

THERMODYNAMIC PROPERTIES OF NANODROPS. MOLECULAR DYNAMICS SIMULATIONS

G.V. Kharlamov^{1*}, A.A. Onischuk², S.V. Vosel², P.A. Purtov²

*Author for correspondence

¹Applied and Theoretical Physics Department,
Novosibirsk State Technical University,
Novosibirsk, 630092,
Russia,

E-mail: g_kharlamov@ngs.ru

²Institute of Chemical Kinetics and Combustion of RAS,
Novosibirsk, 630090,
Russia

ABSTRACT

The results of the structural and thermodynamic properties simulations of the nanodrops, which emerge on the first stage of nanoparticle formation process by supersaturated vapor condensation, are presented. The density profiles, the Irving-Kirkwood pressure tensors, the chemical potentials of the systems, the equimolar radii of the drops and the radii of tension, the mechanical and thermodynamic surface tensions have been calculated.

It is shown that both the mechanical and thermodynamic surface tensions decrease with the decrease of the equimolar radius of the drop, and reach zero at the same R_0 depending on temperature. The radii of tension also reach zero. With the further equimolar radius decrease the surface tension becomes negative, though the drop holds stable in the conditions of the numerical experiment. It means that such droplet is metastable. Apparently, it testifies that the notion of surface tension which is used in macroscopic theory can not be applied to such small drops. The dependence of the ratio of the surface tension of the drop to the surface tension of the flat surface liquid-vapor on the ratio of the equimolar radius of the drop to R_0 is a universal function.

INTRODUCTION

An effective method of nanoparticles production is condensation of the particles from supersaturated vapor. For this purpose some solid material (metal, semiconductor or organic matter) is being evaporated by heating it beyond the melting temperature. Then supersaturated vapor is obtained as a result of quick cooling. The condensation process of the small drops begins in this system. The nanoparticles of various compositions and wide range of sizes can be produced by this

method. The first stage of the nanoparticles formation is nucleation – that is the critical nuclei generation. The nanoparticles grow later from the nuclei by joining atoms from supersaturated vapor. The nucleation is not a unique phenomenon and it takes place in gases, solutions, melts and even in solids when a new phase is being formed inside the host phase. There is the so-called Classical Nucleation Theory (CNT), which enables to calculate the nucleation rates depending on the process conditions [1, 2]. This theory is a statistical thermodynamic one really, because it uses the thermodynamic parameters of clusters and small particles, which are the critical nuclei. The critical nuclei at the moment of their occurrence are the liquid clusters or nanodrops and only later they become the solid nanoparticles as a result of cooling. The surface tension of the critical nucleus is an important parameter in the CNT, because it defines the work of the critical nucleus formation [1, 2]. Though this theory often leads to unsatisfactory results, it and its modifications are still being used for interpretation of experimental data [3, 4].

Thermodynamic description of nanodrops is very important physical problem. There is a problem of applicability of thermodynamic parameters to small drop and cluster as these parameters are the macroscopic thermodynamic notions, but nanodrop is a microscopic system. One of the causes of discrepancy between the CNT and experiment can be that the dependence of the surface tension on the critical nucleus radius is ignored often. However J. Gibbs pointed out, that the surface tension of a droplet (σ) must decrease with the decrease of droplet tension surface radius (R_s), so that at $R_s = 0$ the surface tension be also equal to zero ($\sigma = 0$) [5]. At the same time the equimolar radius of the droplet (R_c) is not equal to

zero. Consequently the notion of the surface tension can not be applied to such droplets.

