Braking penalized receding horizon control of heavy haul trains

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***Abstract*—Incorporated with a receding horizon control (RHC) approach, a penalty method is proposed to reduce energy wasted by braking in a heavy haul train's operation. The train's practical nonlinear model is linearized to design the RHC controller. This controller is then applied to the train practical nonlinear dynamics and its performances are analyzed. In particular, the main focus in this study is on the brake penalty's impact on the train performances. Meantime, a fence method is presented to tackle two issues. The first one is that all the cars in a train cannot be controlled individually due to limit of available transmission channels for control systems in a long train. The other one is that the RHC approach suffers from heavy computation and memory load. Simulations verified that the brake penalty presented in the design can reduce a train's energy consumption and in-train forces remarkably without sacrificing the train's velocity tracking performance. Simulations also verified that the fence method is essential to reduce the related computation load when the RHC approach is applied to a long heavy haul train. Further, it is demonstrated that the fence method can effectively shorten computation time and reduce memory usage without severely jeopardizing the train's performance.**

***Index Terms*—Heavy haul trains, braking penalization, RHC, fence method, optimal control.**

I. Introduction

For a heavy haul train's operation, reducing energy consumption while ensure both the train's operation safety and service quality is an urgent call from the viewpoint of energy shortage and environment conservation. With this purpose, Zhuan et al presented optimal train schedulers that optimize the train's performances in terms of in-train forces, speed tracking and energy consumption [1], [2]. Because the train dynamics are naturally nonlinear, their later work employed nonlinear regulation theory to solve this problem to obtain better results as no approximation is required in the controller design [3]. Also, a series of work in this field have been done by Howlett et al. Open-loop switching controllers were proposed in their papers [4], [5] to operate trains energy efficiently. There are two distinctive differences between Zhuan's and Howlett's works. The first one is whether in-train dynamics of the train are considered. The in-train forces were not considered in Howlett's studies because a single mass point train model (which models a train as a single mass point) was used in controller design. Zhuan et al designed controllers to minimize in-train forces in order to ensure the train's run safety with a cascade mass point model. The second difference is that Howlett's work follows switching control framework and is actually open loop control while Zhuan et al showed closed loop controllers.

In this paper, a closed-loop receding horizon control (RHC) approach is adopted to control a train in a closed loop manner with the objective of reducing the train's energy usage while ensuring its operation safety and service quality. The nonlinear cascade mass point train model [6] is linearized and discretized to design the RHC controller [7]. In comparison with existing train operation control approaches, the closed-loop RHC's ability to handle (both hard and soft) constraints and to compensate the degraded performances caused by model plant mismatch [8] is an advantage from control point of view. As train dynamics are practically nonlinear, model plant mismatch exist inevitably between the train dynamics and the linearized model used in controller design. With respect to this issue, the RHC approach is better than other approaches [2], [5] because it solves the problem in a local horizon where the train dynamics can be considered to be approximately linear. This further implies that using a linearized train model for controller design is acceptable.

The objective of the proposed controller is to minimize energy consumption during the train's operation to save energy and reduce emission, to minimize in-train forces experienced by couplers connecting neighboring cars in the train to ensure safety, and to keep the train's speed as close to the given reference speed profile as possible to ensure the train's punctuality. While the reference speed profile of a train on a given track should and can be optimized as done in [9], [10], it is out of scope of this paper and is not discussed. Only speed tracking with respect to a given reference speed profile is addressed.

The most distinct difference between this study and our previous work is that additional attention is given to energy wasted by the train's braking because it is demonstrated in [11] that statistically, energy wasted by train braking is responsible for 10-20% of the total energy consumption. Study carried out in [11] using actual engineering data from trains run from Eastern Hefei Freight Yard to Chang'anji China showed a 9% energy consumption reduction by purely avoiding unnecessary braking. Specifically, a penalty factor for the train's braking is introduced to prevent the train from applying excessive braking forces.

In addition, as a long heavy haul train normally has more than 150 cars, the RHC approach suffers from heavy computation load and memory demand. Also, the large number of cars in the train made it impossible to control all the cars individually because signal transmission channels available for a control system in a train is limited. For example, 32 channels

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were specified according to the standard of the Association of American Railroads [12], which means that only 32 individual control signals can be used for controlling the train. To tackle this issue, a car fence method is also introduced to facilitate the RHC approach's practical implementation.

The remainder of this paper is organized as follows: the braking penalized RHC of heavy haul trains is presented, followed by the fence methodology in Section II. Simulations are given in Section III to investigate the impacts of fence methodology and braking penalization, respectively. Concluding remarks are drawn in Section IV.

II. Receding horizon control of heavy haul trains

The braking penalized RHC of heavy haul trains is given in this section. A penalty factor is introduced to penalize wagons' braking efforts in a train. The major purpose of this section is to establish equations that represent the train operation and mathematical interpretation of the braking penalization so that the problem could be ready solved.

A. Train model

The cascade mass point model is employed so that in-train dynamics of the train can be considered. Originally, this model for a heavy haul train equipped with an electrically controlled pneumatic braking system was presented in [6]:

$$\begin{cases} m_i \dot{v}_i = u_i + f_{in_{i-1}} - f_{in_i} - f_{a_i}, \quad i = 1, \ldots, n \\ \dot{x}_{in_i} = v_i - v_{i+1}, \quad i = 1, \ldots, n-1 \end{cases} \tag{1}$$

where n is the number of cars in the train, m_i and v_i are respectively the ith car's mass and speed, u_i is the effort of the ith car (driving/braking effort for a locomotive and braking effort for a wagon). The variable $f_{a_i} = f_{r_i} + f_{p_i}$, in which $f_{r_i} = m_i(c_{0_i} + c_{1_i} v_i + c_{2_i} v_i^2)$ is the ith car's rolling resistance and aerodynamic force with coefficients c_{0_i}, c_{1_i} and c_{2_i} determined by experiments, $f_{p_i} = f_{g_i} + f_{c_i}$ is the force due to the track slope and curvature on which the ith car is running. The variable f_{in_i} is in-train forces between the ith and $i+1$th cars, and x_{in_i} is relative displacement of the ith and $i+1$th cars.

