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A COMPARATIVE STUDY OF BUBBLE RISE PHENOMENA IN WATER AND LOW CONCENTRATION POLYMER SOLUTIONS

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ABSTRACT

Bubbles are used in polymer, metallurgy, biotechnology and especially in process industries for improving the heat and mass transfer from a dispersed gaseous phase to viscous liquid phase. A comparative study of the bubble rise characteristics in water and a few selected low concentration polymer solutions is presented in this paper. The characteristics, namely, the bubble velocity, the bubble trajectory, the bubble volume and the drag relationship are investigated. The experiments were conducted in 125 mm cylindrical column at liquid heights of 1 m, 1.2 m, 1.4 m and 1.6 m by introducing different bubble volumes (from 0.1 mL to 5.0 mL) corresponding to each height. The bubble rise velocity and trajectory were measured using a combination of non-intrusive (high speed photographic) method and digital image processing. The parameters that significantly affect the rise of air bubble are identified. The effect of different bubble volumes and liquid heights on the bubble rise velocity and bubble trajectory are analysed and discussed. The correlation between the Reynolds number and the drag coefficient is developed and presented. The results of this study are compared with the results of other analytical and experimental studies available in the literature.

INTRODUCTION

The occurrence of bubbles in natural and industrial processes has encouraged a lot of studies for many years. The knowledge of hydrodynamics behaviour of a single air bubble in Newtonian and non-Newtonian liquids is essential for designing and operating many bioprocesses in pharmaceutical, environmental, and other industries as diverse as geophysics, chemistry and physics [1].

Non-Newtonian fluids have a unique property viscosity that changes with varying shear rate which makes them very complicated in characteristics. Therefore, a precise calculation method of air bubble hydrodynamics in non-Newtonian liquids is necessary. In the design of gas-liquid systems for the process industries where the process fluid has non Newtonian characteristics, it is essential to predict the hydrodynamic behaviour of air bubbles in these types of liquids. The most significant hydrodynamic characteristics of air bubbles are the bubble rise velocity or terminal velocity, trajectory and the drag coefficient. The drag coefficient correlates the drag force exerted on a moving air bubble to its terminal velocity and projected surface area. The terminal velocity of an air bubble is termed as the velocity attained at steady state conditions where all applied forces are balanced. The bubble rise velocity and drag coefficient of an air bubble are mainly depended on the liquid and bubble properties.

The hydrodynamics of the bubble characteristics in a gas-liquid system are still not totally understood. Many researchers have undertaken various studies to predict the actual phenomena of the bubble rise in a column of liquid (Newtonian and non-Newtonian) since last century [1-17]. The relationship between the terminal velocity and volume for larger gas bubbles were investigated by Dewsbury et. al. [2] in non-Newtonian power-law fluids. Tsuge and Hibino [3] reported that the trajectories of rising spherical and ellipsoidal gas bubbles at higher Reynolds numbers were identical. Dewsbury et. al. [4] determined experimentally that the drag coefficient for a rising solid sphere in non-Newtonian pseudo plastic liquids significantly affected by its trajectory and the relationship between the drag coefficient and sphere diameter was non-linear at Re< 135, linear at very low Re , in the Stoke regime and at Re > 135. Miyahara and Yamanaka [5] reported for the case of highly viscous non-Newtonian liquid that the drag coefficient deviated from the Hadamard –Rybczynski type equation if the Reynolds number increased. On the other hand, Dhole et. al [6] investigated that the drag co-efficient always increased with increase in power law index for all values of the Reynolds number.

There have been limited studies available in the literature on comparison of single bubble rise characteristic between water and low concentration polymer. More research and in–depth analysis on bubble rise phenomena in non-Newtonian fluid is necessary as most of the industrial fluids are non-Newtonian in nature. The main aim of this study is to investigate the behaviour of air bubble rising in non-Newtonian liquids and compare the bubble rise characteristics between water and low concentration polymer (Power-Law) shear-thinning fluid. The results of this study are compared with the results of other analytical and experimental studies available in the literature.