NOMENCLATURE

σ	[N/m]	Surface tension
σ_∞	[N/m]	Surface tension of a flat interface of liquid - vapor
R_s	[m]	Droplet tension surface radius
R_e	[m]	Equimolar radius of a droplet
U	[J]	Internal energy of thermodynamic system
V_l	[m ³]	Volume of a liquid drop
V_v	[m ³]	Volume of vapor
A	[m ²]	Surface area of liquid
T	[K]	Temperature
S	[J/K]	Entropy of a system
P_l	[Pa]	Pressure in a drop
P_v	[Pa]	Pressure in vapor
μ	[J/kg]	Chemical potential of a system
m	[kg]	Mass of a system
δ	[m]	The Tolman length
r	[m]	Distance from the centre of a drop
$P(r)$	[Pa]	Pressure tensor
$P_N(r)$	[Pa]	Pressure tensor normal component
$P_T(r)$	[Pa]	Pressure tensor tangential component
e_r, e_θ, e_ϕ	[-]	Unit vectors corresponding to spherical coordinate system
r, θ, ϕ	[m], [rad], [rad]	Spherical coordinates
r_{12}	[m]	Distance between molecules 1 and 2
\mathbf{r}_{12}	[m]	Vector directed from molecule 1 to molecule 2
$n(r)$	[m ⁻³]	Concentration of molecules
$\rho(r)$	[kg/m ³]	Density
ρ_l, ρ_g	[kg/m ³]	Densities of liquid and vapor
$\rho^{(2)}(r_{12}, r - \alpha r_{12})$	[m ⁻⁶]	Pair correlation function of molecules 1 and 2
ε	[J]	Lennard-Jones potential parameter
r_0	[m]	Lennard-Jones potential parameter
m_0	[kg]	molecule mass
μ_l	[J/kg]	Chemical potential of liquid
μ_g	[J/kg]	Chemical potential of vapor
μ_0	[J/kg]	Chemical potential of classical ideal gas
R_0	[m]	Equimolar radius of a droplet at which the surface tension becomes equal to zero

Usually reduced variables are used in computational simulations. In that case the basic units are r_0 , $r_0(\varepsilon/m_0)^{-1/2}$ and m_0 for distance, time and mass respectively.

THERMODYNAMICS OF DROPS

The surface tension concept is proved by J. Gibbs [5] in thermodynamics and it is conventional. This concept provides introduction of additional term σdA (σ is the surface tension, dA is a change of the surface area of a liquid) in fundamental

thermodynamic equations. Such equation for the internal energy of the system consisting of a liquid drop and vapor surrounding it, has the usual form

$$dU = T dS - P_l dV_l - P_v dV_v + \sigma dA + \mu dm. \quad (1)$$

Here dU , dS and dm are changes of the internal energy, of entropy and of mass of all system, dV_l , dV_v and dA are changes of volumes of the liquid drop, of the vapor and of the drop surface area, P_l , P_v and T are pressures in the drop and in the vapor and system temperature, μ is chemical potential of the system.

Thus, such surface tension σ determines additional energy to the thermodynamic potentials, related to change of a surface area of a liquid and it is called the thermodynamic surface tension. In addition to surface tension J. Gibbs introduced a dividing surface which is termed as a tension surface. It is a mathematical surface of a zero thickness to which the forces of surface tension are applied. At the thermodynamic description of systems with flat interfaces of liquid – vapor the position of the tension surface has no importance. For curved interfaces there is a problem of a choice of the tension surface.

For the system consisting of a liquid drop surrounded by natural vapor, the value of surface tension depends on the choice of the tension surface position. J. Gibbs has suggested choosing the tension surface so that the surface tension had the minimum value. Thus the model of Gibbs assumes that in sphere with radius of the tension surface there is a medium with properties of a bulk liquid which is called sometimes comparison phase. The chemical potential of this phase coincides with the chemical potential of the vapor surrounding the sphere. Thus, pressure, density and temperature in the drop are equal to the values of pressure, density and temperature in the bulk liquid at the same chemical potential. The tension surface of the drop generally does not coincide with equimolar surface, therefore the part of mass of the system does not belong to any of two phases and it is the part of substance adsorbed on the interface. The difference between the tension surface radius and the equimolar surface radius is determined by the parameter $\delta = R_e - R_s$ which is called the Tolman length.

The difference of the pressures between the comparison phase and the surrounding vapor obeys the Laplace formula

$$P_l - P_g = 2\sigma/R_s, \quad (2)$$

where P_l and P_g are the pressures of the liquid in the comparison phase and the vapor, respectively. The choice of the pressure and density of the liquid in the comparison phase is arbitrary. According to this choice values of σ , R_s , and also the equimolar radius R_e will be changed.

