FREE SURFACE FLOW: EXPERIMENTS IN A HYDRAULIC CHANNEL WITH THE WATER AT REST AND A MOVING BOTTOM

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ABSTRACT

When considering the problem of water waves, induced by an obstacle, in a fluid domain of finite depth, the main assumption is generally to consider the water as an inviscid fluid, which means that the velocity profile is not sheared but constant upstream from the bottom up to the free-surface.

The objective of the experiments presented in this paper is to produce a free-surface flow over an obstacle, in a hydraulic channel, with a constant velocity profile upstream. To achieve this, we carried out an original device. The channel, with an obstacle lying on its bottom, is filled with a quantity of water at rest. A drive system moves, at a constant adjustable velocity, the bottom of the channel generating then a relative movement of the water with respect to the obstacle. Measurements of the wavelengths are performed and compared with that given by the theory.

INTRODUCTION

The objective of the experiments presented in this work is to produce a free-surface flow over an obstacle in a hydraulic channel with a constant velocity profile upstream, as assumed in the most theoretical studies. To do this, we proceeded to the development of an experimental original device. This has been designed so that the channel can be filled with a quantity of water at rest with a given depth. A drive system can move, at a constant adjustable speed, the bottom of the channel, on which is set the obstacle, generating then a relative motion of the water. In a previous study, the experiments were done in a channel where the obstacle was at rest and the water flowing. The velocity profile is thus sheared due to the fact that it necessarily vanishes at the bottom. When we move the obstacles, at a constant speed in the water otherwise at rest, we can consider, in the relative motion, the flow at uniform velocity far upstream of these obstacles.

NOMENCLATURE

$\bar{b}$	Obstacle height
$b = \bar{b}/h$	Dimensionless obstacle height
F	Froude number
h	Water depth
U	Upstream flow velocity
g	Acceleration due to gravity

Greek Letters

λ	Wavelength
Ψ	Dimensionless streamfunction

DESCRIPTION OF THE EXPERIMENTAL DEVICE

The device is a rectangular cross-section channel, 10m long, 0.30m wide and 0.50m high (photo 1).

Photo 1 View of the hydraulic channel

The channel bottom is a tarpaulin deployed along the channel. The new bottom is then moved with a drive system which consists essentially of three cylinders. The first is called the drum motor on which is wound a cable, of length greater than twice the length of the channel, closed on both sides by a combination of disk and rolling, all crossed by a rod. This drum is attached upstream of the channel walls (photo 2).

Photo 2: The drum motor

The establishment of the other two cylinders is relatively similar to the preceding one; the extremity of the sheet is wound on the first cylinder which is placed at the entrance of the channel. It goes through the bottom of it and then finds the second cylinder placed downstream on the outer walls of the channel where it is wound to its second extremity. To ensure that the sheet follows the movement of the cables, the two cylinders are connected with a fishing line of a length equivalent to twice the length of the channel (Photo 3).

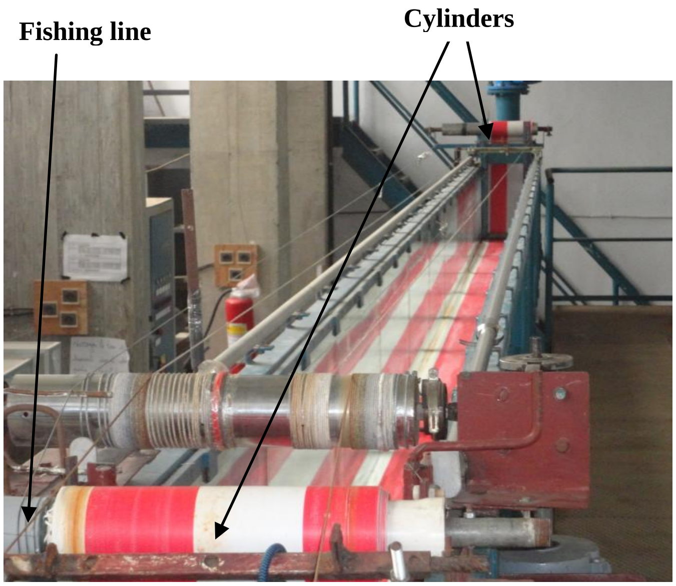


Photo 3: The driving system

The described system is connected to a drum motor attached to the outer wall of the channel. The junction of the motor with the drum is provided by a mechanical speed reducer connecting the vertical axis of the motor to the horizontal axis of the drum.

