

# Performance characteristics of a deep tilling rotavator

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## SUMMARY

### Performance characteristics of a deep tilling rotavator

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The continued increase in the price of fossil-based fuels and lubricants has resulted in tremendous increase in the cost of land preparation. This has resulted in considerable increase in the cost of food. The situation is worsened by the prevalent use of the conventional tillage system in the preparation of seedbeds; particularly for deep tillage. This system of tillage escalates land preparation costs because it requires a series of operations using passive tillage tools to realise an acceptable tilth quality. It also ties down capital in the form of additional machinery and tillage tools; thus increasing significantly the cost of land preparation. Therefore, it is necessary to design better tillage tools that are capable of reducing the number of tillage operations required for the realization of seedbeds of acceptable tilth quality.

The rotavator is one of the tillage tools with the capability for realizing the desired soil tilth quality with significantly reduced number of tillage passes. In comparison to passive tools, the rotavator has a superior soil mixing and pulverisation capability. When rotated in the down-cut direction, it generates a forward thrust that aids traction under difficult field conditions. However, no documented analytical models capable of predicting the performance of rotavators fitted with commercially available blades was found in literature. In addition, there is dearth of information on the behaviour of the magnitude of the horizontal thrust forces generated for a down-cut rotavator for different set tillage depths.

This study was undertaken to develop an analytical model that is capable of predicting the torque requirements of a rotavator fitted with commercially available L-shaped blades. In developing the proposed model, an analytical approach based on the limit equilibrium

analysis was used. An interactive computer program was developed, in MATLAB (*Version 7, Mathworks Inc., USA*), to solve the proposed model. The proposed model was verified by comparing the model and measured torque requirement at predetermined rotavator blade angular positions from the horizontal for a down-cut rotavator.

Field experiments were conducted in a sandy loam soil, using two instrumented research equipment. The research equipment were calibrated in a laboratory and field-tested prior to conducting the field experiments. A torsional shearing apparatus was used to characterize the soil by determining the soil shear strength and soil-metal friction parameters. The rotavator operational parameters, necessary for analyzing its performance, were recorded using an instrumented tool-frame carrier. The experiments were conducted in the down-cut direction of rotation, in the 200 mm – 500 mm set tillage depth range.

The study findings indicated that there was an optimum set tillage depth for each rotavator configuration and operational conditions at which the resultant horizontal thrust generated was greatest. This unique depth was influenced by the bite length. The validation of the proposed model showed that the predicted and measured torque requirements, at different angular blade positions from the horizontal, correlated reasonably well for all the set tillage depths. As the depth of tillage increased, however, the curve for the measured torque requirements exhibited a cyclic behaviour after the peak torque requirements value had been recorded. The cyclic behaviour was probably due to the re-tilling and the instability of the tool-frame carrier, which increased with the set tillage depth.

The knowledge contributed by this research will afford the designers of active tillage tools a better understanding of the operations of the rotavator, particularly in deep tillage. The modelling approach, and instrumentation technique used in this research, can be extended to analyze the performance of rotavators fitted with other types of commercial blades.

**Key terms:** rotavator, deep-tilling, soil-failure modelling, tillage performance, soil shear strength, soil-metal friction, bite length, kinematic parameter, down-cut rotavator, power, specific energy

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UNIVERSITEIT VAN PRETORIA  
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*In memory of my dear mother, the late Mama Meresia Odundo Orony-Marenya (Nyar Aol)*