Tolman [6] has derived dependence of the surface tension on the radius of the tension surface in the form

$$\ln \frac{\sigma}{\sigma_\infty} = \int_\infty^{R_s} \frac{\left(\frac{2\delta}{r^2} \right) \left[1 + \left(\frac{\delta}{r} \right) + \frac{1}{3} \left(\frac{\delta^2}{r^2} \right) \right]}{1 + \left(\frac{2\delta}{r} \right) \left[1 + \left(\frac{\delta}{r} \right) + \frac{1}{3} \left(\frac{\delta^2}{r^2} \right) \right]} dr. \quad (3)$$

Here σ_∞ is the surface tension of a flat interface of liquid - vapor. If we assume that $\delta = \delta_\infty = const$ and $\delta/r \ll 1$ we obtain the approximate formula which is true for big drops

$$\sigma/\sigma_\infty = 1/(1 + 2\delta/R_s). \quad (4)$$

THE MECHANICAL SURFACE TENSION

On the other hand, there is a mechanical definition of surface tension [7, 8] related to the change of continuous profiles of density and pressure by the step ones. The pressure tensor of drop has a spherical symmetry

$$P(r) = e_r e_r P_N(r) + (e_\theta e_\theta + e_\phi e_\phi) P_T(r) \quad (5)$$

The mechanical surface tension is expressed through the pressure tensor [7, 8]:

$$\sigma = \int_0^\infty (r/R_s)^2 (P_N(r) - P_T(r)) dr. \quad (6)$$

Since in this formula R_s is unknown beforehand, it is necessary to have one more equation determining R_s . In papers [8, 9] it is suggested to use the equation

$$\sigma = \int_0^\infty (R_s/r) (P_N(r) - P_T(r)) dr, \quad (7)$$

which is derived from the equations of Buff based on the mechanical equilibrium conditions of the system of drop - vapor. Using the equations (6) and (7), it is possible to calculate R_s and σ , if we know the profiles of the pressure tensor components.

In advance it is not obvious that the surface tension calculated by formulae (6) and (7), will coincide with the thermodynamic surface tension used in (1). Here there is a problem of relation of the surface tension determined by formulae (6) and (7) to Gibbs model of surface tension.

METHODS OF SIMULATION

The molecular dynamics method has been long used to calculate the surface tension for the flat division surface between liquid and gas [10]. It was also used many times to calculate the surface tension of the liquid drops [9, 11 - 15]. From this point of view the most interesting papers are [9, 11], in which the basic methods of molecular dynamics calculations

of small drops surface tension are developed. At the same time all these calculations have been done in a narrow range of system parameters and give no possibility of precise determination of the dependence of the drops surface tension on their radii. Besides, these papers do not concern the problem of applicability of the surface tension concept to the droplets of small radii.

In this work, the thermodynamic properties of droplets were calculated by the molecular dynamics method. The calculations were made for the system containing 60-4500 molecules in a cubic cell with periodic boundary conditions. Interaction between molecules was specified by the Lennard-Jones potential

$$u(r_{12}) = 4\varepsilon \left[\left(\frac{r_0}{r_{12}} \right)^{12} - \left(\frac{r_0}{r_{12}} \right)^6 \right], \text{ if } r_{12} \leq 4.5r_0$$

$$u(r_{12}) = a(r_{12} - 5r_0)^2 + b(r_{12} - 5r_0)^3, \text{ if } 4.5r_0 \leq r_{12} \leq 5r_0$$

$$u(r_{12}) = 0, \text{ if } r_{12} \geq 5r_0$$

(8)