While (1) is nonlinear and complicated, it reveals both overall and in-train dynamics of a train. In order to facilitate controller design, the following procedure is used to deviate a linear discrete state space model from (1) (more details are referred to [13]) to be used for controller design:

(1) Linearization: chose a new control variable $u_i' = u_i - (m_i c_{0_i} + m_i c_{2_i} v_i^2 + f p_i), i = 1, \ldots, n$ to make the train dynamics take the origin as an equilibrium point and ignore coupler damping effect so that the model becomes linear.
(2) Discretization: using zero-order hold method with sampling period T_s to discretize the linearized model.

After the above two steps, the train model used for controller design is as follows

$$x(k+1) = Ax(k) + Bu'(k) \tag{2}$$

where $x(k)$ consists of all car's current velocities and the relative displacements of neighboring cars in a train, $x(k) = [v_1(k), v_2(k), \cdots, v_n(k), x_{in_1}(k), x_{in_2}(k), \cdots, x_{in_{n-1}}(k)]^T$ and $u'(k) = [u_1'(k), u_2'(k), \ldots, u_n'(k)]^T$.

Since the state variables are not all measurable, only the locomotives' speeds, which can be measured in practice [14], [15], are chosen as outputs:

$$y(k) = Cx(k), \tag{3}$$

where C is a matrix with zero row vectors except at the position where a locomotive is used is one. For instance

$$C = \begin{bmatrix} 1, 0, \ldots, 0 \\ 0, 1, \ldots, 0 \end{bmatrix},$$

when two locomotives are connected together at the front of the train.

B. Objective function

In [2], an objective function considering energy consumption, velocity tracking and in-train forces is employed. It is shown that if properly tuned, the designed controller can emphasize on one of those three factors and a trade-off must be done. However, that paper failed to take the following aspects of the optimization into consideration.

Firstly, from the viewpoint of energy consumption, it was claimed that, statistically, the energy wasted to compensate kinetic energy loss caused by braking takes account for 10-20% of the total energy consumption during a train's operation [11]. This implies that remarkable train operation performance improvement can be reached by reducing braking efforts of wagons such that braking loss of kinetic energy is reduced. In this study, the effort is made to reduce the train's energy consumption by using as less braking as possible while following a predefined speed profile.

So a factor K_b is presented here to penalize the braking efforts of wagons. Similar to the work done in [2], a new objective function is presented as follows

$$\begin{aligned} J = \int_{t_0}^{t_f} \big[& K_f \sum_{i=1}^{n-1} f_{in_i}^2 + K_v \sum_{i=1}^{n} (v_i - v_r)^2 \\ & + K_e \big(\sum_{k=1}^{m} u_{l_k}^2 + \sum_{i=1, i \neq l_k}^{n} K_b u_i^2 \big) \big] dt, \end{aligned} \tag{4}$$

where v_r is the reference speed, and K_f, K_e, K_v are weights for in-train forces, energy consumption, and velocity tracking, respectively. The variables t_0 and t_f are the beginning and ending time of the optimization horizon. The variable m is the total number of locomotives and $l_1, l_2, \ldots, l_m$ denotes the relative position of those locomotives in the train. Variables u_{l_k} and u_i $(i = 1, 2, \ldots, n, i \neq l_k)$ are used to distinguish traction/brake forces of locomotives and braking forces of wagons. Moreover, K_b is the penalty factor introduced to punish braking of wagons.

In order to practice predictive control of the train, the equation (4) is transformed into its equivalent one as done in [7] as follows,

$$J = \int_{t_0}^{t_f} (x^T Q x + u'^T R u' + F_1^T x + F_2^T u') dt, \tag{5}$$

where

$$Q = \begin{bmatrix} diag(K_v, \ldots, K_v) & 0 \\ 0 & diag(K_f k_1^2, \ldots, K_f k_{n-1}^2) \end{bmatrix},$$

$F_1^T = -2K_v v_r[\mathbf{1}_{1\times n}, \mathbf{0}_{1\times n-1}]$, $F_2^T = 2K_e[m_1 c_{0_1} + f_{p_1}, \ldots, m_n c_{0_n} + f_{p_n}]$, and R is a diagonal matrix

$$R(i,i) = \begin{cases} K_e, \text{ if } i = l_j, j = 1, 2, \ldots, m, \\ K_b K_e, \text{ otherwise}, \end{cases}$$

where $i = 1, 2, \ldots, n$ and $\mathbf{1}_{1\times n}$ is a row vector with entries one of specified dimension.

C. Receding horizon control

The basic idea of the RHC approach is that one solves fixed horizon optimization problem to compute a sequence of predicted inputs (control variables) over a prediction horizon N_p, and then implements the first control of the calculated sequence. When the next control time arrives, this process is repeated. This strategy has been enormously successful in process control applications [16], [17], [18], [19].

For the train operation problem, the RHC approach uses current train dynamic state, train model, and operational limits to calculate future N_c changes in the control variable so that the train's operation is optimized within the N_p optimization window. After calculation, only the first change in control variable is send out to be implemented, and the calculation procedure is repeated when the next control instant is arrived. The RHC formulation for heavy haul trains is given below.