For varying the rotational speed of the motor and the direction of its movement, the motor is connected to a frequency variator which rotates the motor in one direction or the other one.

In our experiments, we used PVC obstacles. At the bottom of the channel, drive cables and the sheet are attached to the base of the obstacle (photo 4).

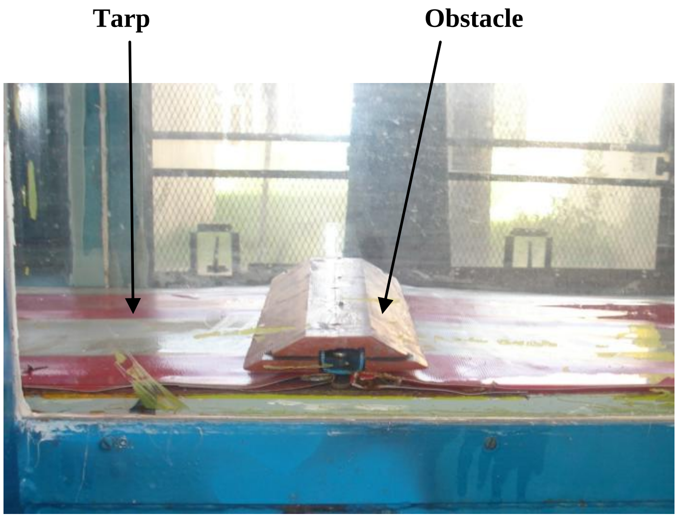


Photo 4: The obstacle and the tarp

The velocity characteristics are measured using a laser Doppler anemometer (photo 5a, 5b) which is shipped with the drive system (photo 5c).

Photo 5: Laser Doppler Anemometer

MATHEMATICAL FORMULATION

It should be remind that in most studies of free surface flows, the fluid is assumed inviscid and incompressible. The hypothesis of an irrotational flow leads to the resolution of the Laplace equation for either the streamfunction or the velocity potential.

Let us recall the case of the streamfunction formulation. We consider the steady 2-D flow over an obstacle of maximum height $\bar{b}$ lying on the bottom of a channel.

Far upstream, the flow is uniform with constant velocity $\vec{U}$ and depth h.

The fluid being incompressible, one can introduce a stream function $\bar{\Psi}(\bar{x}, \bar{y})$ such as:

$$(\bar{u}, \bar{v}) = \left(\frac{\partial \bar{\Psi}}{\partial \bar{y}}, -\frac{\partial \bar{\Psi}}{\partial \bar{x}}\right) \qquad (1)$$

where $\bar{u}$ and $\bar{v}$ represent respectively the horizontal component and the vertical component of the fluid velocity.

The flow being uniform far upstream of the obstacle, the theorem of Lagrange ensures us its irrotationality everywhere in the fluid; thus:

$$\Delta \bar{\Psi} = 0 \qquad (2)$$

The fluid being inviscid, it slips on the bottom of the channel which is then a streamline taken as:

$$\bar{\Psi}(\bar{x}, \bar{y}_b) = 0 \qquad (3)$$

where $\bar{y}_b(\bar{x})$ denotes the equation of the bottom.

The free-surface $\bar{y}_0(x)$ is also a streamline. The value of the streamfunction, on it, is determined by conservation of the flow discharge.

$$\bar{\Psi}(\bar{x}, \bar{y}_0) = Uh \qquad (4)$$

In addition to this kinematic condition, we must express the conservation of the energy of the fluid on this free-surface where the pressure is constant (atmospheric pressure). The Bernoulli's equation gives:

$$g\,\bar{y}_0 + ½\,(V^2 - U^2) = gh \qquad (5)$$

where g is the acceleration due to gravity and V the free-surface velocity.

We note that this dynamic condition is non-linear and that there is no radiation condition downstream for the subcritical regime, except that the velocity of the flow and the height of the free-surface must be bounded.

The problem may be non-dimensionalized with respect to the velocity U and the height h by introducing the following variables:

$(x, y, b, y_b, y_0) = (\bar{x}, \bar{y}, \bar{b}, \bar{y}_b, \bar{y}_0)/h$; $(u, v) = (\bar{u}, \bar{v})/U$; $\Psi = \bar{\Psi}/Uh$.