A special procedure was used to obtain an equilibrium system composed of a liquid drop in the center of the cell and vapor occupying the remaining space. We call such drop obtained in numerical experiment real. The size of the drop depended on the number of particles in the cell and the mean density of the system. The Irving-Kirkwood pressure tensor was calculated by formulae of statistical mechanics [8]

$$P_N(r) = kTn(r) - \frac{1}{2} \int d\mathbf{r}_{12} \frac{(\mathbf{r}_{12} \cdot \mathbf{e}_r)^2}{r_{12}} u'(r_{12}) \times \int_0^1 d\alpha \rho^{(2)}(r_{12}, |\mathbf{r} - \alpha \mathbf{r}_{12}|) \quad (9)$$

$$P_T(r) = kTn(r) - \frac{1}{2} \int d\mathbf{r}_{12} \frac{(\mathbf{r}_{12} \cdot \mathbf{e}_\phi)^2}{r_{12}} u'(r_{12}) \times \int_0^1 d\alpha \rho^{(2)}(r_{12}, |\mathbf{r} - \alpha \mathbf{r}_{12}|)$$

Density profiles, chemical potential of the system, equimolar radii and surface tension radii of drops, mechanical and thermodynamic surface tensions and Tolman length δ were calculated.

Reduced variables were used: distance - $r = r^*/r_0$, temperature - $T = kT^*/\varepsilon$, energy - $U = U^*/\varepsilon$, density - $\rho = \rho^* r_0^3$, time - $t = t^*/r_0 (\varepsilon/m_0)^{1/2}$, pressure - $p = p^* r_0^3/\varepsilon$, surface tension - $\sigma = \sigma^* r_0^2/\varepsilon$, and chemical potential - $\mu = \mu^* m_0/\varepsilon$. Here ε and r_0 are the Lennard-Jones potential

parameters, m_0 is the molecule mass. Variables labeled by an asterisk are dimensional.

RESULTS OF THE SIMULATIONS

Figure 1 presents typical profiles of drop density $\rho(r)$. The equimolar radii of the drops were calculated according to these dependences by the formula [9]

$$R_e^3 = \frac{1}{\rho_g - \rho_l} \int_0^\infty r^3 \frac{d\rho(r)}{dr} dr. \quad (10)$$

Here ρ_l, ρ_g are the comparison phase and gas densities respectively.

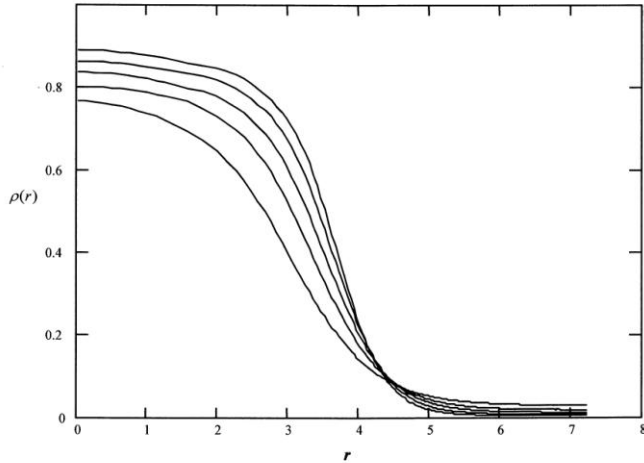


Figure 1 The density profiles for the system, containing $N = 200$ molecules in the cell at the temperatures $T = 0.65; 0.7; 0.75; 0.8; 0.85$ (from the top)

Figure 2 presents typical profiles of the pressure tensor components $P_N(r)$ and $P_T(r)$ used for calculations of the tension surface radius R_s and the surface tension σ .

Figure 3 presents the nanodrops mechanical surface tension dependences on the equimolar radius R_e for various temperatures. One can see, that the mechanical surface tension decreases greatly with the R_e decrease and becomes equal to zero at a certain $R_e = R_0$, which depends on temperature. The mechanical surface tension approaches the value of the surface tension of liquid – vapor flat surface with R_e increase.

To calculate surface tension of drops according to J. Gibbs definition, first, one should know the dependence of chemical potential of bulk liquid phase on pressure, and, second, chemical potential of vapor. To solve the first problem, we have made special series of bulk liquid phase chemical potential calculations by molecular dynamics method at the temperature $T = 0.65$. First, we obtained the equilibrium system consisting of flat liquid phase layer surrounded by vapor from two sides. The layer thickness exceeded the molecules interaction radius three times. The computations have shown that vapor far from the flat separating surface is an ideal gas,

i.e. the ideal gas equations $p = nT$ (n is vapor concentration) holds accurate to 1%. Chemical potential of vapor, and, therefore, liquid layer was calculated by formula $\mu_0 = -(5/2)T \ln(T) + T \ln(p)$ suitable for classical ideal gas.