1) Input constraints: The constraints on inputs are the limitations on the amplitudes and the change rates of a train's traction/braking efforts. With the change rates of a wagon's brake power is assumed to be unbounded, the constraints can be written as

$$\begin{aligned} u_i^l \le u_i \le u_i^u, \quad i = 1, 2, \ldots, n, \\ \Delta u_{l_j}^l \le \Delta u_{l_j} \le \Delta u_{l_j}^u, \quad j = 1, 2, \ldots, m. \end{aligned} \tag{6}$$

where u_i^l, u_i^u are the lower and upper bounds of the ith car's effort (control signal), $\Delta u_{l_j}^l$ and $\Delta u_{l_j}^u$ are the lower and upper limits of the step change of the jth locomotive's effort.

2) State constraints: State constraints concern two aspects. The first one is the in-train forces which should be maintained in a safe range during the train's operation. The second one is a hard speed limit which prevents the train's average speed from exceeding the reference speed for safety reasons.

As for the relationship between in-train forces and the state variable, one has $f_{in_i} = k_i x_{in_i}$, in which k_i is the elastic coefficient of the ith coupler. So the constraints on state variables including speed limit

$$\frac{1}{n}\sum_{i=1}^{n} v_i \le v_r, \tag{7}$$

and in-train forces limits

$$\frac{f^l}{k_i} \le x_{in_i} \le \frac{f^u}{k_i}, \quad i = 1, 2, \ldots, n-1, \tag{8}$$

where f^l and f^u are the lower and upper bounds of in-train forces.

3) Prediction: State prediction is done as in [7]:

$$X = Fx(k) + \Phi U, \tag{9}$$

where $X = [x(k+1|k)^T, x(k+2|k)^T, \ldots, x(k+Np|k)^T]^T$ and $U = [u'(k)^T, u'(k+1)^T, \ldots, u'(k+N_c-1)^T]^T$ are predicted state vector and corresponding optimal control signal based on the current state, and

$$F = \begin{bmatrix} A \\ A^2 \\ A^3 \\ \vdots \\ A^{N_p} \end{bmatrix}, \Phi = \begin{bmatrix} B & 0 & \ldots & 0 \\ AB & B & \ldots & 0 \\ A^2B & AB & \ldots & 0 \\ \vdots & \vdots & \ldots & \vdots \\ A^{N_p-1}B & A^{N_p-2}B & \ldots & A^{N_p-N_c}B \end{bmatrix}.$$

Meanwhile, it is seen in equation (5) that F_1 is related to speed profile v_r and F_2 is related to f_p which two are directly related to the train's position at a given track. Denote the velocity elements in the state variable $x(k+i|k)$ as $v(k+i|k)$, and the position of the train at time instant k as $p(k)$, the train's position at time instant $k+i$ can be approximately calculated as

$$p(k+i|k) = p(k) + \sum_{j=1}^{i} T_s \frac{v(k+j-1|k) + v(k+j|k)}{2}.$$

Note that $v(k+i-1|k)$ is known according to (9).

After that, the F_1 and F_2 matrix can be calculated during the prediction procedure owing to the fact that both v_r and f_p only dependent on p.

As a result, prediction of F_1 and F_2 is done as follows,

$$\begin{aligned} \bar{F}_1 = [F_1(k)^T, \ldots, F_1(k+N_p)]^T \cong \bar{F}_1(v_r) \cong \bar{F}_1(p), \\ \bar{F}_2 = [F_2(k)^T, \ldots, F_2(k+N_p)]^T \cong \bar{F}_2(f_p) \cong \bar{F}_2(p). \end{aligned}$$

where $F_1(k+i)$ and $F_2(k+i)$ are the discrete values of F_1 and F_2 in equation (5), respectively.

Finally, with the above formulation, the train operation optimization problem in the RHC framework, which can be uniformly solved by quadratic programming, is given as follows

minimize

$$J = U^T HU + 2U^T f,$$

subject to train dynamics (2) and constraints

$$MU \le \gamma,$$

where $H = \Phi^T \bar{Q}\Phi + \bar{R}$, $f = \Phi^T \bar{Q} F x(k) + \frac{1}{2}(\Phi^T \bar{F}_1 + \bar{F}_2)$, in which

$$\bar{Q} = diag(Q, \ldots, Q), \bar{R} = diag(R, \ldots, R),$$

$$M = \begin{bmatrix} I_{(nN_c\times nN_c)} \\ -I_{(nN_c\times nN_c)} \\ Z\Phi_x \\ -Z\Phi_x \end{bmatrix}, \gamma = \begin{bmatrix} U^U_{(nN_c\times 1)} \\ -U^L_{(nN_c\times 1)} \\ X^U_{(n-1)N_c\times 1} - ZF_x x(k) \\ -X^L_{(n-1)N_c\times 1} + ZF_x x(k) \end{bmatrix},$$

$$Z = diag\left(\begin{bmatrix} \mathbf{1}_{1\times n} & \mathbf{0}_{1\times n-1} \\ 0 & I_{n-1\times n-1} \end{bmatrix}, \ldots, \begin{bmatrix} \mathbf{1}_{1\times n} & \mathbf{0}_{1\times n-1} \\ 0 & I_{n-1\times n-1} \end{bmatrix}\right),$$

and Φ_x and F_x are the first $(2n-1)N_c$ rows of Φ and F, respectively.

It is seen that the train's current sate $x(k)$ is required to solve the problem formulated above. In order to get this state information, a state observer is designed.

