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ON THE TEACHING OF THE AERODYNAMIC HEATING PROBLEM

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ABSTRACT

The problem of aerodynamic heating, along with some other topics, are lacking in most, if not all, heat transfer textbooks that are used for undergraduate and graduate education. There are many issues in the aerodynamic heating problem that are indeed very important from a convective heat transfer point of view. Although this topic is very important in any low or high speed application, the lack of analytical solutions in heat transfer, compressible flow and/or hypersonic flow textbooks has prompted this study. In practice, poor design and manufacture have led to undesired results, such as space shuttle disasters. Since, over the years, analysis has given way to numerical studies, the instructors do not take the necessary time to go through details. Thus the students just use the results without any awareness of how to get them and the inherent limitations of the analytical solution. The only intent of this paper, therefore, is to present the detailed analytical study that is shown step by step that could be used by heat transfer instructors in their courses and by students in their studies.

INTRODUCTION

The topic of high speed aerodynamic heating in a heat transfer course is important. References to this material are very limited [1] and a sampling of graduate heat transfer textbooks [2-4] do not address this subject effectively or not at all although the fundamental work was done over a century ago [5-8]. Aerospace engineers have utilized these concepts but none have presented detailed analytical results that can be used by the student of heat transfer [9-11]. In some recent heat transfer textbooks, [12-14], limited remarks reference the topic but without a robust mathematical analysis. This paper develops the appropriate high speed aerodynamic heat transfer equations through a rigorous analytical study by simplifying the general forms of the conservation equations and the equation of state for the case under consideration.

NOMENCLATURE

A	[m^2]	Area
a	[-]	Constant; an arbitrary function of x
b	[-]	Constant; an arbitrary function of x
C	[-]	Constant of integration
c	[kJ/(kg-K)]	Specific heat
D	[-]	Substantial or material derivative
e	[kJ/kg]	Specific energy
EC	[-]	ECKERT number, $\left\{U_\infty^2/\left[c_p\left(T_w-T_\infty\right)\right]\right\}$
$\vec{F}$	[kN/kg]	Force vector
f	[-]	Free stream function
g	[m/s^2]	Gravitational acceleration
i	[kJ/kg]	Specific enthalpy
k	[W/(m-K)]	Thermal conductivity
NU	[-]	NUSSELT number, $\left[(hx)/k\right]$
p	[kPa]	Pressure
$PÉ$	[-]	PÉCLET number,[$RE\ PR= (U_\infty x)/\alpha$]
PR	[-]	PRANDTL number, [α/υ]
q	[W]	Heat transfer rate
R	[-]	Recovery factor
RE	[-]	REYNOLDS' number, $\left[(U_\infty x)/\upsilon\right]$
T	[K]	Temperature
t	[s]	Time
U	[m/s]	Velocity
u	[m/s]	x-direction velocity; an arbitrary function of x
v	[m/s]	y-direction velocity; an arbitrary function of x
w	[m/s]	z-direction velocity
$\vec{V}$	[m/s]	Velocity vector
x	[m]	Cartesian axis direction
y	[m]	Cartesian axis direction
z	[m]	Cartesian axis direction
Special characters		
α	[m^2/s]	Thermal diffusivity
β	[-]	Integration coordinate
Φ	[W/m^3]	Viscous dissipation function
η	[-]	Similarity coordinate
μ	[(N-s)/m^2]	Dynamic viscosity
ρ	[kg/m^3]	Density
θ	[-]	Dimensionless temperature for no frictional heating
Θ	[-]	Dimensionless temperature for frictional heating
υ	[m^2/s]	Kinematic viscosity
Φ	[N/(s-m^2)]	Viscous dissipation

ξ	[-]	Integration coordinate
Ψ	[m^2/s]	Similarity free stream function
ζ	[-]]	Derivative of dimensionless temperature, [Θ']

Subscripts and Superscripts

AW	[-]	Adiabatic wall
b	[-]	Body
L	[-]	Based on length
p	[kPa]	Pressure
w	[-]	Wall
∞	[-]	Free stream
'	[1/m]	First derivative
"	[1/m^2]	Second derivative

The four principles that establish a system of determinable equations are:

1. *NEWTON's Second Law of Motion-Conservation of Momentum*

$$\frac{D\vec{V}}{Dt} = \vec{F}_b - \frac{1}{\rho}\nabla p + \upsilon\nabla^2\vec{V} + \frac{1}{3}\upsilon\nabla\left(\nabla \bullet \vec{V}\right) \quad (1)$$

2. *Continuity Equation-Conservation of Mass*

$$\frac{\partial\rho}{\partial t} + \nabla\bullet\left(\rho\vec{V}\right) = 0 \quad (2)$$