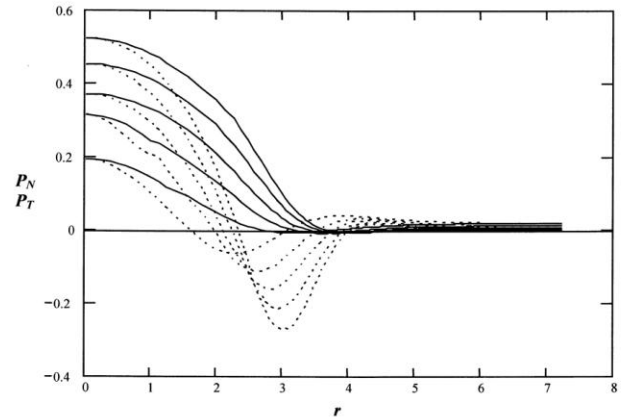


Figure 2 The profiles of the pressure tensor components for system, containing $N = 200$ molecules in cell at temperatures $T = 0.65; 0.7; 0.75; 0.8; 0.85$ (from the top). (solid lines are P_N , dashed lines are P_T)

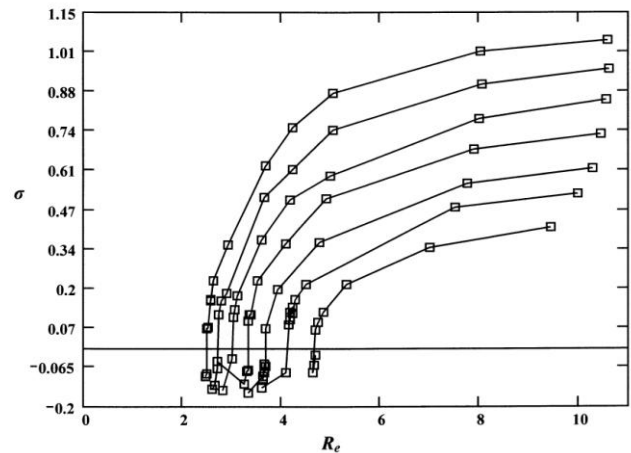


Figure 3 The drop mechanical surface tension dependences (σ) on the equimolar radius (R_e) for various temperatures ($T = 0.65; 0.7; 0.75; 0.8; 0.85; 0.9; 0.95$ from the top).

Then calculations were made of a similar system with two repulsing walls on the opposite sides of the cell that served as plungers compressing the flat liquid layer. Reducing the cell volume, we obtained liquid phases at different pressures. Thus the equation of liquid state at a given temperature has been obtained. Chemical potential of the liquid phase was calculated by the formula

$$\mu_l = \mu_0 + \int_{P_0}^{P_l} dp / \rho. \quad (11)$$

Approximation curve of dependences P_l and ρ_l on μ_l was established.

Chemical potential of vapor has also been calculated by formula (11) with μ_l replaced by μ_g and P_l – by P_g . Equation of vapor state has been obtained from molecular dynamics data for equilibrium systems containing liquid drop surrounded by its own vapor, i.e., from the calculations of surface tension of drops. Vapor pressure and density were taken far away from the drop at the distance exceeding the interaction radius of molecules. After estimation of the chemical potential of the vapor around the drop we equated it to the comparison phase chemical potential and got P_l and ρ_l of the comparison phase. We used approximation curve of the dependence of these values on liquid chemical potential. Further, using formula (2) and the first equation (6), we calculated σ and R_s for thermodynamic surface tension.