It can be verified that the dynamic system with state dynamics (2) with output function (3) is observable, provided that the first or the last car's speed is measured. For instance, if only the first car's speed is measured, *i.e.* $C = [1, 0, \dots, 0]$, the observability matrix of the [A, C] pair is of full rank which implies the system is observable [20]. The condition of knowing the first or last car's speed, is easy to satisfy since in most trains, the first or last car is a locomotive whose speed can be measured in practice [15].

Given this observability analysis, an observer based on discrete Kalman state estimator is designed to estimate the train's state. To be exact, process noise and measurement noise are introduced in the train's model first as follows

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k), \\ y(k) &= Cx(k) + v(k). \end{aligned}$$

where $w(k)$ and $v(k)$ are process noise (disturbance) and measurement noise, respectively.

Then the state estimation from a Kalman filter is given by

$$\hat{x}(k|k) = A\hat{x}(k|k-1) + Bu(k) + L(k)(y(k) - C\hat{x}(k|k-1)), \tag{10}$$

where $\hat{x}(k|k-1)$ is the estimated state variable at sampling instant k using all available measurements up to sampling instant $k-1$. $L(k)$ is the optimal dynamic Kalman gain given by

$$L(k) = AP(k)C^T(R + CP(k)C^T)^{-1},$$

where $P(k)$ satisfies the following discrete-time dynamic Riccati equation [21]

$$P(k+1) = A(P(k) - P(k)C^T(R + CP(k)C^T)^{-1}CP(k))A^T + Q,$$

in which $Q = E(w(k)w(k))^T$ and $R = E(v(k)v(k)^T)$ are the noise covariance data.

D. Fence technology

When the RHC approach is applied directly to a heavy haul train with n cars, the number of control variables of the optimization problem equals nN_c and the number of constraints equals $(6n-2)N_c$. For a train with more than 150 cars, nN_c and $(6n-2)N_c$ can easily reach thousands which composes a very difficult problem for QP solvers. Even the latest developed QP solver [22], [23] cannot always solve this problem within acceptable time. In this application, using more than T_s seconds to solve the problem means that the optimized values of control variables are not available at the next control instant, which prevents the implementation of the controller presented. Also, the problem solving and simulation procedure have a very high memory demand. Therefore, a fixed number car fence method is introduced below.

By presetting a fence factor f, a train is fenced to have n_f virtual cars. Each virtual car includes f or less than f (if the number of adjacent wagons/locomotives is indivisible by f) cars. It should be noticed that locomotives and wagons cannot be fenced into the same virtual car because of the big difference between their characteristics. Firstly, locomotives can pull/push the train as well as brake the train while there is only brake available for wagons. Secondly, the physical parameters of locomotives and wagons are distinctive, like the length and mass of a locomotive is far different from a wagon's.

So, the fixed train fence method works as follows. Assuming that a train with a locomotives at the front, b wagons in the middle, and other c locomotives at the rear. The total number of the virtual cars after fencing with fence factor f can be calculated as

$$n_f = \left[\frac{a}{f}\right] + \left[\frac{b}{f}\right] + \left[\frac{c}{f}\right],$$

where $[x]$ is an operator which gives the integer closest to and no less than x.

Then, the objective function (4) is converted to be as follows,

$$\begin{aligned} J = \int_{t_0}^{t_f} \{ & K_f \sum_{i=1}^{n_f-1} f_{in_i}^2 + K_v \sum_{i=1}^{n_f} (v_i - v_r)^2 \\ & + K_e (\sum_{j=1}^{m_f} u_{l_j} + K_b \sum_{i=1, i\neq l_{f_j}}^{n_f} u_i)\} dt \end{aligned} \tag{11}$$

where m_f is the number of virtual locomotives after fencing and $l_{f_j}, j = 1, 2, \dots, m_f$ is the relative position of the jth virtual locomotive in the train.

Consequently, the numbers of control variables and constraints can be reduced by the fence factor f remarkably: from nN_c to $n_f N_c$ and from $(6n-2)N_c$ to $(6n_f - 2)N_c$, respectively. If the fencing factor f is large, this reduction is really considerable and is contributing to reduce the computation load of the problem.

As for the transmission channel limit for control systems, f can be selected so that n_f is not larger than the number of available control channels.

III. Simulation

The train configuration is the same as in [1]. There are four locomotives in the train with two each located at front and rear of the train. Between those locomotives, 200 wagons are connected one-by-one in the middle. Regarding the safety of the train, allowable maximum absolute value of in-train forces is 2000 kN. The locomotive's and wagon's parameters of the train is listed in Table I and II. Other parameters in simulation are $k_i = 10488$ kN/m, $d_i = 1500$ kNs/m, $i = 1, 2, \dots, n-1$, the maximum wagon's braking force is 180 kN, and the locomotive's traction/braking characteristics are the same as in [2] and not listed here. Besides, the track information GIS data is described in [6] and not listed here for clarity. For the train controller design, $T_s = 20$ s sampling period is used. For the weights in the controller, $K_e = 10$, $K_v = 60$ and $K_f = 10$ are used in all simulations if not explicitly explained to be others.

As for the observer used, a large $Q = 50$ is used owing to the fact that the observer is based on a linearized model which is inaccurate and $R = 0.01$ is set small because the speed can

TABLE I: Locomotive parameters

m (ton)	c_0 (m/s^2)	c_1 ($1/s$)	c_0 ($1/m$)	L (m)
126	7.6685e-3	1.08e-4	2.06e-5	20.47

TABLE II: Wagon parameters

m (ton)	c_0 (m/s^2)	c_1 ($1/s$)	c_0 ($1/m$)	L (m)
66	6.3625e-3	1.08e-4	1.492e-5	12.07

be measured with a high precision with current speed sensor technologies.