Nomenclature

d_b	[m]	bubble diameter
d_h	[m]	bubble height
d_w	[m]	projected diameter onto horizontal plane
d_{eq}	[m]	equivalent sphere diameter
μ	[Pa.s]	apparent viscosity
Re	[-]	Reynolds number
C_d	[-]	drag coefficient
F_d	[N]	drag force
g	[m/s^2]	acceleration due to gravity
U_b	[m/s]	bubble rise velocity
n	[-]	power law index
K	[Pa.s^n/m^2]	consistency index

Greek letters

$\Delta\rho$	[kg/m^3]	density difference between liquid and air bubble
ρ_{liq}	[kg/m^3]	liquid density

EXPERIMENTAL AND CALCULATION

The experimental set up selected in this study was similar to that used by Dewsbury et al. [2]. The experimental apparatus is shown schematically in Figure 1. The rig was used for investigating the bubble rise characteristics in water and polymer solution that consisted of a polycarbonate tube approximately 1.8 m in height and 0.125 m in diameter. The bubble insertion mechanism consisted of a ladle or spoon that had a capability to control the injection of air. The camera lifting apparatus stands approximately 2.0 m high which allows the movement of the camera mount device to move through roughly 1.8 m in height. The variable speed drive of camera lifting apparatus regulates the control of the camera mount device. This drive allows the camera to be raised at approximately the same velocity as the bubble. A high speed digital video camera (Panasonic, NV-GS11, 24X optical Zoom, made in Japan) was mounted on a camera mount device with a small attachment to the side of the camera lifting apparatus.

Bubble rise velocities were computed by a frame by frame analysis of successive images. The bubble images were analysed with the software "Windows Movie Maker" by which the bubble rise time was recorded and velocity was measured. Bubble equivalent diameter was measured from the still frames which were obtained from the video image. The still images were then opened using "SigmaScan Pro 5.0" commercial software and the bubble height (d_h) and the bubble width (d_w) were measured in pixels. The pixel measurements would then be converted to millimetres based on calibration data for the camera.

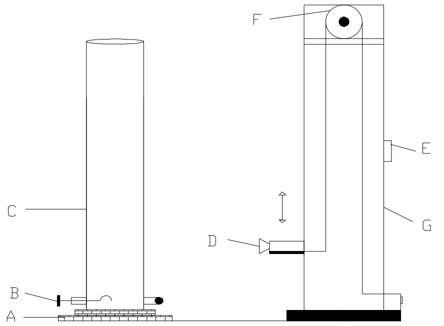

Figure 1 Schematic diagram of experimental apparatus
A = Sturdy Base; B = Rotating Spoon; C = Cylindrical test rig (0.125m or 0.40 m diameter), D = Video camera; E = Variable speed motor; F = Pulley; and G = Camera lifting apparatus.

The bubble equivalent diameter, d_{eq} [9] was computed as

$$d_{eq} = \left(d_h \times d_w{}^2\right)^{1/3} \tag{1}$$

where d_w the long axis length and d_h is the short axis length of the bubble. For this measurement it was assumed that the bubble was axi-symmetric with respect to its short axis direction. Bubble trajectory was determined from the still frames from the video image.

The Reynolds number and drag co-efficient for Newtonian fluid were defined as

$$\mathrm{Re} = \frac{\rho_{liq} U_b d_b}{\mu} \tag{2}$$

and

$$C_d = \frac{8F_d}{\pi \rho_{liq} d_b{}^2 U_b{}^2} \tag{3}$$

The relationship between the drag coefficient of the air bubble and Reynolds number is essential to know for calculating the bubble terminal velocity in non-Newtonian fluids. Since the fluid viscosity varies as a function of the shear rate so the terminal velocity of the bubble also changes with the change in shear rate. The average shear rate over the entire bubble surface is equal to U_b/d_b and the apparent viscosity [1, 8] can then be calculated by

$$\mu = K\left(U_b/d_b\right)^{n-1} \tag{4}$$

In the case of spherical bubble, the Reynolds number for non-Newtonian power law fluid was defined as

$$\mathrm{Re} = \frac{\rho_{liq} d_b^n U_b^{2-n}}{K} \tag{5}$$

For a non-spherical bubble with a vertical axis of symmetry, the Reynolds number was described [1, 2, 8, and 10] by

$$\mathrm{Re} = \frac{d_w^n U_b^{2-n} \rho_{liq}}{K} \tag{6}$$

The drag co-efficient for spherical bubble was calculated by

$$C_d = \frac{4 g d_b \Delta\rho}{3\rho_{liq} U_b^2} \qquad (7)$$

and the drag co-efficient for non-spherical bubble was computed by

$$C_d = \frac{4 g d_{eq}^3 \Delta\rho}{3\rho_{liq} d_w^2 U_b^2} \qquad (8)$$

In equation (8), the drag co-efficient was calculated on the basis of the real bubble geometry where d_{eq} is the equivalent sphere diameter and d_w is the diameter of the horizontal projection of bubble or long axis length of the bubble.