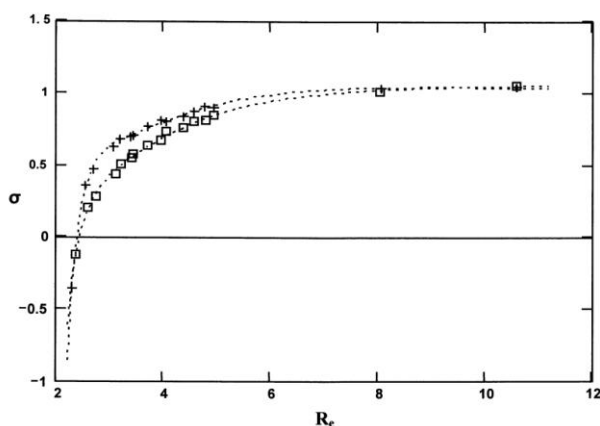


Figure 4 The dependence of mechanical (\square) and thermodynamic (+) surface tensions on the equimolar radius R_e at the temperature $T = 0.65$

Figure 4 presents the mechanical and thermodynamic surface tension dependences on the drop equimolar radius. Significant decrease of both mechanical and thermodynamic surface tension can be observed with R_e decrease. At a certain equimolar radius R_0 both surface tensions go to zero. This radius corresponds to the drop, containing about 50 molecules. The radius of the surface of tension R_s also goes to zero. With the further equimolar radius decrease the surface tension becomes negative, though the drop holds stable in the conditions of the numerical experiment. Figure 4 shows that the thermodynamic surface tension coincides with the mechanical one for relatively big drops containing 2000 and more molecules. For drops containing smaller quantity of molecules the thermodynamic surface tension exceeds the mechanical one up to their disappearance at $R_e = R_0$.

DISCUSSION OF RESULTS

It has been established in our papers [16-18] that the dependence of the nanodrops mechanical surface tension on the

equimolar radius is a universal function relative to the temperature, at which the numerical experiment is conducted (see Figure 5). It has been found that approximation of this dependence by the polynomial of the ratio R_0/R_e can be expressed as follows:

$$\sigma(R_e) = \sigma_\infty \left[1 - 0.519 \cdot R_0/R_e + 0.426 \cdot (R_0/R_e)^2 - 0.907 \cdot (R_0/R_e)^3 \right] \quad (12)$$

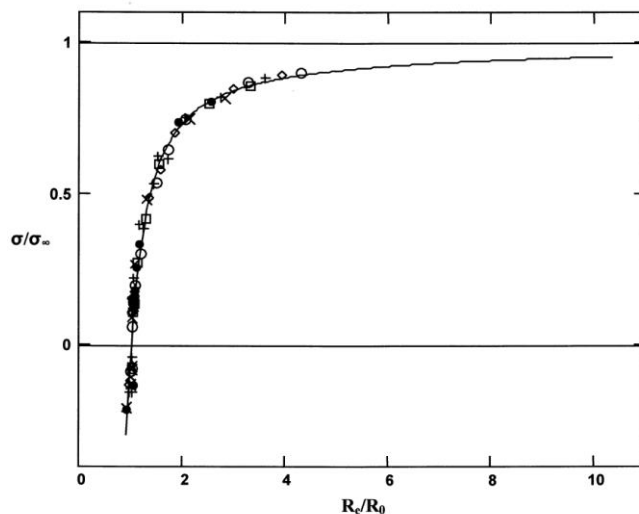


Figure 5 The dependence of the ratio of the mechanical surface tension of the drop to the surface tension of the flat surface liquid-vapour (σ/σ_∞) on the ratio of the equimolar radius of the drop to R_0 for various temperatures ($T = 0.65(\circ)$; $0.7(\diamond)$; $0.75(+)$; $0.8(\square)$; $0.85(\times)$; $0.9(\bullet)$; $0.95(+)$). Solid line is the dependence (12).

In addition, the drop equimolar radius R_0 , at which $\sigma = 0$, greatly depends on the temperature and becomes infinite in the point is equal 1.199 (see Figure 6). The approximation of the dependence of R_0 on the temperature can be expressed by the fit

$$R_0 = 0.768 + \frac{0.963}{1.199 - T} \quad (13)$$

The surface tension of the flat dividing surface liquid – vapor σ_∞ also depends on the temperature and becomes zero in the same point.

