5)*e(i))*(1+(w(i,j)/100)))));
                  !expences regarding inplementation of infrastructure;

@for(species(i):
      @for(year(j):m(i,j)=(((10000/2.5)*e(i))*(1+(w(i,j)/100)))));
                  !expences regarding maintenance;

@for(year(j):f(1,j)=(236.17*(2014+j)-(466829)));
@for(year(j):f(2,j)=(334.12*(2014+j))-666747);
@for(year(j):f(3,j)=(6411.18*(2014+j))-(12809845.42));
@for(year(j):f(4,j)=(147.87*(2014+j))-294382);
@for(year(j):f(5,j)=(0-27750*(2014+j))+(56183250));
                  !forecasting equations for families;

@for(year(j):g(1,j)=(102.47*(2014+j)-(196297)));
@for(year(j):g(2,j)=(402.13*(2014+j))-802229);
@for(year(j):g(3,j)=(16342.62*(2014+j))-(32657316.92));
@for(year(j):g(4,j)=(583.41*(2014+j))-1164154.83);
@for(year(j):g(5,j)=(16500*(2014+j))-(32819666.67));
                  !forecasting equations for male animals;

@for (species(i):
            @for(year(j):@gin(p(i,j))));

@for (species(i):
            @for(year(j):@gin(q(i,j))));

@for (species(i):
            @for(year(j):t(i,j)=y(i,j)*(b(i)-1)));
                  !breeding rate;

@for (species(i):
            @for(year(j)|j#NE#1:y(i,j)=(y(i,(j-1))+(0.5*t(i,j-1))+(q(i,j-
1)))*(1-d(i))*(1-r(i))));
                  !number of female animals on farm;

@for (species(i):
            @for(year(j)|j#NE#1:x(i,j)=(x(i,j-1)+(0.5*t(i,j-1))+(p(i,j-
1)))*(1-d(i))*(1-r(i))));
                  !number of male animals on farm;

@for (species(i):
            @for(year(j):l(i,j)=h(i,j)+k(i,j)+m(i,j)));
```

```
                    !total expences per animal;

@for (species(i):a(i,1)=((x(i,1)+y(i,1))*(l(i,1)))+(2000*36*(1+w(i,1)/100))
            +(0.75*p(i,1)*g(i,1))+((0.25*p(i,1))+q(i,1))*f(i,1));
                    !total expences;

@for (species(i):
            @for(year(j)|j#NE#1:a(i,j)=((t(i,j)+p(i,j)+q(i,j))*(l(i,j)))
            +(2000*36*(1+w(i,j)/100))+(0.75*p(i,j)*g(i,j))+((0.25*p(i,j))
            +q(i,j))*f(i,j)));
                    !total expences;

@for (species(i):
            @for(year(j):v(i,j)=(((y(i,j)+(0.25*x(i,j)))*f(i,j))
            +((0.75*x(i,j))*g(i,j)))));
                    !total value of animals currently on farm;

@for (species(i):
            @for(year(j):y(i,j)<=(s(i)*(x(i,j)))));
                    !relationship between male and female animals;

@for(species(i):
      @for(year(j):y(i,j)>=(0.5*x(i,j))));
                    !at least half the number of females as males;

@sum(species(i):
            @sum(year(j):(x(i,j)+y(i,j))*(e(i)+u(i))))<=1200;
                    !maximum available habitat space;
```