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## NOMENCLATURE

$\tau_{\max}$ =	maximum shear stress at soil-soil failure surface (kPa)
$\rho$ =	angle that the rupture surface makes with the horizontal (degrees)
$\sigma_n$ =	normal stress (kPa)
$\phi$ =	soil internal friction angle (degrees)
$\psi'$ =	the angle formed by the positive direction of the tangent at the given point of the cycloid and the positive direction of the abscissa (degrees)
$\gamma$ =	unit weight (or bulk density) of the soil (kN/m <sup>3</sup> )
$\beta$ =	angle that the tool makes with the horizontal during an instantaneous time moment (degrees).
$\bar{z}_c$ =	depth from the top of the failed soil wedge (m)
$N_\gamma, N_c, N_q$ =	factors dependent on soil frictional strength, soil geometry and tool to soil strength properties (dimensionless)
$N'_c$ =	cohesion N-factor (dimensionless)
$\tau_f$ =	maximum frictional stress at the soil-tool interface (kPa)
$V_{cir}$ =	peripheral velocity of the rotor blades (m/s)
$\tau_o$ =	soil property related to static component of the static shear strength (kPa)
$\tau_i$ =	soil property related to the dynamic component of soil shear strength, proportional to the operating speed (kPa (ms <sup>-1</sup> ) <sup>-1</sup> )
$N'_q$ =	surcharge N-factor (dimensionless)
$\alpha_{c_2}$ =	the angle at the second blade make with the maximum crest height (degrees)
$\frac{2\pi}{z}$ =	the angle between any two adjacent blades on a flange on the rotor (rad.)
$\alpha_{c_1}$ =	the angle the first blade makes with the maximum crest height, $h_c$ (degrees)
$\tau_{\max}$ =	maximum shear stress at failure (kPa)
A =	effective area of the grouser head or the flat steel plate of the torsional shear apparatus in contact with the soil (m <sup>2</sup> )
$a, b$ =	regressions coefficients, which were all significant at 1 % level (Stafford & Tanner, 1983a, 1983b)



$A_1 =$	area of the span in contact with the soil, 'area abed, in Fig. 3.16' ( $m^2$ ),
$A_2 =$	area of the triangular rupture surface, 'area abc or def in Fig. 3.16' ( $m^2$ )
$A_3 =$	areas of the rectangular rupture surface 'area bcfe in Fig. 3.16' ( $m^2$ )
$C_a =$	soil metal adhesion factor ( $kN/m^2$ )
$C_c =$	soil cohesion ( $kN/m^2$ )
$d =$	depth of tillage (m)
$d_1 \dots d_n =$	distance covered in respective plots during a test run
$d_c =$	critical tillage depth (m)
$d_e =$	effective depth of transverse failure (m)
$D_f =$	Resultant draft force (N)
$d_i =$	instantaneous depth of wire from soil surface
$d_z =$	average depth to the centroid of failure wedge (m)
$E_{rot} =$	power required for the rotary work (kW)
$f(v) =$	function containing the soil inertial term (kN)
$F_d =$	draft force under dynamic conditions (kN)
$F_n =$	the applied 'constant' normal load (kN)
$F_s =$	static draft force component (kN)
$g =$	gravitational acceleration, $\approx 9.81 (m/s^2)$
$h_c =$	the peak crescent height at the bottom of the cut furrow (m)
$H_{dc} =$	draft for a blade operated at deep tillage depth (kN)
$H_l =$	draft for the lateral soil failure (kN); Godwin and Spoor (1967) model
$H_s =$	draft for a blade operated at a shallow tillage depth (kN)
$H_t =$	draft for crescent above the critical depth (kN); Godwin and Spoor (1977) model
$k =$	ratio of critical depth to width (dimensionless)
$k_1, k_2, k_3 =$	constants for both soils at all soil water contents (Stafford & Tanner, 1983a)
$K_i =$	tine inclination factor (dimensionless, Hettiaratchi and Reece (1967) model)
$K_o =$	ratio of the horizontal and static vertical stress, $K_o = (1 - \sin \phi)$
$L =$	length of the span of the blade in contact with the soil (m)
$L_b =$	the bite length (m)
$L_{ps} =$	the length of the blade to the point of action of the soil-blade resistance force $P_s$ (m)
$L_{tr} =$	tilling route length (m)