The equations (2) to (5) are then written as follows:

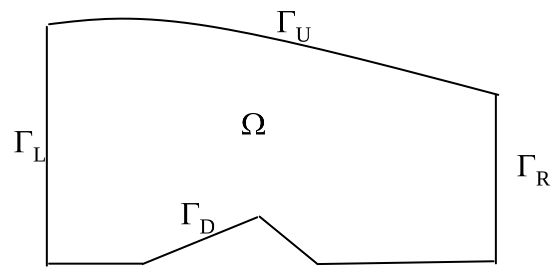
$$\Delta\Psi = 0 \qquad (6.1)$$

$$\Psi(x, y_b) = 0 \qquad (6.2)$$

$$y_0 + ½F^2\left[\left(\frac{\partial\Psi}{\partial n}\right)_{y_0} - 1\right] = 1 \qquad (6.3)$$

$$\Psi(x, y_0) = 1 \qquad (7)$$

where $F = U/\sqrt{gh}$ is the Froude number and $\vec{n}$ the outward normal vector.



In the vertical plane (x,y), the domain Ω is bounded in the lower and upper side by respectively the bottom Γ_D, the unknown free-surface Γ_U and the two verticals Γ_L and Γ_R respectively upstream and downstream of the obstacle. Taking into account the physical conditions above-mentioned, the problem consists on finding a streamfunction Ψ(x,y) which satisfies the Laplace equation and which verifies the boundary conditions (6.2) and (6.3) and that of uniformity of the velocity upstream:

$$\Psi = y \quad \text{on } \Gamma_L \qquad (6.4)$$

$$\partial\Psi/\partial x = 0 \quad \text{on } \Gamma_L \qquad (6.5)$$

The figure 1 shows an example of a solution obtained with this type of boundary condition.

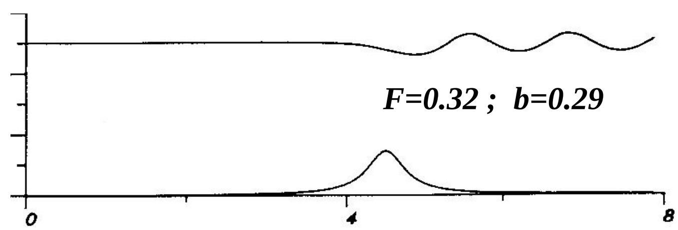


Figure 1: Theoretical free-surface computed

When we consider an open-channel flow, the velocity profile, in any section, is sheared because of the bottom condition; the boundary conditions (6.4) and (6.5) cannot thus be satisfied.

The experiments we conducted and which are described below allow us to satisfy this condition of uniformity of the velocity profile upstream of the obstacle.

THE EXPERIMENTS

The hydraulic channel is first filled up to a given water depth. To choose the desired velocity, we fix the frequency (speed) to drive the obstacle. By triggering the switch, the motor rotates and carries with it the drum. Cables wound on the drum pass through the pulleys, carry the obstacle that moves with the tarp along the channel, then return to the pulleys located at the downstream part of the channel.

Velocity measurements of the “upstream” flow were taken at the centerline of the cross section of the channel using a laser Doppler anemometer. The figure 2 shows that the experimental velocity profile is not sheared and is nearly uniform.

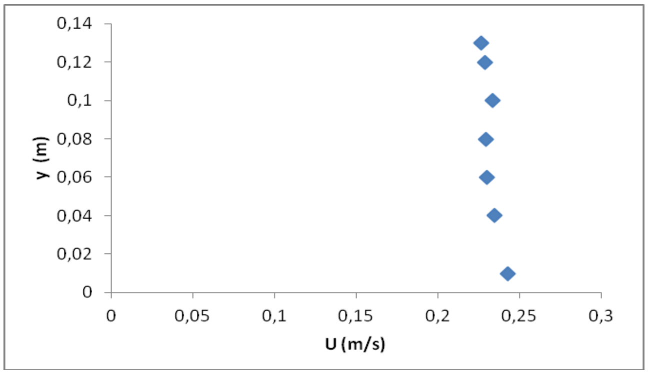


Figure 2: "Upstream" measured velocity profile

Experiments carried out in channels show that, according to the value of the Froude number, the free-surface looks like:
- a unique elevation over the obstacle then gradual going back to the undisturbed level (y = 1) for a supercritical flow upstream (photo 6) .