Regarding hardware platform the simulations are run on, it is a personal computer with Intel Core i7-2600 CPU @ 3.4 GHz and with 4 GB RAM. The operating system installed is Windows 7 32-bit operating system. All simulations are run on that computer with simulation code developed in MATLAB 2011a.

In simulations, the RHC controller designed according to linearized train model (2) is applied to the train practical nonlinear model (1) to verify its performances on the practical nonlinear train plant as shown in Fig. 1.

It is to be noticed that in Fig. 2, the first subplot depicts the train's speed tracking result (mean speed of all cars in the train and reference speed are given), the second one demonstrates the train's locomotives' throttle notches while the third one shows the maximum and minimum in-train forces (fin in the figure represents f_{in}). Besides, Fig. 2, Fig. 3, Fig. 4, Fig. 5 and Fig. 6 share the same track profile as shown in the fourth subplot of Fig. 2.

A. *Effectiveness of the controller*

While the practical train dynamics are nonlinear, the RHC controller presented is based on the linearized version of that nonlinear dynamics. The following simulation is carried out to verify the effectiveness of this controller.

The controller parameters $N_p = 8$ and $N_c = 6$ are used. Further, because of this simulation's purpose, the computation time of the solving this problem is not considered. That is to say, the solution is implemented to the train plant even if the the time used to solve this problem is longer than the sampling period. The solution is shown in Fig. 2, which clearly shows that proposed controller works very well on the practical nonlinear train dynamics. The train's speed tracks the reference speed profile well. Moreover, the train's speed does not exceed the predefined speed profile at most of the time. The only overshoot occurred at position where the speed profile drops instantly, which is actually not achievable in practice. The maximum and minimum in-train forces are within safe range as can be seen from Fig. 2.

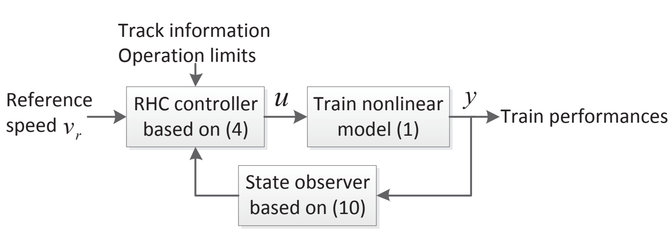


Fig. 1: Simulation diagram

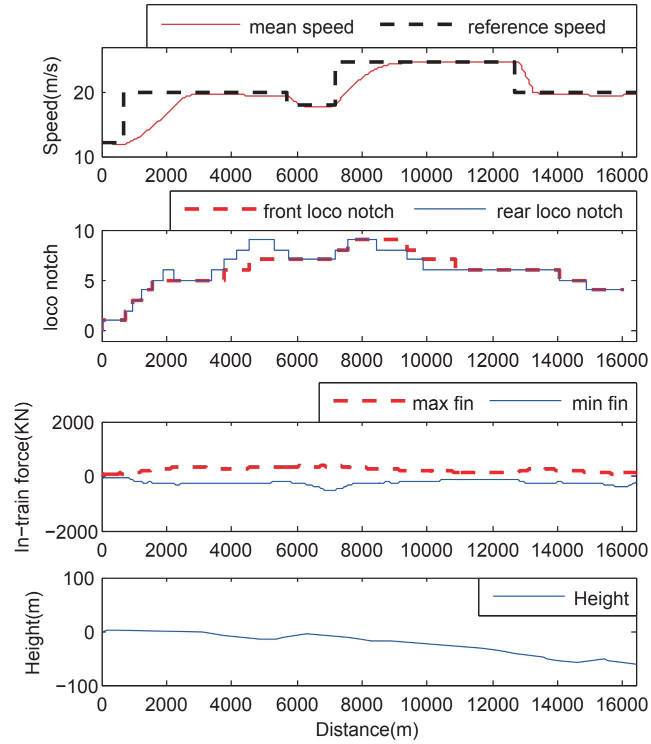


Fig. 2: Controller effectiveness demonstration ($f = 1$, $N_p = 8$, $N_c = 6$)

B. *Impacts of fencing*

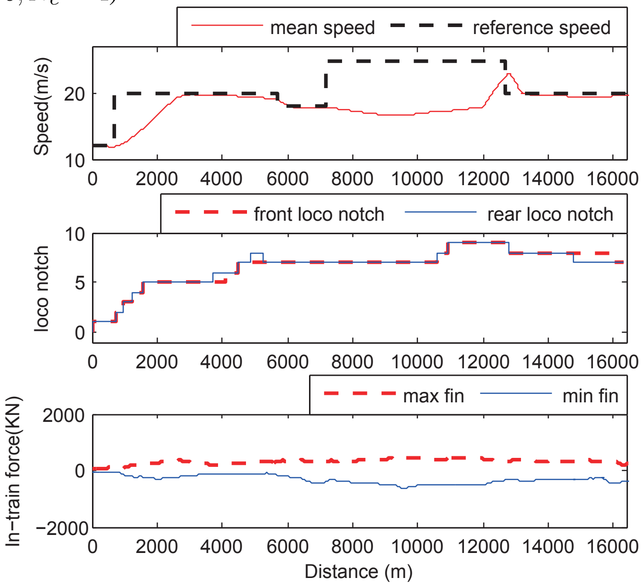
In this section, analysis and simulation verification of the impacts of the fence method is followed by a demonstration example showing the importance of the fence method.