The thermodynamic surface tension dependence on the equimolar radius R_e is the same in its quality as that for mechanical surface tension, though it differs a little in its quantity. For relatively large drops the surface tension approaches the value of the surface tension of the flat dividing surface liquid – vapor. For quite small drops the surface tension

becomes zero or even negative. But the drop does not evaporate at that time. Though the drop will evaporate after all, if we increase the number of molecules in the simulation cell at the same pressure, because that leads to the increase of fluctuations of the force, acting on the nanodrop from the vapor. That's why the state of a nanodrop with a negative surface tension can be considered as a metastable one in respect of small fluctuations.

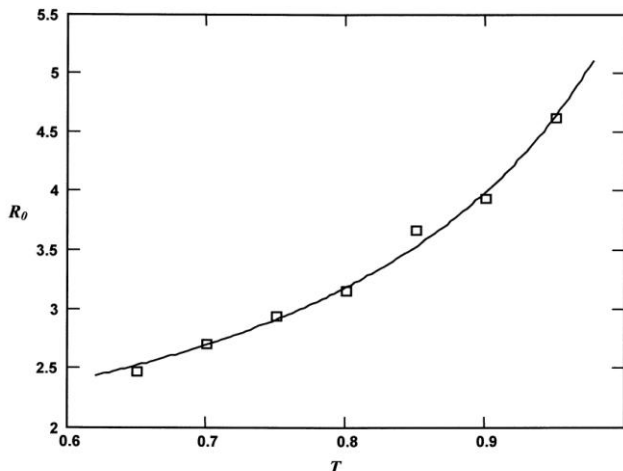


Figure 6 The dependence of R_0 on the temperature. Solid line is the dependence (13)

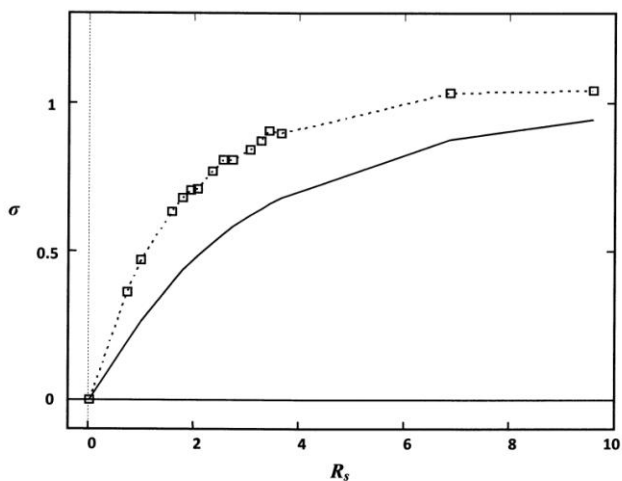


Figure 7 The thermodynamic surface tension (\square) and surface tension calculated by Tolman formula (3) (solid line) as a function of the R_s at the temperature $T = 0.65$.

On the other hand, the negative surface tension and the negative radius of tension surface R_s have no physical meaning and become formal mathematical parameters of the model. Apparently, it testifies that the notion of surface tension which is used in macroscopic theory can not be applied to such small drops. The limit of applicability of the surface tension concept to the droplets changes from about 50 molecules at $T = 0.65$ to about 300 molecules at $T = 0.95$.

Figure 7 shows disagreement between the thermodynamic surface tension calculated by molecular dynamics method and

surface tension calculated by Tolman formula (3). It is possibly that the Tolman equation does not adequate for the small droplets.

CONCLUSION

Thus, it has been established that both mechanical and thermodynamic surface tensions decrease greatly with the decrease of tension surface radius R_s and become equal to zero at a certain $R_e = R_0$. A universal dependence of mechanical surface tension is observed, i.e., the dependence of σ on the temperature is expressed by the dependence of parameters σ_∞ and R_0 on the temperature. The value of the thermodynamic surface tension coincides with value of the mechanical surface tension for relatively large nanodrops (> 2000 molecules) and exceeds it for relatively small nanodrops (< 2000 molecules). The notion of surface tension which is used in macroscopic theory can not be applied to small droplets with equimolar radius less then R_0 .