MATERIAL USED

The polymer solution used in this study was a non-Newtonian (shear thinning pseudoplastic) fluid type. Water and two different polymer solutions (polyacrylamide and xanthan gum) with concentration of 0.025% (by weight) were used. The temperature of all solutions in this study was measured at $25^\circ C$. For every polymer solution, the measured density of the solution was very close to the density of water at $25^\circ C$ since low concentration liquids. Rheological properties of the polymer solutions were measured using an ARES (Advanced Rheometric Expansion System) rheometer. The rheological properties for different polymer solutions are summarized in Table 1. The usual range of shear rates to determine fluid rheology was 1 -650s^{-1}.

Table 1 Rheological and physical properties of polymer solutions

Fluid Type	Concentration (%)	K, $Pa.s^n$	n	Density, kg/m^3
Polyacrylamide	0.025	0.00502	0.8544	998.0
Xanthan Gum	0.025	0.00720	0.7975	999.02

RESULTS AND DISCUSSION

Bubble rise velocity

The velocity profile of water and two polymer solutions for various bubble volumes (0.1 mL- 5.0 mL) at different liquid heights is illustrated in Figure 2. The Figure 2 shows that the bubble velocity increases with the increase in bubble volume for all liquids. The bubble velocity of polymer solution is found relatively lower in comparison of water at different liquid heights. It is seen that the bubble velocity for water at 1.6 m height shows different trend with compared to others. The comparison of the bubble rise velocity with literature data for water is shown in Figure 3. As seen, the current experimental data agree well with these published data [6, 7].

Bubble trajectory

The trajectory results of water and two polymer solutions are shown in Figure 4 for 0.1 mL and 5.0 mL bubble size respectively at 1.0 m height. Very interesting feature was seen for the trajectory of bubbles in the polymer solutions in comparison with the water. Both polymer solutions (in the case of 0.1 mL bubble) had a lower standard deviation and hardly differed from their alignment with the release point which was seen from their reasonably straight standard deviation. For the larger bubbles (5.0 mL), the spread was much broader than 0.1 mL bubbles but still not as broad as the results for bubble in water. This phenomenon is completely opposite to those found in water where the small bubbles deviate more than the larger bubbles. In the trajectory analysis of water, it is seen that small bubbles followed a helical motion while larger bubbles followed a spiral motion. In polymer solution, the viscosity is higher than water; so the horizontal motion of the bubbles is reduced and bubble experiences less resistance to vertical movement.

Drag co-efficient

Bubble drag coefficients as a function of Reynolds number for water are plotted in Figure 5. The experimental data are compared with the well known drag curves which are given as follows:

$$C_d = \frac{24}{\text{Re}} \text{(Stokes model)} \qquad (9)$$

and

$$C_d = \frac{16}{\text{Re}} \text{(Hadamard -Rybczynski model)} \qquad (10)$$

As expected, these models (9, 10) give reasonable fit at low Reynolds number but fail in high Reynolds number.

For larger values of Re, the drag coefficient decreases with the increase in Re that is shown [15] with the following equation,

$$C_d = \frac{14.9}{\text{Re}^{0.78}} \qquad (11)$$

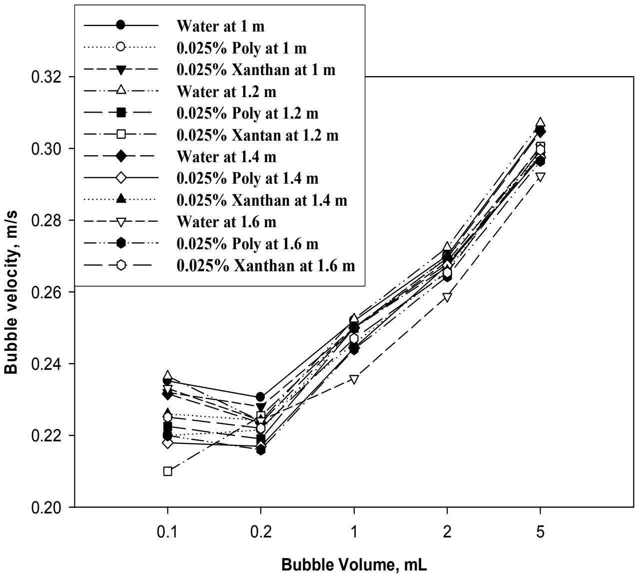

Figure 2 Velocity profile for water, polyacrylamide and xanthan gum solutions at different heights.