1) Necessity of of the fence method: The configuration of the simulation is the same as in Section III-A except the truth that the train will retain its throttle settings unchanged if the computation time required to solve optimal train control problem is longer than the sampling period as the control signal is not available at the control instant. The simulation result is depicted in Fig. 3. It can be observed that the train's speed tracking performance is very poor when the train travels from position 7000 m to 12700 m. The reason for this can be found by comparing Fig. 2 and Fig. 3. It is shown in Fig. 2 that after 7000 m, up gear of the locomotives are required so that the train can follow the faster reference speed, however, the locomotives' throttles are not changed Fig. 3 because the calculation time required to solve the problem is longer than T_s during that section (detailed later). This directly leads to the train's performance become unacceptable.

This demonstration example explicitly shows the necessity of the fence method which can be used to reduce the computation burden of the controller.

2) Fence's impacts on computation and memory load: The fence method's impacts on the computation and memory load of solving the train operation problem can be demonstrated by the following illustrative example.

Fig. 3: Necessity of fence demonstration ($f = 1, N_p = 6, N_c = 4$)



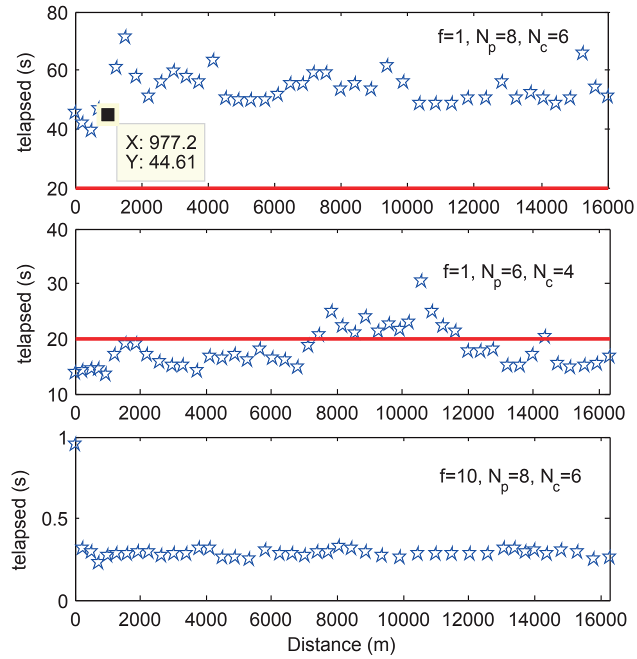


Fig. 4: Computation time

Initially, when $N_p = 8$ and $N_c = 6$ are used without fencing which implies that the number of control variables is $nN_c = 204 \times 6 = 1224$ and number of constraints is $(6n-2)N_c = (6 \times 204 - 2) \times 6 = 7332$. The large number of control variables as well as constraints makes the solving procedure very time consuming. Time required to solving this problem with the latest developed QP solver [23] during the travel of the train with different controller and fence factor settings are compared in Fig. 4, in which the y-axis represents time elapsed during solving the problem at each calculation instant and x-axis represents the train's position where the corresponding calculation occurred. For instance, the fifth point of the first subplot in Fig. 4 shows that the fifth control instant happened when the train run to position 977.2 m and the computation time used to solve the control problem at that point is 44.61 s.

It is manifest from Fig. 4 that when fence method is not used ($f = 1$) the problem solving time is always much longer than the sampling period T_s when $N_p = 8$ and $N_c = 6$. While using short optimization horizon $N_p = 6$ and $N_c = 4$, the computation time decreased to be acceptable at the most of time. However, it still requires more than T_s seconds to solve the problem during the track section 7000 m to 12700 m, which results in the poor speed tracking result as shown in Fig. 3.

Moreover, if one looks at the memory load with the above instances, storing the problem matrices, control variables and train state variables requires more than 250 MB memory.

By contrast, if a fence factor $f = 10$ is used for the same configuration with $N_p = 8$ and $N_c = 6$, the complexity of above problem is reduced a lot: the number of control variables is $n_f N_c = 22 \times 6 = 132$ and number of constraints is $(6n_f - 2)N_c = (6 \times 22 - 2) \times 6 = 780$. The maximum calculating time required to solve the problem during the train's travel drops to 0.95 seconds with an average time used to be 0.30 seconds. Meanwhile, the memory space required to store the same matrices and variables is reduced to 2.91 MB. This comparison between without and with fence factor gives a very straightforward explanation of why the fence method is introduced.

3) Fence's impacts on train performances: As the fence method is useful to bring down the time and memory requirement of the RHC approach, its impacts on the train performances are to be determined in order to show its feasibility when applying to heavy haul trains. During simulations, it is verified that when f increases from one to ten, an upward trend of energy consumption and maximum in-train force is seen while changes regarding velocity tracking is minimal (an example when $f = 10$ is shown in Fig. 5).

Also, the numerical analysis of $f = 1$ and $f = 10$ with same controller parameters are listed in Table III in which $|\delta\overline{v}|$ represent absolute value of the difference between the reference velocity and the mean value of all the cars' velocities, $|f_{in}|$ is absolute value of in-train forces and E stands for energy consumption calculated by $\int_0^T |uv|dt$ where T is the total travel time.

From Table III, it can be seen that energy consumption and in-train forces increase with a large f while speed tracking result remain almost stable. The reason for this is that with a large f many cars are treated as one single unit which hinders optimal operation of each of them, thus energy consumption increases. The in-train forces increase for the same reason.

This means that the fence factor should be selected as small as possible according to the computing resource and available control channels in the train. Also, this verifies that fence method presented can be used in heavy haul train control problem without severely jeopardizing the train's performances when used properly.

