

# **Multidisciplinary Design and Optimisation of Liquid Containers for Sloshing and Impact**

*by*

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## Summary

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The purpose of this study is to perform an investigation of the numerical methods that may contribute to the design and analysis of liquid containers. The study examines several of these methods individually, namely Computational Fluid Dynamics (CFD) analysis of sloshing and Finite Element Methods (FEM) analysis of impact, to evaluate their contribution to the design cycle. Techniques that enhance the use of the various methods are presented and examined to demonstrate effectiveness. In the case of sloshing analysis, experimental tests performed add to the understanding of the phenomena at hand and qualifies the validity of the numerical method used (CFD). As a final contribution, the study presents a method of utilising impact analysis tools, FEM, and CFD in a Multidisciplinary Design Optimisation (MDO) environment. This is an introductory attempt at demonstrating a single coupled multidisciplinary method of designing liquid containers.

The results of the study demonstrate a number of valuable numerical techniques that may be used in the design of liquid containers. The presented Total Deviation Value (TDV) proves to be an effective single quantification of sloshing performance and the CFD tools used to determine the value demonstrate sufficient ability to reproduce the sloshing event itself. More advanced experimental facilities would provide a more in-depth understanding of the limitations of the CFD analysis. The use of numerical optimisation adds a valuable dimension to the use of numerical simulations. Significant design improvements are possible for several design variables without performing exhaustive studies and provide interesting information about design trends. Finally, the use of multiple disciplines, FEM and CFD, in conjunction with the available numerical optimisation routines offers a powerful multidisciplinary design tool that can be adapted to any base geometry and is capable of finding optimal trade offs between the two disciplines according to the designer's needs.

This study provides a platform for further investigations in the use and coupling of sloshing and impact analysis in the design of industrial liquid container applications.

**Keywords:** Liquid Sloshing, Multidisciplinary Optimisation (MDO), Fluid Structure Interaction (FSI), Computational Fluid Dynamics (CFD), Finite Element Methods (FEM), Mathematical Optimisation, Successive Response Surface Method (SRSM), Volume Of Fluids (VOF), Total Deviation Value (TDV), Impact Analysis.

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