3. *First Law of Thermodynamics-Conservation of Energy*

$$\rho\left[\frac{\partial e}{\partial t} + \left(\vec{V}\nabla\right)e\right] = -p\left(\nabla\bullet\vec{V}\right) + \nabla\bullet\left(k\nabla T\right) + \Phi \quad (3)$$

where

$$\Phi = 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \mu\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2\right] - \frac{2}{3}\mu\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)^2 \quad (4)$$

Alternatively, conservation of energy can be expressed as

$$\rho\frac{Di}{Dt} = \frac{Dp}{Dt} + \nabla\bullet(k\nabla T) + \Phi \quad (5)$$

with

$$i = e + p/\rho \quad (6)$$

4. *The Equation of State*

$$p = p(\rho, T) \quad (7)$$

These four equations must be solved simultaneously for an exact solution. Since mathematical analysis will not permit an exact solution, certain assumptions must be made to continue with the analysis.

ASSUMPTIONS AND ANALYSIS

In general, three methods apply to obtaining the fundamental solution to convective heat transfer problems:

1. Solve the governing equations using the concept of the boundary layer and suitable similarity parameters.
2. Perform dimensional analysis coupled with extensive experimental data and curve fitting techniques.
3. Use the approximate integral method.

The boundary layer solution to the convection problem is normally demonstrated under the assumptions of steady flow of an incompressible fluid, constant dynamic viscosity and thermal conductivity, and constant specific heats for the case of an ideal gas. Under these assumptions, conservation of mass, equation (2), reduces to equation (8).

$$\nabla\bullet\vec{V} = 0 \quad (8)$$

Conservation of momentum, equation (1), reduces to equation (9).

$$(\vec{V}\bullet\nabla\vec{V}) = \vec{F}_{BODY} - \frac{1}{\rho}\nabla p + \upsilon\nabla^2\vec{V} \quad (9)$$

Conservation of energy, equation (3), reduces to equation (10)

$$\rho c_p(\vec{V}\bullet\nabla T) = k\nabla^2 T + \Phi' \quad (10)$$

where Φ' is obtained from equation (4) with the last term eliminated as a result of equation (8). The equation of state remains unchanged.

Applying order of magnitude analysis [15] and assuming two-dimensional flow reduce conservation of mass, equation (8), to equation (11).

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (11)$$

Conservation of momentum, equation (9), reduces to equation (12) for the x-direction and to equation (13) for the y-direction, respectively.

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g - \frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} \quad (12)$$

$$\frac{\partial p}{\partial y} = 0 \quad (13)$$

As a consequence of order of magnitude analysis for these, the additional restriction that $RE_L \geq 100$ at the end of the plate follows. Conservation of energy, equation (3), reduces to equation (14).

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{u}{\rho c_p}\frac{\partial p}{\partial x} + \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (14)$$

Again as a consequence of order of magnitude analysis, the additional restrictions that $EC = \frac{U_\infty^2}{c_p(T_w - T_\infty)}$ must have an order of magnitude of one (O(1)) and $P\acute{E} \geq 100$ follow. If the pressure is uniform, then the conservation of energy reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (15)$$

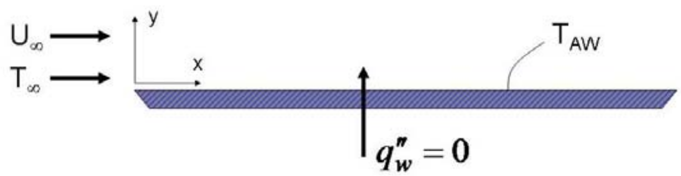

Figure 1. The Fundamental Problem

Consider first an adiabatic flat plate as shown in Figure 1. The desired quantity is the value of the temperature of the adiabatic wall, T_{AW}. Thus, the boundary conditions of the thermal problem are

$$x = 0 \qquad T = T_\infty \qquad (16)$$

$$y = 0 \qquad \frac{\partial T}{\partial y} = 0 \qquad (17)$$

$$y \to \infty \qquad T \to T_\infty \qquad (18)$$

Using similarity variables as

$$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}} \qquad (19)$$

$$\psi(x, y) = \sqrt{U_\infty \upsilon x}\, f(\eta) \qquad (20)$$

$$\Theta = \frac{(T - T_\infty)}{\left(\frac{U_\infty^2}{2c_p}\right)} \qquad (21)$$

equation (15) transforms into

$$\Theta'' + \frac{PR}{2}f\Theta' + 2PR(f'')^2 = 0 \qquad (22)$$

with the transformed boundary conditions of

$$\begin{array}{ll} \eta = 0 & \Theta' = 0 \\ \eta \to \infty & \Theta \to 0 \end{array} \qquad (23)$$