ACKNOWLEDGMENTS

The authors are grateful to the Russian Foundation for Basic Research (RFBR) (project № 11-08-01204a) for financial support.

REFERENCES

- [1] Ya. I. Frenkel. Kinetic Theory of Liquids. Clarendon Press, Oxford, 1946
- [2] Anisimov M.P. Nucleation: theory and experiment, *Russian Chemical Reviews*, Vol. 72, 2003, pp. 591-629
- [3] Iland K., Wölk J., Strey R., Kashchiev D. Argon nucleation in a cryogenic nucleation pulse chamber, *The Journal of Chemical Physics*, Vol. 127, 2007, 154506, 11 p.
- [4] Wedekind J., Wölk J., Reguera D., Strey R. Nucleation rate isotherms of argon from molecular dynamics simulations, *The Journal of Chemical Physics*, Vol.127, 2007, 154515, 11 p.
- [5] J. W. Gibbs. Thermodynamics. The Collected Works Vol. I. Longmans, Green. New York, 1928
- [6] Tolman R.C. The Effect of Droplet Size on Surface Tension, *The Journal of Chemical Physics*, Vol. 17, 1949, pp. 333 – 337
- [7] S. Ono, S. Kondo, Molecular Theory of Surface Tension in Liquids, Springer – Verlag, Berlin – Göttingen – Heidelberg, 1960
- [8] J.S. Rowlinson, B. Widom, Molecular theory of capillarity. Clarendon Press, Oxford, 1982
- [9] Thompson S.M., Gubbins K.E., Walton J.P.R.B., Chantry R.A.R., Rowlinson J.S. A molecular dynamics study of liquid drops, *The Journal of Chemical Physics*, Vol. 81, 1984, pp. 530-542
- [10] V.G. Baidakov, S.P. Protsenko, Z.R. Kozlova, and G.G. Chernykh. Metastable extension of the liquid-vapor phase equilibrium curve and surface tension. *The Journal of Chemical Physics*, Vol. 126, 2007, 214505.
- [11] Brodskaya E.N., Rusanov A.I. Molecular dynamics study of small systems, *Colloidnyi zhurnal*, Vol. 39, 1977, pp. 636 – 656. (in Russian)
- [12] Powles J.G., Fowler R.F., Evans W.A.B. A new method for computing surface tension using a drop of liquid, *Chemical Physics Letters*, V. 96, 1983, pp. 289 -292.
- [13] El Bardouni H., Mareschal M., Lovett R., Baus M. Computer simulation study of the local pressure in a spherical liquid – vapor interface, *The Journal of Chemical Physics*, Vol. 113, 2000, pp. 9804 - 9809

- [14] Arcidiacono S., Poulikakos D., Ventikos Y. Oscillatory behavior of nanodroplets, *Physical Review E*, Vol. 70, 2004, 011505, 7 p.
- [15] Rein ten Wolde P., Frenkel D. Computer simulation study of gas – liquid nucleation in a Lennard-Jones system, *The Journal of Chemical Physics*, Vol. 109, 1998, pp. 9901-9918
- [16] Kharlamov G.V., Onischuk A.A., Purtov P.A., Voxel S.V., Bolesta A.V. Computation of surface tension of small droplets by the molecular dynamics method, *Atmospheric and Oceanic Optics*, Vol. 21, 2008, pp. 679-683 (in Russian)
- [17] G.V. Kharlamov, A.A. Onischuk, P.A. Purtov, S.V. Voxel, A.V. Bolesta. The Problem of Surface Tension Definition of Nanodrops. *e-Journal of Surface Science and Nanotechnology*, Vol. 8, 2010, pp. 197 – 202
- [18] G.V. Kharlamov, A.A. Onischuk, S.V. Voxel, P.A. Purtov. Molecular Dynamics Calculations of Small Drops Surface Tension. *Colloids and Surfaces A: Physicochemical and Engineering Aspects*, Vol. 379, 2011, pp. 10 - 13