The similar trend is seen in Figure 5 when the experimental, C_d compared with the equation (11) but the deviation of these two curves observed quite high.

The prediction of C_d [16] for spherical bubble which is valid for any value of Re is given by

$$C_d = \frac{16}{\mathrm{Re}}\left\{1+\left[\frac{8}{\mathrm{Re}}+\frac{1}{2}\left(1+3.315\,\mathrm{Re}^{0.5}\right)\right]^{-1}\right\} \tag{12}$$

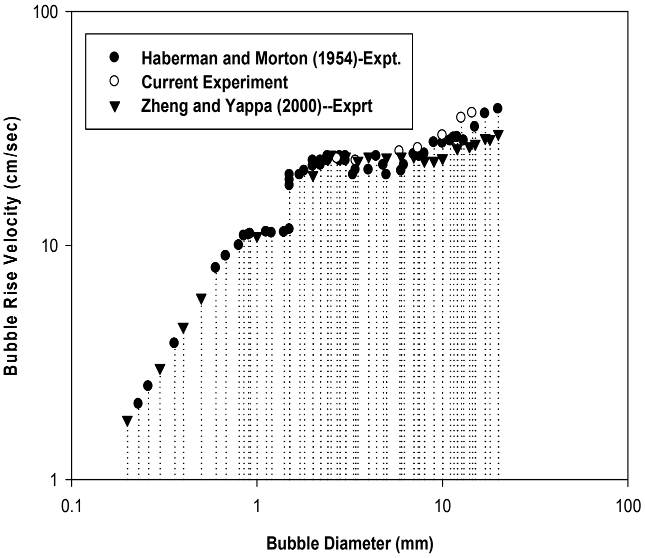

Figure 3 Bubble rise velocity vs. bubble equivalent diameter.

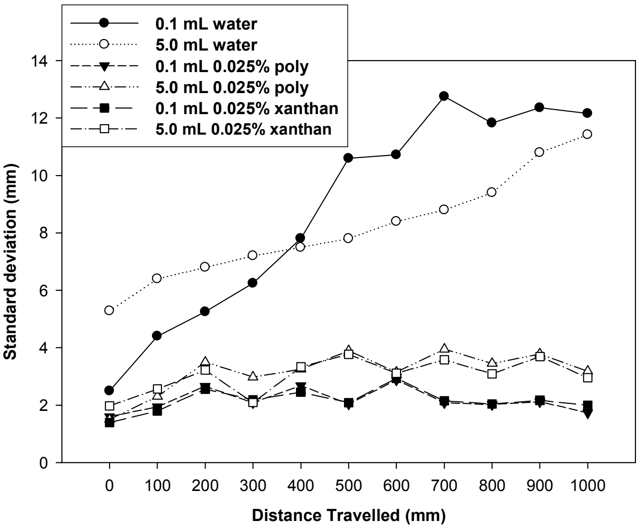

Figure 4 Trajectory profile for water and polymer solutions (0.1 mL and 5.0 mL bubble)

It can be seen from Figure 5 that the experimental values of Cd agree quite well with equation (12) except some scatter observed at low Re.

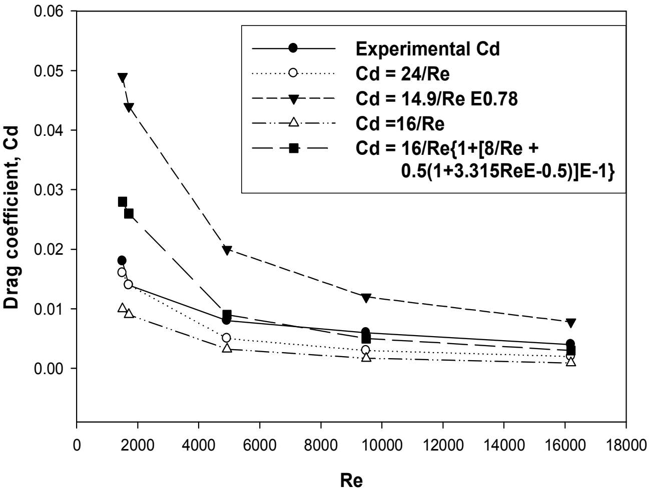

Figure 5 Drag coefficients vs. Reynolds number for rising air bubble in water.