Letting $\zeta = \Theta'$, the order of Equation (22) can be reduced by one to give

$$\zeta' = -\frac{PR}{2}f\zeta - 2PR(f'')^2 \qquad (24)$$

which is a differential equation with linear second term of the type

$$y'(x) = a(x)y(x) + b(x) \qquad (25)$$

To solve this equation, assume a solution of the type

$$y(x) = u(x)v(x) \qquad (26)$$

Then, Equation (25) becomes

$$u'v + v'u = auv + b \qquad (27)$$

or $(u' - au)v + v'u = b$ giving $(u' - au) = 0$ resulting in $\left(\frac{u'}{u}\right) = a$ which integrates to

$$\ln u = \int a dx + C_1'' \qquad (28)$$

Thus,

$$u = C_1' \exp\left[\int a dx\right] = C_1' \exp\left[-\frac{PR}{2}\int_0^\eta f d\xi\right] \qquad (29)$$

Also, $\left(\frac{dv}{dx}\right)u = b$ which gives $v = \int \frac{b(x)}{C_1' \exp\left[-\frac{PR}{2}\int_0^\eta f d\xi\right]}dx + C_2'$ and simplifies to

$$v = \frac{1}{C_1'}\int_0^\eta \exp\left[\frac{PR}{2}\int_0^\beta f d\xi\right]\left\{-2PR(f'')^2\right\}d\beta + C_2' \qquad (30)$$

Therefore, since $\zeta = \frac{d\Theta}{d\eta} = y = uv$, then

$$\frac{d\Theta}{d\eta} = \exp\left[\frac{PR}{2}\int_0^\beta f d\xi\right]\left\{\int_0^\eta \exp\left[\frac{PR}{2}\int_0^\beta f d\xi\right]\left[-2PR(f'')^2\right]d\beta + C_2'\right\} \qquad (31)$$

However, the transformed conservation of momentum equation for no body or pressure forces is $2f''' + ff'' = 0$. The numerical solution for this is given in Table 1 in the APPENDIX. Therefore,

$$\int f d\eta = -2\int \frac{\left(\frac{d^3 f}{d\eta^3}\right)}{\left(\frac{d^2 f}{d\eta^2}\right)} d\eta \tag{32}$$

which simplifies to

$$\int f d\eta = -2\ln\left(\frac{d^2 f}{d\eta^2}\right) = -2\ln(f'') \tag{33}$$

Upon substitution into equation (31),

$$\frac{d\Theta}{d\eta} = (f'')^{PR}\left[C_2 - 2PR\int_0^\eta (f'')^{2-PR} d\xi\right] \tag{34}$$

Using the boundary condition, equation (23), $C_2 = 0$. Thus,

$$\frac{d\Theta}{d\eta} = \left[-2PR(f'')^{PR}\int_0^\eta (f'')^{2-PR} d\xi\right] \tag{35}$$

Upon integration,

$$\Theta = -2PR\int_0^\eta (f'')^{PR}\left[\int_0^\beta (f'')^{2-PR} d\xi\right] d\beta + C_3 \tag{36}$$

Using equation (23) again,

$$C_3 = 2PR\int_0^\infty (f'')^{PR}\left[\int_0^\beta (f'')^{2-PR} d\xi\right] d\beta \tag{37}$$

gives

$$\Theta = 2PR\left[\int_0^\infty (f'')^{PR}\left\{\int_0^\beta (f'')^{2-PR} d\xi\right\} d\beta - \int_0^\eta (f'')^{PR}\left\{\int_0^\beta (f'')^{2-PR} d\xi\right\} d\beta\right]$$

or

$$\Theta = 2PR\int_\eta^\infty (f'')^{PR}\left[\int_0^\beta (f'')^{2-PR} d\xi\right] d\beta \tag{38}$$

For $\eta = 0$, the boundary condition of an adiabatic wall, equation (21) reduces to equation (39).

$$\Theta(0) = \frac{(T_{AW} - T_\infty)}{\left(\frac{U_\infty^2}{2c_p}\right)} \tag{39}$$