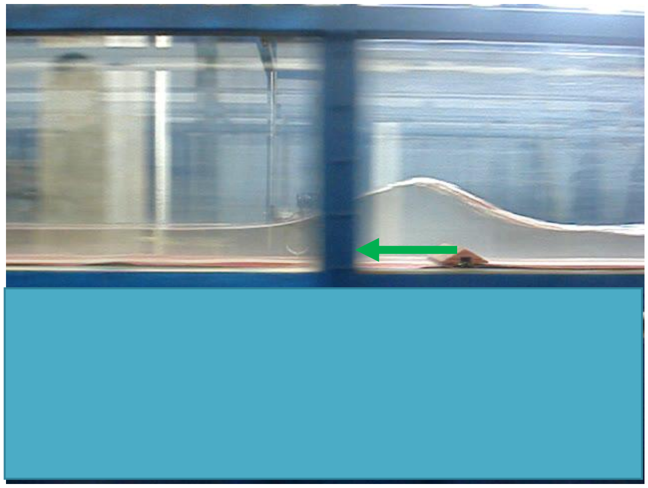


Photo 6

- a depression in the vicinity of the obstacle followed by a supercritical flow downstream of the obstacle (photo 7).

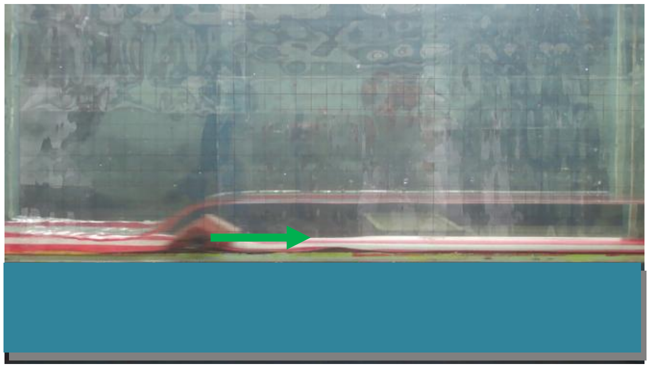


Photo 7

- a depression in the vicinity of the obstacle followed or not by surface waves for a subcritical flow (Photo 8).

Photo 8

- a depression followed by a hydraulic jump (Photo 9).

Photo 9

In most studies dealing with the propagation of surface waves in a hydraulic channel, the dimensionless wavelength considered is generally that deduced from the linear theory:

$$\lambda/h = 2\pi F^2 \qquad (8)$$

$F = \dfrac{U}{\sqrt{gh}}$ is the Froude number where U is the mean velocity.

The experimental wavelength is obtained by measuring, on the photos, the distance between two consecutive peaks. Some of these results are shown in the figure 3 with non-dimensional variables with respect to the undisturbed water depth h which is not kept constant.

It should be reminded that to obtain surface waves depends not only on the value of the Froude number but also on the relative height of the obstacle compared to the water depth.

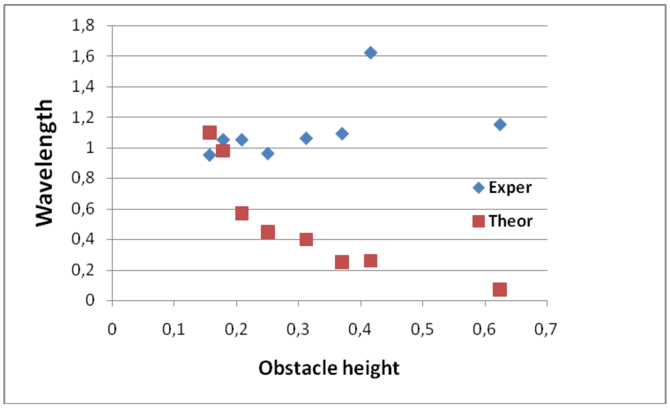


Figure 3 Experimental and theoretical wavelength

CONCLUSION

The device we developed is primarily designed to allow comparison between theoretical results and experimental measurements in similar assumptions. Indeed, while it is impossible to conduct experiments with an inviscid fluid, the condition of nullity of the velocity at the bottom of the channel can be bypassed.

For example, we have measured experimental wavelengths and compared them to the theoretical ones issued from the linear theory which is based on the smallness of the obstacle height relative to the water depth in the channel. The conclusion we reached is that the dimensionless height of the obstacle must be less than 0.2 to estimate reasonably well the experimental wavelength by the linear theory results.

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