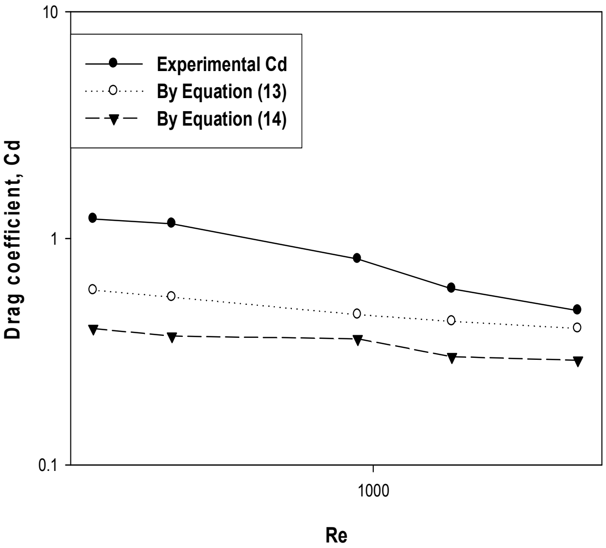

Figure 6 Drag coefficients vs. Reynolds number for rising air bubble in polyacrylamide solution.

No universal drag curve has been developed yet for the case of rising air bubbles in non-Newtonian, particularly, polymer solutions. The drag coefficient for solid particles [17] can be used by the equation (13) for calculation of C_d of bubbles which is as follows:

$$C_d = \frac{24}{\mathrm{Re}}\left(1+0.173\,\mathrm{Re}^{0.657}\right)+\frac{0.413}{1+16{,}300\,\mathrm{Re}^{-1.09}} \tag{13}$$

The above correlation converge to Stokes model at low Re number. A modified correlation was proposed [2] for gas bubbles in non-Newtonian pseudoplasic fluids, given by

$$C_d = \frac{16}{\mathrm{Re}}\left(1+0.173\,\mathrm{Re}^{0.657}\right)+\frac{0.413}{1+16{,}300\,\mathrm{Re}^{-1.09}} \tag{14}$$

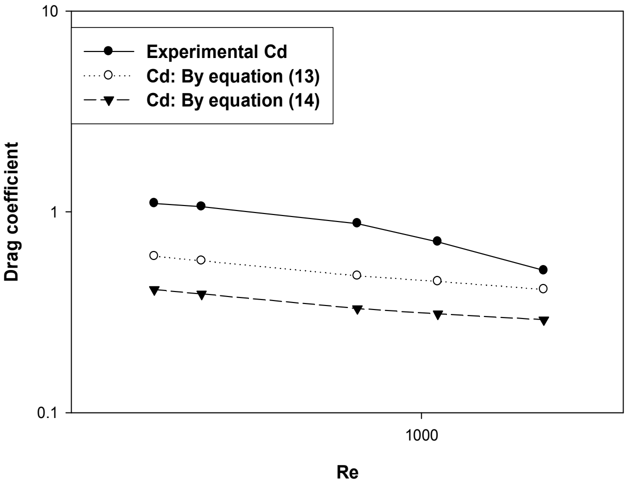

Figure 7 Drag coefficients vs. Reynolds number for rising air bubble in xanthan Gum solution.

The equation (14) converges to the Hadamard -Rybczynski equation, at low Reynolds number. Bubble drag coefficients as a function of Reynolds number for polyacrylamide and xanthan gum solutions are presented in Figure 6 and Figure 7 respectively. It can be seen from Figure 6 that the deviation of the experimental C_d was initially higher in comparison with the equations (13, 14) but this deviation appeared to be less with the increase in Re. This phenomenon is also observed in Figure 7 for xanthan gum solutions.

CONCLUSIONS

The following conclusions can be reached from this study:

- The average bubble rise velocity increases with the increase in bubble volume for water and low concentration polymer solutions.

- For water, smaller bubble travel much more horizontally than larger bubble. In the trajectory analysis, it is seen that small bubbles followed a helical motion while larger bubbles followed a spiral motion (11, 14). As the bubble size increased, the trajectory spread increased for the low concentration polymer solutions. This is due to the increase in viscosity and less resistance on rising of bubbles.

- The relationship between C_d-Re for water produced good agreement with the available experimental studies of the literature. C_d-Re relationship for non-Newtonian polymer solutions also showed acceptable results but it deserves further study for the wide range of Reynolds number.

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