Combining equations (38) and (39), equation (40) results,

$$\Theta(0) = 2PR\int_0^\infty (f'')^{PR}\left[\int_0^\beta (f'')^{2-PR} d\xi\right] d\beta \tag{40}$$

which is called the *recovery factor*, R_{AW}, of the adiabatic wall. Thus, the temperature of the adiabatic wall becomes,

$$T_{AW} = T_\infty + R_{AW}\left(\frac{U_\infty^2}{2c_p}\right) \tag{41}$$

For $0.6 < PR < 15$, $\Theta(0)$ may be approximated by

$$\Theta(0) = R_{AW} = \sqrt{PR} \tag{42}$$

or calculated exactly from equation (40) for a known free stream function *f*. From the steady flow energy equation of an ideal gas,

$$T_{stagnation} = T_\infty + \frac{U_\infty^2}{2c_p} \tag{43}$$

if $U_{stagnation} = 0$. Therefore, R_{AW} is a measure of how close the wall temperature is to the stagnation temperature. Thus, for

$$\begin{array}{ll} PR < 1 & T_{AW} < T_{stagnation} \\ PR = 1 & T_{AW} = T_{stagnation} \\ PR > 1 & T_{AW} > T_{stagnation} \end{array} \tag{44}$$

The usual definition for the convective heat transfer coefficient is

$$h = \frac{q}{A(T_w - T_\infty)} \tag{45}$$

However, in this case $q = 0$ while $(T_{AW} - T_\infty) \neq 0$ which implies that the usual definition of the convective heat transfer coefficient does not hold when frictional heating is present. Therefore, a new definition is necessary. For this consider the case of the heated or cooled plate at a uniform temperature, T_W. For the case of no frictional heating, $\theta = \frac{(T - T_\infty)}{(T_w - T_\infty)}$. Solving for *T*,

$$(T - T_\infty) = \theta(\eta)(T_w - T_\infty) \tag{46}$$

A general solution for *T* may be written as

$$(T - T_\infty) = C(T_w - T_\infty)\theta(\eta) \tag{47}$$

Letting

$$\Theta = \frac{T - T_\infty}{T_{stagnation} - T_\infty} = \frac{(T - T_\infty)}{\left(\frac{U_\infty^2}{2c_p}\right)} \tag{48}$$

for an adiabatic plate, there is a particular solution of the form,

$$(T - T_\infty) = (T_{stagnation} - T_\infty)\Theta(\eta) \tag{49}$$

which gives the complete solution, using equations (47) and (48), as

$$(T - T_\infty) = C(T_w - T_\infty)\theta(\eta) + \frac{U_\infty^2}{2c_p}\Theta(\eta) \quad (50)$$

with

$$\begin{array}{lll} \eta = 0 & \theta = 1 & \Theta = R_{AW} \\ \eta \to \infty & \theta \to 0 & \Theta \to 0 \end{array} \quad (51)$$

Thus at $\eta = 0$,

$$(T_w - T_\infty) = C(T_w - T_\infty) + \left(\frac{U_\infty^2}{2c_p}\right)R_{AW} \quad (52)$$

or using (41)

$$C(T_w - T_\infty) = (T_w - T_\infty) - (T_{AW} - T_\infty) = (T_w - T_{AW}) \quad (53)$$

Thus the general solution becomes,

$$(T - T_\infty) = (T_w - T_{AW})\theta(\eta) + \left(\frac{U_\infty^2}{2c_p}\right)\Theta(\eta) \quad (54)$$

The heat transfer is given by

$$\begin{aligned} q &= -kA\left(\frac{\partial T}{\partial y}\right)_{y=0} = -kA\sqrt{\frac{U_\infty}{\upsilon x}}\left(\frac{\partial T}{\partial \eta}\right)_{\eta=0} \\ &= -kA\sqrt{\frac{U_\infty}{\upsilon x}}(T_w - T_{AW})\theta'(0) \end{aligned} \quad (55)$$

since $\Theta'(0) = 0$. Defining the convective heat transfer coefficient as

$$h = \frac{q}{A(T_w - T_{AW})} \quad (56)$$

then the NUSSELT number becomes

$$\frac{NU_x}{\sqrt{RE_x}} = -\theta'(0) \quad (57)$$

which is the same result as obtained in the case of heat transfer with no friction. Thus, *the solution for no frictional heating applies to the frictional heating problem by replacing T_∞ by T_{AW} in the definition of the convective heat transfer coefficient*. The magnitude of $([T_w - T_\infty]/[T_w - T_{AW}])$ predicts the importance of frictional heating since $(T_w - T_{AW}) == T_w - \left(T_\infty + \frac{U_\infty^2}{2c_p}R_{AW}\right)$. Simplified, this is expressed as

$$\left(\frac{T_w - T_{AW}}{T_w - T_\infty}\right) = 1 - \frac{EC}{2}R_{AW} \quad (58)$$

or

$$\left(\frac{T_w - T_\infty}{T_w - T_{AW}}\right) = \frac{2}{2 - ECR_{AW}} \quad (59)$$

If $EC << O(1)$, then $\left(\frac{T_w - T_\infty}{T_w - T_{AW}}\right) = 1 \Rightarrow T_{AW} = T_\infty$. If $EC = O(1)$, then $\left(\frac{T_w - T_\infty}{T_w - T_{AW}}\right) \gtrless 0$ and must be considered. The heat flux is reduced for $EC > 0$ and increased for $EC < 0$.