$M =$	the maximum torque or the maximum resisting moment (kNm)
$z_n =$	number of blades of the rotavator that pass through the soil in time $t$
$N_a =$	an additional factor comprised in soil cutting forces, which accounts for the acceleration forces in the soil with varying tool speeds, but a fixed soil strength
$N_a =$	an additional factor comprised in the soil cutting force, which accounts for the acceleration forces in the soil for varying tools speed, but a fixed soil strength (dimensionless)
$N_{ca} =$	a dimensionless factor that dependent on the soil-metal adhesion
$N_{ca} =$	a dimensionless factor that depends on soil-metal adhesion
$N_p, N_c, N_q =$	a dimensionless factors depend on soil frictional strength, soil geometry, and tool to soil strength properties
$P =$	total force (kN)
$P_1 =$	force applied to the centre wedge (kN)
$P_2 =$	force applied to the side crescent of the soil (kN)
$P_o =$	geostatic stress (Pa)
$P_r =$	soil resistance force due to the penetration resistance (kN)
$P_{rot} =$	rotary power required to process the soil (kW)
$P_s =$	soil resistance force due to the span and leg of the L-shaped blade (kN)
$q =$	surcharge pressure vertically acting on the soil surface (kNm <sup>-2</sup> )
$Q =$	the normal acting force on the soil-soil interface of the failed soil wedge (N)
$Q_s =$	force upon face of tip of the span (kN)
$R =$	rotor radius (m)
$R_{cr} =$	reaction force acting on the side of the centre wedge (kN)
$r_i =$	inner radius of the annulus shear ring head with grousers (m)
$r_o =$	outer radius of the annulus ring head with grousers (m)
$t =$	time of rotation of the rotor through angle $\alpha$ (s).
$t_1 =$	time taken by the leading blade to turn through angle $\alpha_{c1}$ (s)
$t_2 =$	time taken by the second blade to turn through angle $\alpha_{c2}$ (s)
$t_b =$	time during which the blades rotate through an angle, equal to the angle between the adjacent blades on the same side of a flange (s)
$t_{bt} =$	thickness of the cutting edge of the span of the blade (m)
$t_i =$	time of rotation of a blade through angle, $\alpha_i$ from the horizontal (s)
$T_{meas} =$	experimental torque requirement values at different angular position of the tip of the cutting blade during soil processing (kNm)

$T_{mod} =$	model generated torque requirements values at different angular positions of the tip of the cutting blade during soil processing (kNm)
$T_{pr} =$	instantaneous time moment torque requirement due to force $P_r$ (kNm)
$T_{ps} =$	instantaneous time moment torque requirement due to force $P_s$ (kNm)
$T_{total} =$	total instantaneous time moment torque requirement (kNm)
$V =$	instantaneous peripheral velocity of the tool along the cycloidal path, (m/s)
$V_{ac} =$	actual soil chip volume processed by a single blade ( $m^3$ )
$V_f =$	forward travel speed (m/s)
$V_f =$	forward travel speed of the tractor for rotary tillage or the speed of the tool for a passive tillage implement (m/s)
$V_i =$	the instantaneous peripheral velocity of the tool along the cycloidal path, (m/s.)
$V_s =$	sliding velocity of the blade (m/s)
$V_{wsv} =$	total volume of the soil tilled in time, $t$ ( $m^3$ )
$w =$	tool width (m)
$\alpha =$	the angle of rotation of the blade from the horizontal position (degrees)
$\alpha_i =$	the angle the blade turns through in time, $t_i$ (rad)
$\alpha_r =$	the intercept on the vertical axis for predicted versus measured torque linear regression
$\beta =$	rake angle (degrees)
$\gamma =$	soil bulk density ( $kN/m^3$ )
$\delta =$	soil-metal friction angle (degrees)
$\vartheta =$	thickness of cross-section of the cut soil slice (m)
$\omega =$	rotor speed (rad/s)