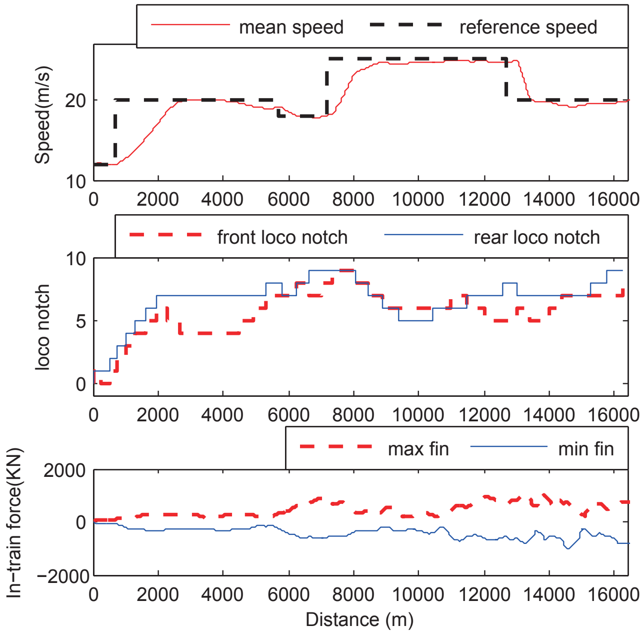


Fig. 5: Train performances with $f = 10$, $N_p = 8$, and $N_c = 6$

TABLE III: Performance with different f while $N_p = 8$ and $N_c = 6$

f	$\lvert\delta\bar{v}\rvert$ (m/s)		$\lvert f_{in}\rvert$ (kN)			E (MJ)
	mean	std	max	mean	std	total
1	1.33	2.15	531.94	113.43	30.71	20050
10	1.41	2.14	1060.38	221.70	68.42	20349

C. Impacts of brake penalization

1) K_b's impacts on train performances: The train's performances with different K_b are summarized in Table IV. It can be seen that when K_b increases from 1 (no braking penalty) to 80, velocity tracking result almost remain unchanged but other two performance indicators improved significantly. Both maximum absolute in-train force and energy consumption are decreased dramatically. To be exact, maximum in-train force drops from 1357.11 kN ($K_b = 1$) to 509.03 ($K_b = 60$), 2.67 times decrease is reached. Total energy consumption diminishes from 23103 MJ to 20064 MJ (13.8% reduction) when K_b increases from 1 to 80. Looking at energy consumed by wagons only, it slides down from 14755 MJ to 12940 MJ (12.3% decrease).

Fundamentally, K_b punishes wagons' braking forces in a train. This leads to the following performance changes directly. The first one is that the energy wasted by braking is reduced with a $K_b > 1$. The second one is that the maximum absolute in-train force decreases as a result of that less frequent braking occurs with a large K_b. These two are direct influences that braking penalization has on the train performances. There is another impact—the train's velocity tracking can be affected since the objective function used in the controller is a weighted one. However, In simulation done here, the weight for velocity tracking K_v is set to 60 (a relatively large weight) which ensures the train's velocity tracking result.

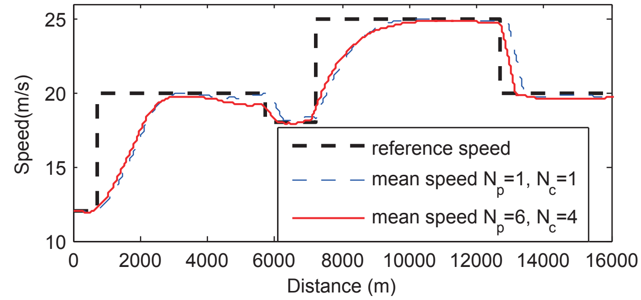


Fig. 6: Velocity tracking comparison with different N_p and N_c

2) K_b incorporated with RHC to improve train performances: Although the train's performances can be improved by brake punishment, there are two points related to this method worth mentioning. On the one hand, it is necessary to apply brake forces to a train appropriately so that it can run down a hill safely. On the other hand, controlling the train with regard to its current state only results in that the train starts to decelerate when it already runs on a downhill track. In such a way, it requires to apply significantly large brake forces in order to operate the train properly, which means that energy consumed by braking at a downhill track is noticeable. Owing to the RHC framework, if the optimization horizon is long enough, the train can reduce as much energy wasted by braking as it can and ensure the train's operation performances by means of slowing down earlier.

With the RHC strategy, the controller can optimize the train's operation by looking ahead. Given the train's current state, track information and reference speed profile, the RHC optimizes the train's operation in a section of track, which implies that, the RHC approach can take advantage of the track information and reduces braking as much as possible in the optimization. For instance, if a train is running over a fluctuating track, the RHC approach uses the uphill section of the track to slow down the train and apply less braking forces at the downhill section compared to approaches that only use the train's brake forces to slow down the train at the downhill section. Also, it can slow the train down earlier before the decline by reducing power rather than applying braking forces at the decline. This feature of RHC gives a way to reduce braking efforts of the train and makes it possible to compensate the speed tracking deterioration resulting from braking penalization.

As shown in Fig. 6, if the optimization and control horizons increase from $N_p = 1, N_c = 1$ to $N_p = 6, N_c = 4$ when $K_b = 60$, the train's velocity tracking result is improved a lot. The train slows down to reference speed at the position exactly as expected in the latter case. During the whole travel, the mean speed tracking error and standard deviation of speed tracking error decrease from 1.30 m/s to 1.20 m/s and from 2.11 m/s to 1.87 m/s, respectively while the other two performances indicators remain stable.