In summary, for high speed flow across a flat plate, the ordinary external laminar and turbulent flow relations

$$\begin{aligned} NU_x &= 0.33206\sqrt{RE_x}(PR)^{1/3} \\ NU_x &= 0.02914(RE_x)^{4/5}(PR)^{1/3} \end{aligned} \quad (60)$$

apply provided the new definition of the convective heat transfer coefficient is used as given in equation (56). For turbulent flow,

$$R_{AW} = (PR)^{1/3} \quad (61)$$

CONCLUSION

The problem of aerodynamic heat transfer is presented in a complete analytical form. The student can understand how this problem is solved and its correct and precise use in the design of systems. It is indeed very important that these details are followed carefully. The mathematical analysis is definitely not beyond the capabilities of students at the graduate level, for sure, and even at the undergraduate level. By not using analysis, we are depriving our students the opportunities of applying their mathematical skills that we so strongly emphasize but neglect to use in our courses. The importance of this problem, of course, cannot be underestimated as tragic events have resulted due to poor design or manufacture.

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APPENDIX

Table 1. Values for solution of $2f'''+ff''=0$

$\eta = y\sqrt{\frac{U_\infty}{\upsilon x}}$	f	$f' = \frac{u}{U_\infty}$	f''	$\frac{1}{2}(\eta f' - f) = \frac{v}{U_\infty}\sqrt{RE_x}$
0	0	0	0.33206	0
0.2	0.00664	0.06641	0.33199	0.00332
0.4	0.02656	0.13277	0.33147	0.01327
0.6	0.05974	0.19894	0.33008	0.02981
0.8	0.10611	0.26471	0.32739	0.05283
1.0	0.16557	0.32979	0.32301	0.08211
1.2	0.23795	0.39378	0.31659	0.11729
1.4	0.32298	0.45627	0.30787	0.15790
1.6	0.42032	0.51676	0.29667	0.20325
1.8	0.52952	0.57477	0.28293	0.25253
2.0	0.65003	0.62977	0.26675	0.30476
2.2	0.78120	0.68132	0.24835	0.35885
2.4	0.92230	0.72899	0.22809	0.41364
2.6	1.07252	0.77246	0.20646	0.46794
2.8	1.23099	0.81152	0.18401	0.52063
3.0	1.39682	0.84605	0.16136	0.57067
3.2	1.56911	0.87609	0.13913	0.61719
3.4	1.74696	0.90177	0.11788	0.65953
3.6	1.92954	0.92333	0.09809	0.69722
3.8	2.11605	0.94112	0.08013	0.73010
4.0	2.30576	0.95552	0.06424	0.75816
4.2	2.49806	0.96696	0.05052	0.78159
4.4	2.69238	0.97587	0.03897	0.80072
4.6	2.88826	0.98269	0.02948	0.81606
4.8	3.08534	0.98779	0.02187	0.82803
4.91755		***0.99000***		***0.83384***
5.0	3.28329	0.99155	0.01591	0.83723
5.2	3.48189	0.99425	0.01134	0.84410
5.4	3.68094	0.99616	0.00793	0.84916
5.6	3.88031	0.99748	0.00543	0.85279
5.8	4.07990	0.99838	0.00365	0.85535
6.0	4.27964	0.99898	0.00240	0.85712
6.2	4.47948	0.99937	0.00155	0.85831
6.4	4.67938	0.99961	0.00098	0.85906
6.6	4.87931	0.99977	0.00061	0.85959
6.8	5.07928	0.99987	0.00037	0.85992
7.0	5.27926	0.99992	0.00022	0.86009
7.2	5.47925	0.99996	0.00013	0.86023
7.4	5.67924	0.99998	0.00007	0.86031
7.6	5.87924	0.99999	0.00004	0.86034
7.8	6.07923	1.00000	0.00002	0.86039
8.0	6.27923	1.00000	0.00001	0.86039
8.2	6.47923	1.00000	0.00001	0.86039
8.4	6.67923	1.00000	0.00000	0.86039
8.6	8.87923	1.00000	0.00000	0.86039
8.8	7.07923	1.00000	0.00000	***0.86039***