IV. Conclusion

A close-loop braking penalized receding horizon control (RHC) approach is presented to optimize operation of heavy

TABLE IV: Performance with different K_b while $N_p = 4$, $N_c = 2$, and $f = 10$

K_b	$\lvert\delta\overline{v}\rvert$ (m/s)		$\lvert f_{in}\rvert$ (kN)			E (MJ)	
	mean	std	max	mean	std	wagon	total
1	1.57	2.12	1357.11	251.97	89.44	14755	23103
40	1.33	2.09	577.72	122.72	31.28	12889	19924
60	1.36	2.08	509.03	122.33	29.15	12743	19655
80	1.36	2.09	512.38	123.55	28.81	12940	20064

haul trains in a long journey with its advantages of constraints and model plant mismatch handling capability. It is verified by simulations that:

(1) The brake penalty presented can reduce the train's energy consumption and in-train forces remarkably. In addition, incorporated with the RHC approach, the brake penalty's side effects on the train's velocity tracking can be compensated by a long optimization horizon.
(2) The fence methodology is effective in bringing down the RHC approach's computation and memory load without severely jeopardizing the train's performances. It is useful when applying the presented control scheme to a train which has a limited control channels.

The RHC controller and the fence method proposed in this brief can be used to control a heavy haul train directly. Or, the whole control and simulation scheme shown in Fig. 1 can be used to run simulations along a train's journey as a driver assistance system.

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References

[1] X. Zhuan and X. Xia, "Cruise control scheduling of heavy haul trains," *IEEE Transactions on Control Systems Technology*, vol. 14, no. 4, pp. 757–766, Jul. 2006.

[2] X. Zhuan and X. Xia, "Optimal scheduling and control of heavy haul trains equipped with electronically controlled pneumatic braking systems," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 6, pp. 1159–1166, Nov. 2007.

[3] X. Zhuan and X. Xia, "Speed regulation with measured output feedback in the control of heavy haul trains," *Automatica*, vol. 44, no. 1, pp. 242–247, 2008.

[4] P. Howlett, "Optimal strategies for the control of a train," *Automatica*, vol. 32, no. 4, pp. 519–532, 1996.

[5] P. Howlett, P. Pudney, and X. Vu, "Local energy minimization in optimal train control," *Automatica*, vol. 45, no. 11, pp. 2692–2698, 2009.

[6] M. Chou, X. Xia, and C. Kayser, "Modelling and model validation of heavy-haul trains equipped with electronically controlled pneumatic brake systems," *Control Engineering Practice*, vol. 15, no. 4, pp. 501–509, 2007.

[7] L. Zhang, X. Zhuan, and X. Xia, "Optimal operation of heavy haul trains using model predictive control methodology," in *2011 IEEE International Conference on Service Operations, Logistics, and Informatics*, Beijing, China, July 2011, pp. 402 –407.

[8] H. Chen and F. Allgower, *Nonlinear Model Based Process Control*. Kluwer Academic Publishers, 1998, ch. Nonlinear model predictive control schemes with guaranteed stability, pp. 465–494.

[9] Y. Wang, B. D. Schutter, T. J. van den Boom, and B. Ning, "Optimal trajectory planning for trains a pseudospectral method and a mixed integer linear programming approach," *Transportation Research Part C: Emerging Technologies*, vol. 29, no. 0, pp. 97 – 114, 2013.

[10] S. Lu, S. Hillmansen, T. K. Ho, and C. Roberts, "Single-train trajectory optimization," *Intelligent Transportation Systems, IEEE Transactions on*, vol. PP, no. 99, pp. 1–8.

[11] Y. Bai, B. Mao, F. Zhou, Y. Ding, and C. Dong, "Energy-efficient driving strategy for freight trains based on power consumption analysis," *Journal of Transportation Systems Engineering and Information Technology*, vol. 9, no. 3, pp. 43 – 50, 2009.

[12] AAR, "Manual of standards and recommended practices, electronically controlled brake system," Association of American Railroads, Technical Report, 2002.

[13] L. Zhang and X. Zhuan, "Optimal operation of heavy haul trains equipped with ecp brake systems using model predictive control methodology," *IEEE Transactions on Control Systems Technology*, vol. Submitted, 2011.

[14] E. Mortlock and G. Hubbs, "Implementing optical speed measurement (OSMES) for communications based train control," in *Proceedings of the 2004 ASME/IEEE Joint Rail Conference*, 2004, pp. 205–211.

[15] R. J. Alton, A. K. Kumar, and B. D. Worden, "Locomotive speed determination," Patent, 2008.

[16] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.

[17] H. Michalska and D. Mayne, "Robust receding horizon control of constrained nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 38, no. 11, pp. 1623 –1633, 1993.

[18] L. Magni, G. De Nicolao, and R. Scattolini, "Output feedback and tracking of nonlinear systems with model predictive control," *Automatica*, vol. 37, no. 10, pp. 1601–1607, 2001.

[19] M. Diehl, H. G. Bock, J. P. Schlloder, R. Findeisen, Z. Nagy, and F. Allgower, "Real-time optimization and nonlinear model predictive control of processes governed by differential-algebraic equations," *Journal of Process Control*, vol. 12, no. 4, pp. 577 – 585, 2002.

[20] G. C. Goodwin, S. F. Graebe, and M. E. Salgado, *Control system design*. Prentice Hall, 2001, ch. Linear state space models, pp. 500–512.

[21] G. C. Goodwin, S. F. Graebe, and M. E. Salgado, *Control system design*. Prentice Hall, 2001, ch. Design via optimal control techniques, pp. 698–713.

[22] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[23] J. Mattingley and S. Boyd, "Real-time convex optimization in signal processing," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 50 –61, 2010.

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